Section 3.1

1. (c)

2. (a)

6. (a) 4 (b) 4

10. (a) (i) 1 (ii) 1 (iii) 1 (iv) 2

(b) (i) 0 (ii) 0 (iii) 0 (iv) 0

11. The limit exists and is equal to 3.

36.

$$\begin{aligned}\lim_{x\to-2}\frac{x^2-4}{x+2} &= \lim_{x\to-2}\frac{(x+2)(x-2)}{x+2} = \lim_{x\to-2}(x-2) \quad \text{(by limit property \#7)}\\ &= -4 \quad \text{(by limit property \#5)}\end{aligned}$$

38.

$$\begin{aligned}\lim_{x\to-3}\frac{x^2-9}{x^2+x-6} &= \lim_{x\to-3}\frac{(x+3)(x-3)}{(x+3)(x-2)} = \lim_{x\to-3}\frac{x-3}{x-2} \quad \text{(by limit property \#7)}\\ &= \frac{\lim_{x\to-3}(x-3)}{\lim_{x\to-3}(x-2)} \quad \text{(by limit property \#4)}\\ &= \frac{6}{5} \quad \text{(by limit property \#5)}\end{aligned}$$

41.

$$\begin{aligned}\lim_{x\to0}\frac{\frac{1}{x+3}-\frac{1}{3}}{x} &= \lim_{x\to0}\frac{\frac{3-(3+x)}{3(x+3)}}{x} = \lim_{x\to0}\frac{-x}{3x(x+3)}\\ &= \lim_{x\to0}\frac{-1}{3(x+3)} \quad \text{(by limit property \#7)}\\ &= \frac{\lim_{x\to0}(-1)}{\lim_{x\to0}(3(x+3))} \quad \text{(by limit property \#4)}\\ &= \frac{-1}{9} = -\frac{1}{9} \quad \text{(by limit properties \#1 and \#5)}\end{aligned}$$

44.

$$\begin{aligned}
\lim_{x\to 36}\frac{\sqrt{x}-6}{x-36} &= \lim_{x\to 36}\frac{\sqrt{x}-6}{(\sqrt{x}-6)(\sqrt{x}+6)} \quad \text{(by “strengthened” limit property \#7)}\\
&= \lim_{x\to 36}\frac{1}{\sqrt{x}+6} \quad \text{(by limit property \#7)}\\
&= \frac{\lim_{x\to 36} 1}{\lim_{x\to 36}(\sqrt{x}+6)} \quad \text{(by limit property \#4)}\\
&= \frac{1}{\lim_{x\to 36}\sqrt{x}+\lim_{x\to 36} 6} \quad \text{(by limit properties \#1 and \#2)}\\
&= \frac{1}{12} \quad \text{(by limit properties \#1, \#5, and \#6)}
\end{aligned}$$

50.

$$\begin{aligned}
\lim_{x\to -\infty}\frac{8x+2}{4x-5} &= \lim_{x\to -\infty}\frac{8+2/x}{4-5/x} = \frac{\lim_{x\to-\infty}(8+2/x)}{\lim_{x\to-\infty}(4-5/x)} \quad \text{(by limit property \#4)}\\
&= \frac{\lim_{x\to-\infty} 8+\lim_{x\to-\infty}(2/x)}{\lim_{x\to-\infty} 4-\lim_{x\to-\infty}(5/x)} \quad \text{(by limit property \#2)}\\
&= \frac{8+0}{4-0} = \frac{8}{4} = 2 \quad \text{(by limit property \#1 and the identities on page 141)}
\end{aligned}$$

54.

$$\begin{aligned}
\lim_{x\to\infty}\frac{2x^2-1}{3x^4+2} &= \lim_{x\to\infty}\frac{2/x^2-1/x^4}{3+2/x^4} \quad \text{(by “strengthened” limit property \#7)}\\
&= \frac{\lim_{x\to\infty}(2/x^2-1/x^4)}{\lim_{x\to\infty}(3+2/x^4)} \quad \text{(by limit property \#4)}\\
&= \frac{\lim_{x\to\infty} 2/x^2-\lim_{x\to\infty} 1/x^4}{\lim_{x\to\infty} 3+\lim_{x\to\infty} 2/x^4} \quad \text{(by limit property \#2)}\\
&= \frac{0}{3} \quad \text{(by limit property \#1 and the limits on page 147)}\\
&= 0
\end{aligned}$$

