MAT – 450: Advanced Linear Algebra

Homework 5

Instructor: Thomas R. Cameron

Due: 4/6/2018

Instructions

There has been some discussion in class regarding the direct sum, and the null space and range of a linear operator. This small homework assignment is intended to clarify our discussion and give you a stronger understanding of the details.

Problem 1. Let V be a finite-dimensional inner product space and $T \in \mathcal{L}(V)$.

a. Suppose that $T = T^*$ and show that $V = R(T) \oplus N(T)$.

b. Suppose that $R(T)$ and $N(T)$ are non-trivial sets. If $V = R(T) \oplus N(T)$ is it necessary that $T = T^*$. If not, then provide a counterexample.

c. Provide an example where $V = \mathbb{R}^2$ is the direct sum of $R(T)$ and $N(T)$, but $N(T) \neq R(T)^{\perp}$. As a connection to part b., note that in this case it is not possible for $T = T^*$.

d. Prove that there exists a $k \in \mathbb{N}$ such that $V = R(T^k) \oplus N(T^k)$.