CSC/MAT-220 **Name:** _____
Fall 2017
Midterm
Handout: 10/6, Due: 10/16 **Pledge:** _____

Each question topic and point value is recorded in the tables below. You may review these topics from any resource at your leisure. Once you decide to start an exam problem, you are on the clock and you must work without any external resources. Each problem can be done one at a time, but must be finished in a single sitting. Answer each question in the space provided, if you run out of room, then you may continue on the back of the page. It is your responsibility to plan out your time to ensure that you can finish all problems within the 7.0 hours allotted. All SML problems should be put in a single file titled *name_midterm.sml*. Use comments so that I may easily see where each problem begins and ends, and drop this file in your dropbox folder by the due date. By writing your name and signing the pledge you are stating that your work adheres to these terms and the Davidson honor code.

Scoring Table

Question	Points	Score
1	10	
2	15	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	15	
Total:	100	

Topics Table

Question	Topic
1	Logical Connectives and Quantifiers
2	Sets
3	Relations
4	Binomial Coefficients
5	Proof by Contradiction
6	Proof by Induction
7	Logic Puzzle
8	SML Concepts: Bindings, Aggregate Data Types, and Functions.
9	SML Programming Clausal Function Expressions, Let Expressions, Binary and Ascii.

Start Time: **End Time:**

1. Let x and y be boolean variables.
   (a) (6 points) Create a truth tables for the following expressions.
      i. $x \implies y$

      ii. $\neg x \vee y$

      iii. $x \wedge \neg y$

   (b) (4 points) A function $f\colon A \to B$ is *strictly decreasing* if and only if for every $x, y \in A$, if $x < y$, then $f(x) > f(y)$.
      i. Write the above statement using logical connectives and quantifiers.

      ii. Write the negation of the above statement using logical connectives and quantifiers.

Start Time: **End Time:**

2. Let A and B be sets.

   (a) (10 points) State the definition of each of the following.

      i. $A \subseteq B$.

      ii. $A = B$.

      iii. $A \cup B$.

      iv. $A \cap B$.

      v. $A - B$.

   (b) (5 points) Prove: $A - (B \cup C) = (A - B) \cap (A - C)$

Start Time: **End Time:**

3. Let A and B be sets.
   (a) (4 points) State the definition of each of the following.
      i. A relation R between A and B.

      ii. An equivalence relation R on A.

   (b) (6 points) Determine which of the following are equivalence relations.
      i. Let A be the set of all lines in a plane and let R be the relation "is perpendicular to."

      ii. Let A be the set of real numbers and let R be the relation "$>$".

      iii. Let A be the set of all triangles in a plane and let R be the relation "is similar to."

Start Time: **End Time:**

4. Let $n, k \in \mathbb{N}$.

   (a) (2 points) State the definition of the binomial coefficient $\binom{n}{k}$.

   (b) (8 points) Give a combinatorial proof for each of the following identities.

   i. $\binom{n}{2} = 1 + 2 + \cdots + (n-1) = \sum_{k=1}^{n-1} k$.

   ii. $\sum_{k=0}^{n} \binom{n}{k} = 2^n$.

Start Time: **End Time:**

5. Prove the following statements using proof by contradiction.

   (a) (5 points) $\log_2(7)$ is irrational.

   (b) (5 points) $7^n - 4^n$ is divisible by 3 for all $n \in \mathbb{N}$.

Start Time: **End Time:**

6. Prove the following statements using induction.

   (a) (5 points) For every natural number $n \geq 1$

$$1 + 2 + \cdots + n = \frac{1}{2}n(n+1).$$

   (b) (5 points) For every natural number $n \geq 1$

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n \cdot (n+1)} = \frac{n}{n+1}.$$

Start Time: **End Time:**

7. This logic puzzle is derived from a famous story entitled "The Lady or the Tiger?". Each of the following signs are placed on a door, behind each door is either a lady *or* a tiger. Suppose you want to choose the room with the lady, use logical reasoning to determine which door you should open.

   (a) (3 points) One of the signs is true, but the other one is false.
   Sign 1: "In this room there is a lady, and in the other room there is a tiger."
   Sign 2: "In one of these rooms there is a lady, and in one of these rooms there is a tiger."

   (b) (3 points) Either both signs are true or both are false.
   Sign 1: "At least one of these rooms contains a lady."
   Sign 2: "A tiger is in the other room."

   (c) (4 points) This time there are three rooms and three signs. One of the three rooms contains a lady, another a tiger, and the third room is empty. The sign on the door of the room containing the lady is true, the sign on the door of the tiger is false, and the sign on the door of the empty room can be either true or false.
   Sign 1: "Room 3 is empty."
   Sign 2: "The tiger is in room 1."
   Sign 3: "This room is empty."
   Determine which room is empty, which room contains the lady, and which room contains the tiger.

Start Time: **End Time:**

8. Answer the following SML concept questions.

   (a) (2 points) Describe the following expression in terms of bindings.

   val logb = fn (x:real,b:real) => Math.ln(x)/Math.ln(b)

   Re-write this expression using the *fun* syntax.

   (b) (4 points) Consider the value bindings below.

   *val x:real*real = (1.0,2.0) and y:int*int = (1,2)*

   i. What is wrong with this expression?

   if x=y then "true" else "false"

   ii. What does this expression return?

   let val (y1:int,y2:int) = y in y = (y2,y1) end

   (c) (4 points) Write pseudocode for creating a record type called *address* that has labels for *state*, *city*, *zipcode*, *street_name*, and *street_number*.

Start Time: **End Time:**

9. In the space provided, include your scratch work to highlight your thought process when solving each problem. Then write the code in your *sml file* (this is what you will be graded on).

   (a) (3 points) Use a clausal function expression to define a function *impl* that returns the logical implication $x \implies y$, where x and y are the two boolean arguments.

   (b) (6 points) Write a function *bin_to_char* that returns the ascii character associated with the binary number argument. Use the following hints and guidelines.

   - Store a binary number as an 8-tuple, where each element has type int.
   - Make use of a clausal function expression and a let expression.
   - Make use of the member function *chr* of the SML Char Signature.
   - Test this on the binary number 01100001, you should get the character *a*.

   (c) (6 points) Write a function *char_to_bin* that returns the binary number associated with the ascii character argument. Use the following hints and guidelines.

   - Store a binary number as an 8-tuple, where each element has type int.
   - Make use of a let expression.
   - Make use of the integer operations mod and int.
   - Make use of the member function *ord* of the SML Char Signature.
   - Test this on the character *a*, you should get the binary number 01100001.