Section 4.5

7. $y = \ln[(x+5)^{1/2}] = \dfrac{1}{2}\ln(x+5) \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2(x+5)}$

9. $y = \dfrac{3}{2}\ln(x^4+5x^2) \Rightarrow \dfrac{dy}{dx} = \dfrac{3}{2}\cdot\dfrac{4x^3+10x}{x^4+5x^2} = \dfrac{6x^2+15}{x^3+5x}$

11. $\dfrac{dy}{dx} = -5x\cdot\dfrac{3}{3x+2} - 5\ln(3x+2) = -\dfrac{15x}{3x+2} - 5\ln(3x+2)$

14. $\dfrac{dy}{dx} = x\cdot\dfrac{-2x}{2-x^2} + \ln|2-x^2| = -\dfrac{2x^2}{2-x^2} + \ln|2-x^2|$

16. $\dfrac{d\nu}{du} = \dfrac{\frac{1}{u}\cdot u^3 - 3u^2\ln u}{u^6} = \dfrac{1-3\ln u}{u^4}$

23. $\dfrac{dy}{dx} = \dfrac{1/x}{\ln x} = \dfrac{1}{x\ln x}$

27. $\dfrac{dy}{dx} = \dfrac{e^x\ln x - e^x/x}{(\ln x)^2} = \dfrac{xe^x\ln x - e^x}{x(\ln x)^2}$

34. $\dfrac{dy}{dx} = \dfrac{3}{(\ln 10)3x} = \dfrac{1}{(\ln 10)x}$

36. $y = \dfrac{1}{2}\log_7(4x-3) \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2}\cdot\dfrac{4}{(\ln 7)(4x-3)} = \dfrac{2}{(\ln 7)(4x-3)}$

37. $y = \dfrac{3}{2}\log_3(x^2+2x) \Rightarrow \dfrac{dy}{dx} = \dfrac{3}{2}\cdot\dfrac{2x+2}{(\ln 3)(x^2+2x)} = \dfrac{3x+3}{(\ln 3)(x^2+2x)}$

47. If two functions differ by a constant then they will have the same derivative since the derivative of a constant is 0. Since $\ln 6x = \ln 6 + \ln x$ differs from $\ln x$ by the constant $\ln 6$, the two functions have the same derivative even though they are not equal.

58. $P'(t) = \dfrac{t+100}{t+2} + \ln(t+2)$ so that $P'(2) \approx 26.9$ and $P'(8) \approx 13.1$ where in both cases the units are ants/day.

Section 4.6

1. $\frac{dy}{dx} = \frac{1}{2} \cdot \cos 8x \cdot 8 = 4 \cos 8x$

3. $\frac{dy}{dx} = 12 \cdot \sec^2(9x+1) \cdot 9 = 108 \sec^2(9x+1)$

4. $\frac{dy}{dx} = -4 \cdot -\sin(7x^2-4) \cdot 14x = 56x \sin(7x^2-4)$

6. $y = -9(\sin x)^5 \Rightarrow \frac{dy}{dx} = -9 \cdot 5(\sin x)^4 \cdot \cos x = -45 \sin^4 x \cos x$

9. $\frac{dy}{dx} = -6x \cdot (\cos 2x \cdot 2) - 6 \sin 2x = -12x \cos 2x - 6 \sin 2x$

12. $\frac{dy}{dx} = \frac{(\sec^2 x)(x-1) - \tan x}{(x-1)^2}$

13. $\frac{dy}{dx} = \cos e^{4x} \cdot (e^{4x} \cdot 4) = 4e^{4x} \cos e^{4x}$

15. $\frac{dy}{dx} = (-\sin x)e^{\cos x}$

17. $\frac{dy}{dx} = \cos(\ln 3x^4) \cdot \frac{12x^3}{3x^4} = \frac{4}{x} \cos(\ln 3x^4)$

21. $\frac{dy}{dx} = \frac{2\cos x(3-2\sin x) - 2\sin x \cdot (-2\cos x)}{(3-2\sin x)^2} = \frac{6\cos x}{(3-2\sin x)^2}$

