Section 2.1

3. E

4. D

6. F

8. D

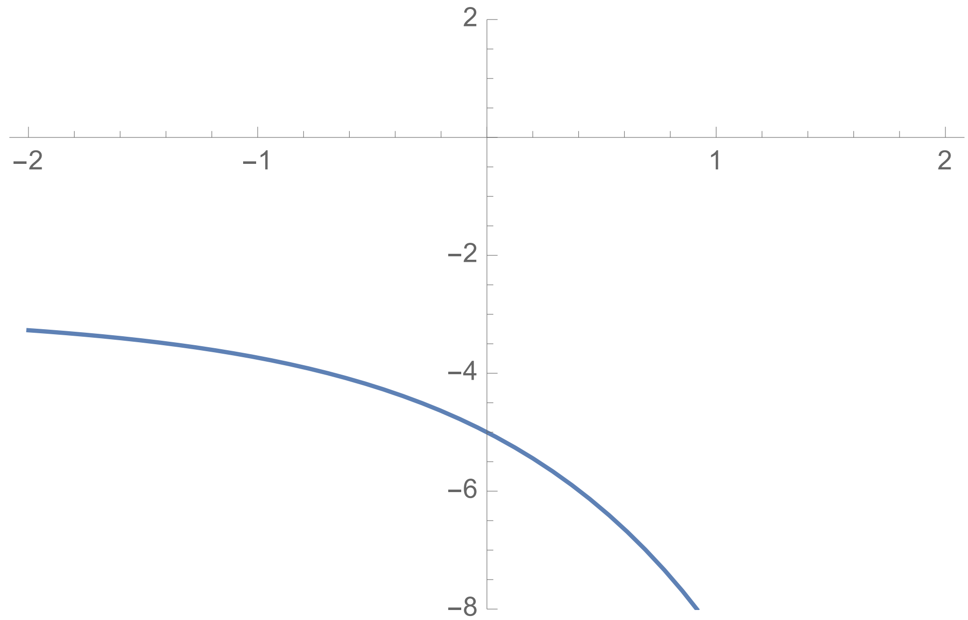
19. Since $16^{x+3} = (2^4)^{x+3} = 2^{4x+12}$ and $64^{2x-5} = (2^6)^{2x-5} = 2^{12x-30}$ the equation $16^{x+3} = 64^{2x-5}$ is equivalent to the equation $2^{4x+12} = 2^{12x-30}$ which, in turn, is equivalent to the equation $4x + 12 = 12x - 30$ which has solution $x = 21/4$.

21. The equation $e^{-x} = (e^4)^{x+3} = e^{4x+12}$ is equivalent to the equation $-x = 4x + 12$ which has solution $-12/5$.

24. The equation $2^{x^2-4x} = \left(\frac{1}{16}\right)^{x-4} = (2^{-4})^{x-4} = 2^{-4x+16}$ is equivalent to the equation $x^2 = 16$ which has solutions $x = \pm 4$.

26. The equation $8^{x^2} = (2^3)^{x^2} = 2^{3x^2} = 2^{x+4}$ is equivalent to the equation $3x^2 = x + 4$ which, in turn, is equivalent to the equation $3x^2 - x - 4 = (3x - 4)(x + 1) = 0$ which has solutions $x = 4/3, -1$.

30.



38. (a) $f(1) = 500 \cdot 2^3 = 4000$

(b) $f(0) = 500 \cdot 2^0 = 500$

(c) Solution 1. By the answer to part (b), there were 500 bacteria present initially. We first find how much time must pass until there are 1000 present by solving $1000 = f(t)$ for t:

$$1000 = f(t) = 500 \cdot 2^{3t} \Rightarrow 2 = 2^1 = 2^{3t} \Rightarrow 1 = 3t \Rightarrow t = 1/3 \quad \text{(hr)}$$

Next, note that

$$f(t + 1/3) = 500 \cdot 2^{3(t+1/3)} = 500 \cdot 2^{3t+1} = 500 \cdot 2^{3t} \cdot 2 = 2f(t)$$

which shows that no matter what the value of t, the number of bacteria doubles 1/3 of an hour (20 minutes) after that.

Solution 2. We solve directly for a time t_0 such that for all times t the equation $f(t + t_0) = 2f(t)$ is satisfied. Since

$$f(t + t_0) = 500 \cdot 2^{3(t+t_0)} = 500 \cdot 2^{3t+3t_0} = 500 \cdot 2^{3t} \cdot 2^{3t_0} = 2^{3t_0} f(t)$$

if $f(t + t_0) = 2f(t)$ then this means that

$$2^{3t_0} = 2 = 2^1 \Rightarrow 3t_0 = 1 \Rightarrow t_0 = 1/3 \text{ (hr)}$$

(or 20 minutes) as before.

