Multiple linear regression

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Grading the professor

Many college courses conclude by giving students the opportunity to evaluate the course and the instructor anonymously. However, the use of these student evaluations as an indicator of course quality and teaching effectiveness is often criticized because these measures may reflect the influence of non-teaching related characteristics, such as the physical appearance of the instructor. The article titled, "Beauty in the classroom: instructors' pulchritude and putative pedagogical productivity" by Hamermesh and Parker found that instructors who are viewed to be better looking receive higher instructional ratings.

Here, you will analyze the data from this study in order to learn what goes into a positive professor evaluation.

Getting Started

Load packages

In this lab, you will explore and visualize the data using the **tidyverse** suite of packages. The data can be found in the companion package for OpenIntro resources, **openintro**.

Let's load the packages.

```
library(tidyverse)
library(openintro)
library(GGally)
```

This is the first time we're using the `GGally` package. You will be using the `ggpairs` function from this package later in the lab.

The data

The data were gathered from end of semester student evaluations for a large sample of professors from the University of Texas at Austin. In addition, six students rated the professors' physical appearance. The result is a data frame where each row contains a different course and columns represent variables about the courses and professors. It's called `evals`.

```
glimpse(evals)
```

```
## Rows: 463
## Columns: 23
## $ course_id     <int> 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 1~
## $ prof_id       <int> 1, 1, 1, 1, 2, 2, 2, 3, 3, 4, 4, 4, 4, 4, 4, 4, 4, 5, 5,~
## $ score         <dbl> 4.7, 4.1, 3.9, 4.8, 4.6, 4.3, 2.8, 4.1, 3.4, 4.5, 3.8, 4~
## $ rank          <fct> tenure track, tenure track, tenure track, tenure track, ~
## $ ethnicity     <fct> minority, minority, minority, minority, not minority, no~
## $ gender        <fct> female, female, female, female, male, male, male, male, ~
## $ language      <fct> english, english, english, english, english, english, en~
## $ age           <int> 36, 36, 36, 36, 59, 59, 59, 51, 51, 40, 40, 40, 40, 40, ~
## $ cls_perc_eval <dbl> 55.81395, 68.80000, 60.80000, 62.60163, 85.00000, 87.500~
## $ cls_did_eval  <int> 24, 86, 76, 77, 17, 35, 39, 55, 111, 40, 24, 24, 17, 14,~
## $ cls_students  <int> 43, 125, 125, 123, 20, 40, 44, 55, 195, 46, 27, 25, 20, ~
## $ cls_level     <fct> upper, upper, upper, upper, upper, upper, upper, upper, ~
## $ cls_profs     <fct> single, single, single, single, multiple, multiple, mult~
## $ cls_credits   <fct> multi credit, multi credit, multi credit, multi credit, ~
## $ bty_f1lower   <int> 5, 5, 5, 5, 4, 4, 4, 5, 5, 2, 2, 2, 2, 2, 2, 2, 2, 7, 7,~
## $ bty_f1upper   <int> 7, 7, 7, 7, 4, 4, 4, 2, 2, 5, 5, 5, 5, 5, 5, 5, 5, 9, 9,~
## $ bty_f2upper   <int> 6, 6, 6, 6, 2, 2, 2, 5, 5, 4, 4, 4, 4, 4, 4, 4, 4, 9, 9,~
## $ bty_m1lower   <int> 2, 2, 2, 2, 2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 7, 7,~
## $ bty_m1upper   <int> 4, 4, 4, 4, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3, 6, 6,~
## $ bty_m2upper   <int> 6, 6, 6, 6, 3, 3, 3, 3, 3, 2, 2, 2, 2, 2, 2, 2, 2, 6, 6,~
## $ bty_avg       <dbl> 5.000, 5.000, 5.000, 5.000, 3.000, 3.000, 3.000, 3.333, ~
## $ pic_outfit    <fct> not formal, not formal, not formal, not formal, not form~
## $ pic_color     <fct> color, color, color, color, color, color, color, color, ~
```

We have observations on 21 different variables, some categorical and some numerical. The meaning of each variable can be found by bringing up the help file:

```
`?`(evals)
```

