GUIDELINES FOR WRITING MATHEMATICAL PROOFS

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The writing of mathematical proof is an art form that can never be mastered, only improved upon during one's lifetime. Of course, a part of writing is producing high-quality documents for the reader; for mathematical writing this is almost exclusively done using LaTeX. An introduction to mathematical writing is presented in [1][§1.2]. Below is a summary of the writing guidelines that we find most useful.

1. **Know your audience.** Every writer should have a clear idea of the intended audience for a piece of writing. In that way, the writer can give the right amount of information at the proper level of sophistication to communicate effectively. This is especially true for mathematical writing. For example, if a mathematician is writing a solution to a textbook problem for a solutions manual for instructors, the writing would be brief with many details omitted. However, if the writing was for a students solution manual, more details would be included.

2. **Begin with a carefully worded statement of the result to be proven.** The statement should be a simple declarative statement of the problem. Do not simply rewrite the problem as stated in the textbook or given on a handout. Problems often begin with phrases such as Show that or Prove that. This should be reworded as a simple declarative statement within the theorem environment, then state your proof within the proof environment. These environments are available within specific LaTeX packages. In the example below, we use the document class "amsart", along with packages "amsmath" and "amsthm". Then `\newtheorem` is used to create the theorem environment (do a Google search to learn more about this), and we define the title and author of the document and then use `\maketitle`.

```
\documentclass{amsart}
\usepackage{amsmath}
\usepackage{amsthm}

\newtheorem{theorem}{Theorem}

\title{My Title}
\author{My Name}
\begin{document}
\maketitle

\end{document}
```

3. **Use the pronoun "we".** If a pronoun is used in a proof, the usual convention is to use we instead of I. The idea is to stress that you and the reader are doing the mathematics together. It will help encourage the reader to continue working through the mathematics.

4. **The Math Mode.** In LaTeX, any characters bound between two \$ symbols are considered to be in math mode. Math mode is important because it italicizes variables and displays mathematical equations in a clear, easy to read, fashion. Consider the LaTeX example below.

```
1 The roots, $r_{1}$ and $r_{2}$, of the quadratic polynomial $ax^{2}+bx+c$ are
2 \[
3 r_{1},~r_{2}=\frac{-b\pm\sqrt{b^{2}-4ac}}{2a}
4 \]
```

The roots, r_1 and r_2, of the quadratic polynomial $ax^2 + bx + c$ are

$$r_1, \ r_2 = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

```
 1 \begin{theorem}
 2 The sum of any two even integers is even.
 3 \end{theorem}
 4 \begin{proof}
 5 Let $n_{1}$ and $n_{2}$ be any two even integers, then there exists two integers $k
       _{1}$ and $k_{2}$ such that
 6 \[
 7 n_{1}=2k_{1}~\text{ and }~n_{2}=2k_{2}.
 8 \]
 9 Therefore, $n_{1}+n_{2}=2\left(k_{1}+k_{2}\right)$, and it follows that the sum is
       even.
10 \end{proof}
```

Theorem 1. *The sum of any two even integers is even.*

Proof. Let n_1 and n_2 be any two even integers, then there exists two integers k_1 and k_2 such that

$$n_1 = 2k_1 \ \text{ and } \ n_2 = 2k_2.$$

Therefore, $n_1 + n_2 = 2(k_1 + k_2)$, and it follows that the sum is even. □

References

1. E.R. Scheinerman, *Mathematics A Discrete Introduction*, 3rd ed., Brooks/Cole, 2013.