Section 2.2

3. $\log_3 81 = 4$

5. $\log_3 \frac{1}{9} = -2$

11. $10^5 = 100{,}000$

12. $10^{-3} = 0.001$

16. We seek $r = \log_3 27$. In exponential form this equation becomes $3^r = 27$. Since $27 = 3^3$ we seek r such that $3^r = 3^3$ and it follows that $r = 3$. Therefore, $\log_3 27 = 3$.

19. We seek $r = \log_2 \sqrt[3]{\dfrac{1}{4}}$. In exponential form this equation becomes $2^r = \sqrt[3]{\frac{1}{4}}$. Since $2^{-2} = \dfrac{1}{4}$ we have $\sqrt[3]{\dfrac{1}{4}} = \left(\dfrac{1}{4}\right)^{1/3} = (2^{-2})^{1/3} = 2^{-2/3}$ and our equation becomes $2^r = 2^{-2/3}$ from which it follows that $r = -2/3$. Therefore, $\log_2 \sqrt[3]{\dfrac{1}{4}} = -2/3$.

34. $\log_b 18 = \log_b (2 \cdot 3^2) = \log_b 2 + \log_b 3^2 = \log_b 2 + 2\log_b 3 = a + 2c$

36. $\log_b (9b^2) = \log_b (3^2 b^2) = 2\log_b 3 + 2\log_b b = 2c + 2$

41. $\log_x 36 = -2 \Rightarrow x^{-2} = \dfrac{1}{x^2} = 36 \Rightarrow x^2 = \dfrac{1}{36} \Rightarrow x = \pm\dfrac{1}{6} \Rightarrow x = \dfrac{1}{6}$ (The last equality is due to the fact that the base of a logarithm must be positive.)

42. The equation $\log_9 27 = m$ has exponential form $9^m = 27$. Since $9^m = (3^2)^m = 3^{2m}$ and $27 = 3^3$ our equation becomes $3^{2m} = 3^3$ from which it follows that $2m = 3$ and $m = 3/2$.

43. The equation $\log_8 16 = z$ has exponential form $8^z = 16$. Since $8^z = (2^3)^z = 2^{3z}$ and $16 = 2^4$ our equation becomes $2^{3z} = 2^4$ from which it follows that $3z = 4$ and $z = 4/3$.

44. $\log_y 8 = \dfrac{3}{4} \Rightarrow y^{3/4} = 8 \Rightarrow y = (y^{3/4})^{4/3} = 8^{4/3} = 16$