CSC/MAT-220 **Name:** ______________________

Fall 2017

Final

Handout: 12/6, Due: 12/14 **Pledge:** ______________________

Each question topic and point value is recorded in the tables below. You may review these topics from any resource at your leisure. Once you decide to start an exam problem, you are on the clock and you must work without any external resources. Each problem can be done one at a time, but must be finished in a single sitting. Answer each question in the space provided, if you run out of room, then you may continue on the back of the page. It is your responsibility to plan out your time to ensure that you can finish all problems within the 7.0 hours allotted. All SML problems should be put in a single file titled *name_final.sml*. Use comments so that I may easily see where each problem begins and ends, and drop this file in your dropbox folder by the due date. By writing your name and signing the pledge you are stating that your work adheres to these terms and the Davidson honor code.

Scoring Table

Question	Points	Score
1	10	
2	10	
3	10	
4	10	
Total:	40	

Topics Table

Question	Topic
1	Recurrence Relations
2	Functions
3	Cardinality and Infinity
4	Permutations
5	Probability
6	Graphs
7	Trees
8	SML Concepts
9	SML Programming

Time Table

Question	1	2	3	4	5	6	7	8	9
Time									

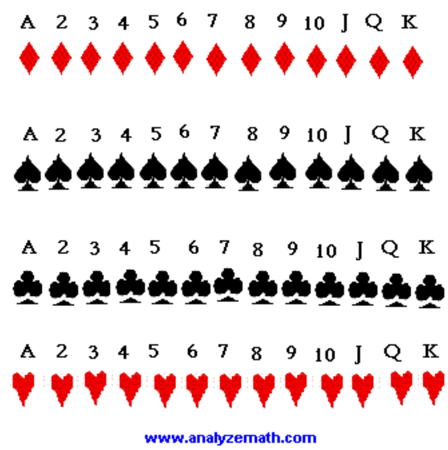
1. (a) (2 points) State the definition of a permutation and the symmetric group on n elements (denoted S_n).

   (b) (4 points) Prove that the cardinality of S_n is $n!$.

   (c) (4 points) Consider the permutation $\pi = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 3 & 1 & 2 & 4 & 5 & 7 & 6 \end{bmatrix}$. Draw and graph of π and then write π as a collection of pairwise disjoint cycles and as a composition of transpositions. Find π^{-1}.

2. (a) (4 points) State the definition of a sample space, event, random variable, and expectation value.

(b) (6 points) Consider the sample space determined by randomly drawing two cards (one at a time without replacement) from a standard deck of 52, see figure to the right. Please answer each of the following.

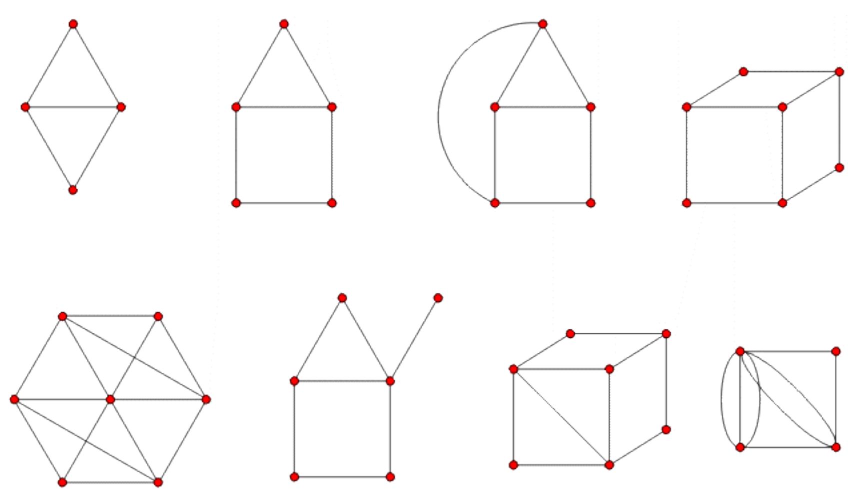


- How many outcomes are in this sample space?
- Let X be a random variable defined on the sample space, where X is 2 if both cards have the same rank, 1 if both cards have the same suit, and 0 otherwise. What is $P(X = 2)$, $P(X = 1)$, and $P(X = 0)$?
- What is the expectation value $E(X)$?

3. (a) (2 points) State the definition of a Simple Graph and a Multi Graph.

   (b) (3 points) Draw Euler's Graph representation of the Seven Bridges of Königsberg. Explain why this graph does not have an Euler walk.

   (c) (5 points) Put Yes/No next to each graph below, denoting whether or not it has an Euler walk.



4. (a) (3 points) State the definition of a tree, forest, and leaf.

   (b) (3 points) Show that a tree can also be defined recursively. State the base case and recursive step.

   (c) (4 points) Use proof by mathematical induction to show that a tree on $n \geq 2$ vertices has $n \geq 2$ leaves.