Computational Photography Assignment 3

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Task 1

Def(Linear) Given two measurable function f and g and α, $\beta \in \mathbb{C}$. An operator $\mathcal{F}$ is called *linear* if the following identity holds true:

$$\mathcal{F}\left\{\alpha f + \beta g\right\}(x) = \alpha \mathcal{F}\left\{f\right\}(x) + \beta \mathcal{F}\left\{g\right\}(x) \tag{1}$$

Def(Shift Invariant) Given a measurable function f. An operator $\mathcal{F}$ is called *shift invariant* if the following identity holds true:

$$\mathcal{F}\left\{S_\delta f\right\}(x) = S_\delta(\mathcal{F}\left\{f\right\})(x) \tag{2}$$

where the shift operator S_δ is defined as the following:

$$S_\delta(f)(x) = f(x+\delta) \tag{3}$$

For any given function f and small number $\delta \in \mathbb{R}$.

Note that notational convention the operation notion for any operator F, we write $F(f)(x)$ instead of $F(f(x))$. In words we apply the operator F (i.e. the functional F) to the function f. Thus is would even be possible to omit the argument x. However to make the reasoning clear, I will keep it.

Therefore, directly applying the definition from equation 3 to equation 2 we can also define the property *shift invariant* as the follwoing:

$$\mathcal{F}\left\{S_\delta(f(x))\right\} := \mathcal{F}\left\{f(x+\delta)\right\} = S_\delta(\mathcal{F}\left\{f\right\}(x) \tag{4}$$

Relying on the definitions of the equations before, equation 4 and equation 1 let I solved this task,

(a) $\mathcal{F}\left\{f\right\}(x) = e^{f(x)}$ is *not linear* and is *shift invariant*.

Show: $\mathcal{F}$ is not *linear*

Proof.

$$\begin{aligned}\mathcal{F}\left\{\alpha f + \beta g\right\}(x) &= e^{\alpha f(x)+\beta g(x)} \\ &= e^{\alpha f(x)} e^{\beta g(x)} \\ &\neq \alpha e^{f(x)} + \beta e^{g(x)} \\ &= \alpha \mathcal{F}\left\{f\right\}(x) + \beta \mathcal{F}\left\{g\right\}(x)\end{aligned}$$

□

Show $\mathcal{F}$ is *shift invariant*

Proof.

$$\begin{aligned}\mathcal{F}\left\{S_\delta(f(x))\right\} &= \mathcal{F}\left\{f(x+\delta)\right\} \\ &= e^{f(x+\delta)} \\ &= S_\delta(e^{f(x)}) \\ &= S_\delta(\mathcal{F}\left\{f\right\}(x)\end{aligned}$$

□

(b) $\mathcal{F}\left\{f\right\}(x) = f(x)f(x-1)$ is *not linear* and is *shift invariant*.

Show $\mathcal{F}$ is *linear*

Proof.

$$\begin{aligned}\mathcal{F}\left\{\alpha f + \beta g\right\}(x) &= (\alpha f(x) + \beta g(x))(\alpha f(x-1) + \beta g(x-1)) \\ &= \alpha^2 f(x) f(x-1) + \beta^2 g(x) g(x-1) + \alpha\beta(f(x)g(x-1) + f(x-1)g(x)) \\ &\neq \alpha f(x) f(x-1) + \beta g(x) g(x-1) \\ &= \alpha \mathcal{F}\left\{f\right\}(x) + \beta \mathcal{F}\left\{g\right\}(x)\end{aligned}$$

□

Show $\mathcal{F}$ is *shift invariant*

Proof.

$$a = a$$

□

(c) $\mathcal{F}\left\{f\right\}(x) = s$ is *not linear* and is *shift invariant.*

Show $\mathcal{F}$ is *linear*

Proof.

$$a = a$$

□

Show $\mathcal{F}$ is *shift invariant*

Proof.

$$a = a$$

□

(d) $\mathcal{F}\left\{f\right\}(x) = s$ is *not linear* and is *shift invariant.*

Show $\mathcal{F}$ is *linear*

Proof.

$$a = a$$

□

Show $\mathcal{F}$ is *shift invariant*

Proof.

$$a = a$$

□

(e) $\mathcal{F}\left\{f\right\}(x) = s$ is *not linear* and is *shift invariant.*

Show $\mathcal{F}$ is *linear*

Proof.

$$a = a$$

□

Show $\mathcal{F}$ is *shift invariant*

Proof.

$$a = a$$

□

(f) $\mathcal{F}\left\{f\right\}(x) = s$ is *not linear* and is *shift invariant.*

Show $\mathcal{F}$ is *linear*

Proof.

$$a = a$$

□

Show $\mathcal{F}$ is *shift invariant*

Proof.

$$a = a$$

□

(g) $\mathcal{F}\{f\}(x) = s$ is *not linear* and is *shift invariant*.

Show $\mathcal{F}$ is *linear*

Proof.

$$a = a$$

□

Show $\mathcal{F}$ is *shift invariant*

Proof.

$$a = a$$

□

(h) $\mathcal{F}\{f\}(x) = s$ is *not linear* and is *shift invariant*.

Show $\mathcal{F}$ is *linear*

Proof.

$$a = a$$

□

Show $\mathcal{F}$ is *shift invariant*

Proof.

$$a = a$$

□

Task 2

Given an $m \times n$ monochromatic (i.e. there is only one color-channel) Image I. Give an algorithm how to apply box-filtering on this image. Furthermore analyse the asymptotic complexity of this algorithm.

Algorithm 1 Moving Average box filter

Input: Grayscale *Image* I with resolution $m \times n$
Output: Box filtered Image *Image* $\hat{I}$

Procedures: $getDimensions(Image)$, $zeros(height, width)$

$[h, w] = getDimensions(I)$
$\hat{I} = zeros(h, w)$
$r = \lceil \frac{w-1}{2} \rceil$
Foreach $Pixel\, p \in Image\, I$ **do**
    $contribution = 0$
    Foreach $Pixel\, p_n \in r - Neighborhood\ \mathcal{N}_r(p)$ **do**
        $contribution = contribution + I(p + p_n)$
    end for
    $\hat{I}(p) = \frac{contribution}{m \cdot n}$
end for

Remarks:

- By pixels in the Algorithm 1 we are referring to the coordinates of the pixel in the image. Therefore p corresponds to the x and y coordinates of pixel p in the Image I.
- $I(p)$ denotes accessing the pixel-(color)-values in the images at the position of the pixel p in the image I.
- $\mathcal{N}_r(p)$ denotes the neighborhood with radius r around a given pixel p. In the context of pixel-coordinates, think of it as a box-grid, centred at the pixel coordinates of p. This grid has a radius of r. This means there are r neighbors (pixel-coordinates in the grid) below, on top, on the left and on the right of p.
- Our algorithm can easily be extended for color Images by simply applying the same algorithm to each color-channel separately.
- The assumption of being provided by a m by n can easily be extended for the case when $n \neq m$. This only will affect the computation of the radius r in algorithm 1.
  Computing $\lceil 0.5 \cdot \left(\lceil \frac{m-1}{2} \rceil + \lceil \frac{n-1}{2} \rceil \right) \rceil$ would be a valid option in order to compute r.
- If w (i.e. n) is odd, then $\lceil \frac{w-1}{2} \rceil$ is equal to $\frac{w-1}{2}$.

- The procedure *getDimensions* returns the width-and height resolution of a provided Image.
- The procedure *zeros* creates a new image with the provided resolutions.

Aysmptotic Complexity

Task 3

Task 4

Task 5

Task 6