91. Since

$$\lim_{n\to\infty}\overline{C}(n) = \lim_{n\to\infty}\frac{C(n)}{n} = \lim_{n\to\infty}\frac{15000+60n}{n} = \lim_{n\to\infty}\frac{15000/n+60}{1} = \frac{0+60}{1} = 60$$

This means that when the test is done on a very large number of patients, the cost per patient of the test will be about \$60.

One scenario in which this could happen is when the initial cost overhead (purchase of a big piece of testing equipment, training of technicians who will perform the test, etc.)

is \$15,000 and the cost of the actual test itself (medications, materials, the wages of the technician, etc.) is \$60. Then, if n is the number of tests performed, the total cost (overhead plus the tests themselves) would be $C(n) = 15000 + 60n$. Note that as the number of tests grows very large, the average cost approaches the cost of the actual test itself.

Section 3.2

2. Discontinuous at $x = -1$. (a) 2 (b) 2 (c) 4 (d) Does not exist (e) The two–sided limit doesn't exist and therefore conditions 2 & 3 for continuity at $x = -1$ are not satisfied.

3. Discontinuous at $x = 1$. (a) 2 (b) -2 (c) -2 (d) -2 (e) Condition 3 for continuity is not satisfied. (BTW: The continuity is removeable.)

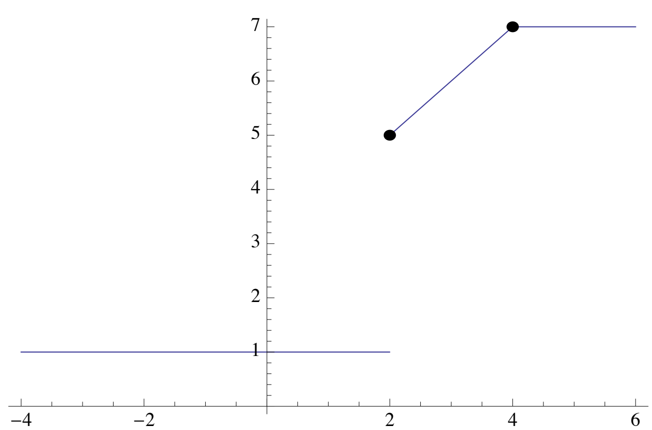
6. Discontinuous at $x = 0, 2$. $x = 0$: (a) Does not exist (b) $-\infty$ (Does not exist.) (c) $-\infty$ (Does not exist.) (d) $-\infty$ (Does not exist.) (e) All three conditions fail to be satisfied.

$x = 2$: (a) Does not exist (b) -2 (c) -2 (d) -2 (e) Conditions 1 & 3 fail to be satisfied. (This discontinuity is removeable.)

9. Discontinuous at $x = 2$; $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2}(x+2) = 4$

10. Discontinuous at $x = -5$; $\lim_{x\to -5} \frac{x^2-25}{x+5} = \lim_{x\to -5} \frac{(x-5)(x+5)}{x+5} = \lim_{x\to -5}(x-5) = -10$

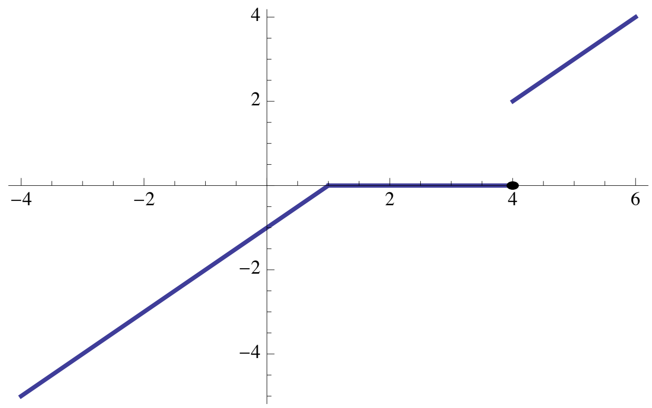
21.