(d) Solution 1. (This uses the answers from parts (b) and (c).) Note that $32{,}000/500 = 64 = 2^6$. So, to get from the initial number 500 to 32,000 we need to double 500 six times. Each "doubling" takes 1/3 of an hour, so for 500 to be doubled six times takes 2 hours.

Solution 2. (Starting from scratch.) We solve $32{,}000 = 500 \cdot 2^{3t}$ for t:

$$32{,}000 = 500 \cdot 2^{3t} \Rightarrow 64 = 2^6 = 2^{3t} \Rightarrow 6 = 3t \Rightarrow 2 = t$$

So it takes 2 hours.

47. (a) $Q(6) = 1000(5^{-1.8}) \approx 55.2$ g

(b) If $8 = 1000(5^{-0.3t})$ then $0.008 = 5^{-3} = 5^{-0.3t}$ which implies $-3 = -0.3t$ or $t = 10$ months.

Section 2.2

3. $\log_3 81 = 4$

5. $\log_3 \frac{1}{9} = -2$

11. $10^5 = 100{,}000$

12. $10^{-3} = 0.001$

16. We seek $r = \log_3 27$. In exponential form this equation becomes $3^r = 27$. Since $27 = 3^3$ we seek r such that $3^r = 3^3$ and it follows that $r = 3$. Therefore, $\log_3 27 = 3$.

19. We seek $r = \log_2 \sqrt[3]{\dfrac{1}{4}}$. In exponential form this equation becomes $2^r = \sqrt[3]{\frac{1}{4}}$. Since $2^{-2} = \dfrac{1}{4}$ we have $\sqrt[3]{\dfrac{1}{4}} = \left(\dfrac{1}{4}\right)^{1/3} = (2^{-2})^{1/3} = 2^{-2/3}$ and our equation becomes $2^r = 2^{-2/3}$ from which it follows that $r = -2/3$. Therefore, $\log_2 \sqrt[3]{\dfrac{1}{4}} = -2/3$.

34. $\log_b 18 = \log_b (2 \cdot 3^2) = \log_b 2 + \log_b 3^2 = \log_b 2 + 2\log_b 3 = a + 2c$

36. $\log_b (9b^2) = \log_b (3^2 b^2) = 2\log_b 3 + 2\log_b b = 2c + 2$

41. $\log_x 36 = -2 \Rightarrow x^{-2} = \dfrac{1}{x^2} = 36 \Rightarrow x^2 = \dfrac{1}{36} \Rightarrow x = \pm\dfrac{1}{6} \Rightarrow x = \dfrac{1}{6}$ (The last equality is due to the fact that the base of a logarithm must be positive.)

42. The equation $\log_9 27 = m$ has exponential form $9^m = 27$. Since $9^m = (3^2)^m = 3^{2m}$ and $27 = 3^3$ our equation becomes $3^{2m} = 3^3$ from which it follows that $2m = 3$ and $m = 3/2$.

43. The equation $\log_8 16 = z$ has exponential form $8^z = 16$. Since $8^z = (2^3)^z = 2^{3z}$ and $16 = 2^4$ our equation becomes $2^{3z} = 2^4$ from which it follows that $3z = 4$ and $z = 4/3$.

44. $\log_y 8 = \dfrac{3}{4} \Rightarrow y^{3/4} = 8 \Rightarrow y = (y^{3/4})^{4/3} = 8^{4/3} = 16$

Section 2.3

There are three basic types of applied problems in this section: (i) Express y as a function of t in the form $y = y_0 e^{kt}$ where y_0 and k are *known* numbers.

(ii) Given *three* of the parameters y, y_0, k, and t, find the fourth.

(iii) Given *two* of k, t and the ratio y/y_0 (the last often expressed as a percentage), find the third.

4. The easiest approach is to solve our half–life equation, $T = -(\ln 2)/k$, for k. Another approach, working from scratch, is as follows. If $y = y_0 e^{kt}$ then $y_0/2 = y_0 e^{kT}$ which implies that $1/2 = e^{kT}$. Hence, $\ln(1/2) = -\ln 2 = kT$ or that $k = -(\ln 2)/T$.

