Section 6.3

3.

$$16x - 10xy' - 10y + 6yy' = 0 \;\Rightarrow\; [6y - 10x]y' = 10y - 16x$$
$$\Rightarrow\; y' = \frac{10y - 16x}{6y - 10x} = \frac{5y - 8x}{3y - 5x} = \frac{8x - 5y}{5x - 3y}$$

9. $\dfrac{1}{\sqrt{x}} + \dfrac{2y'}{\sqrt{y}} = 5y' \Rightarrow \dfrac{1}{\sqrt{x}} = 5y' - \dfrac{2y'}{\sqrt{y}} = \dfrac{5\sqrt{y}y' - 2y'}{\sqrt{y}} = y'\left(\dfrac{5\sqrt{y} - 2}{\sqrt{y}}\right) \Rightarrow y' = \dfrac{\sqrt{y}}{\sqrt{x}(5\sqrt{y} - 2)}$

14. $x^2y'e^y + 2xe^y + y' = 3x^2 \Rightarrow [x^2e^y + 1]y' = 3x^2 - 2xe^y \Rightarrow y' = \dfrac{3x^2 - 2xe^y}{x^2e^y + 1}$

15. $1 + \dfrac{y'}{y} = 3x^2y^2y' + 2xy^3 \Rightarrow \left[\dfrac{1}{y} - 3x^2y^2\right] y' = 2xy^3 - 1 \Rightarrow \left[\dfrac{1 - 3x^2y^3}{y}\right] y' = 2xy^3 - 1$
$\Rightarrow y' = \dfrac{y(2xy^3 - 1)}{1 - 3x^2y^3}$

17. $(xy' + y)\cos(xy) = 1 \Rightarrow xy' + y = \sec(xy) \Rightarrow y' = \dfrac{\sec(xy) - y}{x}$

18. $y'\sec^2 y + 1 = 0 \Rightarrow y' = -\cos^2 y$

19. $2x + 2yy' = 0 \Rightarrow -6 + 8y' = 0$ at $(-3, 4) \Rightarrow y' = 3/4 \Rightarrow y = (3/4)x + 25/4$

21. $2x^2yy' + 2xy^2 = 0 \Rightarrow 2y' - 2 = 0$ at $(-1, 1) \Rightarrow y' = 1 \Rightarrow y = x + 2$

32 [redacted]. If we set $x = 2$ in the equation we get $y^3 + 8y - 8y = 27 \Rightarrow y^3 = 27 \Rightarrow y = 3$. We have $3y^2y' + 2x^2y' + 4xy - 8y' = 3x^2 \Rightarrow 27y' + 8y' + 24 - 8y' = 12$ at$(2, 3) \Rightarrow y' = -12/27 = -4/9$
$\Rightarrow y = (-4/9)x + 35/9$

37 [redacted]. $(2/3)x^{-1/3} + (2/3)y^{-1/3}y' = 0 \Rightarrow 2/3 + (2/3)y' = 0$ at $(1, 1) \Rightarrow y' = -1 \Rightarrow y = -x + 2$

38 [redacted]. $6(x^2 + y^2)(2x + 2yy') = 25(2x - 2yy') \Rightarrow 120 + 60y' = 100 - 50y'$ at $(2, 1) \Rightarrow y' = -2/11$
$\Rightarrow y = (-2/11)x + 15/11$

39 [redacted]. $x^2y^2 + y^4 = 20x^2 \Rightarrow 2x^2yy' + 2xy^2 + 4y^3y' = 40x \Rightarrow 4y' + 8 + 32y' = 40$ at $(1, 2)$
$\Rightarrow y' = 8/9 \Rightarrow y = (8/9)x + 10/9$

Section 6.4

Note. In each solution the "prime" notation indicates derivatives with respect to t. For the applied exercises, t is understood to be time.

3. $2xy' + 2x'y - 5x' + 9y^2y' = 0 \Rightarrow 6y' + 2(-6)(-2) + 30 + 9(-2)^2y' = 0 \Rightarrow y' = -9/7$

8. $yx'/x + y' \ln x + xy'e^y + x'e^y = 0 \Rightarrow y' + 5 = 0 \Rightarrow y' = -5$

21. Using the notation from Figure 18 with the length of the ladder equal to 17 feet we have

$x^2+y^2 = 17^2 \Rightarrow 2xx'+2yy' = 0 \Rightarrow xx'+yy' = 0 \Rightarrow 72+\sqrt{17^2-64}y' = 0 \Rightarrow y' = -24/5$ ft/min

Hence, the top of the ladder is sliding down the side of the building at a rate of 24/5 ft/min.

