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M.Sc. in Statistics - Financial Analytics

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Contents

Abstract

The following paper contains the analysis of the returns of USA mutual funds for the period 07/1963 – 07/2019, also combined with selected explanatory factors, which are: {S&P500 excess returns, SMB, HML, RMW, CMA, MOM, BAB, CAR} again for the period 07/1963 - 07/2019. The analysis was performed in two parts, the Performance Evaluation and the Portfolio Construction and in both cases, the datasets were separated into the in-sample period (07/1963 - 07/2015) and the out-of-sample period (08/2015 - 07/2019).

Introduction

The following analysis is part of the course evaluation of Financial Analytics of the Master's program in Statistics of the Athens University of Economics and Business (A.U.E.B.). The given datasets are provided through an .xlsx file and the analysis is performed through the programming language R. This implies that the data must be entered in the R environment and be manipulated in such a way that will represent the time series. After that, the first step will be to start analyzing the data by evaluating the performance of the funds. This can be done by using several econometric methods, and the ones who will be initially applied in this paper are the Sharpe ratio, Treynor ratio, Sortino ratio, Jensen's alpha. However, we can extend the analysis and continue by adding other methods to estimate the Jensen's alpha apart from the single factor model (S&P500), such as the multiple regression models by applying model selection methods or by applying GARCH type models.

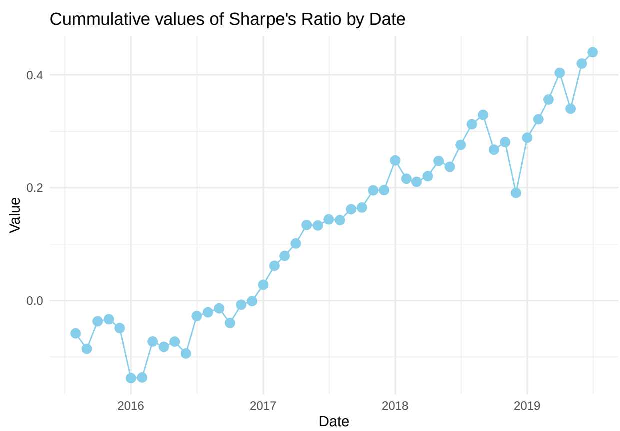
Part A: Performance Evaluation

Sharpe Ratio

In order to calculate the top performing fund based on Sharpe Ratio we need to calculate the following formula.

$$\text{Sharpe Ratio} = \frac{E(R_i) - r_f}{\sigma_i}$$

Then, we select the top 20% funds in terms of performance, we construct an equally weighted portfolio and then we can see and determine whether this ratio is effective or not based on the out-of-sample values and more specifically, the cumulative function that they create. If the graph has an upward trend, then it is effective.



As we can see, the Sharpe Ratio is quite effective since in 4 years the portfolio has gained more than 40% of the initial value.

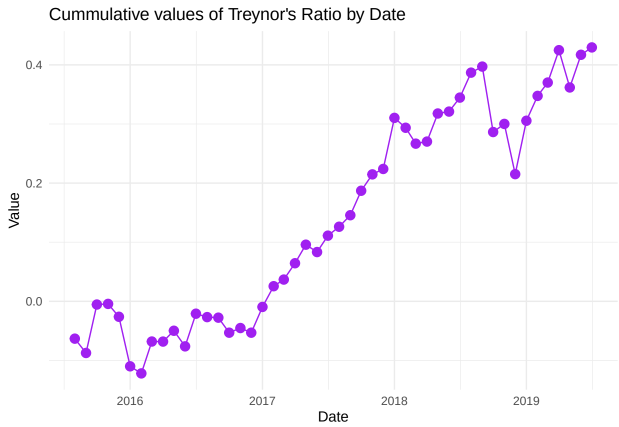
Treynor's Ratio

In order to calculate the top performing fund based on Treynor's Ratio we need to calculate the following formula.

$$\text{Treynor's Ratio} = \frac{E(R_i) - r_f}{\beta_i}$$

Here, we will have to create a model with each fund "i" and then keep the coefficient that corresponds to the S&P500 variable.

