CSC/MAT-220: Class Activiity

Cantor's Theorem

November 1, 2017

Theorem. *Let A be a set. If $f\colon A \to \mathcal{P}(A)$, then f is not onto.*

Proof. To show that f cannot be onto, we must find a set $B \in \mathcal{P}(A)$, such that there is no $a \in A$ that satisfies $f(a) = B$. Define

$$B = \{x \in A\colon x \notin f(x)\}$$

and suppose there is an $a \in A$ such that $f(a) = B$.

Show that

- If $a \in B$, then we arrive at a contradiction.
- If $a \notin B$, then we arrive at a contradiction.

Then, make a concluding argument to complete the proof. □