

MAT-150: Linear Algebra

EFY 5

September 25, 2017

Part 1: Volume and Orientation of Parallelepipeds

Please do each of the following.

- i. State the definition of a parallelepiped \mathcal{P} in \mathbb{R}^n .
 - What is the dimension of \mathcal{P} ?
 - What is the ambient space of \mathcal{P} ?
- ii. Let $a \in \mathbb{R}$
 - Give a geometric description of the parallelepiped \mathcal{P} determined by a .
 - What is $V(\mathcal{P})$?
 - What is $O(\mathcal{P})$?
- iii. State the definition of a proper parallelepiped \mathcal{P} in \mathbb{R}^n .
 - Which of the following Parallelepipeds are proper? Why?

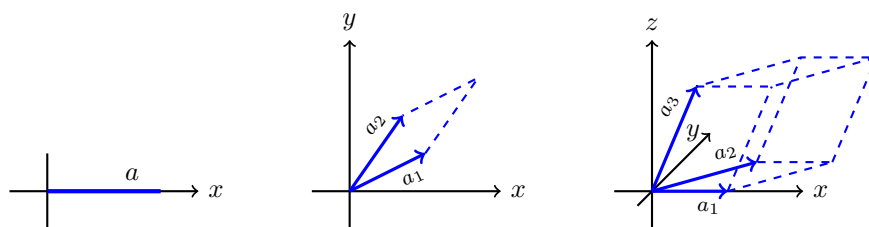


Figure 1: Parallelepipeds in \mathbb{R} , \mathbb{R}^2 , and \mathbb{R}^3 .

- State the definition of the base of a proper parallelepiped \mathcal{P} in \mathbb{R}^n
 - State the definition of the height of a proper parallelepiped \mathcal{P} in \mathbb{R}^n
- iv. State the definition of the image of the parallelepiped \mathcal{P} under the linear transformation T .

v. Let

$$a_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{and} \quad a_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}.$$

- Denote by \mathcal{P} the parallelepiped determined by a_1 and a_2 . Explain why \mathcal{P} is not proper.
- Find the matrix representation of the linear transformation T such that the image $T(\mathcal{P})$ will be proper.
- Compute the vectors $T(a_1)$ and $T(a_2)$ that determine the proper parallelepiped $T(\mathcal{P})$.
- Compute $V(\mathcal{P})$ and $O(\mathcal{P})$.

vi. Let

$$a_1 = \begin{bmatrix} a \\ c \end{bmatrix} \quad \text{and} \quad a_2 = \begin{bmatrix} b \\ d \end{bmatrix},$$

where $a, b, c, d \in \mathbb{R}$.

- Define

$$\cos(\theta) = \frac{a}{\sqrt{a^2 + c^2}} \quad \text{and} \quad \sin(\theta) = \frac{c}{\sqrt{a^2 + c^2}},$$

and

$$Q = \begin{bmatrix} \cos(\theta) & \sin(\theta) \\ -\sin(\theta) & \cos(\theta) \end{bmatrix}.$$

- Show that Qa_1 and Qa_2 determine a proper parallelepiped.
- Compute the volume and orientation of this parallelepiped.