

MAT – 112: Calculus I and Modeling

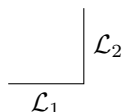
Solution 1

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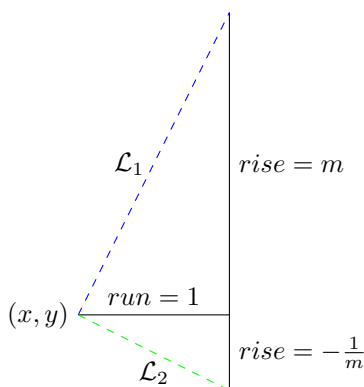
1/26/2018

Other Problems

Problem 1. By definition, two lines \mathcal{L}_1 and \mathcal{L}_2 are perpendicular if \mathcal{L}_1 is horizontal and \mathcal{L}_2 is vertical, or the slope of \mathcal{L}_1 is $m \neq 0$, and the slope of \mathcal{L}_2 is $-\frac{1}{m}$. The horizontal and vertical case is clear, since the intersection of the two lines creates a right angle (side of a square) as seen below.



The case where \mathcal{L}_1 has a slope of $m \neq 0$ and \mathcal{L}_2 has a slope of $-\frac{1}{m}$ is shown below. Starting at the point (x, y) , the line \mathcal{L}_1 denotes going over 1 and up m , whereas the line \mathcal{L}_2 denotes going over 1 and down $\frac{1}{m}$.



Problem 2. To show that $h = g \circ f$ is a function from A to C , we must show that h takes each element of A to exactly one element of C . To this end, let α be an element of A . Then, $f(\alpha)$ is a unique element in B , which is the domain of the function g . It follows that g takes $f(\alpha)$ to exactly one element of C . Therefore, $h(\alpha) = g(f(\alpha))$ is a unique element in C .

Problem 3. Using the method of completing the square, we can transfer the quadratic $ax^2 + bx + c$, where $a \neq 0$, into vertex form as follows

$$\begin{aligned} ax^2 + bx + c &= a \left(x^2 + \frac{b}{a}x \right) + c \\ &= a \left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2} \right) + c - \frac{b^2}{4a} \\ &= a \left(x + \frac{b}{2a} \right)^2 + \frac{4ac - b^2}{4a} \end{aligned}$$

Once in vertex form, we can identify the vertex, axis of symmetry, x -intercept, and y -intercept.

Vertex: $\left(-\frac{b}{2a}, \frac{4ac-b^2}{4a}\right)$

Axis of Symmetry: $x = -\frac{b}{2a}$

y-intercept: $(0, c)$

x-intercept: The x -intercepts occur when the quadratic equals zero. Thus, we can solve for x as follows

$$\begin{aligned} a \left(x + \frac{b}{2a} \right)^2 + \frac{4ac - b^2}{4a} &= 0 \\ a \left(x + \frac{b}{2a} \right)^2 &= \frac{b^2 - 4ac}{4a} \\ \left(x + \frac{b}{2a} \right)^2 &= \frac{b^2 - 4ac}{4a^2} \\ x + \frac{b}{2a} &= \pm \frac{\sqrt{b^2 - 4ac}}{2a} \\ x &= -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \end{aligned}$$

Note that we have derived the quadratic formula.