Solution 2

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Assignment

Copy and past all problems below into a blank notebook. Include a title, your name, and date at the top of the notebook. Furthermore, be sure to clearly label and evaluate your solutions in such a way that when printed it is easy to grade your work.

1. Define A={{1,5,-2,0},{-3,1,9,-5},{4,-8,-1,7}} and b={-7,9,0}. Use the LinearSolve and NullSpace commands to compute the general solution vector as a function of t. Be sure to clearly display the solution vector at the end.

- Solution

In[1]:=
```
A = {{1, 5, -2, 0}, {-3, 1, 9, -5}, {4, -8, -1, 7}}; b = {-7, 9, 0};
```

In[2]:=
```
p = LinearSolve[A, b];
vh = NullSpace[A][[1]];
w[t_] := p + t * vh
```

In[5]:=
```
w[t]
```

Out[5]= $\left\{-\frac{11}{7} - 8\,t,\ -\frac{6}{7} + 2\,t,\ \frac{4}{7} + t,\ 7\,t\right\}$

2. Define the vectors v1={0,-1,-2,1}, v2={-3,-2,-3,4}, v3={-6,-1,0,5}, v4={4,3,3,-9}, v5={9,1,-1,-7}. Show that these vectors are linearly dependent, and provide explanation.

- Solution

In[6]:=
```
v1 = {0, -1, -2, 1};
v2 = {-3, -2, -3, 4};
v3 = {-6, -1, 0, 5};
v4 = {4, 3, 3, -9};
v5 = {9, 1, -1, -7};
A = Transpose[Append[Append[Append[Append[{v1}, v2], v3], v4], v5]];
RowReduce[A]
```

Out[8]=
```
{{1, 0, -3, 0, 5}, {0, 1, 2, 0, -3}, {0, 0, 0, 1, 0}, {0, 0, 0, 0, 0}}
```

From the row reduction of the matrix A, we can see that column vectors 1, 2, and 4 are pivot columns, and columns 3 and 5 are not.Therefore, these vectors are linearly dependent.

3. Let v1,v2,v3,v4,v5 be the vectors from Problem 2. Define A=[v1,v2,v3,v4,v5] and use the NullSpace command to find solutions to the homogenous equation Ax=0. Use the solutions to the homogenous equation to write each non-pivot column in terms of the previous pivot columns. Display each linearly combination as an equality (==), not an assignment (=). That way, Mathematica will return true or false.

- Solution

In[9]:=
```
NullSpace[A]
```
Out[9]=
```
{{-5, 3, 0, 0, 1}, {3, -2, 1, 0, 0}}
```
In[10]:=
```
v5 == 5 v1 - 3 v2
```
Out[10]=
```
True
```
In[11]:=
```
v3 == 2 v2 - 3 v1
```
Out[11]=
```
True
```

4. The reflection of a point in the plane about the line L through the origin which makes an angle t with the x-axis can be represented by the matrix {{Cos(2*t),Sin(2*t)},{Sin(2*t),-Cos(2*t)}}. Use this matrix to define the rotation transformation as a function of the point (x1,x2) and the angle t. Then, provide of a graphic which displays arrows pointing to the original point (x1,x2) and its reflection about the line L. In addition, use the Line command to add the line L to the graphic.

- Solution

In[12]:=
```
ClearAll["Global`*"];
A[t_] := {{Cos[2 * t], Sin[2 * t]}, {Sin[2 * t], -Cos[2 * t]}};
x[x1_, x2_, t_] := A[t].{x1, x2}
```

In[15]:=
```
Animate[Graphics[{Blue, Arrow[{{0, 0}, x[x1, x2, t]}], Dashed, Arrow[{{0, 0}, {x1, x2}}],
   Green, Line[{{-Cos[t], -Sin[t]}, {Cos[t], Sin[t]}}]}, Axes → True,
  PlotRange → {{-2, 2}, {-2, 2}}], {x1, -1, 1}, {x2, -1, 1}, {t, 0, 2 * Pi}]
```

Out[15]=