33. $\frac{d}{dx}(\cot x) = \frac{d}{dx}\left(\frac{\cos x}{\sin x}\right) = \frac{-\sin x \cdot \sin x - \cos x \cdot \cos x}{\sin^2 x} = -\frac{1}{\sin^2 x} = -\csc^2 x$

34.

$$\begin{aligned}
\frac{d}{dx}(\sec x) &= \frac{d}{dx}\left(\frac{1}{\cos x}\right) = \frac{d}{dx}\left((\cos x)^{-1}\right) = -(\cos x)^{-2} \cdot (-\sin x) \\
&= \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} = \sec x \tan x
\end{aligned}$$

Section 5.1

3. (a) $(-\infty, -2)$ (b) $(-2, +\infty)$

4. (a) $(3, +\infty)$ (b) $(-\infty, 3)$

6. (a) $(1, 5)$ (b) $(-\infty, 1)$ and $(5, +\infty)$

16. (a) $x = -1, 2$ (b) $(-\infty, -1)$ and $(2, +\infty)$ (c) $(-1, 2)$

The steps in the solution are below.

(i) Domain of f: $(-\infty, \infty)$ (ii) $f'(x) = 2x^2 - 2x - 4 = 2(x-2)(x+1)$ is defined everywhere and 0 at the (only) critical numbers $x = -1, 2$. These critical numbers partition the number line as below.

$(-\infty, -1)$: Both factors of $f'(x)$ are negative so $f'(x) > 0$ and f is increasing on this interval.

$(-1, 2)$: First factor negative, second positive, so $f'(x) < 0$ and f is decreasing on this interval.

$(2, +\infty)$: Both factors are positive, so $f'(x) > 0$ and f is increasing on this interval.

19. (a) $x = -2, -1, 0$ (b) $(-2, -1)$ and $(0, +\infty)$ (c) $(-\infty, -2)$ and $(-1, 0)$

The steps in the solution are below.

(i) Domain of f: $(-\infty, \infty)$ (ii) $f'(x) = 4x^3 + 12x^2 + 8x = 4x(x+1)(x+2)$ is defined everywhere and 0 at the (only) critical numbers $x = -2, -1, 0$. These critical numbers partition the number line as below.

$(-\infty, -2)$: All three factors of $f'(x)$ are negative so $f'(x) < 0$ and f is decreasing on this interval.

$(-2, -1)$: First two factors negative, third positive, so $f'(x) > 0$ and f is increasing on this interval.

$(-1, 0)$: First factor negative, next two positive so $f'(x) < 0$ and f is decreasing on this interval.

$(0, +\infty)$: All three factors are positive so $f'(x) > 0$ and f is increasing on this interval.

24. (a) None (b) None (c) $(-\infty, 4)$ and $(4, +\infty)$

The steps in the solution are below.

(i) Domain of f: $(-\infty, 4)$ and $(4, +\infty)$ (ii) $f'(x) = \dfrac{(1)(x-4)-(x+3)(1)}{(x-4)^2} = \dfrac{-7}{(x-4)^2}$ is undefined at $x = 4$, but since $x = 4$ is not in the domain of f it is not a critical number. The derivative $f'(x)$ is nowhere zero, so there are no critical numbers. Since $f'(x) < 0$ for all x in the domain of f it follows that f is decreasing on $(-\infty, 4)$ and $(4, +\infty)$.

25. (a) $x = 0$ (b) $(0, +\infty)$ (c) $(-\infty, 0)$.

The steps in the solution are below.

(i) Domain of f: $(-\infty, +\infty)$ (ii) $y = f(x) = (x^2+1)^{1/2} \Rightarrow f'(x) = \dfrac{1}{2}(x^2+1)^{-1/2} \cdot 2x = \dfrac{x}{\sqrt{x^2+1}}$ is defined everywhere and is 0 only at $x = 0$. This critical number splits the domain of f into two intervals.

