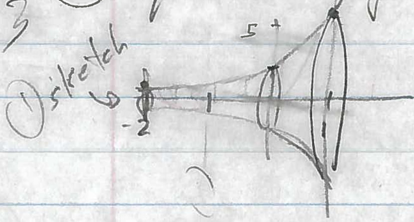
#~~3~~ 30

8.3

12. $f(x) = 2e^x,\ y=0,\ x=-2,\ x=1$

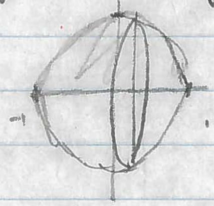
1 sketch



$$V = \pi\int_{-2}^{1}(2e^x)^2\,dx = 4\pi\int_{-2}^{1} e^{2x}\,dx \quad \left(\overset{TI}{=}\ 2(e^6-1)e^{-4}\pi\right) \text{ ok!}$$

$$= 4\pi\left(\tfrac{1}{2}\right)e^{2x}\Big|_{-2}^{1} = 2\pi\left(e^2 - e^{-4}\right) \approx 46.3117$$

17. $f(x) = 1-x^2,\ y=0$



$$V = \pi\int_{-1}^{1}(1-x^2)^2\,dx = \pi\int_{-1}^{1}(1-2x^2+x^4)\,dx \quad \overset{TI}{=} \tfrac{16\pi}{15}$$

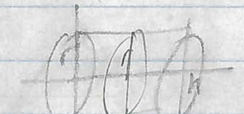
$$= \pi\left[x - \tfrac{2}{3}x^3 + \tfrac{1}{5}x^5\Big|_{-1}^{1}\right] = \pi\left[1-\tfrac{2}{3}+\tfrac{1}{5} - \left(-1+\tfrac{2}{3}-\tfrac{1}{5}\right)\right]$$

$$= \pi\left[2 - \tfrac{4}{3} + \tfrac{2}{5}\right] = 16\pi/15$$

24. $$V = \pi\int_{-a}^{a}\left(\tfrac{b}{a}\sqrt{a^2-x^2}\right)^2 dx = \frac{\pi b^2}{a^2}\int_{-a}^{a} a^2 - x^2\,dx = \frac{\pi b^2}{a^2}\left(a^2x - \frac{x^3}{3}\right)\Bigg|_{-a}^{a} =$$

$$= \frac{\pi b^2}{a^2}\left[a^3 - \frac{a^3}{3} - \left(-a^3 + \frac{a^3}{3}\right)\right] = \frac{\pi b^2}{a^2}\left(2a^3 - \frac{2}{3}a^3\right) = \frac{\pi b^2}{a^2}\left(\frac{4}{3}a^3\right) = \frac{4}{3}\pi b^2 a$$

25.



r is constant ($f(x) = r$)

$$V = \pi\int_0^h r^2\,dx = \pi r^2\int_0^h dx = \pi r^2(x)\Big|_0^h = \pi r^2(h-0) = \pi r^2 h$$

31. ave $e^{x/5}$ over $[0,5]$

$$\frac{1}{5}\int_0^5 e^{x/5}\,dx = \frac{1}{5}5e^{x/5}\Big|_0^5 = e^1 - e^0 = e-1 \approx 1.718$$

or $$\left(\frac{1}{5}\int_0^5 e^{x/5}\,du = \frac{1}{5}\int_0^1 e^u 5\,du = e^u\Big|_0^1 = e^1 - e^0 = e-1 \approx 1.718\right)$$

$u = \frac{x}{5}$, $du = \frac{1}{5}dx$, $5du = dx$

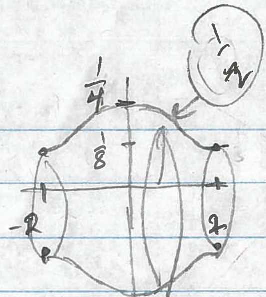
x	u
0	0
5	1

34. ave $\sin x$ over $[0,\pi]$

$$\frac{1}{\pi}\int_0^\pi \sin x\,dx = \frac{1}{\pi}(-\cos x)\Big|_0^\pi = \frac{1}{\pi}\left[-\cos\pi + \cos 0\right] = \frac{1}{\pi}[1+1] = \frac{2}{\pi} \approx .63662$$

TI ok

1 1

(36.) 

$$V = \pi \int_{-2}^{2} \left(\frac{1}{4+x^2}\right)^2 dx \overset{TI}{=} \frac{\pi(\pi+2)}{32} \approx .504775$$

$\frac{1}{2}$ either $\frac{1}{2}$ fnInt 1

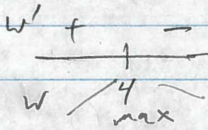
(42.) $W(t) = -3.75t^2 + 30t + 40$ words/min typing. over $[0,5]$ min?

a) $W(0) = 40$ words/min

b) max W?

