COMP2022: Formal Languages and Logic

2017, Semester 1, Week 4

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Adapted from slides by A/Prof Kalina Yacef

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Outline

- Grammars
- Context-Free Grammars (CFG)
- Context-Free Languages (CFL)
- Parsing (introduction)
- Ambiguity
- Recursive Grammars
- Clean Grammars
- Types of Grammar

Introduction

So far we have seen two different, but equivalent, methods of describing languages: finite automata and regular expressions, which describe *regular languages*

We have already proven that some languages, such as $\{0^n 1^n \mid n \geq 0\}$, cannot be described using FA or RE.

Today we will introduce *context-free grammars (CFG)*, which describe the next category of languages, the *context-free languages*

Later, will see grammars called *regular grammars*, which describe exactly *regular languages*

Grammars

Grammars are another way to describe a language

A *grammar* is a set of rules which can be used to *generate* a language

The language generated is the set of all strings which can be *derived* from the grammar

Introductory example (G_1)

$S \to 01$ Base case: $01 \in L$

$S \to 0S1$ Recursive case: if $S \in L$ then $0S1 \in L$

G_1 generates the language $L = \{0^n1^n \mid n > 0\}$, which we already know is not regular

How does it derive 000111?

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\end{aligned}
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Introductory example (G_2)

$$
\begin{aligned}
S &\rightarrow \mathit{NounPhrase}\ \mathit{VerbPhrase} \\
\mathit{NounPhrase} &\rightarrow \text{the } \mathit{Noun} \\
\mathit{VerbPhrase} &\rightarrow \mathit{Verb}\ \mathit{NounPhrase} \\
\mathit{Noun} &\rightarrow \text{girl} \mid \text{ball} \\
\mathit{Verb} &\rightarrow \text{likes} \mid \text{sees}
\end{aligned}
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What language does G_2 generate?

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$$

What language does G_2 generate?

{ the girl likes the girl, the girl likes the ball,
the girl sees the girl, the girl sees the ball,
the ball likes the girl, the ball likes the ball,
the ball sees the girl, the ball sees the ball }

Definitions

Terminals

- The finite set of symbols which make up strings of the language

Non-terminals / Variables

- A finite set of symbols used to generate the strings.
- They never appear in the language.

Start symbol

- The variable used to start every derivation

Definitions

Production rules

- Sometimes called substitution or derivation rules
- Define strings of *variables* and *terminals* which can be substituted for a *variable*:

$$Variable \rightarrow <\text{string of } Variables \text{ and } Terminals>$$

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A variable can have many rules:

$$Noun \rightarrow \text{girl}$$
$$Noun \rightarrow \text{ball}$$
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$$Noun \rightarrow \text{quokka}$$

They can be written together:

$$Noun \rightarrow \text{girl} \mid \text{ball} \mid \text{quokka}$$

Some common notational conventions

If not stated otherwise:

- $A, B, C, \ldots$ and S are variables
- S is the start variable
- $a, b, c, \ldots$ are terminals
- $\ldots, X, Y, Z$ are either terminals or variables
- $\ldots w, x, y, z$ are strings of terminals *only*
- $\alpha, \beta, \gamma, \ldots$ are strings of terminals and/or variables

Context-free Grammar (CFG)

A *context-free grammar* is a grammar where every production rule has the form $A \to \alpha$

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- α is a string of terminals and/or variables (possibly ϵ)

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Context-free grammars describe *context-free languages*

Example:
$\{a^n b^n \mid n \in N\}$ is not a regular language (no finite automata exists recognising it), but we can prove that it is a context-free language, because the following grammar generates it:

$$S \to aSb \mid \varepsilon$$

CFG: Formal Definition

A *context-free grammar* G is a 4-tuple (V, T, P, S) where:

- V is a finite set of *variables*
- T is a finite set of *terminals*
- P is a finite set of *production rules* in the form $\alpha \to \beta$ where $\alpha \in V$ and $\beta \in \{V \cup T \cup \{\varepsilon\}\}^\star$
- $S \in V$ is a special variable called the *Start Symbol*

Example G_1

$$S \to 01$$
$$S \to 0S1$$

More formally, $G_1 = (T, V, S, P)$ where:

$$T =$$
$$V =$$
$$S =$$
$$P =$$

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More formally, $G_2 = (T, V, S, P)$ where:

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\begin{aligned}
T &= \\
V &= \\
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S &= S \\
P &= (\textsf{set of seven rules above})
\end{aligned}
$$

Language of a Grammar

Let w be a string over T.
$w \in L(G)$ if and only if it is possible to derive w from S

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$\alpha \Rightarrow \beta$ denotes that α derives β in one step
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If two grammars generate the same langauge, then they are *equivalent*.

