MAT-150: Linear Algebra
Homework 3

Due: 9/29/2017

Book Problems

Please turn in your solution for each of the following exercises.

§2.8: 31-36

§2.9: 19-24

Other Problems

Problem 1

Let A be an $m \times n$ matrix. Prove that the column space of A is a subspace of $\mathbb{R}^m$ and the null space of A is a subspace of $\mathbb{R}^n$.

For Problems 2 - 4, you must use the definition of the determinant given in class.

Problem 2

Let $T: \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation. Prove that T is invertible if and only if $\det(T) \neq 0$.

Problem 3

Let $T: \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation. Prove that if T is invertible, then

$$\det(T^{-1}) = \frac{1}{\det(T)}.$$

Problem 4

Let $T,\ U: \mathbb{R}^n \to \mathbb{R}^n$ be linear transformations. Prove that the composition, $TU(x) = T\left(U(x)\right)$ for all $x \in \mathbb{R}^n$, satisfies

$$\det(TU) = \det(T)\det(U)$$