Section 7.1

5. $\int 6\ dk = 6k + C$

13. $\int (5z^{1/2} + \sqrt{2})\ dz = 5\left(\frac{2}{3}z^{3/2}\right) + \sqrt{2}z + C = \frac{10}{3}z^{3/2} + \sqrt{2}z + C$

16. $\int x^2(x^4 + 4x + 3)\ dx = \int (x^6 + 4x^3 + 3x^2)\ dx = \frac{1}{7}x^7 + x^4 + x^3 + C$

21. $\int 7z^{-2}\ dz = -7z^{-1} + C = -\frac{7}{z} + C$

28. $\int \frac{2}{3x^4}\ dx = \int \frac{2}{3}x^{-4}\ dx = \frac{2}{3}\int x^{-4}\ dx = \frac{2}{3} \cdot \left(-\frac{1}{3}x^{-3}\right) + C = -\frac{2}{9x^3} + C$

38. $\int (2y-1)^2\ dy = \int (4y^2 - 4y + 1)\ dy = \frac{4}{3}y^3 - 2y^2 + y + C$

39. $\int \frac{\sqrt{x}+1}{\sqrt[3]{x}}\ dx = \int \frac{x^{1/2}+1}{x^{1/3}}\ dx = \int (x^{1/6} + x^{-1/3})\ dx = \frac{6}{7}x^{7/6} + \frac{3}{2}x^{2/3} + C$

41. $\int 10^x\ dx = \frac{10^x}{\ln 10} + C$

45. $f(x) = \int x^{2/3}\ dx = \frac{3}{5}x^{5/3} + C \Rightarrow \frac{3}{5} = f(1) = \frac{3}{5} + C \Rightarrow C = 0 \Rightarrow f(x) = \frac{3}{5}x^{5/3}$

48. $\int \frac{a - bx}{x}\ dx = \int (\frac{a}{x} - b)\ dx = a \ln x - bx + C$

55. $\nu(t) = \int (5t^2 + 4)\ dt = \frac{5}{3}t^3 + 4t + C \Rightarrow 6 = \nu(0) = C \Rightarrow \nu(t) = \frac{5}{3}t^3 + 4t + 6$

56. $s(t) = \int (9t^2 - 3t^{1/2})\ dx = 3t^3 - 2t^{3/2} + C \Rightarrow 8 = s(1) = 3 - 2 + C \Rightarrow C = 7$
$\Rightarrow s(t) = 3t^3 - 2t^{3/2} + 7$

Section 7.2

2.

(a) $u = 3x^2 - 5, \quad du = 6xdx$

(b) $u = 1 - x, \quad du = -dx$

(c) $u = 2x^3 + 1, \quad du = 6x^2dx$

(d) $u = x^4, \quad du = 4x^3dx$

3. Using $u = 2x + 3, \quad du = 2dx, \int 4(2x+3)^4\, dx = \int 2u^4\, du = \frac{2}{5}u^5 + C = \frac{2}{5}(2x+3)^5 + C$

8. Using $u = 2x^3 + 7, \quad du = 6x^2dx$

$$\int \frac{6x^2dx}{(2x^3+7)^{3/2}} = \int u^{-3/2}\, du = -2u^{-1/2} + C = -2(2x^3+7)^{-1/2} + C = -\frac{2}{\sqrt{2x^3+7}} + C$$

9. Using $u = 4z^2 - 5, \quad du = 8zdz$

$$\int z\sqrt{4z^2-5}\, dz = \frac{1}{8}\int u^{1/2}\, du = \frac{1}{8}\cdot\frac{2}{3}u^{3/2} + C = \frac{1}{12}(4z^2-5)^{3/2} + C$$

12. Using $u = -r^2, \quad du = -2rdr, \quad \int re^{-r^2}\, dr = -\frac{1}{2}\int e^u\, du = -\frac{1}{2}e^u + C = -\frac{1}{2}e^{-r^2} + C$

15. Using $u = \frac{1}{z}, \quad du = -\frac{1}{z^2}dz, \quad \int \frac{e^{1/z}}{z^2}\, dz = -\int e^u\, du = -e^u + C = -e^{1/z} + C$

18. Using $u = x^2 + 3, \quad du = 2xdx,$

$$\int \frac{-4x}{x^2+3}\, dx = -2\int \frac{1}{u}\, du = -2\ln|u| + C = -2\ln|x^2+3| + C = -2\ln(x^2+3) + C$$

29. Using $u = 1 + 3\ln x, \quad du = \frac{3}{x}dx$

$$\int \frac{(1+3\ln x)^2}{x}\, dx = \frac{1}{3}\int u^2\, du = \frac{1}{9}u^3 + C = \frac{1}{9}(1+3\ln x)^3 + C$$

