CSC/MAT-220: Discrete Structures
EFY 4

Thomas R. Cameron

September 18, 2017

The Lion and the Unicorn. This Logic Puzzle is due to Raymond Smullyan and can be found in his book entitled *What is the name of this book?*.

In this puzzle there are three main characters: a girl named Alice, the Lion, and the Unicorn. Further, the Lion lies on Mondays, Tuesdays, and Wednesdays, and tells the truth on the other days of the week. The Unicorn lies on Thursdays, Fridays, and Saturdays, but tells the truth on the other days of the week. Consider the following statements and questions.

I. The Lion says to Alice"Yesterday was one of my lying days." and the Unicorn says "Yesterday was one of my lying days too." What day is it?

II. This time Alice met the Lion alone. He makes two statements, "I lied yesterday" and "I will lie again two days after tomorrow." What day is it?

III. On what days of the week is it possible for the Lion to make the following two statements: "I lied yesterday" and "I will lie again tomorrow."

IV. On what days of the week is it possible for the Lion to make the following single statement: "I lied yesterday and I will lie again tomorrow."

Make a truth table for the Lion and the Unicorn and use this table and basic logic to answer each question with a brief explanation.

Solution. Consider the following table

	Su	M	Tu	W	Th	F	Sa
Lion	1	0	0	0	1	1	1
Unicorn	1	1	1	1	0	0	0

Figure 1: Truth = 1 and Lie = 0.

I. Let p_1 denote the Lion's statement, and p_2 the Unicorn's statement. Note that on no day of the week can both the Lion and the Unicorn be lying. Therefore, all that remains is the following compound statements and their implications

$$p_1 \wedge p_2 \implies \text{Day} = \text{Su} \implies \neg p_1 \wedge p_2 \implies \text{contradiction}$$
$$\neg p_1 \wedge p_2 \implies \text{Day} = \text{M or Tu or W} \implies \underbrace{(\neg p_1 \wedge \neg p_2)}_{\text{M}} \vee \underbrace{(p_1 \wedge \neg p_2)}_{\text{Tu or W}} \implies \text{contradiction}$$
$$p_1 \wedge \neg p_2 \implies \text{Day} = \text{Th or F or Sa} \implies \underbrace{(p_1 \wedge \neg p_2)}_{\text{Th}} \vee \underbrace{(\neg p_1 \wedge p_2)}_{\text{F or Sa}} \implies \text{Day} = \text{Th}\ .$$

Therefore, the day is Thursday.

II. Let p_1 denote the Lion's first statement, and p_2 his second statement. Note that both of these statements are either true or false. Therefore, we only need to consider the following two compound statements and their implications

$$p_1 \wedge p_2 \implies \text{Day} = \underset{\text{Su or F or Sa}}{\text{Su or Th or F or Sa}} \implies \underset{\text{Su or F or Sa}}{(\neg p_1 \wedge p_2)} \vee \underset{\text{Th}}{(p_1 \wedge \neg p_2)} \implies \text{contradiction}$$

$$\neg p_1 \wedge \neg p_2 \implies \text{Day} = \underset{\text{M}}{\text{M}} \text{ or } \underset{\text{Tu or W}}{\text{Tu or W}} \implies \underset{\text{M}}{(\neg p_1 \wedge \neg p_2)} \vee \underset{\text{Tu or W}}{(p_1 \wedge \neg p_2)}.$$

Therefore, the day is Monday.

In a similar fashion, we can show that there is no day of the week that holds for the two statements in III, and the single statement in IV could occur on both Monday and Wednesday.