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- Begin with the start symbol
- Repeatedly replace one variable with the right hand side of one of it's productions
- ... until the string is composed only of terminal symbols

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Example, derivation of 000111 from this grammar:

$$S \to 0S1 \mid \varepsilon \qquad\qquad S \Rightarrow$$

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Leftmost and Rightmost Derivations

Leftmost derivation: always derive the leftmost variable first
Rightmost derivation: always derive the rightmost variable first

Example: "the girl sees the ball"

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$\Rightarrow$ the girl sees $\mathit{NounPhrase}$
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$\Rightarrow$ the girl sees the ball

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&\Rightarrow \text{the girl sees } NounPhrase \\
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Context-Free Languages

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The syntax of most programming languages are context-free.

$$S \to \text{while } E \text{ do } S$$
$$S \to \text{if } E \text{ then } S \text{ else } S$$
$$S \to I := E$$
$$S \to \{SL\}$$
$$L \to ;SL \mid \varepsilon$$
$$E \to \text{ ... (description of an expression)}$$
$$I \to \text{ ... (description of an identifier)}$$

Context-Free Languages

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$\{$Regular Languages$\} \subset \{$Context-Free Languages$\}$

- The *union* of two CFL is also context-free
- The *concatenation* of two CFL is also context-free
- The *star closure* of a CFL is also context-free

Example

Consider the grammar G:

$$S \to AB \qquad\qquad S \Rightarrow AB$$
$$A \to \varepsilon \mid aA$$
$$B \to \varepsilon \mid bB$$

What is $L(G)$?

Example

Consider the grammar G:

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\begin{aligned}
S &\rightarrow AB \\
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\end{aligned}
$$

$$
\begin{aligned}
S &\Rightarrow AB \\
&\Rightarrow aAB \\
&\Rightarrow^{+} aaaaAB \\
&\Rightarrow aaaaB
\end{aligned}
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\begin{aligned}
S &\Rightarrow AB \\
&\Rightarrow aAB \\
&\Rightarrow^{+} aaaaAB \\
&\Rightarrow aaaaB \\
&\Rightarrow aaaabB \\
&\Rightarrow^{+} aaaabbbbbB \\
&\Rightarrow aaaabbbbb
\end{aligned}
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&\Rightarrow aaaabbbbb
\end{aligned}
$$

i.e. $L(G) = L(a^{\star}b^{\star}) = \{a^n b^m \mid n \geq 0, m \geq 0\}$

More examples

Describe the language generated

1. $S \to aSa \mid bSb \mid \varepsilon$
2. $S \to aS \mid bS \mid a$
3. $S \to SS \mid bS \mid a$
4. $S \to aT \mid bT \mid \varepsilon$
   $T \to aS \mid bS$
5. $S \to aSa \mid bSb \mid a \mid b$

More examples

Give grammars generating these languages

1. $\{ba^{n+1}b \mid n \geq 0\}$
2. Odd-length strings in $\{a, b\}^{\star}$ with middle symbol a
3. Even-length strings in $\{a, b\}^{\star}$ with matching middle symbols
4. Binary strings containing more 0's than 1's
5. Strings over $\{a, b\}$ with at least three a's

Constructing grammars

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- Star closure: the grammar for $M^\star$ starts with $S \to S_M S \mid \varepsilon$

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Using the union rule

Let $L = \{\varepsilon, a, b, aa, bb, ..., a^n, b^n, ...\}$

Then $L = M \cup N$ where $M = \{a^n \mid n \geq 0\}, N = \{b^n \mid n \geq 0\}$

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Let $L = \{a^m b^n \mid m \geq 0, n \geq 0\}$

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Using the star closure rule we get:
$S \rightarrow S_M S \mid \varepsilon$
$S_M \rightarrow aa \mid bb$

Parsing

Given a sentence, the problem of *parsing* is determining *how* the grammar generates it.

i.e. To discover the *correct* derivation of the sentence, or the correct parse tree

Parse Tree

A *parse tree* is a tree labelled by symbols from the CFG

- root $=$ the start symbol
- interior node $=$ a variable
- leaf node $=$ a terminal or ε
- children of X $=$ the right hand side of a production rule for X, in order

