CSC/MAT-220: Discrete Structures

Solution 1

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Problem 3.9

Let A, B, C be points in the plane. We say that C is *between* A and B if

$$d(A, B) = d(A, C) + d(C, B),$$

where $d(x, y)$ is the distance between two points x, y in the plane. Furthermore, we define the points A, B, C as *collinear* if A is between B and C, B is between A and C, and C is between A and B.

Problem 4.9

Two errors in this sentence are as follows:

- Points are not said to be in the plane.
- A line is an infinitely long collection of points, not a distance.

We can rewrite this sentence as follows: "The length of the line segment connecting two points in the plane is the shortest distance between them."

Problem 5.9

Proposition. *Suppose a, b, c are integers such that $a|b$ and $a|c$. Then, $a|(b+c)$.*

Proof. Since $a|b$ and $a|c$, there exists integers k_1 and k_2 such that $ak_1 = b$ and $ak_2 = c$. Adding these two equations gives

$$a(k_1 + k_2) = b + c$$

and the result follows. □

Problem 5.21

Proposition. *The difference between distinct, nonconsecutive perfect squares is composite.*

Proof. Suppose that $a < b$ are nonconsecutive integers. Then $b = a+k$ for some integer $k > 1$, and it follows that

$$\begin{aligned} b^2 - a^2 &= (a + k)^2 - a^2 \\ &= 2ak + k^2. \end{aligned}$$

Therefore, the difference between the perfect squares a^2 and b^2 is divisible by $k > 1$, and is therefore composite. □