(1) $W'(t) = -7.5t + 30 = 0$

$t = \frac{-30}{-7.5} = 4$



$W(4) = -3.75(16) + 30(4) + 40 = 100$

($\frac{1}{2}$) at $t = 4$ min the maximum is 100 words/min ←

c) ave W? (1)

$$\frac{1}{5}\int_0^5 -3.75t^2 + 30t + 40\,dt = \frac{1}{5}\left[\frac{-3.75}{3}t^3 + \frac{30}{2}t^2 + 40t\right]_0^5$$

$$= \frac{1}{5}\left[\frac{-3.75}{3}(5)^3 + 15(25) + 40(5) - 0\right] = \frac{1}{5}[418.75] = 83.75 \text{ words/min}$$

is average

Section 7.4

3. $\displaystyle\int_{-1}^{2}(5t-3)\,dt=(5t^2/2-3t)\Big|_{-1}^{2}=(10-6)-(5/2+3)=-\frac{3}{2}$

7. Letting $w=4u+1, dw=4du$;

$$\int_0^2 3\sqrt{4u+1}\,du=\frac{3}{4}\int_1^9 w^{1/2}\,dw=\frac{1}{2}w^{3/2}\Big|_1^9=\frac{1}{2}(27-1)=13$$

14. Letting $u=2p+1, du=2dp$; $\displaystyle\int_1^4\frac{-3}{(2p+1)^2}\,dp=-\frac{3}{2}\int_3^9 u^{-2}\,du=\frac{3}{2u}\Big|_3^9=\frac{1}{6}-\frac{1}{2}=-\frac{1}{3}$

20. $\displaystyle\int_{0.5}^{1}(p^3-e^{4p})\,dp=(p^4/4-e^{4p}/4)\Big|_{0.5}^{1}=(1/4-e^4/4)-(1/64-e^2/4)\approx-11.57$

21. Letting $u=2y^2-3, du=4ydy$;

$$\int_{-1}^{0}y(2y^2-3)^5\,dy=\frac{1}{4}\int_{-1}^{-3}u^5\,du=\frac{1}{24}u^6\Big|_{-1}^{-3}=\frac{(-3)^6}{24}-\frac{(-1)^6}{24}=\frac{91}{3}$$

26. Letting $u=\ln x, du=\dfrac{1}{x}dx$;

$$\int_1^3\frac{(\ln x)^{1/2}}{x}\,dx=\int_0^{\ln 3}u^{1/2}\,du=\frac{2}{3}u^{3/2}\Big|_0^{\ln 3}=\frac{2}{3}(\ln 3)^{3/2}\approx 0.7677$$

30. Letting $u=1+e^{2z}, du=2e^{2z}dz$ we have

$$\int_0^1\frac{e^{2z}}{\sqrt{1+e^{2z}}}\,dz = \frac{1}{2}\int_2^{1+e^2}u^{-1/2}\,du=u^{1/2}\Big|_2^{1+e^2}=\sqrt{1+e^2}-\sqrt{2}\approx 1.482$$

31. $\displaystyle-\cos x\Big|_0^{\pi/4}=1-\frac{\sqrt{2}}{2}$

63. The total change in the pollution concentration over the 4–year period $0\le t\le 4$ is

$$\begin{aligned}P(4)-P(0) &= \int_0^4 P'(t)\,dt=\int_0^4 140t^{5/2}\,dt=140\int_0^4 t^{5/2}\,dt\\ &= 140\left(\frac{2}{7}t^{7/2}\Big|_0^4\right)=140\cdot\left(\frac{2^8}{7}-0\right)=5120>4850\end{aligned}$$

so the answer is “no”.