Similarly, a positive trend indicates a good metric for this portfolio.



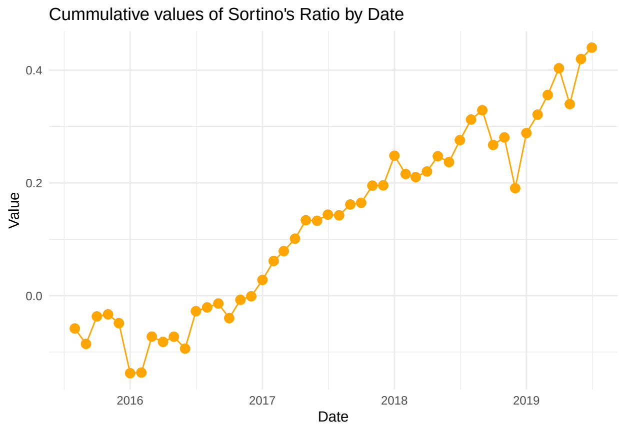
Even though in the beginning there is some variability around 0, eventually after 2017 the upward trend is clear.

Sortino's Ratio

In order to calculate the top performing fund based on Sortino's Ratio we need to calculate the following formula.

$$\text{Sortino's Ratio} = \frac{E(R_i) - r_f}{\delta_\text{i}}$$

Here, we have to calculate the "delta" for each fund "i" and then continue the analysis with the top 20% funds.



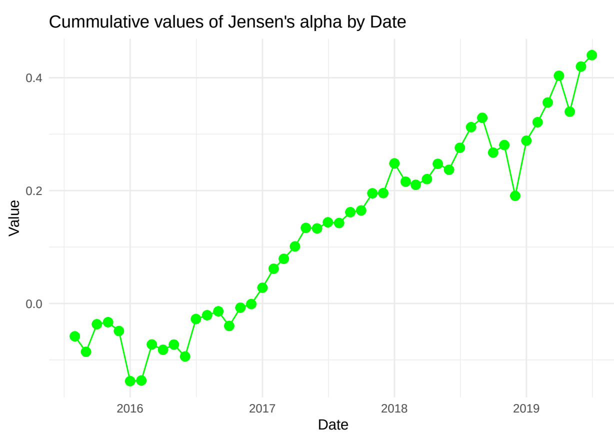
Similar image, the trend starts to launch after 2017.

Jensen's alpha (Single Factor model)

Last but not least, the Jensen's alpha. This will be the first part where we will use only the single factor model with the S&P500 variable.

In order to calculate the top performing fund based on Sortino's Ratio we need to calculate the following formula.

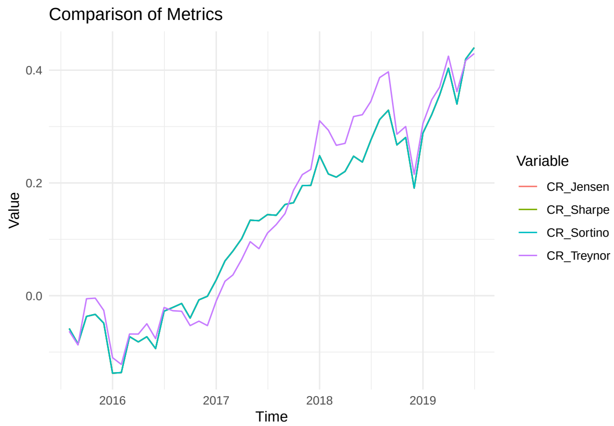
$$\alpha_i = E(R_i) - [r_f + \beta_p \cdot (E(R_M) - r_f)]$$



Once more, we get a positive result.

Aggregated

Now, if we try to plot all the above models into one plot, we will notice something rather interesting.



It seems that 3 out of 4 metrics have exactly the same values and there is an overlapping scene. The one that is different than the others is the Treynor's Ratio which doesn't seem to be very far away from the other values.

Jensen's alpha (Multiple Regression - Model selection)

Now, the continuation has to do with Jensen's alpha again, but with different modelling approaches.