Discontinuous at $x = 2$

(c) limit from the left is 1 and limit from the right is 5

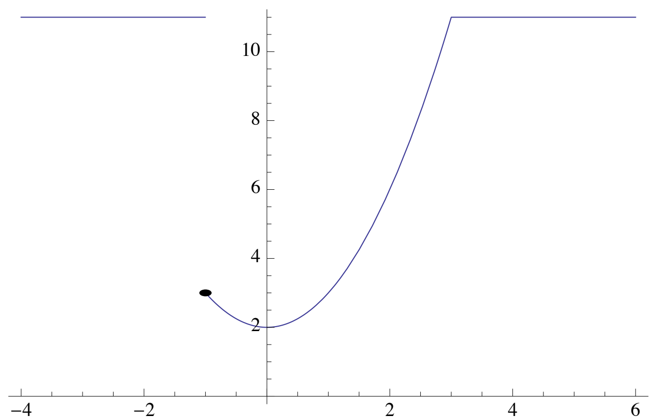
22.



Discontinuous at $x = 4$

(c) limit from the left is 0 and limit from the right is 2

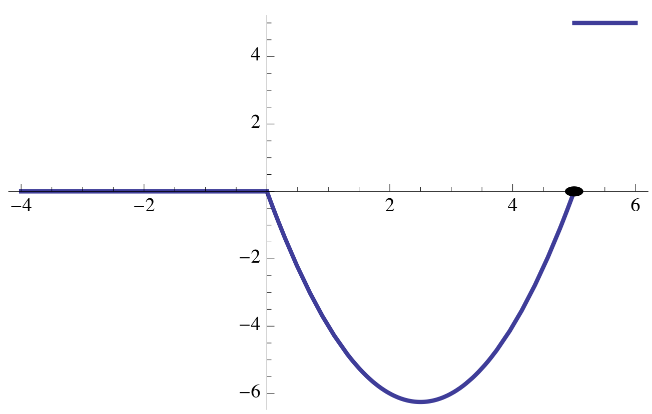
23.



Discontinuous at $x = -1$

(c) limit from the left is 11 and limit from the right is 3

24.



Discontinuous at $x = 5$

(c) limit from the left is 0 and limit from the right is 5

28. No matter the choice of k, f will be continuous for all $x < 3$ and for all $x > 3$ since on each of these open intervals f is equal to a polynomial and polynomials are continuous everywhere. So, we only need to worry about continuity at $x = 3$. Note that the one–sided limits at $x = 3$ are

$$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-}(x^3 + k) = 27 + k \quad \text{and} \quad \lim_{x\to 3^+} f(x) = \lim_{x\to 3^+}(kx - 5) = 3k - 5$$

Since continuity at $x = 3$ requires these two one–sided limits to be equal we must have

$$27 + k = 3k - 5 \Rightarrow 32 = 2k \Rightarrow k = 16$$

For $k = 16$ both one–sided limits are equal to 43 and thus $\lim\limits_{x\to 3} f(x) = 43 = f(3)$ and f is continuous at $x = 3$ (and therefore continuous for all x).

30. The function f is continuous on the open intervals $x < 2$ and $2 < x$ since on these open intervals f is equal to a rational function whose denominator is not 0. (See the description of continuity for a rational function in the table on page 149.) So, we only need to worry about continuity at $x = -2$. We check the limit at $x = -2$:

$$\lim_{x\to -2} f(x) = \lim_{x\to -2} \frac{3x^2 + 2x - 8}{x + 2} = \lim_{x\to -2} \frac{(x + 2)(3x - 4)}{x + 2} = \lim_{x\to -2}(3x - 4) = -10$$

Continuity of f at $x = -2$ requires that $f(-2) = k - 6 = -10$ which implies $k = -4$.

