



Visual Computing Institute  
Computer Vision

# Unsupervised Learning of Depth and Ego-Motion from Monocular Video Using 3D Geometric Constraints

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August 02, 2018

# Roadmap

Monocular Visual Odometry and SLAM

Scene Model

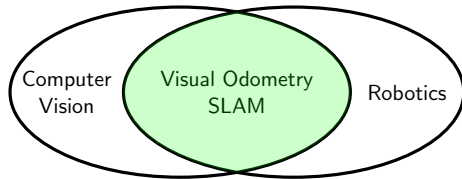
Related Work

Method

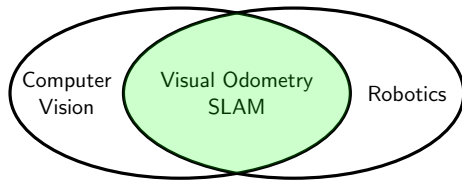
Evaluation

Conclusion and Future Work

# Monocular Visual Odometry and SLAM



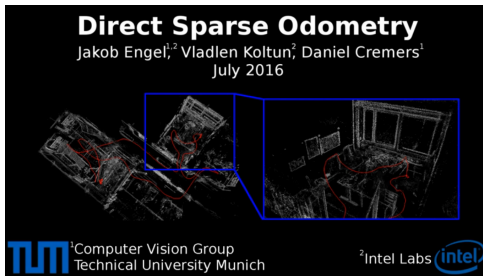
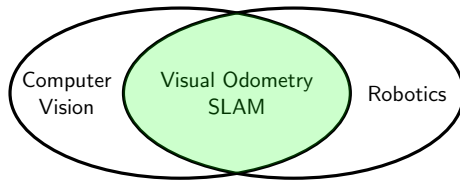
# Monocular Visual Odometry and SLAM



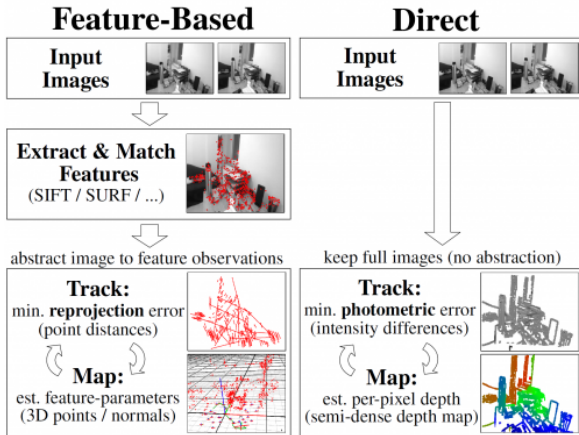
## Main applications

- ▶ Virtual and augmented reality
- ▶ Unknown surface exploration
- ▶ Autonomous navigation

# Monocular Visual Odometry and SLAM



# Analytic Visual Frameworks



[Engel et al., ECCV 2014]

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# Euclidean Transformations

Rotation  $R \in \mathbb{R}^{3 \times 3}$ , followed by a translation  $t \in \mathbb{R}^3$

$$T = \begin{pmatrix} R & t \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



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$$T_i^j = T_j^{-1} T_i$$

## Point Reprojection

$$\Pi : \mathbb{R}^4 \rightarrow \mathbb{R}^2, \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix} \mapsto \begin{pmatrix} x/z \\ y/z \end{pmatrix}$$

$$\Pi^{-1} : \mathbb{R}^2 \times \mathbb{R} \rightarrow \mathbb{R}^4, \left( \begin{pmatrix} u \\ v \end{pmatrix}, d_p \right) \mapsto \begin{pmatrix} u/d_p \\ v/d_p \\ 1/d_p \\ 1 \end{pmatrix}$$

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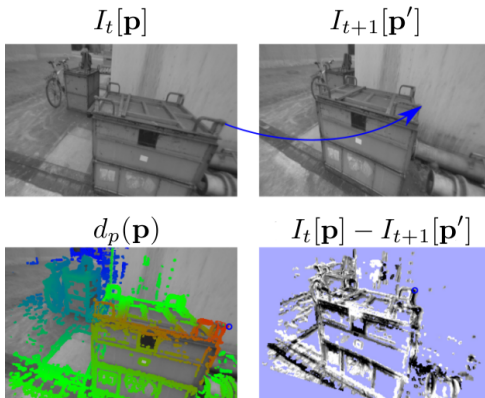
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# Point Reprojection



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$$E_{total} = \sum_{i \in F} \sum_{\mathbf{p}^* \in sp(i)} \|\mathbf{p}^* - \mathbf{p}\|_2^2$$