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- root = the start symbol
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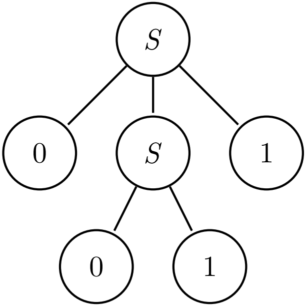
- children of X = the right hand side of a production rule for X, in order

Example parse tree for "0011" in
$S \to 0S1 \mid 01$
An in-order traversal of the leaf nodes retrieves the string



Parse Tree or Derivation Tree

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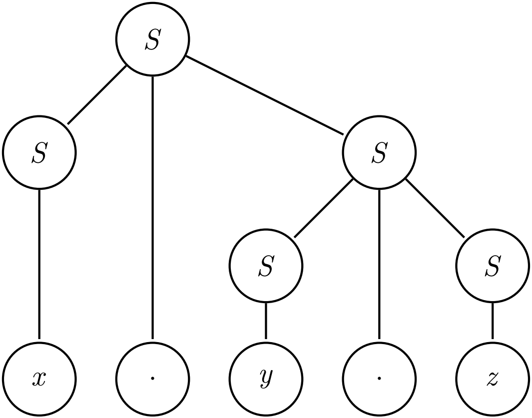

$$S \to S \cdot S$$
$$S \to x \mid y \mid z$$

This parse tree implies that the expression means $x \cdot (y \cdot z)$

Natural Language Processing (NLP) example

$$
\begin{aligned}
S &\rightarrow \mathit{NounPhrase}\ \mathit{VerbPhrase} \\
\mathit{NounPhrase} &\rightarrow \mathit{ComplexNoun} \mid \mathit{ComplexNoun}\ \mathit{PrepPhrase} \\
\mathit{VerbPhrase} &\rightarrow \mathit{ComplexVerb} \mid \mathit{ComplexVerb}\ \mathit{PrepPhrase} \\
\mathit{PrepPhrase} &\rightarrow \mathit{Prep}\ \mathit{ComplexNoun} \\
\mathit{ComplexNoun} &\rightarrow \mathit{Article}\ \mathit{Noun} \\
\mathit{ComplexVerb} &\rightarrow \mathit{Verb} \mid \mathit{Verb}\ \mathit{NounPhrase} \\
\mathit{Article} &\rightarrow \mathsf{a} \mid \mathsf{the} \\
\mathit{Noun} &\rightarrow \mathsf{girl} \mid \mathsf{dog} \mid \mathsf{stick} \mid \mathsf{ball} \\
\mathit{Verb} &\rightarrow \mathsf{chases} \mid \mathsf{sees} \\
\mathit{Prep} &\rightarrow \mathsf{with}
\end{aligned}
$$

Simple example

Is the string "a ball" accepted by this grammar?

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All the production rules for $VerbPhrase$ produce a $ComplexVerb$, which in turn must produce a $Verb$.

Therefore all strings in the language contain a verb. “a ball” does not contain a verb, so it cannot be accepted by the grammar.

Ambiguity: example

Ambiguity: several meanings for the same sentence.

"The girl chases the dog with a stick" has *two leftmost derivations*

$$
\begin{aligned}
Sentence &\Rightarrow NounPhrase\ VerbPhrase \\
&\Rightarrow ComplexNoun\ VerbPhrase \\
&\Rightarrow Article\ Noun\ VerbPhrase \\
&\Rightarrow \text{the } Noun\ VerbPhrase \\
&\Rightarrow \text{the girl } VerbPhrase \\
&\Rightarrow \underline{\text{the girl } ComplexVerb} \\
&\Rightarrow \text{the girl } Verb\ NounPhrase \\
&\Rightarrow \text{the girl chases } NounPhrase \\
&\Rightarrow \text{the girl chases } ComplexNoun\ PrepPhrase \\
&\Rightarrow \text{the girl chases } Article\ Noun\ PrepPhrase \\
&\Rightarrow \text{the girl chases the } Noun\ PrepPhrase \\
&\Rightarrow \text{the girl chases the dog } PrepPhrase \\
&\Rightarrow \text{the girl chases the dog } Prep\ ComplexNoun \\
&\Rightarrow \text{the girl chases the dog with } ComplexNoun \\
&\Rightarrow \text{the girl chases the dog with } Article\ Noun \\
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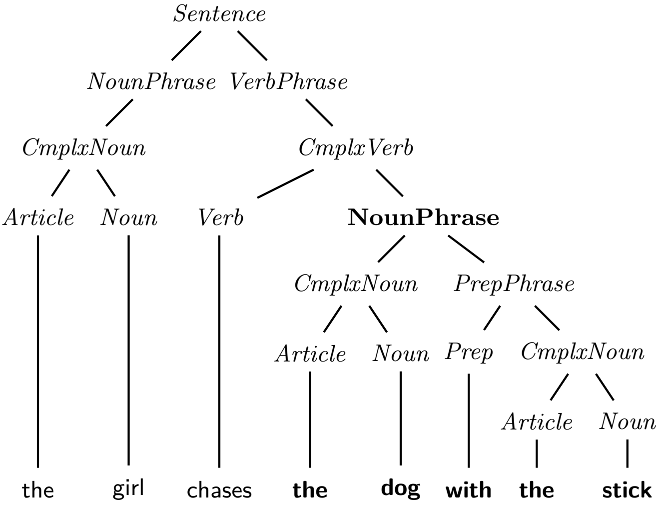
Who has the stick?