Jensen's alpha (GARCH model)

```
##  [1] "11" "40" "78" "13" "58" "62" "43" "82" "55" "33" "15" "77" "7"  "19" "3"
## [16] "51" "36" "37"
```

```
##
##  ***** ESTIMATION WITH ANALYTICAL GRADIENT *****
##
##
##      I     INITIAL X(I)        D(I)
##
##      1     1.497089e-05     1.000e+00
##      2     5.000000e-02     1.000e+00
##      3     5.000000e-02     1.000e+00
##
##     IT   NF      F         RELDF    PRELDF    RELDX   STPPAR   D*STEP   NPRELDF
##      0    1 -2.356e+02
##      1    8 -2.356e+02  3.05e-05  2.34e-04  1.0e-05  5.5e+10  1.0e-06  6.48e+06
##      2    9 -2.356e+02  3.78e-05  3.88e-05  9.0e-06  2.0e+00  1.0e-06  1.87e-01
##      3   10 -2.356e+02  6.57e-08  2.40e-07  1.0e-05  2.0e+00  1.0e-06  1.88e-01
##      4   11 -2.356e+02  3.01e-08  4.05e-08  1.0e-05  2.0e+00  1.0e-06  1.88e-01
##      5   20 -2.365e+02  3.67e-03  5.18e-03  5.7e-01  1.9e+00  1.3e-01  1.87e-01
##      6   22 -2.366e+02  2.67e-04  2.99e-04  3.5e-02  2.0e+00  1.3e-02  4.07e-03
##      7   24 -2.366e+02  8.74e-05  8.81e-05  3.5e-02  1.8e+00  1.3e-02  3.61e-03
##      8   26 -2.366e+02  2.39e-05  2.38e-05  6.5e-03  2.0e+00  2.6e-03  2.12e-03
##      9   29 -2.366e+02  2.45e-04  2.44e-04  4.7e-02  9.9e-01  2.1e-02  2.42e-03
##     10   32 -2.366e+02  4.34e-06  4.33e-06  9.4e-04  2.0e+00  4.2e-04  2.24e-03
##     11   34 -2.367e+02  9.09e-06  9.09e-06  1.8e-03  2.0e+00  8.4e-04  2.21e-03
##     12   36 -2.367e+02  1.75e-06  1.75e-06  3.7e-04  2.0e+00  1.7e-04  2.29e-03
##     13   38 -2.367e+02  3.51e-06  3.51e-06  7.5e-04  2.0e+00  3.4e-04  2.26e-03
##     14   40 -2.367e+02  7.01e-07  7.01e-07  1.5e-04  2.0e+00  6.7e-05  2.30e-03
##     15   42 -2.367e+02  1.40e-06  1.40e-06  3.0e-04  2.0e+00  1.3e-04  2.21e-03
##     16   44 -2.367e+02  2.80e-07  2.80e-07  6.0e-05  2.0e+00  2.7e-05  2.32e-03
```

```
##      17   46 -2.367e+02  5.61e-07  5.61e-07  1.2e-04  2.0e+00  5.4e-05  2.24e-03
##      18   48 -2.367e+02  1.12e-06  1.12e-06  2.4e-04  2.0e+00  1.1e-04  2.25e-03
##      19   51 -2.367e+02  2.24e-08  2.24e-08  4.8e-06  2.0e+00  2.1e-06  2.25e-03
##      20   53 -2.367e+02  4.48e-08  4.48e-08  9.5e-06  2.0e+00  4.3e-06  2.25e-03
##      21   55 -2.367e+02  8.96e-08  8.96e-08  1.9e-05  2.0e+00  8.6e-06  2.26e-03
##      22   57 -2.367e+02  1.79e-08  1.79e-08  3.8e-06  2.0e+00  1.7e-06  2.25e-03
##      23   59 -2.367e+02  3.59e-09  3.59e-09  7.6e-07  2.0e+00  3.4e-07  2.26e-03
##      24   61 -2.367e+02  7.17e-09  7.17e-09  1.5e-06  2.0e+00  6.9e-07  2.26e-03
##      25   63 -2.367e+02  1.44e-09  1.44e-09  3.1e-07  2.0e+00  1.4e-07  2.26e-03
##      26   66 -2.367e+02  1.15e-08  1.15e-08  2.4e-06  2.0e+00  