

CSC/MAT-220: Discrete Structures

EFY 10

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One-to-One and Onto Let A be any non-empty set and let S be a non-empty subset of A . Define the function $f: A \rightarrow \{0, 1\}$ by $f(x) = 1$ if $x \in S$ and $f(x) = 0$ if $x \notin S$. Under what conditions is

- i. f one-to-one?
- ii. f onto?
- iii. f bijective?

Solution.

- i. The function f is one-to-one, when the sets S and $A - S$ have cardinality at most 1.
- ii. The function f is onto, when S is a proper subset of A .
- iii. The function f is bijective when (i.) and (ii.) hold, which implies that $|S| = 1$ and $|A - S| = 1$.

Function images of sets Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined by $f(x) = x^2$. Find $f^{-1}(T)$ for each of the following

- i. $T = \{9\}$
- ii. $T = [4, 9]$
- iii. $T = [-4, 9]$

Solution. The pre-image of a set $D \subseteq \mathbb{R}$ is defined by

$$f^{-1}(D) = \{x \in A: f(x) \in D\}.$$

Therefore, the answers are as follows:

- i. $f^{-1}(T) = \{-3, 3\}$,
- ii. $f^{-1}(T) = [-3, 2] \cup [2, 3]$,
- iii. $f^{-1}(T) = [-3, 3]$.