CSC/MAT-220: Discrete Structures
Solution 3

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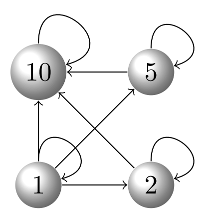
Book Problems

Problem 14.16

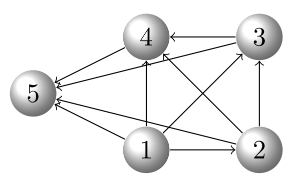
All this proof has shown is that $xRy \implies xRx$; that is, if there is a y such that xRy, then xRx under symmetry and transitivity. However, if x is not related to any y, then the result does not follow. Hence, R is not necessarily reflexive.

Problem 14.17

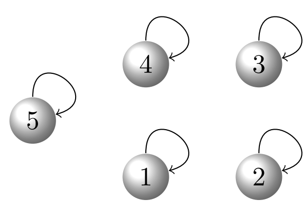
a.



b.

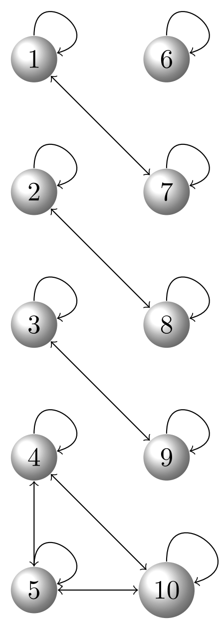


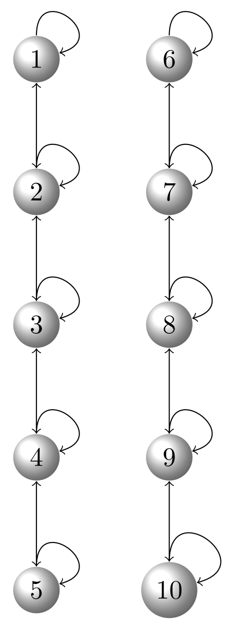
c.



Problem 15.14

a. **Equivalence Relation 1** **Equivalence Relation 2**





b. The equivalence classes for Relation 1 are as follows

$$[1] = \{1, 7\},\ [2] = \{2, 8\},\ [3] = \{3, 9\},\ [4] = \{4, 5, 10\},\ [6] = \{6\}.$$

The equivalence classes for Relation 2 are as follows

$$[1] = \{1, 2, 3, 4, 5\},\ [6] = \{6, 7, 8, 9, 10\}.$$

c. The equivalence relations look like single loops (reflexive) and cycles (transitive) that go both ways (symmetric).

Other Problems

I. To show that R is an equivalence relation, we must show that R that is reflexive, symmetric, and transitive. To this end, let $(a, b), (c, d), (e, f) \in \mathbb{R} \times \mathbb{R}$, and note the following:

- We have $(a, b)R(a, b)$, since $a = a$.
- If $(a, b)R(c, d)$, then $(c, d)R(a, b)$, since $c = a$.
- If $(a, b)R(c, d)$ and $(c, d)R(e, f)$, then $(a, b)R(e, f)$, since $a = c = e$.

The equivalence class $[(a, b)]$ is a vertical line through the point (a, b).

II. Based on the partition $\mathcal{P}$ and the fact that R is an equivalence relation, we arrive at the following

$$R = \{(a, a),\ (b, b),\ (c, c),\ (b, c),\ (c, b),\ (d, d)\}$$

III.

Proposition. *A relation R is an equivalence relation if and only if it is reflexive and circular.*

Proof. Suppose that R is an equivalence relation, then R is reflexive, symmetric, and transitive. Suppose that aRb and bRc, then aRc via the transitive property, and cRa via the symmetric property. Therefore, R is reflexive and circular.

Suppose that R is reflexive and circular. If aRb, then, since bRb, it follows that bRa. Therefore, R is symmetric. Further, if aRb and bRc, then cRa and by the symmetric property aRc. Therefore, R is transitive, and the result follows. $\square$