MAT – 112: Calculus I and Modeling

Solution 6

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Other Problems

Problem 1. The Volume of a right circular cone is given by

$$V = \frac{\pi}{3} r^2 h,$$

where r is the radius of the circle and h is the height of the cone. In our current scenario we have $r = x$ and $h = y + 3$. Furthermore, since the cone is inscribed by a sphere of radius 3 we have $x^2 + y^2 = 3^2$. Therefore, we can write the volume as a function of y:

$$\begin{aligned} V(y) &= \frac{\pi}{3}(9 - y^2)(y + 3) \\ &= \frac{\pi}{3}(27 + 9y - 3y^2 - y^3), \end{aligned}$$

where $0 \leq y \leq 3$. Therefore, we can find the absolute max volume of the cone by finding the critical points and comparing the volume at the critical points and end points. Note that

$$V^{'}(y) = \frac{\pi}{3}(9 - 6y - 3y^2),$$

which is zero when $y = -3$ and $y = 1$. Furthermore,

$$\begin{aligned} V(0) &= 9\pi, \\ V(1) &= \frac{32}{3}\pi, \\ V(3) &= 0. \end{aligned}$$

Therefore, the max volume of the cone is $\frac{32}{3}\pi$ and occurs when $y = 1$ and $x = \sqrt{8}$.