5. Since a radioactive substance decays exponentially we have $y = y_0 e^{kt}$ for some $k < 0$. From Exercise 4 we know that

$$k = -(\ln 2)/T = \frac{(-1) \cdot \ln 2}{T} = \frac{\ln 2^{-1}}{T} = \frac{\ln(1/2)}{T}$$

Using properties of exponents and substituting this expression for k gives

$$y = y_0 e^{kt} = y_0 (e^k)^t = y_0 (e^{\ln(1/2)/T})^t = y_0 (e^{\ln(1/2) \cdot (1/T)})^t = y_0 (e^{\ln(1/2)})^{t/T} = y_0 \left(\frac{1}{2}\right)^{t/T}$$

11. Let y_0 denote the number of women at the beginning of the study. Since the exercise gives us the survival rate as the proportion y/y_0 (expressed as a percentage) we write the equation for exponential decay in the form $y/y_0 = e^{kt}$. The 37% 5–year survival rate means that if we set $t = 5$ then $0.37 = y/y_0 = e^{5k}$. Therefore,

$$0.37 = e^{5k} \Rightarrow \ln 0.37 = 5k \Rightarrow k = \frac{\ln 0.37}{5} \approx -0.1989$$

in agreement with the mortality rate given in the statement of the exercise. (Note that we have a minus sign in addition to the numerical value.)

13. Let y_0 denote the initial amount of C–14 in the shrub. We showed in class that the half–life of C–14 is 5600 years. Using the formula in Exercise 4 gives $k = -\dfrac{\ln 2}{T} = -\dfrac{\ln 2}{5600} \approx -0.0001238$. Hence, $y = y_0 e^{-0.0001238t}$. Since the exercise asks us about a proportion ("percent of the original carbon–14") we write this equation in the form $y/y_0 = e^{-0.0001238t}$. Setting $t = 43{,}000$ yields

$$y/y_0 = e^{-(0.0001238)\cdot(43{,}000)} = e^{-5.3234} \approx 0.00488$$

So about 0.488% of the original C–14 was present in the charcoal.

20. We use the formula from Exercise 5.

(a) $y = 4 \cdot \left(\dfrac{1}{2}\right)^{t/13}$. When $t = 100$ this yields $y = 4 \cdot \left(\dfrac{1}{2}\right)^{100/13} \approx 0.0193$ g.

(b) We need to solve the equation $0.1 = 4 \cdot \left(\dfrac{1}{2}\right)^{t/13}$ for t. This equation is equivalent to $0.025 = \left(\dfrac{1}{2}\right)^{t/13}$. Taking the natural logarithm of both sides of this equation give

$$\ln(0.025) = \frac{t}{13}\ln(1/2) \Rightarrow t = \frac{13\ln(0.025)}{\ln(1/2)} \approx 69.19 \text{ years}$$

24. (a) $y = 40e^{-0.004\cdot 180} \approx 19.47$ watts.

(b) Substituting $k = -0.004$ into our formula $T = -(\ln 2)/k$ for the half–life T yields $T \approx 173.29$ days. (You can also solve $\dfrac{1}{2} = 0.5 = e^{-0.004T}$ for $\ln(0.5) = -0.004T$ or $T \approx 173.29$ days.)

(c) According to this model the power will never be completely gone since no matter what the value of t the value of y will always be positive (but will be incredibly small when t is very large).

25. In this exercise t denotes temperature instead of time and $y_0 = 10$ is the amount that dissolves at temperature $t = 0$. Hence our formula has the form $y = 10e^{kt}$.

(a) We are given that $11 = 10e^{10k}$. But then, $1.1 = e^{10k}$ or $\ln(1.1) = 10k$ from which it follows that $k = [\ln(1.1)]/10 \approx 0.009531$. Hence, $y = 10e^{0.009531t}$.

(b) Solve $15 = 10e^{0.009531t}$ for $t = \ln(1.5)/0.009531 \approx 42.5417$ degrees Celsius.

28. We have $f(t) = 18 - 14.6e^{-0.6t}$ and we need to solve the equation $10 = 18 - 14.6e^{-0.6t}$ for t. This equation is equivalent to

$$\frac{8}{14.6} = e^{-0.6t} \Rightarrow \ln\left(\frac{8}{14.6}\right) = -0.6t \Rightarrow t = -\frac{1}{0.6}\ln\left(\frac{8}{14.6}\right) \approx 1.00263 \text{ hours}$$

That is, it takes about 1 hour.

