

# MAT – 112: Calculus I and Modeling

## Product Rule

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### Instructions

Write out your own proof of the product using the outline below. Fill in the blanks and follow the prompts given.

**Product Rule.** Let  $f$  and  $g$  be functions that are differentiable at  $x$ , then

$$\frac{d}{dx}(fg)(x) = f'(x)g(x) + f(x)g'(x).$$

*Proof.* Note that

$$\begin{aligned}\frac{d}{dx}(fg)(x) &= \lim_{h \rightarrow 0} \frac{(fg)(x+h) - fg(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x)}{h} \\ &= \lim_{h \rightarrow 0} \frac{f(x+h)g(x+h) - f(x)g(x+h) + f(x)g(x+h) - f(x)g(x)}{h} \quad (\text{Justify}) \\ &= \underline{\hspace{2cm}} \quad (\text{Use sum limit rule}) \\ &= \lim_{h \rightarrow 0} g(x+h) \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} + \lim_{h \rightarrow 0} f(x) \lim_{h \rightarrow 0} \frac{g(x+h) - g(x)}{h} \quad (\text{Justify}) \\ &= g(x)f'(x) + f(x)g'(x) \\ &= f'(x)g(x) + f(x)g'(x).\end{aligned}$$

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