MAT-150: Linear Algebra
EFY 7

Due: October 30, 2017

Problem Statement. Let

$$A = \begin{bmatrix} 1 & 2 & 1 \\ 2 & 2 & 2 \\ 0 & 1 & 0 \end{bmatrix}.$$

Find the "obvious" eigenvalue and a corresponding eigenvector. Use the eigenvector to deflate the problem of finding the eigenvalues to a smaller 2×2 problem. Then, find the eigenvalues of the smaller 2×2 problem.

Solution. Since the first and third columns of A are the same, it follows that A is not invertible. Therefore, the "obvious" eigenvalue is $\lambda = 0$, with a corresponding eigenvector of $v = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$. Define $V = [v\ e_2\ e_3]$, and note that the columns of V are linearly independent (in fact they are orthogonal). It is also interesting to note that $V^{-1} = V$.

Next, we use the similarity transformation to deflate our problem as follows

$$V^{-1}AV = \begin{bmatrix} 0 & 2 & 1 \\ 0 & 2 & 2 \\ 0 & 3 & 1 \end{bmatrix}.$$

Recall that similarity transformations preserve eigenvalues. Furthermore, the remaining two eigenvalues left to be found are the eigenvalues of the smaller 2×2 matrix

$$\hat{A} = \begin{bmatrix} 2 & 2 \\ 3 & 1 \end{bmatrix}.$$

The eigenvalues are the roots of the characteristic equation $(2-\lambda)(1-\lambda)-6 = 0$, which can be written as

$$(\lambda - 4)(\lambda + 1) = 0.$$

Therefore, the eigenvalues of the original matrix A are $\{0,\ 4,\ -1\}$.