1.1e-06  2.26e-03
##      27   69 -2.367e+02  2.31e-10  2.31e-10  4.9e-08  2.0e+00  2.2e-08  2.26e-03
##      28   71 -2.367e+02  4.69e-11  4.69e-11  9.6e-09  2.0e+00  4.4e-09  2.26e-03
##      29   73 -2.367e+02  9.35e-11  9.35e-11  1.9e-08  2.0e+00  8.8e-09  2.26e-03
##      30   75 -2.367e+02  1.94e-11  1.94e-11  3.7e-09  2.0e+00  1.8e-09  2.26e-03
##      31   77 -2.367e+02  3.85e-11  3.85e-11  7.8e-09  2.0e+00  3.5e-09  2.26e-03
##      32   79 -2.367e+02  7.45e-11  7.45e-11  1.5e-08  2.0e+00  7.0e-09  2.26e-03
##      33   81 -2.367e+02  1.64e-11  1.64e-11  3.1e-09  2.0e+00  1.4e-09  2.26e-03
##      34   83 -2.367e+02  3.43e-12  3.43e-12  5.7e-10  2.0e+00  2.8e-10  2.26e-03
##      35   85 -2.367e+02  1.59e-12  1.59e-12  1.2e-10  2.9e+00  5.6e-11  2.26e-03
##      36   86 -2.367e+02  1.70e-12  1.70e-12  2.4e-10  2.7e+00  1.1e-10  2.26e-03
##      37   87 -2.367e+02  2.97e-12  2.97e-12  4.1e-10  2.0e+00  2.3e-10  2.26e-03
##      38   89 -2.367e+02  1.06e-12  1.06e-12  8.8e-11  3.4e+00  4.5e-11  2.26e-03
##      39   91 -2.367e+02  3.35e-12  3.35e-12  7.8e-10  2.0e+00  3.4e-10  2.26e-03
##      40   93 -2.367e+02  2.57e-12  2.57e-12  1.6e-10  2.9e+00  6.7e-11  2.26e-03
##      41   95 -2.367e+02  3.40e-13  3.40e-13  2.4e-11  9.4e+00  1.3e-11  2.26e-03
##      42   97 -2.367e+02  1.53e-12  1.53e-12  2.1e-10  2.2e+00  1.1e-10  2.26e-03
##      43   99 -2.367e+02  4.80e-13  4.80e-13  4.1e-11  7.0e+00  2.1e-11  2.26e-03
##      44  101 -2.367e+02  8.11e-13  8.11e-13  9.6e-11  2.3e+00  4.3e-11  2.26e-03
##      45  103 -2.367e+02  1.60e-12  1.60e-12  2.1e-10  1.3e+02  8.6e-11  2.26e-03
##      46  106 -2.367e+02  3.18e-14  3.20e-14  4.3e-12  2.1e+04  1.7e-12  2.26e-03
##      47  108 -2.367e+02  6.41e-14  6.40e-14  8.6e-12  2.8e+03  3.4e-12  2.26e-03
##      48  110 -2.367e+02  1.26e-14  1.28e-14  1.7e-12  6.7e+04  6.9e-13  2.26e-03
##      49  112 -2.367e+02  2.59e-14  2.56e-14  3.4e-12  8.6e+03  1.4e-12  2.26e-03
##      50  114 -2.367e+02  5.04e-15  5.12e-15  6.9e-13  1.8e+05  2.8e-13  2.26e-03
##      51  117 -2.367e+02 -1.20e-16  1.02e-16  1.4e-14  9.4e+06  5.5e-15  2.26e-03
##
```

```
##  ***** FALSE CONVERGENCE *****
##
##  FUNCTION    -2.366537e+02   RELDX        1.372e-14
##  FUNC. EVALS     117         GRAD. EVALS      51
##  PRELDF       1.024e-16      NPRELDF      2.256e-03
##
##      I      FINAL X(I)        D(I)          G(I)
##
##      1    1.240848e-05     1.000e+00     1.240e-01
##      2    1.992597e-01     1.000e+00    -4.376e+00
##      3    3.883100e-15     1.000e+00     4.917e-01

## [1] "Jensen's alpha for the equally weighted portfolio in the out-of-sample period is: 1e-05"
```