22. (a) Let y denote the distance north and x the distance west. We are given that $y' = 30$ and $x' = 40$. If z is the distance between the two cars then $x^2 + y^2 = z^2$ which implies $2xx' + 2yy' = 2zz'$ or $xx' + yy' = zz'$. After two hours $x = 80$, $y = 60$ and $z = 100$ and we have

$$80 \cdot 40 + 60 \cdot 30 = 100z' \Rightarrow z' = 50 \text{ mph}$$

(b) Now at the end of two hours $x = 40$ and $y = 60$ which implies $z = \sqrt{40^2 + 60^2} = 20\sqrt{13}$. Now

$$40 \cdot 40 + 60 \cdot 30 = 20\sqrt{13}z' \Rightarrow z' = \frac{3400}{20\sqrt{13}} \approx 47.15 \text{ mph}$$

23. Like Example 2 on page 343 we have $A' = 2\pi rr' = 2\pi \cdot 4 \cdot 2 = 16\pi \approx 50.27$ ft^2/min

24. $V = (4/3)\pi r^3$ Thus, $V' = 4\pi r^2 r' = 4\pi \cdot 16(-1/4) = -16\pi$ in^3/hr

26. The volume of a cone of radius r and height h is $V = (1/3)\pi r^2 h$. Since we are told that $h = 2r$ this volume formula becomes $V = (2/3)\pi r^3$ and thus $V' = 2\pi r^2 r'$. Substituting $r = 6$ and $r' = 0.75$ we get $V' = 54\pi$ cubic inches per minute.

29. Let x denote the horizontal distance between the stern of the boat and the dock and let y denote the length of the rope. We are given that $y' = -1$ and wish to find x' when $x = 8$. Since the stern of the boat is 8 feet below the pulley, it follows from the Pythagorean Theorem that $x^2 + 64 = y^2$. Differentiating both sides of this equation with respect to time

t gives $2xx' = 2yy'$ or $xx' = yy'$. When $x = 8$ it follows from the equation $x^2 + 64 = y^2$ that $y = 8\sqrt{2}$. Therefore, when $x = 8$ we have $8x' = -8\sqrt{2}$ which implies $x' = -\sqrt{2}$ ft/s. Hence, when $x = 8$, the boat is approaching the dock at a speed of $\sqrt{2} \approx 1.414$ ft/s.

35. (a) We have $x = 50 \tan\theta$ and $\theta' = 4\pi$ radians per minute. (We have to convert the rate "twice per minute" to radians per minute since our derivative formulas for trig functions assume angles are measured in radians.) Hence, $x' = 50(\sec^2\theta) \cdot \theta' = 200\pi \sec^2\theta$. At the closest point, $\theta = 0$ which gives $x' = 200\pi$ m/min.

(b) When $x = 50$ we have $\theta = \pi/4$ and $\sec^2(\pi/4) = 2$. Hence, at this moment $x' = 400\pi$ m/min.

Section 6.5

1. $dy = (6x^2 - 5)dx = (6(-2)^2 - 5)(0.1) = 1.9$

8. $dy = \dfrac{12}{(2x+1)^2}dx = \dfrac{12}{49}(-0.04) = -\dfrac{0.48}{49} \approx -0.0098$

10. Use $f(x+\Delta x) \approx f(x) + f'(x)\Delta x$ with $f(x) = \sqrt{x}, f'(x) = \dfrac{1}{2\sqrt{x}}, x = 25, \Delta x = -2$ to get

$$\sqrt{23} \approx 5 + \frac{1}{10}(-2) = \frac{24}{5} = 4.8$$

14. Use $f(x+\Delta x) \approx f(x) + f'(x)\Delta x$ with $f(x) = e^x, f'(x) = e^x, x = 0, \Delta x = -0.002$ to get

$$e^{-0.002} \approx 1 + 1(-0.002) = 0.998$$

23. $V = \dfrac{4}{3}\pi r^3 \Rightarrow \Delta V \approx dV = 4\pi r^2 dr = 4\pi(14)^2(2) = 1568\pi \text{ mm}^3$

25. $A = \pi r^2 \Rightarrow \Delta A \approx dA = 2\pi r dr = 2\pi(20)2 = 80\pi \text{ mm}^2$

32. $A = s^2 \Rightarrow \Delta A \approx dA = 2sds = 2(3.45)(\pm 0.002) = \pm 0.0138 \text{ in}^2$

34. $A = \pi r^2 \Rightarrow \Delta A \approx dA = 2\pi r dr = 2\pi(4.87)(\pm 0.04) \approx \pm 1.224 \text{ in}^2$

39. Converting to inches the radius of each beach ball is 6 inches. For one beach ball we have

$$V = \frac{4}{3}\pi r^3 \Rightarrow \Delta V \approx dV = 4\pi r^2 dr = 4\pi(6^2)(0.03) = 4.32\pi \text{ in}^3$$

We need to multiply this by 5000 getting

$$21{,}600\pi \approx 67{,}858.4 \text{ in}^3$$