Section 3.5

3. Y_2 is the function and Y_1 is the derivative.

6. Y_2 is the function and Y_1 is the derivative.

Section 4.1

3, 7, 13. See the book's answers.

18. $y = -2x^{-1/3}$ so $\dfrac{dy}{dx} = (2/3)x^{-4/3} = \dfrac{2}{3x^{4/3}}$.

20. Dividing the square root in the denominator into the two terms in the numerator yields

$$g(x) = x^{5/2} - 4x^{1/2} \Rightarrow g'(x) = (5/2)x^{3/2} - 2x^{-1/2} = \frac{5}{2}\sqrt{x^3} - \frac{2}{\sqrt{x}}$$

32. $\dfrac{dy}{dx} = -15x^4 - 24x^2 + 8x$ and setting $x = 1$ yields $\dfrac{dy}{dx} = -31$. Hence, the equation of the tangent line has the form $y = -31x + b$. Setting $x = 1$ in the equation for y yields $y = -7$. Therefore, the tangent line passes through the point $(1, -7)$, which gives the equation $-7 = -31 \cdot 1 + b$, from which it follows that $b = 24$. We conclude that $y = -31x + 24$.

37. $\dfrac{dy}{dx} = 6x^2 + 18x - 60 = 6(x^2 + 3x - 10) = 6(x+5)(x-2)$ The tangent line is horizontal when the derivative has value 0 which yields $x = 2, -5$.

44, $f'(x) = 3x^2 + 12x + 21$ Setting $f'(x) = 9$ gives the equation $3x^2 + 12x + 21 = 9$ or

$$0 = 3x^2 + 12x + 12 = 3(x^2 + 4x + 4) = 3(x+2)^2 \Rightarrow x = -2$$

Since $f(-2) = -24$ the point in question is $(-2, -24)$.

45. $f'(x) = 3g'(x) - 2h'(x)$ so that $f'(5) = 3g'(5) - 2h'(5) = 36 + 6 = 42$.

47. (a) 2 (b) $\dfrac{2-1}{1-(-1)} = \dfrac{1}{2}$ (c) $[-1, \infty)$ (d) $[0, \infty)$

Section 4.2

1. $\frac{dy}{dx} = (3x^2+2)\cdot 2 + 6x(2x-1) = 18x^2 - 6x + 4$

2. $\frac{dy}{dx} = (5x^2-1)\cdot 4 + 10x(4x+3) = 60x^2 + 30x - 4$

6. $\frac{dg}{dt} = \frac{d}{dt}[(3t^2+2)(3t^2+2)] = (3t^2+2)\cdot 6t + 6t(3t^2+2) = 36t^3 + 24t$

8. $\frac{dy}{dx} = (2x-3)\cdot \frac{1}{2\sqrt{x}} + 2(\sqrt{x}-1) = 3\sqrt{x} - \frac{3}{2\sqrt{x}} - 2$

11. $\frac{dy}{dx} = \frac{6(3x+10) - 3(6x+1)}{(3x+10)^2} = \frac{57}{(3x+10)^2}$

15. $\frac{dy}{dx} = \frac{(2x+1)(x-1) - 1\cdot(x^2+x)}{(x-1)^2} = \frac{x^2-2x-1}{(x-1)^2}$

18. $\frac{dy}{dx} = \frac{(-2x+8)(4x^2-5) - 8x\cdot(-x^2+8x)}{(4x^2-5)^2} = \frac{-32x^2+10x-40}{(4x^2-5)^2}$

22. $\frac{dr}{dt} = \frac{\frac{1}{2\sqrt{t}}\cdot(2t+3) - 2\sqrt{t}}{(2t+3)^2} = \frac{-\sqrt{t} + \frac{3}{2\sqrt{t}}}{(2t+3)^2} = \frac{3-2t}{2\sqrt{t}(2t+3)^2}$

29. $h'(3) = f(3)g'(3) + f'(3)g(3) = 9\cdot 5 + 8\cdot 4 = 77$

30. $h'(3) = \frac{f'(3)g(3) - f(3)g'(3)}{(g(3))^2} = \frac{8\cdot 4 - 9\cdot 5}{16} = -\frac{13}{16}$

33. $f'(x) = \frac{1\cdot(x-2) - 1\cdot x}{(x-2)^2} = -\frac{2}{(x-2)^2}$ so $f'(3) = -2$. Hence, the tangent line has an equation of the form $y = -2x + b$. Since the point (3,3) is on this line we have $3 = -2\cdot 3 + b$ which implies $b = 9$. Therefore, the equation of the tangent line is $y = -2x + 9$.

41. (a) $s'(x) = \frac{1\cdot(m+nx) - x\cdot n}{(m+nx)^2} = \frac{m}{(m+nx)^2}$

(b) (a) $s'(50) = \frac{10}{(10+3\cdot 50)^2} = \frac{1}{2{,}560} \approx 0.00039$ (mm/ml)

Section 4.3

11. $f[g(x)] = \sqrt{(8x^2-6)+2} = \sqrt{8x^2-4} = 2\sqrt{2x^2-1}$ and
$g[f(x)] = 8(\sqrt{x+2})^2 - 6 = 8(x+2) - 6 = 8x + 10$

12. $f[g(x)] = 9(2\sqrt{x+2})^2 - 11(2\sqrt{x+2}) = 9(4(x+2)) - 22\sqrt{x+2} = 36x + 72 - 22\sqrt{x+2}$
and
$g[f(x)] = 2\sqrt{9x^2 - 11x + 2}$

