# MAT – 112: Calculus I and Modeling

Solution 3

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## Other Problems

**Problem 1.** Here we use the limit definition of the derivative to derive formula for computing the derivative of $x^2$, $\sqrt{x}$, and $1/x$.

- Let $f(x) = x^2$ and note that

$$\begin{aligned}\frac{d}{dx}f(x) &= \lim_{h\to 0}\frac{(x+h)^2 - x^2}{h}\\ &= \lim_{h\to 0}\frac{[x^2+2xh+h^2]-[x^2]}{h}\\ &= \lim_{h\to 0}\frac{2xh+h^2}{h}\\ &= \lim_{h\to 0}(2x+h) = 2x.\end{aligned}$$

- Let $f(x) = \sqrt{x}$ and note that

$$\begin{aligned}\frac{d}{dx}f(x) &= \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\\ &= \lim_{h\to 0}\frac{\sqrt{x+h}-\sqrt{x}}{h}\left(\frac{\sqrt{x+h}+\sqrt{x}}{\sqrt{x+h}+\sqrt{x}}\right)\\ &= \lim_{h\to 0}\frac{(x+h)-(x)}{h\left(\sqrt{x+h}+\sqrt{x}\right)}\\ &= \lim_{h\to 0}\frac{1}{\sqrt{x+h}+\sqrt{x}} = \frac{1}{2\sqrt{x}}.\end{aligned}$$

- Let $f(x) = 1/x$ and note that

$$\begin{aligned}\frac{d}{dx}f(x) &= \lim_{h\to 0}\frac{\left(\frac{1}{x+h}\right)-\left(\frac{1}{x}\right)}{h}\\ &= \lim_{h\to 0}\frac{x-(x+h)}{hx(x+h)}\\ &= \lim_{h\to 0}\frac{-1}{x(x+h)} = -\frac{1}{x^2}.\end{aligned}$$