MAT – 112: Calculus I and Modeling
EFY 1

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Instructions

Please complete each of the following problems. You should work in groups of three, or at most four, and hand in only one submission per group. Be sure that your arguments are well justified and presented clearly.

Problem 1. Consider the flat-tax example given in class. Given a tax rate of r and gross income g, the amount taxed is

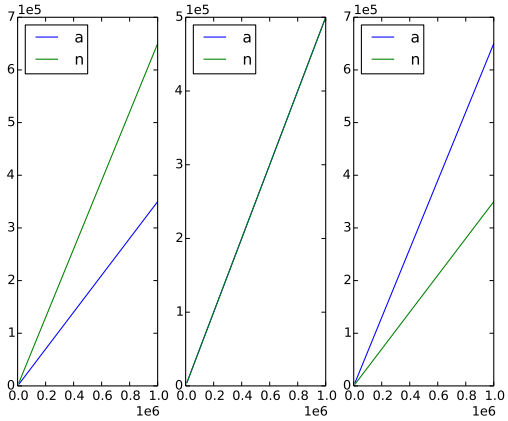
$$a = r \cdot g,$$

and net income is given given by

$$n = (1 - r)g.$$

For $r < 50\%$, $r = 50\%$, and $r > 50\%$, sketch a plot of n and a as a function of g on a single graph, where the x-axis denotes gross income and the y-axis denotes dollars.

Solution:



Problem 2. Given the line $\mathcal{L}$ described by the equation $y = 2x + 1$. Find the equation of a line perpendicular to $\mathcal{L}$ that goes through the point $(1, 3)$.

Solution: Since the line $\mathcal{L}$ has a slope of 2, we know that the perpendicular line must have a slope of $-\frac{1}{2}$. Therefore, the equation of the line is of the form

$$y - 3 = -\frac{1}{2}(x - 1).$$

Solving for y, we get $y = -\frac{1}{2}x + \frac{7}{2}$.

Problem 3. Consider a closed rectangular box with length l, width w, and height h. Suppose that $w = l$ and the total surface area of the box must equal 100 ft^2. Find a function for the volume of the box in terms of its length. Then, sketch a plot of this function over the appropriate domain.
You may use desmos or any other plotting software as an aide.

Solution: Given the constraints of the problem, we have the following equations for the volume V and surface area A of our rectangular box:

$$V = l^2 h \quad \text{and} \quad A = 2l^2 + 4lh.$$

Since $A = 100$, we can solve for h, which gives $h = \frac{100 - 2l^2}{4l}$. Therefore, the volume may be written as

$$\begin{aligned} V &= l^2 \left(\frac{100 - 2l^2}{4l} \right) \\ &= 25l - \frac{1}{2} l^3. \end{aligned}$$

Since the length must be positive, we have $l > 0$. Furthermore, the volume must be positive, which implies that $25l - \frac{1}{2}l^3 > 0$. We rewrite this inequality as $l^2 < 50$. Therefore, $0 < l < \sqrt{50}$. A plot of V as a function of l is below.