CSC/MAT-220: Discrete Structures
EFY 1

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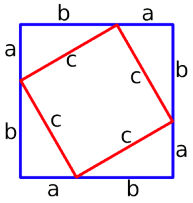
Develop a proof of the Pythagorean theorem and its converse. Be sure to state your starting assumptions, and to guide the reader from those assumptions to your conclusion in a clear and precise manner.

Solution.

Throughout this development, we assume knowledge of simple geometric shapes, such as the triangle and square, and their areas.

Theorem 1 (Pythagorean Theorem)**.** *Let a, b be the legs, and c the hypotenuse of a right-triangle, then $c^2 = a^2 + b^2$.*

Proof. Take 4 copies of the right triangle and join them together, thereby forming a square with sides $(a + b)$ as shown below.



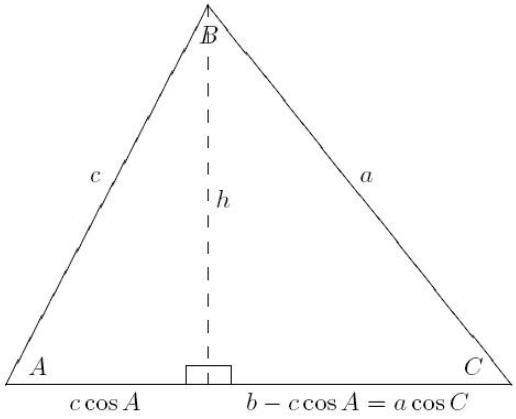
The area of the above square is the sum of the area of 4 right triangles with sides a, b, c, and a square with sides c. Therefore,

$$(a + b)^2 = c^2 + 4\left(\frac{1}{2}ab\right)$$

The result follows by expanding and canceling out like terms from the above equation. □

We are now ready to prove the Law of Cosines, as a direct corollary of the Pythagorean theorem.

Proposition 2 (Law of Cosines)**.**


B
c
h
a
A
C
$c\cos A$
$b - c\cos A = a\cos C$