MAT-150: Linear Algebra

EFY 2

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In this EFY we show that the matrix vector product preserves vector addition and scalar multiplication. In addition, we solve a simple linear equation to illustrate the particular and homogeneous solution.

Problem 1

Let A be an $m \times n$ matrix, u and v vectors in $\mathbb{R}^n$, and c a real scalar. Then

- $A(u+v) = Au + Av$,
- $A(cu) = cAu$.

Proof. Let $a_1, \ldots, a_n$ denote the column vectors of A. Then

$$\begin{aligned} A(u+v) &= (u_1+v_1)a_1 + \cdots + (u_n+v_n)a_n \\ &= (u_1a_1 + \cdots + u_na_n) + (v_1a_1 + \cdots + v_na_n) \\ &= Au + Av. \end{aligned}$$

Furthermore, we have

$$\begin{aligned} A(cu) &= (cu_1)a_1 + \cdots + (cu_n)a_n \\ &= c(u_1a_1 + \cdots + u_na_n) \\ &= cAu. \end{aligned}$$

□

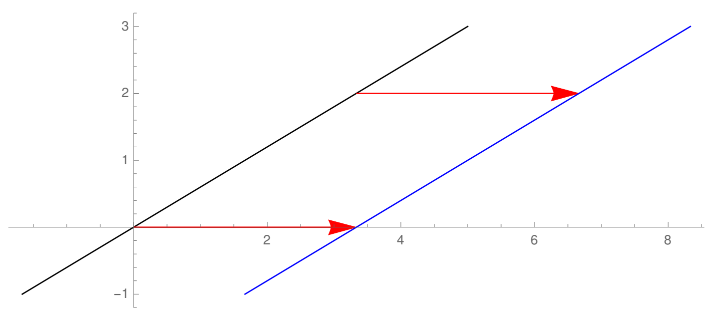
Problem 2

Solve $3x_1 - 5x_2 = 10$. Write the solution as a sum of the homogeneous and particular solutions, and draw the general solution as a sum of the homogeneous and particular solutions.

Solution. It is immediately clear that $x_2 := t$ is a free variable, and $x_1 = \frac{10+5t}{3}$. Therefore, we can write the general solution vector as follows

$$w = \begin{bmatrix} \frac{10}{3} \\ 0 \end{bmatrix} + t \begin{bmatrix} \frac{5}{3} \\ 1 \end{bmatrix}.$$

In the figure on the next page, the black line denotes the homogeneous solution set, the red vector the particular solution, and the blue line the general solution set.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