Section 2.1

3. E

4. D

6. F

8. D

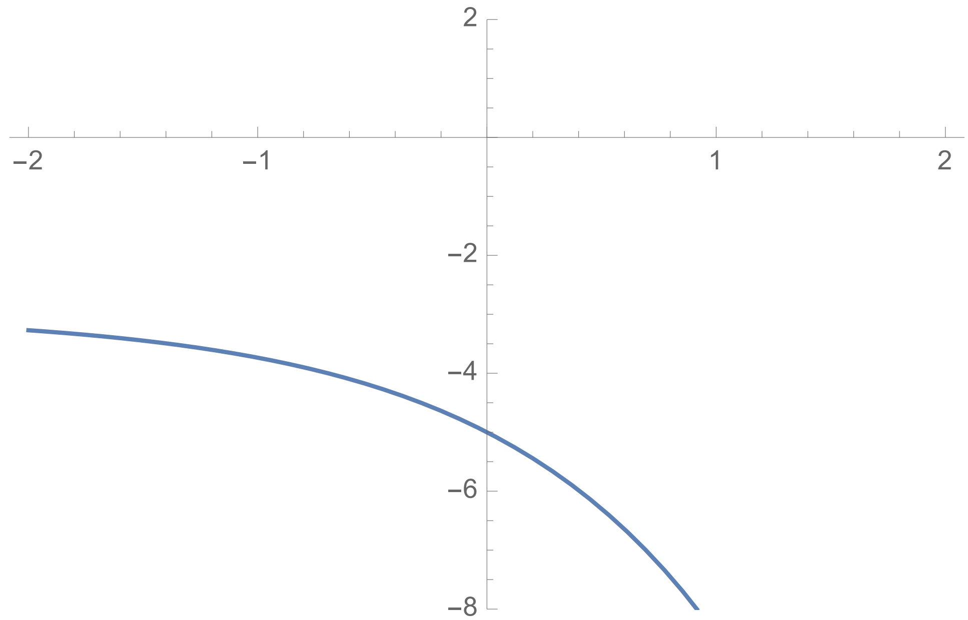
19. Since $16^{x+3} = (2^4)^{x+3} = 2^{4x+12}$ and $64^{2x-5} = (2^6)^{2x-5} = 2^{12x-30}$ the equation $16^{x+3} = 64^{2x-5}$ is equivalent to the equation $2^{4x+12} = 2^{12x-30}$ which, in turn, is equivalent to the equation $4x + 12 = 12x - 30$ which has solution $x = 21/4$.

21. The equation $e^{-x} = (e^4)^{x+3} = e^{4x+12}$ is equivalent to the equation $-x = 4x + 12$ which has solution $-12/5$.

24. The equation $2^{x^2-4x} = \left(\dfrac{1}{16}\right)^{x-4} = (2^{-4})^{x-4} = 2^{-4x+16}$ is equivalent to the equation $x^2 = 16$ which has solutions $x = \pm 4$.

26. The equation $8^{x^2} = (2^3)^{x^2} = 2^{3x^2} = 2^{x+4}$ is equivalent to the equation $3x^2 = x+4$ which, in turn, is equivalent to the equation $3x^2 - x - 4 = (3x - 4)(x + 1) = 0$ which has solutions $x = 4/3, -1$.

30.



38. (a) $f(1) = 500 \cdot 2^3 = 4000$

(b) $f(0) = 500 \cdot 2^0 = 500$

(c) Solution 1. By the answer to part (b), there were 500 bacteria present initially. We first find how much time must pass until there are 1000 present by solving $1000 = f(t)$ for t:

$$1000 = f(t) = 500 \cdot 2^{3t} \Rightarrow 2 = 2^1 = 2^{3t} \Rightarrow 1 = 3t \Rightarrow t = 1/3 \quad \text{(hr)}$$

Next, note that

$$f(t + 1/3) = 500 \cdot 2^{3(t+1/3)} = 500 \cdot 2^{3t+1} = 500 \cdot 2^{3t} \cdot 2 = 2f(t)$$

which shows that no matter what the value of t, the number of bacteria doubles 1/3 of an hour (20 minutes) after that.

Solution 2. We solve directly for a time t_0 such that for all times t the equation $f(t + t_0) = 2f(t)$ is satisfied. Since

$$f(t + t_0) = 500 \cdot 2^{3(t+t_0)} = 500 \cdot 2^{3t+3t_0} = 500 \cdot 2^{3t} \cdot 2^{3t_0} = 2^{3t_0} f(t)$$

if $f(t + t_0) = 2f(t)$ then this means that

$$2^{3t_0} = 2 = 2^1 \Rightarrow 3t_0 = 1 \Rightarrow t_0 = 1/3 \text{ (hr)}$$

(or 20 minutes) as before.

(d) Solution 1. (This uses the answers from parts (b) and (c).) Note that 32,000/500 = $64 = 2^6$. So, to get from the initial number 500 to 32,000 we need to double 500 six times. Each "doubling" takes 1/3 of an hour, so for 500 to be doubled six times takes 2 hours.

Solution 2. (Starting from scratch.) We solve $32{,}000 = 500 \cdot 2^{3t}$ for t:

$$32{,}000 = 500 \cdot 2^{3t} \Rightarrow 64 = 2^6 = 2^{3t} \Rightarrow 6 = 3t \Rightarrow 2 = t$$

So it takes 2 hours.

47. (a) $Q(6) = 1000(5^{-1.8}) \approx 55.2$ g

(b) If $8 = 1000(5^{-0.3t})$ then $0.008 = 5^{-3} = 5^{-0.3t}$ which implies $-3 = -0.3t$ or $t = 10$ months.