22. $\dfrac{dy}{dx} = 5(2x^3+9x)^4(6x^2+9)$

27. $g(t) = -3(7t^3-1)^{1/2} \Rightarrow g'(t) = -\dfrac{3}{2}(7t^3-1)^{-1/2} \cdot 21t^2 = -\dfrac{63t^2}{2\sqrt{7t^3-1}}$

33. $q'(y) = 4y^2 \cdot \dfrac{5}{4}(y^2+1)^{1/4} \cdot 2y + 8y(y^2+1)^{5/4} = 10y^3(y^2+1)^{1/4} + 8y(y^2+1)^{5/4} =$
$2y(y^2+1)^{1/4}(5y^2+4y^2+4) = 2y(y^2+1)^{1/4}(9y^2+4)$

34. $p'(z) = 6z \cdot \dfrac{4}{3}(6z+1)^{1/3} + (6z+1)^{4/3} = (6z+1)^{1/3}(14z+1)$

38. $p'(t) = \dfrac{3(2t+3)^2 \cdot 2 \cdot (4t^2-1) - (2t+3)^3 \cdot 8t}{(4t^2-1)^2} = \dfrac{2(2t+3)^2[3(4t^2-1) - (2t+3)\cdot 4t]}{(4t^2-1)^2}$
$= \dfrac{2(2t+3)^2(4t^2-12t-3)}{(4t^2-1)^2}$

44. (a) $g'[f(1)]f'(1) = g'(2) \cdot (-6) = (3/7) \cdot (-6) = -18/7$

(b) $g'[f(2)]f'(2) = g'(4) \cdot (-7) = (5/7) \cdot (-7) = -5$

48. $f'(x) = x^2 \cdot \dfrac{4x^3}{2\sqrt{x^4-12}} + 2x\sqrt{x^4-12}$ so $f'(2) = 40$ and the tangent line has an equation of the form $y = 40x + b$. Since $f(2) = 8$ the tangent line passes through the point $(2, 8)$. It follows that $8 = 80 + b$ and thus $b = -72$. The equation of the tangent line is $y = 40x - 72$.

50. $f'(x) = \dfrac{1 \cdot (x^2+4)^4 - x \cdot 4(x^2+4)^3 \cdot 2x}{(x^2+4)^8} = \dfrac{(x^2+4)^3(x^2+4-8x^2)}{(x^2+4)^8} = \dfrac{(x^2+4)^3(4-7x^2)}{(x^2+4)^8}$
so that $f'(x) = 0$ for $x = \pm\sqrt{4/7} = \pm 2/\sqrt{7}$.

Section 6.4

15. (a)

$$\frac{dm}{dt} = \frac{dm}{dw}\frac{dw}{dt} = 85.65 \cdot 0.54w^{0.54-1}\frac{dw}{dt} = 46.251w^{-0.46}\frac{dw}{dt}$$

(b)

$$\frac{dm}{dt} = 46.251(0.25)^{-0.46}(0.01) \approx 0.875 \text{ kcal/day}^2$$

18.

$$\frac{dE}{dt} = \frac{dE}{dw}\frac{dw}{dt} = 22.8 \cdot (-0.34)w^{-1.34}\frac{dw}{dt} = -7.752w^{-1.34}\frac{dw}{dt} = -7.752(10)^{-1.34} \cdot 0.1 \approx -0.0354$$

where the units are kcal/kg/km/day. (I don't pretend to understand the meaning of this set of units.)

Section 4.4

1. $\dfrac{dy}{dx} = 4e^{4x}$

3. $\dfrac{dy}{dx} = -24e^{3x}$

8. $\dfrac{dy}{dx} = -2xe^{-x^2}$

15. $\dfrac{dy}{dx} = 4(x+3)^2e^{4x} + 2(x+3)e^{4x} = 2(x+3)(2x+7)e^{4x}$

18. $\dfrac{dy}{dx} = \dfrac{e^x(2x+1) - 2e^x}{(2x+1)^2} = \dfrac{e^x(2x+1-2)}{(2x+1)^2} = \dfrac{e^x(2x-1)}{(2x+1)^2}$

19. $\dfrac{dy}{dx} = \dfrac{(e^x - e^{-x}) \cdot x - (e^x + e^{-x})}{x^2}$

21. $y = 10{,}000(9+4e^{-0.2t})^{-1}$; $\dfrac{dy}{dx} = -10{,}000(9+4e^{-0.2t})^{-2} \cdot 4(-0.2)e^{-0.2t} = \dfrac{8{,}000e^{-0.2t}}{(9+4e^{-0.2t})^2}$

28. $\dfrac{dy}{dx} = -6(\ln 10)x10^{3x^2-4}$

30. $\dfrac{ds}{dt} = 5 \cdot (\ln 2)\dfrac{1}{2}(t-2)^{-1/2}2^{\sqrt{t-2}} = \dfrac{5(\ln 2)2^{\sqrt{t-2}}}{2\sqrt{t-2}}$

35. $\dfrac{dy}{dt} = y_0 \cdot ke^{kt} = k \cdot y_0e^{kt} = ky$

41. (a) $h(15) = 37.79(1.021)^{15} \approx 51{,}613{,}470$

(b) $\dfrac{dy}{dt} = 37.79(\ln 1.021)(1.021)^t \Rightarrow \left.\dfrac{dy}{dt}\right|_{t=15} \approx 1.07$ This means that when $t = 15$ the population is increasing at about 1,070,000 people per year.

67. (a) $I_C = \dfrac{dQ}{dt} = CV \cdot -\left(-\dfrac{1}{RC}e^{-t/RC}\right) = \dfrac{V}{R}e^{-t/RC}$

(b) $I_C = \dfrac{10}{10^7}e^{-200/(10^7 \cdot 10^{-5})} \approx 1.35 \times 10^{-7}$ amps