MAT – 112: Calculus I and Modeling
Solution 1

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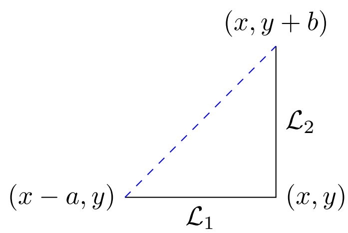
1/26/2018

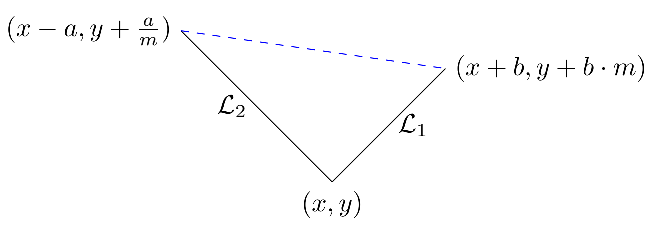
Other Problems

Problem 1. By definition, two lines $\mathcal{L}_1$ and $\mathcal{L}_2$ are perpendicular if $\mathcal{L}_1$ is horizontal and $\mathcal{L}_2$ is vertical, or the slope of $\mathcal{L}_1$ is $m \neq 0$, and the slope of $\mathcal{L}_2$ is $-\frac{1}{m}$. The horizontal and vertical case is shown below. The angle intersecting



the two lines $\mathcal{L}_1$ and $\mathcal{L}_2$ is equal to 90 degrees if and only if the above triangle is a right triangle and therefore satisfies Pythagorean's theorem. The length of the blue dashed line is equal to the distance between the points $(x-a, y)$ and $(x, y+b)$ which is equal to $\sqrt{a^2+b^2}$. Since the legs of the triangle are of length a and b, it follows that Pythagorean's theorem holds.

The case where $\mathcal{L}_1$ has a slope of $m \neq 0$ and $\mathcal{L}_2$ has a slope of $-\frac{1}{m}$ is shown below. Again, we must show that Pythagorean's theorem holds. The length of



the blue dashed line is equal to

$$\sqrt{(a+b)^2+(\frac{a}{m}-b\cdot m)^2} = \sqrt{a^2+\frac{a^2}{m^2}+b^2+b^2m^2}.$$

Since the legs of the triangle are of length $\sqrt{a^2 + \frac{a^2}{m^2}}$ and $\sqrt{b^2 + b^2m^2}$, it follows that Pythagorean's theorem holds.

Problem 2. To show that $h = g \circ f$ is a function from A to C, we must show that h takes each element of A to exactly one element of C. To this end, let α be an element of A. Then, $f(\alpha)$ is a unique element in B, which is the domain of the function g. It follows that g takes $f(\alpha)$ to exactly one element of C. Therefore, $h(\alpha) = g(f(\alpha))$ is a unique element in C.

Problem 3. Using the method of completing the square, we can transfer the quadratic $ax^2 + bx + c$, where $a \neq 0$, into vertex form as follows

$$\begin{aligned}
ax^2 + bx + c &= a\left(x^2 + \frac{b}{a}x\right) + c \\
&= a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - \frac{b^2}{4a} \\
&= a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a}
\end{aligned}$$

Once in vertex form, we can identify the vertex, axis of symmetry, x-intercept, and y-intercept.
Vertex: $(-\frac{b}{2a}, \frac{4ac-b^2}{4a})$
Axis of Symmetry: $x = -\frac{b}{2a}$
y-intercept: $(0, c)$
x-intercept: The x-intercepts occur when the quadratic equals zero. Thus, we can solve for x as follows

$$\begin{aligned}
a\left(x + \frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} &= 0 \\
a\left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a} \\
\left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\
x + \frac{b}{2a} &= \pm\frac{\sqrt{b^2 - 4ac}}{2a} \\
x &= -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}
\end{aligned}$$

Note that we have derived the quadratic formula.