MAT-150: Linear Algebra
EFY 9

Due: November 10, 2017

Problem 1.

    a. Let W be a subspace of $\mathbb{R}^n$. State the definition of the orthogonal complement of W.

    b. Let

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}.$$

    Show, by example, that Theorem 3 of Section 6.1 holds for the matrix A.

Problem 2. The vectors $\{u_1, u_2, \ldots, u_n\}$ in $\mathbb{R}^n$ are said to be orthonormal if

$$\langle u_i, u_j \rangle = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}.$$

Let $U = [u_1, u_2, \ldots, u_n]$ be a matrix with orthonormal columns. Prove that $U^T U = I$, where I is the $n \times n$ identity. Then justify the following statements.

- U^T is the inverse of U.

- The fact that the matrix Q in the QR Algorithm (and Francis's Algorithm) is orthonormal makes the algorithm far more efficient.