

# MAT-450: Advanced Linear Algebra

## EFY 1

Thomas R. Cameron

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### Instructions

Please complete each of the following problems. You should work in groups of two and hand in only one submission per group. Be sure that your arguments are well justified and presented clearly.

**Problem 1.** Show that the set  $Z_2 = \{0, 1\}$  is a field under addition and multiplication modulo 2. It is clear that the commutative, associative, and distributive properties hold. Therefore, all that remains is for you to show that the identity and inverse properties hold.

**Problem 2.** Let  $S$  be a nonempty set and  $\mathbb{F}$  be a field. Show that the set of all functions from  $S$  to  $\mathbb{F}$ , denoted by  $\mathcal{F}(S, \mathbb{F})$ , is a vector space over  $\mathbb{F}$ .

**Problem 3.** As we will discuss in later weeks, the field  $\mathbb{C}$  over the real numbers is isomorphic to  $\mathbb{R}^2$ . Let

$$a + bi \quad \text{and} \quad c + di$$

be two complex numbers. Show that the complex multiplication  $(a + bi)(c + di)$  can be represented by the following matrix multiplication

$$\begin{bmatrix} a & -b \\ b & a \end{bmatrix} \begin{bmatrix} c \\ d \end{bmatrix}.$$