CSI problem. Assume that we start measuring time t from the moment $t = 0$ at which death occurred. At the moment of death the body temperature was 98.6 degrees. Since the thermostat was set at 68 we have $T_0 = 68$. According to Newton's Law of Cooling the temperature t hours after death will be $T = 68 + Ce^{-kt}$ for some constants C and k. Since at time $t = 0$ we have $98.6 = T = 68 + C$, we find that $C = 30.6$ and thus $T = 68 + 30.6e^{-kt}$. Now, let t denote the specific number of hours since death at 10:30 am. We are told that $80 = 68 + 30.6e^{-kt}$ and that $78.5 = 68 + 30.6e^{-k(t+1)}$. We can solve these two equations simultaneously either "by hand" or by using the "solve" feature of the TI89. In either case we get $k = 0.1335314$ and $t = 7.0103$. So 10:30 am is about 7 hours since death occurred, and thus the moment of death was around 3:30 am.

Here's the "by hand" solution. Since $80 = 68 + 30.6e^{-kt}$ and $78.5 = 68 + 30.6e^{-k(t+1)}$ we have $12 = 30.6e^{-kt}$ and $10.5 = 30.6e^{-k(t+1)}$ and thus

$$\frac{12}{10.5} = \frac{30.6e^{-kt}}{30.6e^{-k(t+1)}} = \frac{e^{-kt}}{e^{-kt-k}} = e^{k} \Rightarrow k = \ln\left(\frac{12}{10.5}\right) \approx 0.1335314$$

Therefore,

$$12 = 30.6e^{-kt} \Rightarrow \frac{12}{30.6} = e^{-kt} \Rightarrow \ln\left(\frac{12}{30.6}\right) = -kt \Rightarrow t = \frac{\ln\left(\frac{12}{30.6}\right)}{-k} = \frac{\ln\left(\frac{12}{30.6}\right)}{-0.1335314} \approx 7.0103$$

Section 2.4

2. $\pi/2$

6. $16\pi/9$

10. $120°$

14. $100°$

34. $\sqrt{3}/2$

36. $1/\sqrt{3} = \sqrt{3}/3$

40. -1

43. $-\sqrt{2}$

78.

(a) Expanding the expression inside the cosine yields

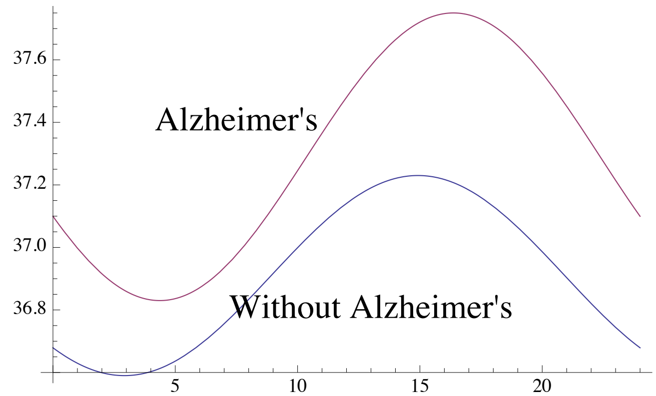
$$\frac{(t-6)\pi}{14.77} = \left(\frac{\pi}{14.77}\right)t - \frac{6\pi}{14.77}$$

Then, using the formula for the period on page 111 (with $b = \pi/14.77$) gives the period $2 \cdot 14.77 = 29.54$ (days). There is a lunar cycle every 29.54 days.

(b). Since the maximum value of the cosine function is $1 = \cos(0)$, the maximum number of consultations occurs when $t = 6$ (days) since January 16, 2014; that is, on January 22, 2014. The corresponding y value is then 101.8, which corresponds to a percentage increase of 1.8% over the daily mean.

(c) Setting $t = 31 - 16 = 15$ yields $y \approx 99.39$ (percent of the daily mean).

81. (a)



The graphs do not intersect.

(b) $t = 14.92$ hours or around 2:55 PM

(c) $t = 16.37$ hours or around 4:22 PM

91. If h denotes the height of the building in meters then $\tan 42.8^\circ = \dfrac{h}{65}$ and thus $h = 65 \tan 42.8^\circ \approx 60.2$ m.

92. Let x denote the horizontal distance between the two sides of the canyon. Using the information in the problem and the right triangle in the upper right–hand portion of the figure, we see that $\cot 27^\circ = \dfrac{x}{105}$ and thus $x = 105 \cot 27^\circ \approx 206$ ft.