Part B: Portfolio Construction

Sample estimate of mean and covariance matrix

Estimating the mean vector and covariance matrix using a sample period involves straightforward calculations based on historical data. This approach assumes that the historical returns provide a reasonable estimate of future expected returns and risk.

To estimate the mean vector, we compute the average return for each asset over the historical period. This is calculated as:

$$\mu_i = \frac{1}{T} \sum_{t=1}^{T} r_{i,t}$$

The covariance matrix estimates the pairwise relationships (covariances) between assets, reflecting their joint variability over time. It is computed as:

$$\sum = \frac{1}{T-1} \sum_{t=1}^{T} (r_{i,t} - \mu_i) * (r_{j,t} - \mu_j)$$

Advantages:

- Simplicity: This method is straightforward to implement and understand, requiring basic calculations of means and variances.
- Use of Historical Data: It utilizes all available historical data, making it a comprehensive approach.
- Flexibility: It can be easily adapted to different subsets of data or varying time periods. Limitations:
- Assumption of Stationarity: Assumes that the mean and covariance structure of returns are stationary over time, which may not always hold true in practice.
- Sensitive to Outliers: Historical data may include extreme events (outliers) that can distort estimates of mean and variance.
- Short-Term Bias: Recent data may disproportionately influence estimates, potentially not reflecting long-term trends accurately.

In summary, the sample estimate approach provides a practical way to estimate mean returns and covariances based on historical data, offering simplicity and comprehensiveness but requiring careful consideration of its assumptions and limitations.

The outcome of this analysis is the following:

Single Factor Model

The Single Index Model (SIM) is a method for estimating the mean vector and covariance matrix of asset returns using a market index as a proxy for systematic risk (market risk). It assumes that the returns of individual assets can be explained by their exposure to systematic risk factors, primarily represented by the returns of a market index.

In the context of the Single Index Model:

Mean Vector Estimation: The mean return of an asset

$$\mu_i = \alpha_i + \beta_i * \mu_{market}$$

where μ_i is the mean return of asset i, α_i is the asset-specific expected excess return (alpha), β_i is the asset's sensitivity (beta) to the market index, and μ_{market} is the expected return of the market index.

Covariance Matrix Estimation: The covariance between the returns of two assets

$$\sigma_{ij} = \beta_i * \beta_j * \sigma^2_{market}$$

Advantages:

- Systematic Risk Focus: Captures the relationship between asset returns and the market index, emphasizing systematic risk factors.
- Portfolio Construction: Facilitates portfolio optimization by focusing on factors that influence the entire market.
- Risk Management: Provides insights into how individual assets contribute to overall portfolio risk.

Limitations:

- Model Assumptions: Relies on the assumption that asset returns can be adequately explained by market factors, which may oversimplify complex market dynamics.
- Data Requirements: Requires accurate estimation of beta coefficients, which can be sensitive to the choice of market index and time period.
- Dynamic Nature: Market conditions may change over time, affecting the stability of beta coefficients and model predictions.

In conclusion, the Single Index Model offers a structured approach to estimate mean returns and covariances by relating individual asset returns to market index returns. It provides valuable insights into systematic risk factors but requires careful consideration of its underlying assumptions and limitations in practical applications.

```
##         min_var    mean_var cumul_min_var cumul_mean_var
## 626 0.009371311 0.009371311   -0.09280165    -0.09280165
## 627 0.009371311 0.009371311   -0.10201176    -0.10201176
## 628 0.009371311 0.009371311   -0.08790556    -0.08790556
## 629 0.009371311 0.009371311   -0.08451503    -0.08451503
## 630 0.009371311 0.009371311   -0.09937627    -0.09937627
## 631 0.009371311 0.009371311   -0.28211808    -0.28211808
```