CSC/MAT-220: Discrete Structures
EFY 3

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An Interesting Relation. Let R be a relation from $\mathbb{N} \times \mathbb{N}$ to $\mathbb{N}$ defined by

$$R = \{((72, 99), 27),\ ((27, 45), 18),\ ((18, 39), 21),\ ((21, 36), x),\ ((x, 28), 13),\ ((13, 21), 7)\}.$$

Give a formal description of the pattern in this relation and use your description to find the value of the variable x.

Solution: Recall that every natural number can be represented by a sequence of *digits*, numbers between 0 and 9 in order from most to least significant. For example, the number 72 has digits 7 and 2. Based on the explicitly defined ordered pairs in R, the pattern may be stated as follows: if $(a, b)Rc$, then the digits of a and b sum to c. Therefore, $x = 12$.

More on Even and Odd Integers. In the game of chess, is it possible for the knight to go (by allowable moves) from the lower left-hand corner of the board to the upper right-hand corner, and in the process to land exactly once on each square?

Give a detailed explanation of your answer that includes mathematical variables to make your argument both clear and concise.

Solution: We denote the color of each square s on the chess board by

$$\begin{cases} C(s) = 1 & \text{if the square } s \text{ is black} \\ C(s) = 0 & \text{if the square } s \text{ is white} \end{cases} \tag{1}$$

Next, we denote by the function $K\colon \{0, 1\} \to \{0, 1\}$ a single move of the kinght. We define this function as follows

$$\begin{cases} K(C(s)) = 1 \text{ if C(s)=0} \\ K(C(s)) = 0 \text{ if C(s)=1.} \end{cases} \tag{2}$$

Let us build a list of square colors, starting with the square s_1, as follows

$$(C(s_1),\ C(s_2),\ \ldots,\ C(s_n))$$