CSC/MAT-220: Discrete Structures
EFY 10

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One-to-One and Onto Let A be any non-empty set and let S be a non-empty subset of A. Define the function $f\colon A \to \{0,1\}$ by $f(x) = 1$ if $x \in S$ and $f(x) = 0$ if $x \notin S$. Under what conditions is

i. f one-to-one?

ii. f onto?

iii. f bijective?

Solution.

i. The function f is one-to-one, when the sets S and $A - S$ have cardinality at most 1.

ii. The function f is onto, when S is a proper subset of A.

iii. The function f is bijective when (i.) and (ii.) hold, which implies that $|S| = 1$ and $|A - S| = 1$.

Function images of sets Let $f\colon \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2$. Find $f^{-1}(T)$ for each of the following

i. $T = \{9\}$

ii. $T = [4, 9]$

iii. $T = [-4, 9]$

Solution. The pre-image of a set $D \subseteq \mathbb{R}$ is defined by

$$f^{-1}(D) = \{x \in A\colon f(x) \in D\}.$$

where A is the domain of f, in this case $A = \mathbb{R}$.
Therefore, the answers are as follows:

i. $f^{-1}(T) = \{-3, 3\}$,

ii. $f^{-1}(T) = [-3, -2] \cup [2, 3]$,

iii. $f^{-1}(T) = [-3, 3]$.