Solution 1

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Assignment

1. Form an augmented matrix to represent the following system. Use RowRedue to determine if the system is consistent and, if so, report the general solution vector.

$$\begin{aligned} x_1 \qquad\qquad\qquad - 2\,x_4 &= -3 \\ 2\,x_2 + 2\,x_3 \qquad &= 0 \\ x_3 + 3\,x_4 &= 1 \\ -2\,x_1 + 3\,x_2 + 2\,x_3 + x_4 &= 5 \end{aligned}$$

   - Solution:

In[1]:=
```
aug = {{1, 0, 0, -2, -3}, {0, 2, 2, 0, 0}, {0, 0, 1, 3, 1}, {-2, 3, 2, 1, 5}};
RowReduce[aug]
```

Out[2]=
```
{{1, 0, 0, -2, -3}, {0, 1, 0, -3, -1}, {0, 0, 1, 3, 1}, {0, 0, 0, 0, 0}}
```

The above system is consistent and the 4th variable is free. Therefore, the general solution vector is

In[3]:=
```
f[t_] := {-3 + 2 t, -1 + 3 t, 1 - 3 t, t};
```

2. Manually perform the row operations and back-substitution necessary to solve the system corresponding to the following augmented matrix.

In[4]:=
```
aug = {{1, 0, 3, 0, 2}, {0, 1, 0, -3, 3}, {0, -2, 3, 2, 1}, {3, 0, 0, 7, -5}};
```

   - Solution:

In[5]:=
```
R = aug
R[[4]] = R[[4]] - 3 * R[[1]]; R
R[[3]] = R[[3]] + 2 * R[[2]]; R
R[[4]] = R[[4]] + 3 * R[[3]];
R
```

Out[5]=
```
{{1, 0, 3, 0, 2}, {0, 1, 0, -3, 3}, {0, -2, 3, 2, 1}, {3, 0, 0, 7, -5}}
```

Out[6]=
```
{{1, 0, 3, 0, 2}, {0, 1, 0, -3, 3}, {0, -2, 3, 2, 1}, {0, 0, -9, 7, -11}}
```

Out[7]=
```
{{1, 0, 3, 0, 2}, {0, 1, 0, -3, 3}, {0, 0, 3, -4, 7}, {0, 0, -9, 7, -11}}
```

Out[8]=
```
{{1, 0, 3, 0, 2}, {0, 1, 0, -3, 3}, {0, 0, 3, -4, 7}, {0, 0, 0, -5, 10}}
```

In[9]:=
```
sol = {0, 0, 0, 0};
sol[[4]] = R[[4, 5]] / R[[4, 4]];
sol[[3]] = (R[[3, 5]] - R[[3, 4]] * sol[[4]]) / R[[3, 3]];
sol[[2]] = (R[[2, 5]] - R[[2, 4]] * sol[[4]] - R[[2, 3]] * sol[[3]]) / R[[2, 2]];
sol[[1]] =
  (R[[1, 5]] - R[[1, 4]] * sol[[4]] - R[[1, 3]] * sol[[3]] - R[[1, 2]] * sol[[2]]) / R[[1, 1]];
sol
```

Out[14]= $\left\{3, -3, -\frac{1}{3}, -2\right\}$

3. Use RowReduce to solve the system below and represent the infinite set of solutions using Graphics3D and Animate.

In[15]:=
```
aug = {{1, 2, 1, 3}, {-2, -4, -2, -6}, {3, 6, 3, 9}};
```

- Solution:

