CSC/MAT-220: Discrete Structures

EFY 5

September 22, 2017

Step 1. Start by tallying the number of problems that you did not complete as of 1:30 pm (ET) on 9/22/2017. Record the number below.

The Lion and the Unicorn. The point of this puzzle was to test your logic skill, and perhaps more importantly to employ truth tables, boolean variables, and logical connectives to elegantly present your solution to this puzzle. As an example, consider my solution to the first part of this puzzle below.

```latex
\begin{figure}[h]
\centering
\begin{tabular}{c | c | c | c | c | c | c | c |}
& Su & M & Tu & W & Th & F & Sa\\
\hline
Lion & 1 & 0 & 0 & 0 & 1 & 1 & 1\\
Unicorn & 1 & 1 & 1 & 1 & 0 & 0 & 0\\
\end{tabular}
\caption{Truth = $1$ and Lie = $0$.}
\end{figure}

Let $p_{1}$ denote the Lion's statement, and $p_{2}$ the Unicorn's statement. Note that on no day of the week can both the Lion and the Unicorn be lying. Therefore, all that remains is the following statements and their implications
\begin{align*}
&p_{1}\land p_{2}\implies\text{ Day = Su }\implies \lnot p_{1}\land p_{2}\implies\text{ contradiction}\\
&\lnot p_{1}\land p_{2}\implies\text{ Day = M or Tu or W }\implies \underset{\text{M}}{(\lnot p_{1}\land \lnot p_{2})}\lor\underset{\text{Tu or W}}{(p_{1}\land\lnot p_{2})}\implies\text{ contradiction}\\
&p_{1}\land\lnot p_{2}\implies\text{ Day = Th or F or Sa }\implies \underset{\text{Th}}{(p_{1}\land\lnot p_{2})}\lor\underset{\text{F or Sa}}{(\lnot p_{1}\land p_{2})}\implies\text{ Day = Th }.
\end{align*}
Therefore, the day is Thursday.
```

	Su	M	Tu	W	Th	F	Sa
Lion	1	0	0	0	1	1	1
Unicorn	1	1	1	1	0	0	0

Figure 1: Truth = 1 and Lie = 0.

Let p_1 denote the Lion's statement, and p_2 the Unicorn's statement. Note that on no day of the week can both the Lion and the Unicorn be lying. Therefore, all that remains is the following statements and their implications

$$\begin{aligned}
&p_1 \land p_2 \implies \text{ Day = Su } \implies \lnot p_1 \land p_2 \implies \text{ contradiction}\\
&\lnot p_1 \land p_2 \implies \text{ Day = M or Tu or W } \implies \underset{\text{M}}{(\lnot p_1 \land \lnot p_2)} \lor \underset{\text{Tu or W}}{(p_1 \land \lnot p_2)} \implies \text{ contradiction}\\
&p_1 \land \lnot p_2 \implies \text{ Day = Th or F or Sa } \implies \underset{\text{Th}}{(p_1 \land \lnot p_2)} \lor \underset{\text{F or Sa}}{(\lnot p_1 \land p_2)} \implies \text{ Day = Th }.
\end{aligned}$$

Therefore, the day is Thursday.

Question: What improvements can you make to your solution of this puzzle?

Problem 14.16. Try to identify the underlying assumption the author is making. To help, consider a relation R on the set $A = \{1, 2, 3\}$, that is both symmetric and transitive. Imagine that 3 is not related to either 1 nor 2, is it possible that R is not reflexive? Now, imagine that all $3R1$ and $3R2$, is it possible that R is not reflexive?

Question: What is the connection between Problem 14.16 and Other Problems III?

Problem 14.17 and 15.14 Discuss your Relation Graphs. What do you notice about the graphs of equivalence relations versus those that are not equivalence relations? Use this to help you describe what an equivalence relation looks like.

Problems 16.2 and 16.18. Despite the fact there are many ways to solve these problems, the key is to use your knowledge of equivalence relations, classes, and partitions. So, your outline should include the following steps:

- Identify the set A that you are working over, and its cardinality $|A|$.
- Identify the relation R on the set A, verify that it is an equivalence relation.
- Justify why every equivalence class has the same number of elements m.
- Compute the number of equivalence classes

$$\frac{|A|}{m}.$$

Other Problems I and II. In these problems you are asked to describe the equivalence relation/classes. Note that some times a geometric definition is best; whereas, other times it only makes sense to write out the set definition.