MAT – 450: Advanced Linear Algebra EFY 3

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February 1, 2018

Instructions

Write an exposition that develops the vector space $\mathcal{L}(V, W)$, where V and W are vector spaces over the same field \mathbb{F} . Your exposition should be lively and include the following points:

- Definition of $\mathcal{L}(V, W)$ and convincing justification for why it is a vector space.
- Description of the association we have between $\mathcal{L}(V,W)$ and $\mathbb{F}^{m\times n}$, when $\dim(V)=n$ and $\dim(W)=m$, and argue why this association preserves addition and scalar multiplication (here you are assuming the reader knows the definition of matrix addition and scalar multiplication).
- Composition of linear operators is linear, and use this result to motivate the definition of matrix multiplication (here you are defining matrix multiplication for the reader).
- Relate the previous point back to $\mathcal{L}(V)$, when dim V=n, and specifically mention closure.
- Any matrix multiplication properties that you find particularly interesting.