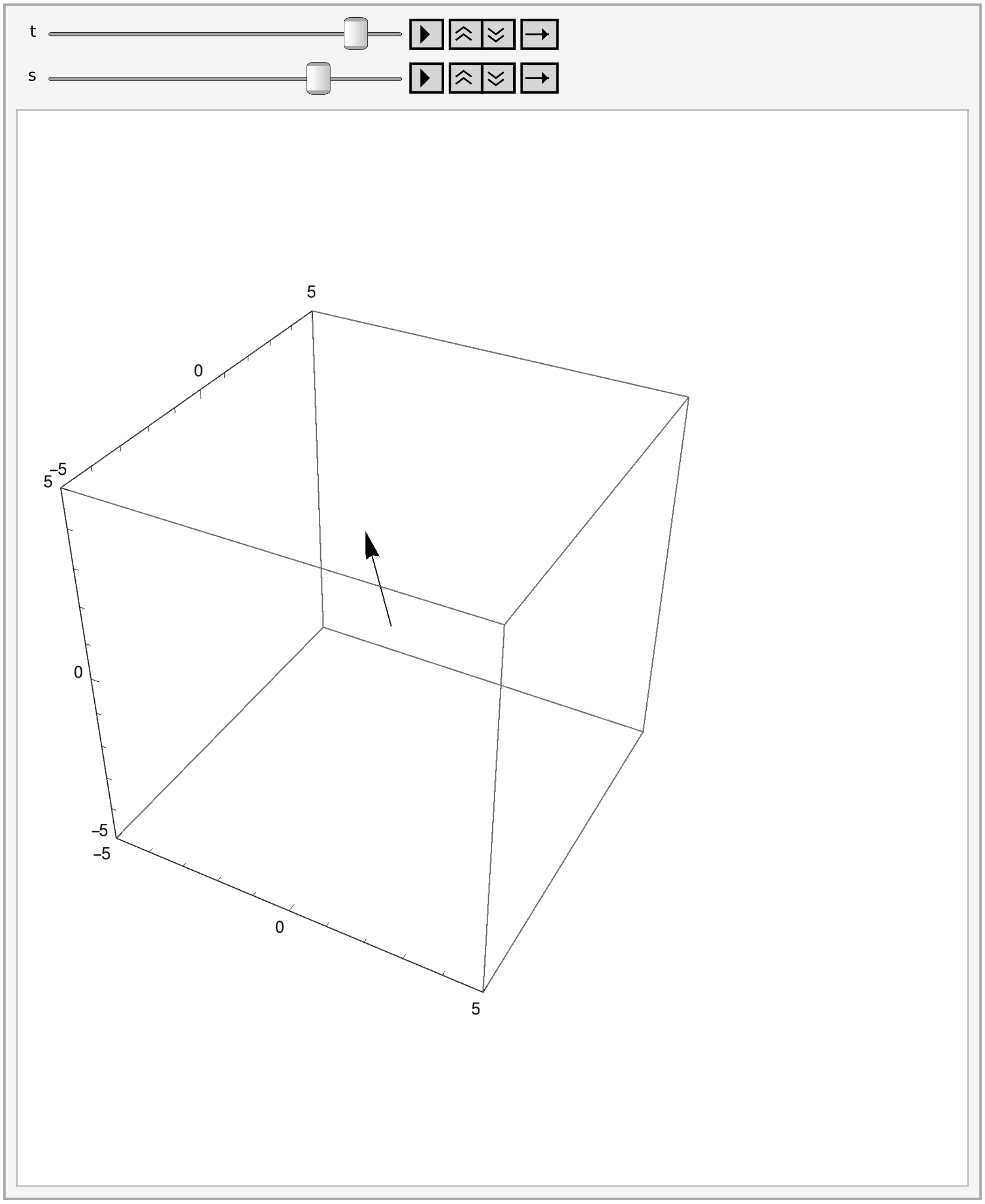
In[16]:=
```
RowReduce[aug]
```

Out[16]=
```
{{1, 2, 1, 3}, {0, 0, 0, 0}, {0, 0, 0, 0}}
```

In[17]:=
```
f[t_, s_] := {3 - 2 * t - s, t, s};
```

In[18]:=
```
Animate[Graphics3D[{Arrow[{{0, 0, 0}, f[t, s]}]}, Axes → True,
  PlotRange → {{-5, 5}, {-5, 5}, {-5, 5}}], {t, -2, 2}, {s, -2, 2}]
```

Out[18]=



4. Use Graphics3D to display the linear combination of the following vectors that results in the vector y={8,-7,2}. Additionally, display the commutativity of the aforementioned linear combination.

In[19]:=
```
u1 = {1, 1, 1}; u2 = {-2, 1, 1}; u3 = {1, -2, 1};
```

- Solution:

First, we will solve the corresponding system of equations

```
In[20]:= y = {8, -7, 2};
aug = ConstantArray[0, {3, 4}];
aug[[All, 1]] = u1;
aug[[All, 2]] = u2;
aug[[All, 3]] = u3;
aug[[All, 4]] = y; aug
RowReduce[aug]

Out[25]= {{1, -2, 1, 8}, {1, 1, -2, -7}, {1, 1, 1, 2}}

Out[26]= {{1, 0, 0, 1}, {0, 1, 0, -2}, {0, 0, 1, 3}}

In[27]:= Graphics3D[{Blue, Arrow[{{0, 0, 0}, u1}], Red, Arrow[{{0, 0, 0}, -2 * u2}],
  Green, Arrow[{{0, 0, 0}, 3 * u3}], Purple, Arrow[{{0, 0, 0}, y}], Blue,
  Dashed, Arrow[{-2 * u2, -2 * u2 + u1}], Arrow[{3 * u3, 3 * u3 + u1}], Red,
  Dashed, Arrow[{u1, u1 - 2 * u2}], Arrow[{3 * u3, 3 * u3 - 2 * u2}], Green,
  Dashed, Arrow[{u1, u1 + 3 * u3}], Arrow[{-2 * u2, -2 * u2 + 3 * u3}],
  Blue, Thick, Arrow[{-2 * u2 + 3 * u3, -2 * u2 + 3 * u3 + u1}], Red,
  Thick, Arrow[{u1 + 3 * u3, u1 + 3 * u3 - 2 * u2}], Green, Thick,
  Arrow[{u1 - 2 * u2, u1 - 2 * u2 + 3 * u3}]}, Axes → True, Boxed → False]
```

Out[27]=