MAT – 450: Advanced Linear Algebra
Homework 6

Instructor: Thomas R. Cameron

Due: 4/20/2018

Instructions

You must complete all other problems and type your solutions in LaTeX. The book problems are listed for your edification and I *strongly* encourage you to work through them. You will find that some of the book problems will be helpful in completing the other problems. In addition, the book problems may show up on a EFY or Review. Note that the other problems are graded rigorously with high expectations on clear and concise mathematical writing as outlined in the mathematical writing handout. Lastly, you may work with other students and ask me any questions, but you must write your solutions independently so I may interpret your understanding while grading. Any sources you use, including internet sources must be cited using `\thebibliography` environment.

Book Problems

§6.1: 1, 10, 12

§6.2: 1, 15, 16

§6.3: 1, 18, 21

Other Problems

Problem 1. Let $A \in \mathbb{C}^{n\times m}$ and recall the Frobenius matrix norm defined by

$$\begin{aligned}\|A\|_F &= \sqrt{\text{trace}\left(AA^*\right)} \\ &= \left(\sum_{i=1}^{n}\sum_{j=1}^{m}|a_{ij}|^2\right)^{1/2}.\end{aligned}$$

Let $\sigma_1 \geq \cdots \geq \sigma_r$ denote the singular values of A. Show that

$$\|A\|_F = \sqrt{\sigma_1^2 + \cdots + \sigma_r^2}.$$