35. Since $g(x) = \dfrac{x + 4}{x^2 + 2x - 8} = \dfrac{x + 4}{(x + 4)(x - 2)}$ is a rational function it follows from the description in the table on page 149 that g is continuous on the open intervals $x < -4$, $-4 < x < 2$, and $2 < x$. Hence, g is continuous everywhere except at $x = -4, 2$ where the function is undefined. For x close to (but not equal to) -4 we have $g(x) = \dfrac{x + 4}{x^2 + 2x - 8} = \dfrac{x + 4}{(x + 4)(x - 2)} = \dfrac{1}{x - 2}$. Therefore,

$$\lim_{x\to -4} g(x) = \lim_{x\to -4} \frac{1}{x - 2} = -\frac{1}{6}$$

and if we define $g(-4) = -1/6$ then the new g becomes continuous at $x = -4$. However, at $x = 2$ both one–sided limits of g are infinite and thus $\lim\limits_{x\to 2} g(x)$ doesn't exist. Since this remains the case no matter how we define $g(2)$, there is no way to remove the discontinuity at $x = 2$. Hence, the correct choice is (a).

Section 3.3

1. $\dfrac{3^2+6-(1^2+2)}{3-1} = \dfrac{12}{2} = 6$

4. $\dfrac{2\cdot 4^3 - 4\cdot 4^2 + 6\cdot 4 - (2\cdot(-1)^3 - 4\cdot(-1)^2 + 6\cdot(-1))}{4-(-1)} = \dfrac{88-(-12)}{5} = \dfrac{100}{5} = 20$

6. $\dfrac{\sqrt{4}-\sqrt{1}}{2-1} = \dfrac{2-1}{2-1} = 1$

9. $\dfrac{e^0 - e^{-2}}{0-(-2)} = \dfrac{1-e^{-2}}{2} \approx 0.4323$

10. $\dfrac{\ln 4 - \ln 2}{4-2} = \dfrac{\ln 2}{2} \approx 0.3466$

17. $\lim\limits_{h\to 0} \dfrac{(1+h)^3 + 2(1+h) + 9 - 12}{h} = \lim\limits_{h\to 0} \dfrac{1+3h+3h^2+h^3+2+2h+9-12}{h}$

$= \lim\limits_{h\to 0} \dfrac{5h+3h^2+h^3}{h} = \lim\limits_{h\to 0}(5+3h+h^2) = 5$

20. $\lim\limits_{h\to 0} \dfrac{-4(2+h)^2 - 6 - (-22)}{h} = \lim\limits_{h\to 0} \dfrac{-4(4+4h+h^2)+16}{h} = \lim\limits_{h\to 0} \dfrac{-16h-4h^2}{h}$

$= \lim\limits_{h\to 0}(-16-4h) = -16$

25. $\dfrac{2.001^{2.001}-4}{0.001} = 6.7793269821$ and $\dfrac{1.999^{1.999}-4}{-0.001} = 6.7658599872$ Averaging these two estimates yields 6.7726 (to 4 decimal places).

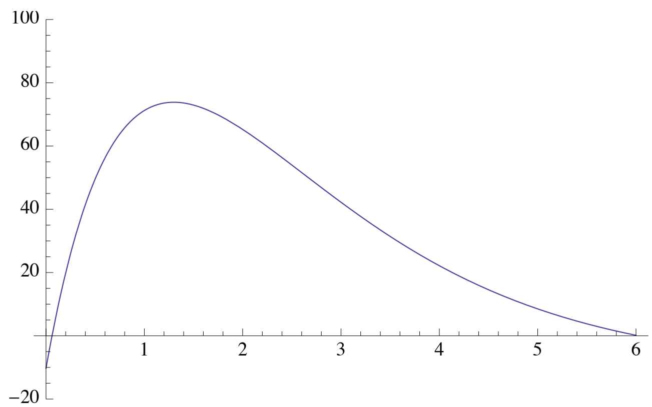
31. (a) $\dfrac{20-2}{4-1} = \dfrac{18}{3} = 6\%$ per day