64. Letting $u = \ln(t+1), du = \dfrac{dt}{t+1}$ we have

(a) $\displaystyle\int_0^{24} \frac{80\ln(t+1)}{t+1}\ dt = 80\int_0^{\ln 25} u\ du = 40u^2\Big|_0^{\ln 25} = 40(\ln 25)^2 - 0 = 40(\ln 25)^2 \approx 414$ barrels

(b) $\displaystyle\int_{24}^{48} \frac{80\ln(t+1)}{t+1}\ dt = 80\int_{\ln 25}^{\ln 49} u\ du = 40u^2\Big|_{\ln 25}^{\ln 49} = 40(\ln 49)^2 - 40(\ln 25)^2 \approx 191$ barrels

Section 7.4

49. The area over the interval $[0, 2]$ is $\int_0^2 (4 - x^2)\ dx = (4x - x^3/3)\Big|_0^2 = 8 - \frac{8}{3} = \frac{16}{3}$. The area below the interval $[2, 3]$ is

$$-\int_2^3 (4 - x^2)\ dx = -(4x - x^3/3)\Big|_2^3 = -(12 - 9) + (8 - \frac{8}{3}) = \frac{7}{3}$$

Hence, the total shaded area is $\frac{16}{3} + \frac{7}{3} = \frac{23}{3}$.

55. The two quarter circles cancel out, since they are the same shape and size and one is above the x–axis while the other is below. The trapezoid over $[0, 2]$ has area 4 and the triangle below $[8, 16]$ has area 12. Therefore, $\int_0^{16} f(x)\ dx = 4 - 12 = -8$.

Section 11.1

1. $\frac{dy}{dx} = -4x + 6x^2 \Rightarrow y = \int (-4x + 6x^2) dx = -2x^2 + 2x^3 + C$

2. $\frac{dy}{dx} = 4e^{-3x} \Rightarrow y = \int 4e^{-3x}\ dx = -\frac{4}{3}e^{-3x} + C$

3. $4x^3 - 2\frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = 2x^3 \Rightarrow y = \int 2x^3\ dx = \frac{1}{2}x^4 + C$

5. $y\frac{dy}{dx} = x^2 \Rightarrow y\ dy = x^2\ dx \Rightarrow \int y\ dy = \int x^2\ dx \Rightarrow \frac{1}{2}y^2 = \frac{1}{3}x^3 + C$

6. $y\frac{dy}{dx} = x^2 - x \Rightarrow y\ dy = (x^2 - x)\ dx \Rightarrow \int y\ dy = \int (x^2 - x)\ dx \Rightarrow \frac{1}{2}y^2 = \frac{1}{3}x^3 - \frac{1}{2}x^2 + C$

10. $(y^2 - y)\frac{dy}{dx} = x \Rightarrow (y^2 - y)\ dy = x\ dx \Rightarrow \int (y^2 - y)\ dy = \int x\ dx \Rightarrow \frac{1}{3}y^3 - \frac{1}{2}y^2 = \frac{1}{2}x^2 + C$

14. $\frac{dy}{dx} = \frac{e^{y^2}}{y} \Rightarrow \frac{y}{e^{y^2}}\ dy = dx \Rightarrow ye^{-y^2}\ dy = dx \Rightarrow \int ye^{-y^2}\ dy = \int dx \Rightarrow -\frac{1}{2}e^{-y^2} = x + C$

16. $\frac{dy}{dx} = \frac{e^x}{e^y} \Rightarrow e^y\ dy = e^x\ dx \Rightarrow \int e^y\ dy = \int e^x\ dx \Rightarrow e^y = e^x + C \Rightarrow y = \ln(e^x + C)$

19. $\frac{dy}{dx} + 3x^2 = 2x \Rightarrow \frac{dy}{dx} = 2x - 3x^2 \Rightarrow y = \int (2x - 3x^2)\ dx = x^2 - x^3 + C$
We set $x = 0$ and $y = 5$ to get $5 = C$ and therefore $y = x^2 - x^3 + 5$.

23. $\dfrac{dy}{dx} = \dfrac{x^3}{y} \Rightarrow y\ dy = x^3\ dx \Rightarrow \displaystyle\int y\ dy = \int x^3\ dx \Rightarrow \frac{1}{2}y^2 = \frac{1}{4}x^4 + C$

We set $x = 0$ and $y = 5$ to get $\frac{25}{2} = C$ and therefore $\dfrac{1}{2}y^2 = \dfrac{1}{4}x^4 + \dfrac{25}{2}$ or $y^2 = \dfrac{1}{2}x^4 + 25$

27. $\dfrac{dy}{dx} = \dfrac{2x+1}{y-3} \Rightarrow (y-3)\ dy = (2x+1)\ dx \Rightarrow \displaystyle\int (y-3)\ dy = \int (2x+1)\ dx$

$\Rightarrow \dfrac{1}{2}y^2 - 3y = x^2 + x + C$. We set $x = 0$ and $y = 4$ to get $8 - 12 = -4 = C$ and therefore $\dfrac{1}{2}y^2 - 3y = x^2 + x - 4$.