MAT – 112: Calculus I and Modeling

Solution 2

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Other Problems

Problem 1. Let (x, y) be a point in the plane other than the origin. Let r be the distance of the radii from the origin to (x, y) and let θ be the angle measured counterclockwise from the positive x-axis to the radii. Then, we define

$$\cos\theta = \frac{x}{r}, \qquad \sin\theta = \frac{y}{r}, \qquad \tan\theta = \frac{y}{x}.$$

The reciprocal trigonometric functions are defined as follows:

$$\begin{aligned} \sec\theta &= \frac{r}{x} = \frac{1}{r/x} = \frac{1}{\cos\theta} \qquad (y \neq 0), \\ \csc\theta &= \frac{r}{y} = \frac{1}{r/y} = \frac{1}{\sin\theta} \qquad (x \neq 0), \\ \cot\theta &= \frac{x}{y} = \frac{1}{y/x} = \frac{1}{\tan\theta} \qquad (y \neq 0). \end{aligned}$$

Note further, since $\tan\theta = \frac{y}{x}$ and $\cot\theta = \frac{x}{y}$, we can substitute $x = r\cos\theta$ and $y = r\sin\theta$ to get

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \quad \text{and} \quad \cot\theta = \frac{\cos\theta}{\sin\theta}.$$

Lastly, it follows from Pythagorean's theorem that $r = \sqrt{x^2 + y^2}$. Therefore,

$$\begin{aligned} \sin^2\theta + \cos^2\theta &= \left(\frac{y}{r}\right)^2 + \left(\frac{x}{r}\right)^2 \\ &= \frac{y^2 + x^2}{r^2} \\ &= \frac{y^2 + x^2}{y^2 + x^2} = 1. \end{aligned}$$