(b) $\lim\limits_{h\to 0} \dfrac{(3+h)^2 + (3+h) - 12}{h} = \lim\limits_{h\to 0} \dfrac{9+6h+h^2+3+h-12}{h} = \lim\limits_{h\to 0} \dfrac{7h+h^2}{h}$

$= \lim_{h\to 0}(7+h) = 7\%$

This means that at exactly 3 days after the flu begins, the percent of the population infected with the flu is growing at an instantaneous rate of change of 7% per day.

34.(a)



The thermic effect of food

(b) $\dfrac{f(1)-f(0)}{1-0} = \dfrac{71.2266720478-(-10.28)}{1} = 81.5066720478$ kilojoules per hour per hour

(c) $\displaystyle\lim_{h\to 0}\frac{f(1+h)-f(1)}{h} = \lim_{h\to 0}\frac{-10.28+175.9(1+h)e^{-(1+h)/1.3}-(-10.28+175.9e^{-1/1.3})}{h}$

≈ 18.81 kilojoules per hour per hour

(d) ≈ 1.3 hours

Section 3.4

5. 2

6. -1

7. 1/4

8. $-4/5$

9. 0

10. Undefined

21. (a) Setting $x = 3$ and $x = 5$ we obtain the points $(3, 15)$ and $(5, 35)$ respectively on the graph of f. The slope of the secant line through these two points is $\dfrac{35 - 15}{5 - 3} = 10$. Using the point $(5, 35)$ we obtain the point–slope equation $y - 35 = 10(x - 5)$ which can be written in slope–intercept form as $y = 10x - 15$. (The point $(3, 15)$ could have been used instead and the same slope–intercept equation would have resulted.)

(b) The slope m of the tangent line through the point $(3, 15)$ is

$$\begin{aligned} m &= \lim_{h\to 0} \frac{f(3+h) - f(3)}{h} = \lim_{h\to 0} \frac{(3+h)^2 + 2(3+h) - 15}{h} = \lim_{h\to 0} \frac{9 + 6h + h^2 + 6 + 2h - 15}{h} \\ &= \lim_{h\to 0} \frac{8h + h^2}{h} = \lim_{h\to 0} (8 + h) = 8 \end{aligned}$$

Hence, the point–slope equation of the tangent line through $(3, 15)$ is $y - 15 = 8(x - 3)$ which can be written in the slope intercept form as $y = 8x - 9$.

22. (a) Setting $x = -1$ and $x = 3$ we obtain the points $(-1, 5)$ and $(3, -3)$ respectively on the graph of f. The slope of the secant line through these two points is $\dfrac{5 - (-3)}{-1 - 3} = -2$. Using the point $(3, -3)$ we obtain the point–slope equation $y + 3 = -2(x - 3)$ which can be written in slope–intercept form as $y = -2x + 3$.

(b) The slope m of the tangent line through the point $(-1, 5)$ is

$$\begin{aligned} m &= \lim_{h\to 0} \frac{f(-1+h) - f(-1)}{h} = \lim_{h\to 0} \frac{6 - (-1+h)^2 - 5}{h} = \lim_{h\to 0} \frac{6 - (1 - 2h + h^2) - 5}{h} \\ &= \lim_{h\to 0} \frac{2h - h^2}{h} = \lim_{h\to 0} (2 - h) = 2 \end{aligned}$$

Hence, the point–slope equation of the tangent line through $(-1, 5)$ is $y - 5 = 2(x + 1)$ which can be written in the slope intercept form as $y = 2x + 7$.

23. (a) Setting $x = 2$ and $x = 5$ we obtain the points $(2, 5/2)$ and $(5, 1)$ respectively on the graph of f. The slope of the secant line through these two points is $\dfrac{1 - 5/2}{5 - 2} = -1/2$. Using the point $(5, 1)$ we obtain the point–slope equation $y - 1 = -\frac{1}{2}(x - 5)$ which can be written in slope–intercept form as $y = -\frac{1}{2}x + \frac{7}{2}$.

