

MAT – 112: Calculus I and Modeling

Solution 3

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Other Problems

Problem 1. Here we use the limit definition of the derivative to derive formula for computing the derivative of x^2 , \sqrt{x} , and $1/x$.

- Let $f(x) = x^2$ and note that

$$\begin{aligned}\frac{d}{dx}f(x) &= \lim_{h \rightarrow 0} \frac{(x+h)^2 - x^2}{h} \\ &= \lim_{h \rightarrow 0} \frac{[x^2 + 2xh + h^2] - [x^2]}{h} \\ &= \lim_{h \rightarrow 0} \frac{2xh + h^2}{h} \\ &= \lim_{h \rightarrow 0} (2x + h) = 2x.\end{aligned}$$

- Let $f(x) = \sqrt{x}$ and note that

$$\begin{aligned}\frac{d}{dx}f(x) &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \\ &= \lim_{h \rightarrow 0} \frac{\sqrt{x+h} - \sqrt{x}}{h} \left(\frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}} \right) \\ &= \lim_{h \rightarrow 0} \frac{(x+h) - (x)}{h(\sqrt{x+h} + \sqrt{x})} \\ &= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \frac{1}{2\sqrt{x}}.\end{aligned}$$

- Let $f(x) = 1/x$ and note that

$$\begin{aligned}\frac{d}{dx}f(x) &= \lim_{h \rightarrow 0} \frac{\left(\frac{1}{x+h}\right) - \left(\frac{1}{x}\right)}{h} \\ &= \lim_{h \rightarrow 0} \frac{x - (x+h)}{hx(x+h)} \\ &= \lim_{h \rightarrow 0} \frac{-1}{x(x+h)} = -\frac{1}{x^2}.\end{aligned}$$