MAT – 450: Advanced Linear Algebra

Homework 3

Instructor: Thomas R. Cameron

Due: 3/2/2018

Instructions

You must complete all other problems and type your solutions in LaTeX. The book problems are listed for your edification and I *strongly* encourage you to work through them. You will find that some of the book problems will be helpful in completing the other problems. In addition, the book problems may show up on a EFY or Review. Note that the other problems are graded rigorously with high expectations on clear and concise mathematical writing as outlined in the mathematical writing handout. Lastly, you may work with other students and ask me any questions, but you must write your solutions independently so I may interpret your understanding while grading. Any sources you use, including internet sources must be cited using `\thebibliography` environment.

Book Problems

§5.A: 3, 18, 20, 21, 24,

§5.B: 1, 5, 6, 9

§5.C: 1, 5, 6, 7, 9

Other Problems

Problem 1. Let $T \in \mathcal{L}(V)$, where V is a vector space over the field $\mathbb{F}$.

a. Let $v_1, \ldots, v_k$ be eigenvectors of T, show that $S = \mathit{span}(v_1, \ldots, v_k)$ is T-invariant.

b. Let $\beta = \{v_1, v_2, v_3\}$ be any basis for V and suppose that

$$[T]_\beta = \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

Find an invariant subspace of T that is not spanned by eigenvectors of T.

Problem 2. Let $T \in \mathcal{L}(V)$, where V is a vector space over the field $\mathbb{F}$. Suppose that the sum

$$\sum_{k=0}^{\infty} (I - T)^k$$

converges. Show that this sum is equal to T^{-1}.

Problem 3. Let $T \in \mathcal{L}(V)$, where V is a finite dimensional vectors space over the field $\mathbb{F}$. Show that T is diagonalizable if and only if every T-invariant subspace of V can be spanned by eigenvectors of T.

Problem 4. Do problem 16 in Section 5.C.