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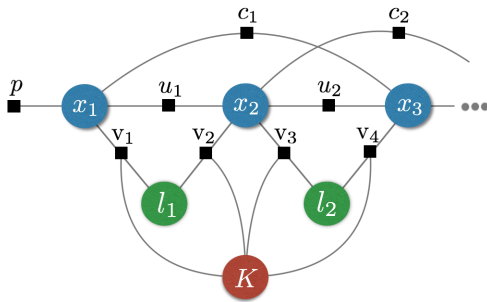
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- ▶ Maximum a Posteriori (MAP) estimation
- ▶ Non-linear non-convex least-squares optimization problem
- ▶ Good initialization required

## SLAM Represented as a Factor Graph



● robot poses ● landmarks  
● camera matrix ■ factors

[Cadena et al., IEEE T-RO 2016]

## Probabilistic Interpretation of SLAM

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Advantages in comparison with bundle adjustment:

- ▶ Simultaneous incorporation of various sensors
- ▶ Incremental solution possible

# Visual SLAM

Visual SLAM rendered possible by means of a simplified scene model:

- ▶ Rigid Lambertian world
- ▶ Temporal coherence and constant illumination
- ▶ Pinhole camera model and epipolar geometry  
⇒ 6 degrees of freedom for motion, 1 for depth



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Open problems

- ▶ Life-long operation
- ▶ High-level geometry understanding
- ▶ Resilience in a variety of environments

## Deep Learning for SfM

Paradigm shift from analytic to statistical solutions

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Success stories

- ▶ Monocular depth estimation  
(Garg et al., Godard et al., Kuznietsov et al.)
- ▶ Joint monocular depth and ego-motion estimation  
(Zhou et al., Vijayanarasimhan et al.)
- ▶ Rigid body detection and motion tracking  
(Byravan et al.)

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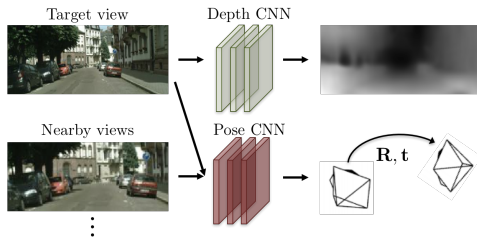
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# SfMLearner Overview



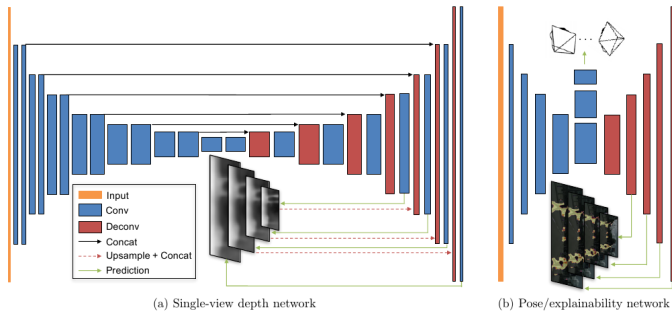
(a) Training: unlabeled video clips.



(b) Testing: single-view depth and multi-view pose estimation.

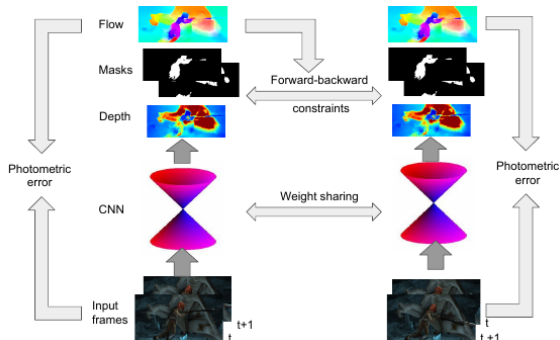
[Zhou et al., CVPR 2017]

# SfMLearner Network



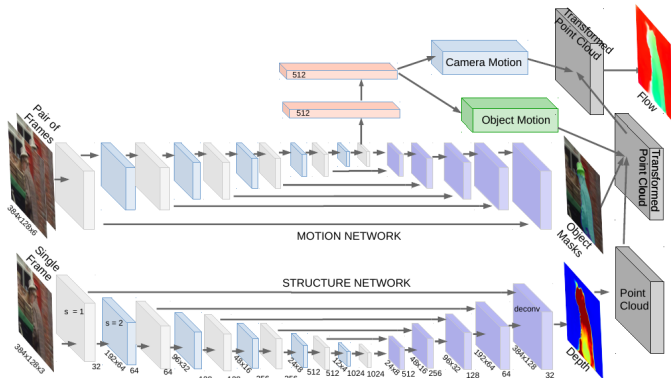
[Zhou et al., CVPR 2017]

# SfM-Net Motion Subnetwork



[Vijayanarasimhan et al., ArXiv 2017]

