MAT-150: Linear Algebra

Homework 6

Due: 12/1/2017

Book Problems.

Please turn in your solution for each of the following exercises.

§7.1: 17, §7.2: 13, 17 (Mathematica), §7.3: 4, 17 (Mathematica)

Other Problems.

Problem 1. Given Schur's Theorem which states that for every $n \times n$ matrix A there exists a unitary Q such that

$$Q^* AQ = T,$$

where T is upper-triangular, $Q^*Q = I$, and the * denotes the conjugate transpose. Prove that if A is Hermitian ($A = A^*$), then the following holds true

- $Q^*AQ = D$, where D is a diagonal matrix. Thus, A is orthogonally diagonalizable.
- A has n real eigenvalues.
- There exists an orthogonal basis for $\mathbb{C}^n$ that consists of eigenvectors of A.

Throughout, make mention of what the result looks like over the real numbers. For instance, over the real numbers a Hermitian matrix is simply symmetric, since $A = A^T$.

Problem 2. Use results of Problem 1 to justify part b. and part c. of the Spectral Theorem (Theorem 3 of Section 7.1).

Problem 3. Use results of Problem 1 to prove the Principle Axes Theorem (Theorem 4 of Section 7.2).