First Leftmost Derivation Tree



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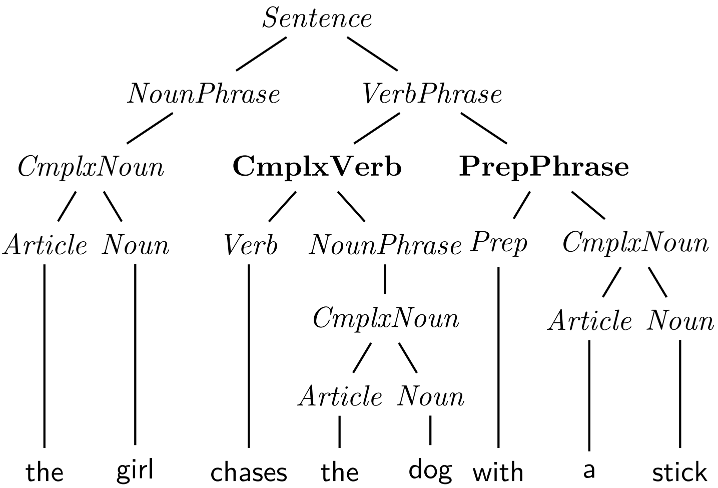

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A string is *ambiguous* on a given grammar if it has two different parse trees. Otherwise, it is unambigous.

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Similarly for rightmost derivations.

Is this grammar ambiguous?

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A grammar for an infinite language must contain at least one recursive variable

Balanced Parentheses

This grammar generates the language of balanced parentheses:

$$B \to (B) \mid BB \mid \varepsilon$$

Show that it is ambiguous.

Remove left recursion

Original grammar is left-recursive: $B \to (B) \mid BB \mid \varepsilon$

An equivalent grammar without left-recursion: $B \to (B)B \mid \varepsilon$

Left parsing of $()()()$ is now deterministic:

Remaining input	Derivation steps	
$()()()$	B	start symbol
$()()()$	$(B)B$	$B \to (B)B$
$)()()$	$B)B$	matching terminals
$)()()$	$)B$	$B \to \varepsilon$
$()()$	B	matching terminals
...	...	...

Clean grammars

- No circular definitions: $A_1 \Rightarrow A_2 \Rightarrow ... \Rightarrow A_n \Rightarrow A_1$
  All the A's can generate the same set of strings, therefore there is no reason to distinguish between them. They should be reduced to a single variable.

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- No null productions (except for the start symbol)

Types of grammars

We are interested in 4 classes of grammars, depending on the type of production rules that they allow:

Type 0 (unrestricted)	$\chi \to \alpha$
Type 1 (context-sensitive)	$\chi \to \alpha$ where $1 \leq \lvert\chi\rvert \leq \lvert\alpha\rvert$
Type 2 (context-free)	$A \to \alpha$
Type 3 (regular)	$A \to \omega B$ and $A \to \omega$

χ	arbitrary string of one or more symbols
α	arbitrary string of symbols, possibly null
A, B	non-terminal symbols
ω	arbitrary string of terminal symbols

Recall the Chomsky Hierarchy from week 1!

Context-free Grammars

- Generate Context-Free Languages
  - Very important class of languages in CS (compilers, NLP, etc.)
- All rules are in the form $A \to \alpha$
- Closed under Union, Concatenation and Star Closure
- String derivation (left-most, right-most)
- Ambiguous grammars
- Clean grammars

Next lecture:

- Push-Down Automata
- Parsing