MAT – 112: Calculus I and Modeling

Quotient Rule

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Instructions

Write out your own proof of the quotient rule using the outline below. Fill in the blanks and follow the prompts given.

Quotient Rule. Let f and g be functions that are differentiable at x, where $g(x) \neq 0$, then

$$\frac{d}{dx}\left(\frac{f}{g}\right)(x) = \frac{f^{'}(x)g(x) - f(x)g^{'}(x)}{[g(x)]^2}$$

Proof. Note that

$$\begin{aligned}
\frac{d}{dx}\left(\frac{f}{g}\right)(x) &= \lim_{h\to 0}\frac{\left(\frac{f}{g}\right)(x+h) - \left(\frac{f}{g}\right)(x)}{h} \\
&= \lim_{h\to 0}\frac{\left(\frac{f(x+h)}{g(x+h)}\right) - \left(\frac{f(x)}{g(x)}\right)}{h} \\
&= \lim_{h\to 0}\frac{g(x)f(x+h) - f(x)g(x+h)}{h(g(x)g(x+h)} \quad \text{(Justify)} \\
&= \lim_{h\to 0}\frac{g(x)f(x+h) - g(x)f(x) + g(x)f(x) - f(x)g(x+h)}{hg(x)g(x+h)} \quad \text{(Justify)} \\
&= \underline{\hspace{6cm}} \quad \text{(Use sum limit rule)} \\
&= \lim_{h\to 0} g(x) \lim_{h\to 0}\frac{f(x+h) - f(x)}{hg(x)g(x+h)} - \lim_{h\to 0} f(x) \lim_{h\to 0}\frac{g(x+h) - g(x)}{hg(x)g(x+h)} \quad \text{(Justify)} \\
&= \lim_{h\to 0}\frac{1}{g(x)g(x+h)}\left(g(x)\lim_{h\to 0}\frac{f(x+h) - f(x)}{h} - f(x)\lim_{h\to 0}\frac{g(x+h) - g(x)}{h}\right) \quad \text{(Justify)} \\
&= \frac{1}{[g(x)]^2}\left(g(x)f^{'}(x) - f(x)g^{'}(x)\right) \\
&= \frac{f^{'}(x)g(x) - f(x)g^{'}(x)}{[g(x)]^2}.
\end{aligned}$$

□