# SfM-Net Architecture



[Vijayanarasimhan, arXiv 2017]



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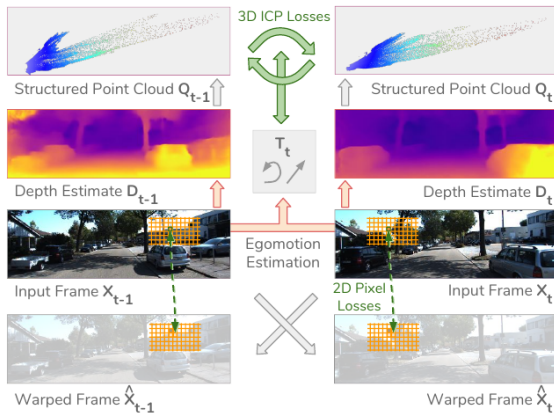
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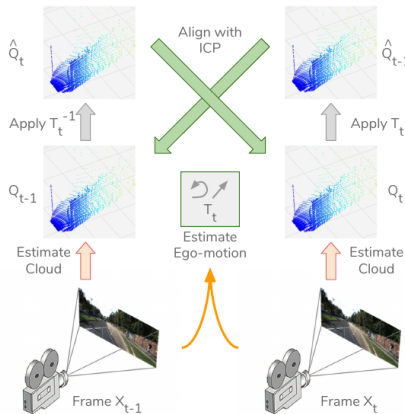
Conclusion and Future Work

# Method Overview



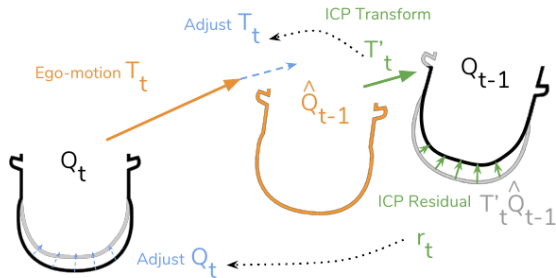
[Mahjourian et al., CVPR 2018]

# Three-Dimensional Point Cloud Alignment



[Mahjourian et al., CVPR 2018]

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[Mahjourian et al., CVPR 2018]

## Three-Dimensional Geometry Loss Term

Point cloud of frame  $i$  at time  $t$ :

$$Q_t^i = \{M_i[\mathbf{p}_t^i](K^{-1}\Pi^{-1}(\mathbf{p}_t^i, d_i(\mathbf{p}_t^i))) | \mathbf{p}_t^i \in P\}$$

Warped towards the next local coordinate frame  $i + 1$ :

$$Q_{t-1}^{i+1} = T_t Q_t^i$$

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Objective function of the Iterative Closest Point (ICP) algorithm used for point cloud alignment:

$$\operatorname{argmin}_{T'} \frac{1}{2} \|T' Q_{t-1}^{i+1} - Q_t^{i+1}\|_2^2$$

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Loss term:

$$L_{3D} = |T'_t - I| + |r_t|$$

## Differentiable Loss Function

Photometric consistency term:

$$L_{ph} = \sum_{\mathbf{p}_t^i \in P} \|M_i[\mathbf{p}_t^i](I_t[\mathbf{p}_t^i] - I_{t+1}[\mathbf{p}_{t+1}^i])\|_2$$



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Structured similarity term:

$$SSIM(x, y) = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{(\mu_x^2 + \mu_y^2 + c_1)(\sigma_x + \sigma_y + c_2)}$$

$$L_{SSIM} = \sum_{\mathbf{p}_t^i \in P} M_i[\mathbf{p}_t^i](1 - SSIM(I_t[\mathbf{p}_t^i], I_{t+1}[\mathbf{p}_{t+1}^i]))$$

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Depth gradient smoothness term:

$$L_{sm} = \sum_{\mathbf{p}_t^i \in P} \|\partial_x d_i(\mathbf{p}_t^i)\| e^{-\|\partial_x I_t[\mathbf{p}_t^i]\|} + \|\partial_y d_i(\mathbf{p}_t^i)\| e^{-\|\partial_y I_t[\mathbf{p}_t^i]\|}$$

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Weighted sum:

$$L = \sum_s \alpha L_{ph}^s + \beta L_{3D}^s + \gamma L_{sm}^s + \omega L_{SSIM}^s$$

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# Depth Estimation Evaluation

$$RMSE = \sqrt{\frac{1}{|P|} \sum_{\mathbf{p}_t^i \in P} \|d_i(\mathbf{p}_t^i) - d_i^{gt}(\mathbf{p}_t^i)\|_2^2}$$