$(-\infty, 0)$: $f'(x) < 0$ so f is decreasing on this interval

$(0, +\infty)$: $f'(x) > 0$ so f is increasing on this interval

26. (a) ± 3 and $\pm 3\sqrt{2}/2 \approx \pm 2.12$ (b) $(-3\sqrt{2}/2, 3\sqrt{2}/2)$ (c) $(-3, -3\sqrt{2}/2)$ and $(3\sqrt{2}/2, 3)$

The steps in the solution are below.

(i) Domain of $y = f(x) = x(9-x^2)^{1/2}$: $[-3, 3]$

(ii) $f'(x) = x \cdot \dfrac{1}{2}(9-x^2)^{-1/2} \cdot (-2x) + (9-x^2)^{1/2} = -\dfrac{x^2}{\sqrt{9-x^2}} + \sqrt{9-x^2} = \dfrac{9-2x^2}{\sqrt{9-x^2}}$ is undefined for $x = \pm 3$ and is 0 for $x = \pm 3\sqrt{2}/2$. These critical numbers divide the domain of f into three intervals and we use test points as follows.

$(-3, -3\sqrt{2}/2)$: $f'(-2.5) \approx -2.11 < 0$ so f is decreasing on this interval.

$(-3\sqrt{2}/2, 3\sqrt{2}/2)$: $f'(0) = 3 > 0$ so f is increasing on this interval.

$(3\sqrt{2}/2, 3)$: $f'(2.5) \approx -2.11 < 0$ so f is decreasing on this interval.

27. (a) $x = 0$ (b) $(0, +\infty)$ (c) $(-\infty, 0)$ The steps in the solution are below.

(i) Domain of f: $(-\infty, +\infty)$ (ii) $f'(x) = \dfrac{2}{3}x^{-1/3} = \dfrac{2}{3\sqrt[3]{x}}$ is undefined at $x = 0$ and is 0 nowhere. The critical number $x = 0$ divides the domain of f into $(-\infty, 0)$ and $(0, +\infty)$. On the first interval $f'(x) < 0$ and on the second interval $f'(x) > 0$ so f is decreasing on

$(-\infty, 0)$ and is increasing on $(0, +\infty)$.

32. (a) $x = 1/2, 1$ (b) $(-\infty, 1/2)$ and $(1, +\infty)$ (c) $(1/2, 1)$

The steps in the solution are below.

(i) Domain of $y = f(x) = xe^{x^2-3x}$: $(-\infty, +\infty)$

(ii) $f'(x) = x \cdot (2x-3)e^{x^2-3x} + e^{x^2-3x} = (2x^2-3x+1)e^{x^2-3x} = (2x-1)(x-1)e^{x^2-3x}$ is defined everywhere and is 0 at $x = 1/2, 1$. These critical numbers divide the domain of f into 3 intervals and we use test point as follows.

$(-\infty, 1/2)$: $f'(0) = 1 > 0$ so f is increasing on this interval.

$(1/2, 1)$: $f'(0.6) \approx -0.019 < 0$ so f is decreasing on this interval.

$(1, +\infty)$: $f'(3) = 3/e^2 > 0$ so f is increasing on this interval.

41. You never "go to the left ..." when making a decision about increasing/decreasing behavior. You always ask what happens to the graph of f as you move to the right.

54. Since t is hours after the drug is administered the domain of f is $[0, +\infty)$. Since $K'(t) = \dfrac{4(3t^2+27) - (4t)\cdot(6t)}{(3t^2+27)^2} = \dfrac{108 - 12t^2}{(3t^2+27)^2} = \dfrac{12(3-t)(3+t)}{(3t^2+27)^2}$ is defined on the domain of f and is 0 for $t = \pm 3$ the only critical number of K is $t = 3$. (We discard $t = -3$ since this number does not belong to the domain of f.) Since $K'(t) > 0$ on the interval $(0, 3)$, f is increasing on this interval. Since $K'(t) < 0$ on the interval $(3, +\infty)$, K is decreasing on this interval.