MAT-150: Linear Algebra

Homework 4

Due: 11/3/2017

Book Problems.

Please turn in your solution for each of the following exercises.

§5.1: 16

§5.2: 12

§5.3: 30

Other Problems.

Problem 1.

Let A be an $n \times n$ matrix. Use the definition of an eigenvalue to show that λ is an eigenvalue of A if and only if $\det(A - \lambda) = 0$. As a corollary, show that $\lambda = 0$ is an eigenvalue of A if and only if A is not invertible.

Problem 2.

Let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{bmatrix}$. Find the eigenvalues of A and the corresponding eigenspaces. Is there a basis for $\mathbb{R}^3$ that consists of only eigenvectors of A?

Problem 3.

Let $p(\lambda) = a_0 + a_1\lambda + a_2\lambda^2 + \cdots + \lambda^n$ be a monic polynomial of degree n and define the $n \times n$ matrix

$$A = \begin{bmatrix} -a_{n-1} & -a_{n-2} & -a_{n-3} & \cdots & -a_0 \\ 1 & & & & \\ & 1 & & & \\ & & \ddots & & \\ & & & 1 & \end{bmatrix}.$$

a. Let λ be an eigenvalue of A with associated eigenvector v. Write down and examine the equation $Av = \lambda v$. Demonstrate that v must be a multiple of the vector

$$\begin{bmatrix} \lambda^{n-1} \\ \vdots \\ \lambda^2 \\ \lambda \\ 1 \end{bmatrix} \tag{1}$$

and λ must satisfy $p(\lambda) = 0$.

b. Conversely, show that if $p(\lambda) = 0$, then λ is an eigenvalue of A with eigenvector v given by (1).