

CSC/MAT-220: Discrete Structures

EFY 13

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Problem Statement. You are working as an Uber driver for a city with a North Side, Downtown, and South Side, which we will denote by N , D , and S , respectively. When you pick up a customer, we denote by $P(A|B)$, the probability you go to A given that you were in B . Furthermore, we have a set of data which describes the following conditional probabilities:

$$\begin{array}{lll} P(N|N) = 0.5 & P(D|N) = 0.3 & P(S|N) = 0.2 \\ P(N|D) = 0.2 & P(D|D) = 0.8 & P(S|D) = 0.0 \\ P(N|S) = 0.3 & P(D|S) = 0.3 & P(S|S) = 0.4 \end{array}$$

- i. Describe the process of you picking up n customers as a sequence of independent events. Why is this a Markov Process?
- ii. Let $x = [x_1, x_2, x_3]$ denote a probability distribution, where x_1 is the probability you are on the North Side, x_2 is the probability you are Downtown, and x_3 is the probability you are on the South Side. Show that the process in (i) can be described as a sequence of matrix multiplications

$$x, Px, P^2x, \dots, P^n x.$$

What is the matrix P ?

- iii. Write an ML function *Uber* that takes as arguments an initial state x and an integer n and returns the state after picking up n customers. Be sure to comment your code and print out a copy, and attach with our writeup when you turn it in.
- iv. Use the *Uber* function to answer the following question. Record your results up to 4 digits after the decimal place, and display them in a table.
 - Given that you start on the North Side, what is the probability that you end up on the North Side, Downtown, or South Side; after 10, 20, and 30 pick-ups?