31. Using $u = e^{2x} + 5, du = 2e^{2x}dx,$

$$\int \frac{e^{2x}}{e^{2x}+5}\, dx = \frac{1}{2}\int \frac{1}{u}\, du = \frac{1}{2}\ln|u| + C = \frac{1}{2}\ln|e^{2x}+5| + C = \frac{1}{2}\ln(e^{2x}+5) + C$$

60. If we expand Stan's answer we get $\frac{x^4}{2} + 2x^2 + 2 + C$. Since C is an arbitrary constant, so is 2+C which means that Stan's answer is of the form

$$\frac{x^4}{2} + 2x^2 + \text{ ``arbitrary constant''}$$

which is what Ollie has. Equivalently, the functions $\frac{x^4}{2} + 2x^2 + 2$ and $\frac{x^4}{2} + 2x^2$ differ by a constant (namely 2) and thus are both antiderivatives for the same function $2x(x^2 + 2)$.

62. (a) To recover $P(t)$ from its derivative $P'(t) = 500te^{-t^2/5}$ we need to integrate the function $te^{-t^2/5}$. Using $u = -t^2/5$, $du = -(2t/5)dt$ we have

$$P(t) = \int 500te^{-t^2/5}dt = -1250 \int e^u \, du = -1250e^u + C = -1250e^{-t^2/5} + C$$

Using the initial condition $P(0) = 2000$ gives

$$2000 = P(0) = -1250 + C \Rightarrow C = 3250$$

and therefore

$$P(t) = 3250 - 1250e^{-t^2/5}$$

(b) $P(3) = 3250 - 1250e^{-9/5} \approx 3043$.

Section 7.2

39. Let $u = x^2, du = 2xdx$; $\displaystyle\int x \sin x^2 \; dx = \frac{1}{2}\int \sin u \; du = -\frac{1}{2}\cos u + C = -\frac{1}{2}\cos x^2 + C$

42. Let $u = 8x, du = 8dx$; $\displaystyle -2\int \csc^2 8x \; dx = -\frac{1}{4}\int \csc^2 u \; du = -\frac{1}{4}(-\cot u) + C$
$\displaystyle = \frac{1}{4}\cot 8x + C$

43. Let $u = \sin x, du = \cos x dx$; $\displaystyle\int \sin^7 x \cos x \; dx = \int u^7 \; du = \frac{1}{8}u^8 + C = \frac{1}{8}\sin^8 x + C$

47. Let $u = 1 + \cos x, du = -\sin x dx$; $\displaystyle\int \frac{\sin x}{1+\cos x} \; dx = -\int \frac{du}{u} = -\ln|u| + C$
$= -\ln|1 + \cos x| + C = -\ln(1 + \cos x) + C$

55. Let $u = e^x, du = e^x dx$; $\int e^x \sin e^x \; dx = \int \sin u \; du = -\cos u + C = -\cos e^x + C$

58. Let $u = x^5, du = 5x^4 dx$; $\int x^4 \sec x^5 \tan x^5 \; dx = \frac{1}{5}\int \sec u \tan u \; du = \frac{1}{5}\sec u + C =$
$\frac{1}{5}\sec x^5 + C$

Section 7.3

4. (a) $\displaystyle\frac{1}{1/2}\cdot\frac{1}{2} + \frac{1}{1}\cdot\frac{1}{2} + \frac{1}{3/2}\cdot\frac{1}{2} + \frac{1}{2}\cdot\frac{1}{2} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = 2\frac{1}{12}$

(b) $\displaystyle\int_{1/2}^{5/2} \frac{1}{x} \; dx$

8. (a) $1^2 \cdot 1 + 2^2 \cdot 1 + 3^2 \cdot 1 + 4^2 \cdot 1 = 30$

(b) $2^2 \cdot 1 + 3^2 \cdot 1 + 4^2 \cdot 1 + 5^2 \cdot 1 = 54$

(c) $\displaystyle\frac{30 + 54}{2} = 42$

(d) $(3/2)^2 \cdot 1 + (5/2)^2 \cdot 1 + (7/2)^2 \cdot 1 + (9/2)^2 \cdot 1 = 41$

15. (a) $\displaystyle\frac{1}{4}\cdot 1 + \frac{3}{4}\cdot 1 + \frac{5}{4}\cdot 1 + \frac{7}{4}\cdot 1 = 4$

(b) Since the graph of $f(x) = x/2$ is on or above the x–axis over the interval $0 \le x \le 4$, the value of the integral $\displaystyle\int_0^4 f(x) \; dx$ is equal to the area below the graph of f and above the x-axis. This region is a triangle of base 4 and height 2, so its area is 4. Hence, $\displaystyle\int_0^4 f(x) \; dx = 4$.

(This agrees with the “approximation” in part (a).)

17. (a) The integral in question is equal to the area of a triangle with base 4 and height 2, namely 4.

