

# MAT-150: Linear Algebra

EFY 9

Due: November 10, 2017

## Problem 1.

- a. Let  $W$  be a subspace of  $\mathbb{R}^n$ . State the definition of the orthogonal complement of  $W$ .
- b. Let

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}.$$

Show, by example, that Theorem 3 holds for the matrix  $A$ .

**Problem 2.** The vectors  $\{u_1, u_2, \dots, u_n\}$  in  $\mathbb{R}^n$  are said to be orthonormal if

$$\langle u_i, u_j \rangle = \begin{cases} 0, & \text{if } i \neq j \\ 1, & \text{if } i = j \end{cases}.$$

Let  $U = [u_1, u_2, \dots, u_n]$  be a matrix with orthonormal columns. Prove that  $U^T U = I$ , where  $I$  is the  $n \times n$  identity. Then justify the following statements.

- $U^T$  is the inverse of  $U$ .
- The fact that the matrix  $Q$  in the QR Algorithm (and Francis's Algorithm) is orthonormal makes the algorithm far more efficient.