MAT – 112: Calculus I and Modeling

EFY 3

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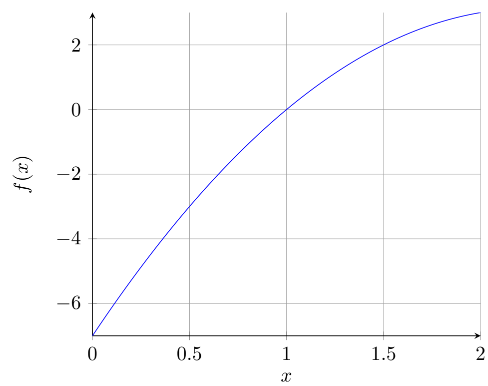
Instructions

Please complete each of the following problems. You should work in groups of three, or at most four, and hand in only one submission per group. Be sure that your arguments are well justified and presented clearly.

Problem 1. Use the plot below to compute the average rate of change of f over the intervals $[1, 2]$, $[1, 1.5]$, and $[1, 1.25]$.



Use these average rate of change values to estimate the instantaneous rate of change of f at the point $x = 1$.

Solution: The average rate of change (ARC) over the three intervals is given below.

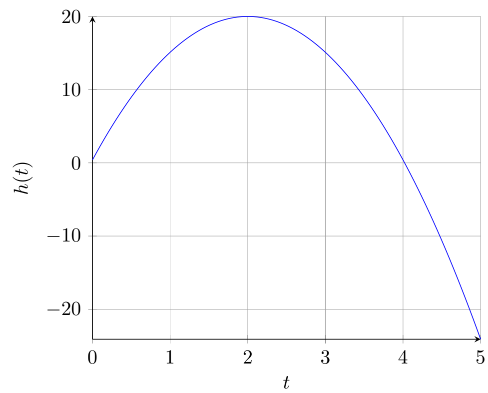
- Over $[1, 2]$: $ARC = \frac{3-0}{2-1} = 3$.
- Over $[1, 1.5]$: $ARC = \frac{2-0}{1.5-1} = 4$.
- Over $[1, 1.25]$: $ARC = \frac{1-0}{1.25-1} = 4$.

It appears that the limit is approaching 4. This is estimate is off by 1, the instantaneous rate of change of f at 1 is equal to 5, but the best we can do given the data we are given.

Problem 2. A ball is thrown in the air with initial velocity 19.6 m/s and at an initial height of 0.4 m. The height of the ball as a function of time is given by the equation

$$h(t) = -4.9t^2 + 19.6t + 0.4$$

and is plotted below.



(i.) Use the problem description and intuition to find the instantaneous velocity at $t = 0$ and $t = 2$. Provide explanation.

(ii.) Find the time when the ball hits the ground.

(iii.) Find the instantaneous velocity of the ball when it hits the ground.

Solution:

(i.) At $t = 0$, the velocity must be equal to the initial velocity of 19.6 m/s. At $t = 2$, the ball has reached its peak and the instantaneous velocity is 0 m/s.

(ii.) The ball hits the ground when $h(t) = 0$. We can find the possible values using the quadratic formula

$$t = \frac{-19.6 \pm \sqrt{(19.6)^2 - 4(-4.9)(0.4)}}{2(-4.9)} = -0.02,\ 4.02.$$

We are only interested in the positive value. Thus, the ball hits the ground after 4.02 s.

(iii.) To find the instantaneous velocity of $h(t)$ at the point a, we must find the following limit

$$\lim_{x \to 0} \frac{h(a+x) - h(a)}{x}.$$

Since, we cannot simply plug-in zero, we must simplify this limit. Note that

$$\begin{aligned} h(a+x) - h(a) &= [-4.9(a+x)^2 + 19.6(a+x) + 0.4] - [-4.9a^2 + 19.6a + 0.4] \\ &= [-4.9(a^2 + 2ax + x^2) + 19.6(a+x)] - [-4.9a^2 + 19.6a] \\ &= -4.9(2ax + x^2) + 19.6x. \end{aligned}$$

Therefore, the instantaneous velocity of $h(t)$ at the point a is given by

$$\begin{aligned} \lim_{x \to 0} \frac{-4.9(2ax + x^2) + 19.6x}{x} &= \lim_{x \to 0} -4.9(2a + x) + 19.6 \\ &= -4.9(2a) + 19.6. \end{aligned}$$

Plugging in $a = 4.02$, we find that the instantaneous velocity of $h(t)$ at 4.02 is equal to $-19.796\ m/s$. Note that this is only slight faster than the ball's initial speed, since the ball started at a mere height of 0.4 m.