(b) The integral in question is equal to the sum of the areas of two triangles, the first with base and height both 3 and thus area $9/2$, and the second with base and height both 1 and thus area $1/2$. Hence, the total area is $9/2 + 1/2 = 5$.

31. With left endpoints we get

$$3650 \cdot 2 + 3701 \cdot 2 + 3793 \cdot 2 + 3113 \cdot 2 = 28514$$

With right endpoints we get

$$3701 \cdot 2 + 3793 \cdot 2 + 3113 \cdot 2 + 2520 \cdot 2 = 26254$$

The average of these two estimates is

$$\frac{28514 + 26254}{2} = 27384$$

39. With left endpoints we get

$$0 \cdot 1 + 8 \cdot 1 + 13 \cdot 1 + 17 \cdot 1 = 38 \text{ (ft)}$$

With right endpoints we get

$$8 \cdot 1 + 13 \cdot 1 + 17 \cdot 1 + 18 \cdot 1 = 56 \text{ (ft)}$$

The average of these two estimates is

$$\frac{38 + 56}{2} = 47 \text{ (ft)}$$

Section 7.4

3. $\displaystyle\int_{-1}^{2}(5t-3)\ dt = (5t^2/2-3t)\Big|_{-1}^{2} = (10-6)-(5/2+3) = -\frac{3}{2}$

7. Letting $w = 4u+1, dw = 4du$;

$$\int_0^2 3\sqrt{4u+1}\ du = \frac{3}{4}\int_1^9 w^{1/2}\ dw = \frac{1}{2}w^{3/2}\Big|_1^9 = \frac{1}{2}(27-1) = 13$$

14. Letting $u = 2p+1, du = 2dp$; $\displaystyle\int_1^4 \frac{-3}{(2p+1)^2}\ dp = -\frac{3}{2}\int_3^9 u^{-2}\ du = \frac{3}{2u}\Big|_3^9 = \frac{1}{6}-\frac{1}{2} = -\frac{1}{3}$

20. $\displaystyle\int_{0.5}^{1}(p^3-e^{4p})\ dp = (p^4/4-e^{4p}/4)\Big|_{0.5}^{1} = (1/4-e^4/4)-(1/64-e^2/4)\approx -11.57$

21. Letting $u = 2y^2-3, du = 4ydy$;

$$\int_{-1}^{0} y(2y^2-3)^5\ dy = \frac{1}{4}\int_{-1}^{-3} u^5\ du = \frac{1}{24}u^6\Big|_{-1}^{-3} = \frac{(-3)^6}{24}-\frac{(-1)^6}{24} = \frac{91}{3}$$

26. Letting $u = \ln x, du = \dfrac{1}{x}dx$;

$$\int_1^3 \frac{(\ln x)^{1/2}}{x}\ dx = \int_0^{\ln 3} u^{1/2}\ du = \frac{2}{3}u^{3/2}\Big|_0^{\ln 3} = \frac{2}{3}(\ln 3)^{3/2}\approx 0.7677$$

30. Letting $u = 1+e^{2z}, du = 2e^{2z}dz$ we have

$$\int_0^1 \frac{e^{2z}}{\sqrt{1+e^{2z}}}\ dz \;=\; \frac{1}{2}\int_2^{1+e^2} u^{-1/2}\ du = u^{1/2}\Big|_2^{1+e^2} = \sqrt{1+e^2}-\sqrt{2}\approx 1.482$$

31. $\displaystyle -\cos x\Big|_0^{\pi/4} = 1-\frac{\sqrt{2}}{2}$

63. The total change in the pollution concentration over the 4–year period $0\le t\le 4$ is

$$\begin{aligned} P(4)-P(0) &= \int_0^4 P'(t)\ dt = \int_0^4 140t^{5/2}\ dt = 140\int_0^4 t^{5/2}\ dt \\ &= 140\left(\frac{2}{7}t^{7/2}\Big|_0^4\right) = 140\cdot\left(\frac{2^8}{7}-0\right) = 5120 > 4850 \end{aligned}$$

so the answer is “no”.

64. Letting $u = \ln(t+1), du = \dfrac{dt}{t+1}$ we have

(a) $\displaystyle\int_0^{24} \frac{80\ln(t+1)}{t+1}\ dt = 80\int_0^{\ln 25} u\ du = 40u^2\Big|_0^{\ln 25} = 40(\ln 25)^2 - 0 = 40(\ln 25)^2 \approx 414$ barrels

(b) $\displaystyle\int_{24}^{48} \frac{80\ln(t+1)}{t+1}\ dt = 80\int_{\ln 25}^{\ln 49} u\ du = 40u^2\Big|_{\ln 25}^{\ln 49} = 40(\ln 49)^2 - 40(\ln 25)^2 \approx 191$ barrels