CSC/MAT-220: Discrete Structures
EFY 6

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Binomials

Prove the following Theorems.

Binomial Theorem: Let $n \in \mathbb{N}$ and x and y be variables. Then,

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}.$$

Hint: In a class of n students, each student is given the choice of solving either one of x different algebra problems or one of y different geometry problems. How many different outcomes are possible?

Solution: Since each student has $(x+y)$ choices for which problem to solve, there are $(x+y)^n$ possible outcomes.

Additionally, we can count the number of outcomes based on the condition on the number of students who choose to solve an algebra problem. For $0 \leq k \leq n$, there are $\binom{n}{k}$ ways to select which k of the n students chose an algebra problem, then x^k ways for those students to decide which algebra problems to do, and y^{n-k} ways for the remaining $(n-k)$ students to decide which geometry problems to do. Altogether, there are $\sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$ possible outcomes.

Binomial Formula: Let $0 \leq k \leq n$. Then,

$$n! = \binom{n}{k} k!(n-k)!$$

Hint: How many ways can the numbers 1 through n be arranged in a list?

Solution: There are $n!$ arrangements, since the first number can be chosen n ways, the next number can be chosen $(n-1)$ ways, and so on.

Additionally, we can count the number of arrangements based on the condition on which numbers are among the first k in our arrangement. There are, by definition, $\binom{n}{k}$ ways to choose which of the n numbers appear among the first k. Once there are chosen, there are $k!$ ways to arrange them, followed by $(n-k)!$ ways to arrange the remaining elements.