CSC/MAT-220: Discrete Structures
EFY 3

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An Interesting Relation. Let R be a relation from $\mathbb{N} \times \mathbb{N}$ to $\mathbb{N}$ defined by

$R = \{((72, 99), 27)\,,\ ((27, 45), 18)\,,\ ((18, 39), 21)\,,\ ((21, 36), x)\,,\ ((x, 28), 13)\,,\ ((13, 21), 7)\}.$

Give a formal description of the pattern in this relation and use your description to find the value of the variable x.

Solution: Recall that every natural number can be represented by a sequence of *digits*, numbers between 0 and 9, in order from most to least significant. For example, the number 72 has digits 7 and 2. Based on the explicitly defined ordered pairs in R, the pattern may be stated as follows: if $(a, b)Rc$, then the digits of a and b sum to c. Therefore, $x = 12$.

More on Even and Odd Integers. In the game of chess, is it possible for the knight to go (by allowable moves) from the lower left-hand corner of the board to the upper right-hand corner, and in the process to land exactly once on each square?

Give a detailed explanation of your answer that includes mathematical variables to make your argument both clear and concise.

Solution: We denote the color of each square on the chess board by s, where $s = 1$, if the square is black, and $s = 0$, if the square is white. Then, we define the function $K \colon \{0, 1\} \to \{0, 1\}$ by

$$\begin{cases} K(s) = 1 \text{ if s=0} \\ K(s) = 0 \text{ if s=1} \end{cases}, \tag{1}$$

and note that this function represents the color of the resulting square after one move of the knight. Lastly, we let $K^n(s)$ denote the composition $K(K(\cdots(K(s))))$, which represents the color of the resulting square after n moves of the knight. It is clear from (1), that $K^n(s) = s$, if n is even, and $K^n(s) \neq s$, if n is odd.

Suppose it is possible for the knight to move from the lower left-hand corner of the board to the upper right-hand corner, while landing exactly once on each square. Then, the knight will undergo $n = 63$ moves in this process, and it follows from the above discussion that $K^n(s) \neq s$. Therefore, the night must be on a different colored square from the one it started on. However, this is not possible, since the lower left-hand and upper right-hand squares of a chess board are of the same color.