## Depth Estimation Evaluation

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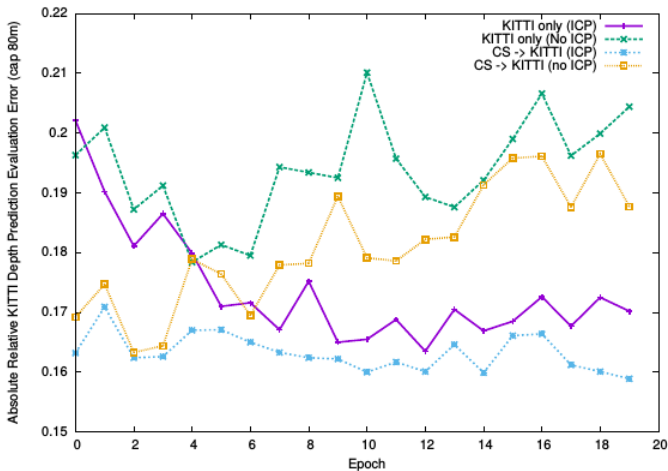
$$\Delta_i = \ln d_i(\mathbf{p}_t^i) - \ln d_i^{gt}(\mathbf{p}_t^i)$$

$$RMSE_{scale-invariant}^{log} = \sqrt{\frac{1}{|P|} \sum_{\mathbf{p}_t^i \in P} \Delta_i^2 - \frac{1}{|P|^2} \left( \sum_{\mathbf{p}_t^i \in P} \Delta_i \right)^2}$$

# Depth Estimation Evaluation

Method	Supervision	Dataset	Depth Cap	RMSE	$RMSE_{scale-invariant}^{log}$
All losses	–	Cityscapes + KITTI	0-80m	<b>5.912</b>	<b>0.243</b>
All losses	–	KITTI	0-80m	6.220	0.250
No ICP loss	–	KITTI	0-80m	6.267	0.252
Zhou et al.	–	Cityscapes + KITTI	0-80m	6.565	0.275
Zhou et al.	–	KITTI	0-80m	6.856	0.283
Eigen et al. Coarse	Depth	KITTI	0-80m	6.563	0.292
Eigen et al. Fine	Depth	KITTI	0-80m	6.307	0.282
All losses	–	Bike dataset	0-80m	7.741	0.309
No ICP loss	–	Bike dataset	0-80m	7.750	0.305
SfM-Net	–	Stereo KITTI 2012	0-80m	N/A	0.45
SfM-Net	–	Stereo KITTI 2015	0-80m	N/A	0.41
All losses	–	Cityscapes + KITTI	1-50m	<b>4.383</b>	<b>0.227</b>
All losses	–	KITTI	1-50m	4.549	0.231
Garg et al.	Stereo	KITTI	1-50m	5.104	0.273

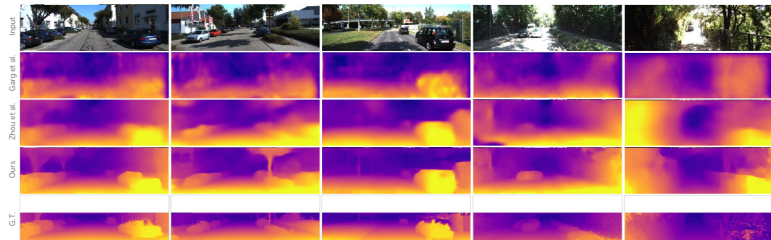
## 3D Loss Term Ablation and Pre-Training on Cityscapes



[Mahjourian et al., CVPR 2018]



# Qualitative Results



[Mahjourian et al., CVPR 2018]

## Ego-Motion Estimation Evaluation

$$ATE = \sqrt{\frac{1}{|T|} \sum_{P_i \in T} \|trans(Q_i^{-1} S P_i)\|_2^2}$$

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Method	Sequence 09	Sequence 10
Full ORB-SLAM	0.014 $\pm$ 0.008	<b>0.012 <math>\pm</math> 0.011</b>
Zhou et al.	0.021 $\pm$ 0.017	0.020 $\pm$ 0.015
No ICP loss	0.014 $\pm$ 0.010	0.013 $\pm$ 0.011
All losses	<b>0.013 <math>\pm</math> 0.010</b>	<b>0.012 <math>\pm</math> 0.011</b>

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- ▶ Main contribution: Novel differentiable three-dimensional geometry loss term
- ▶ Attained precision: equal to a full SLAM system
- ▶ Robustness: competitive results even after training on a highly irregular custom dataset and evaluation on an unrelated well-calibrated benchmark

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Future research directions:

- ▶ Dynamic object detection and tracking
- ▶ Optimization over an extended time lapse
- ▶ Scene model generalization: non-rigidity, specular reflections
- ▶ Learning and evaluation on a richer dataset incorporating all 6 DOF for ego-motion

# References I



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