CSC/MAT-220: Discrete Structures

Homework 5

Due: 11/10/2017

Book Problems

Please do each of the following problems from your book:

23.2 (a, f, i), 24.19 (you may skip part c, since it is EFY 12), and 25.17.

Other Problems

Problem 1

Show that the following pairs of sets, S and T, have the same cardinality by finding a specific bijection between them.

(i.) $S = [0, 1]$ and $T = [0, 2]$,

(ii.) $S = \{(x, y) \colon\ x = \cos(2\theta - \pi/2), y = 1 + \sin(2\theta - \pi/2),\ \theta \in (-\pi/2, \pi/2)\}$ and $T = \mathbb{R}$.
Hint: Start by drawing a picture of the sets S and T, in order to gain some intuition for the problem.

Problem 2

A real number is said to be algebraic if it is a root of a polynomial equation

$$a_n x^n + \cdots + a_1 x + a_0 = 0$$

with integer coefficients. Note that the *algebraic* numbers include the rationals and all roots of rationals (such as $\sqrt{2}$, $\sqrt{5}$, etc.). If a number is not algebraic, then it is *transcendental*.

(i.) Show that the set of polynomials with integer coefficients is countable.

(ii.) Show that the set of algebraic numbers is countable.

(iii.) Are there more algebraic numbers or transcendental numbers?