(b) The slope m of the tangent line through the point $(2, 5/2)$ is

$$\begin{aligned} m &= \lim_{h \to 0} \frac{f(2+h) - f(2)}{h} = \lim_{h \to 0} \frac{\frac{5}{2+h} - \frac{5}{2}}{h} = \lim_{h \to 0} \frac{\frac{10 - 5(2+h)}{2(2+h)}}{h} \\ &= \lim_{h \to 0} \frac{-5h}{2h(2+h)} = \lim_{h \to 0} \frac{-5}{2(2+h)} = -\frac{5}{4} \end{aligned}$$

Hence, the point–slope equation of the tangent line through $(2, 5/2)$ is $y - 5/2 = -\frac{5}{4}(x - 2)$ which can be written in the slope intercept form as $y = -\frac{5}{4}x + 5$.

26. (a) Setting $x = 25$ and $x = 36$ we obtain the points $(25, 5)$ and $(36, 6)$ respectively on the graph of f. The slope of the secant line through these two points is $\dfrac{6 - 5}{36 - 25} = 1/11$. Using the point $(25, 5)$ we obtain the point–slope equation $y - 5 = \frac{1}{11}(x - 25)$ which can be written in slope–intercept form as $y = \frac{1}{11}x + \frac{30}{11}$.

(b) The slope m of the tangent line through the point $(25, 5)$ is

$$\begin{aligned} m &= \lim_{h \to 0} \frac{f(25+h) - f(25)}{h} = \lim_{h \to 0} \frac{\sqrt{25+h} - 5}{h} \\ &= \lim_{h \to 0} \frac{\sqrt{25+h} - 5}{h} \cdot \frac{\sqrt{25+h} + 5}{\sqrt{25+h} + 5} \\ &= \lim_{h \to 0} \frac{25 + h - 25}{h(\sqrt{25+h} + 5)} = \lim_{h \to 0} \frac{h}{h(\sqrt{25+h} + 5)} \\ &= \lim_{h \to 0} \frac{1}{\sqrt{25+h} + 5} = \frac{1}{10} \end{aligned}$$

Hence, the point–slope equation of the tangent line through $(25, 5)$ is $y - 5 = \frac{1}{10}(x - 25)$ which can be written in the slope intercept form as $y = \frac{1}{10}x + \frac{5}{2}$.

39. (a) $[a, 0)$ and $(b, c]$ (b) $(0, b)$ (c) only at $x = 0$ and $x = b$

51. (a) If we let m denote the instantaneous rate of change of I with respect to time t at $t = 5$ then

$$\begin{aligned} m &= \lim_{h\to 0} \frac{I(5+h) - I(5)}{h} = \lim_{h\to 0} \frac{27 + 72(5+h) - 1.5(5+h)^2 - 349.5}{h} \\ &= \lim_{h\to 0} \frac{27 + 360 + 72h - 1.5(25 + 10h + h^2) - 349.5}{h} \\ &= \lim_{h\to 0} \frac{72h - 15h - 1.5h^2}{h} = \lim_{h\to 0} \frac{57h - 1.5h^2}{h} \lim_{h\to 0}(57 - 1.5h) = 57 \end{aligned}$$

This means that exactly 5 minutes into the meal the person is eating at a rate of 57 grams per minute.

(b) If we let m denote the instantaneous rate of change of I with respect to time t at $t = 24$ then

$$\begin{aligned} m &= \lim_{h\to 0} \frac{I(24+h) - I(24)}{h} = \lim_{h\to 0} \frac{27 + 72(24+h) - 1.5(24+h)^2 - 891}{h} \\ &= \lim_{h\to 0} \frac{27 + 1728 + 72h - 1.5(576 + 48h + h^2) - 891}{h} \\ &= \lim_{h\to 0} \frac{-1.5h^2}{h} = \lim_{h\to 0} -1.5h = 0 \end{aligned}$$