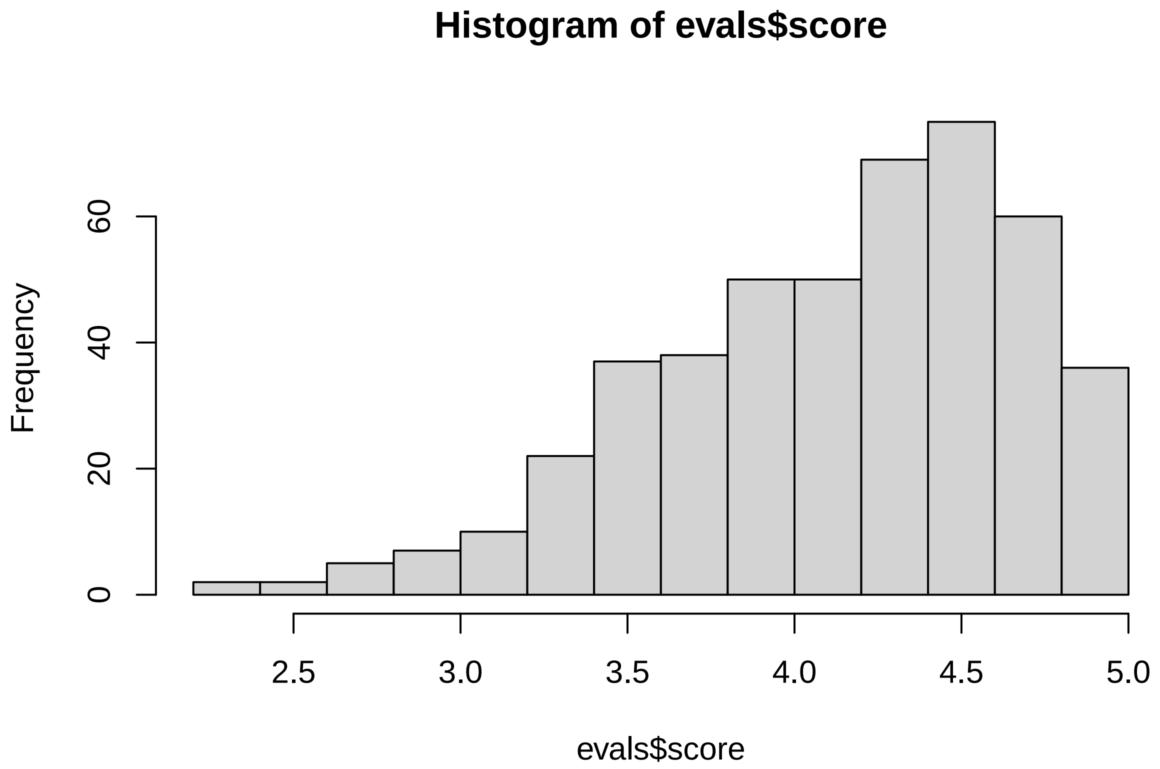
Exploring the data

1. Is this an observational study or an experiment? The original research question posed in the paper is whether beauty leads directly to the differences in course evaluations. Given the study design, is it possible to answer this question as it is phrased? If not, rephrase the question.

This appears to be observational study since the data its using a collected from actual students ratings for professors for the class and their physical appearance. Thus, there will be some bias for individual students preferences. A possible way to rephrase the question is "Is there correlation between the physical appearance and course evaluations?"

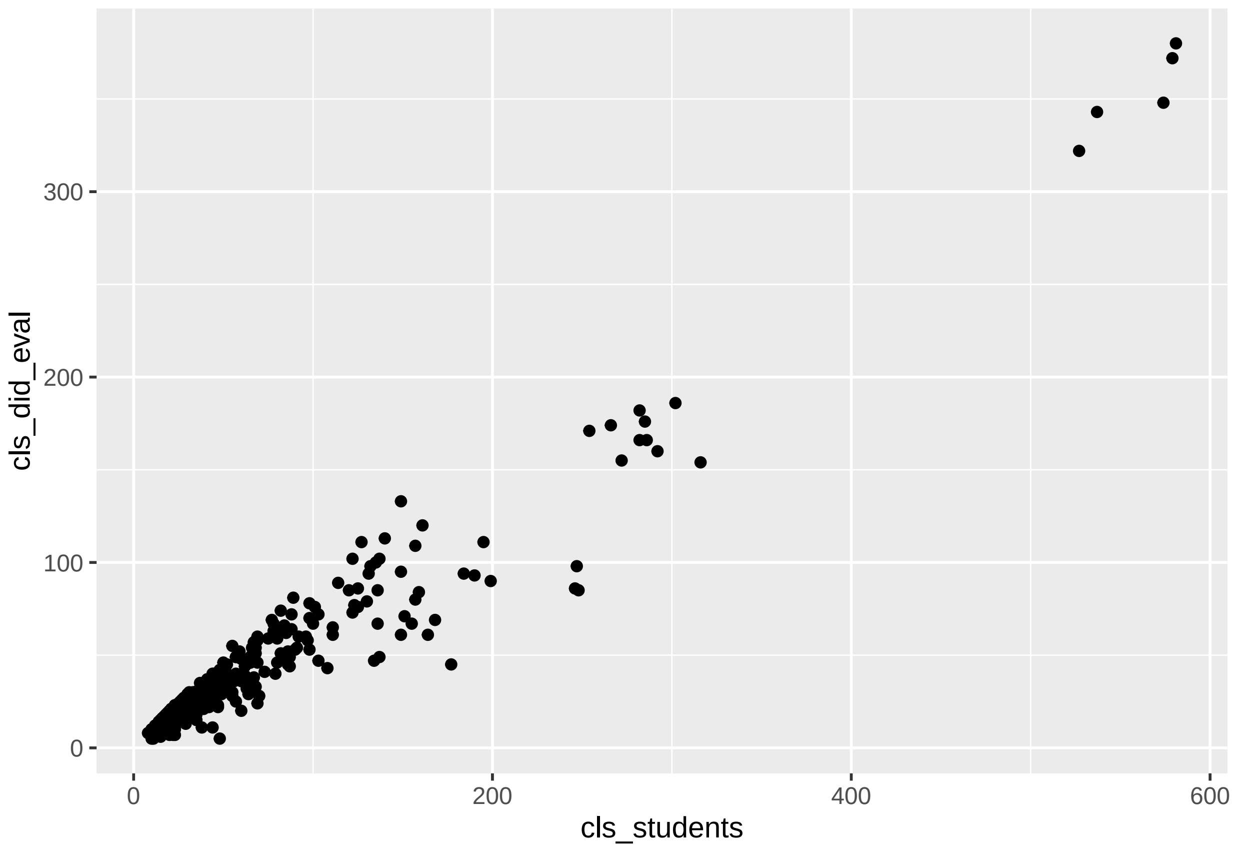
2. Describe the distribution of `score`. Is the distribution skewed? What does that tell you about how students rate courses? Is this what you expected to see? Why, or why not?

```
hist(evals$score)
```



3. Excluding `score`, select two other variables and describe their relationship with each other using an appropriate visualization.

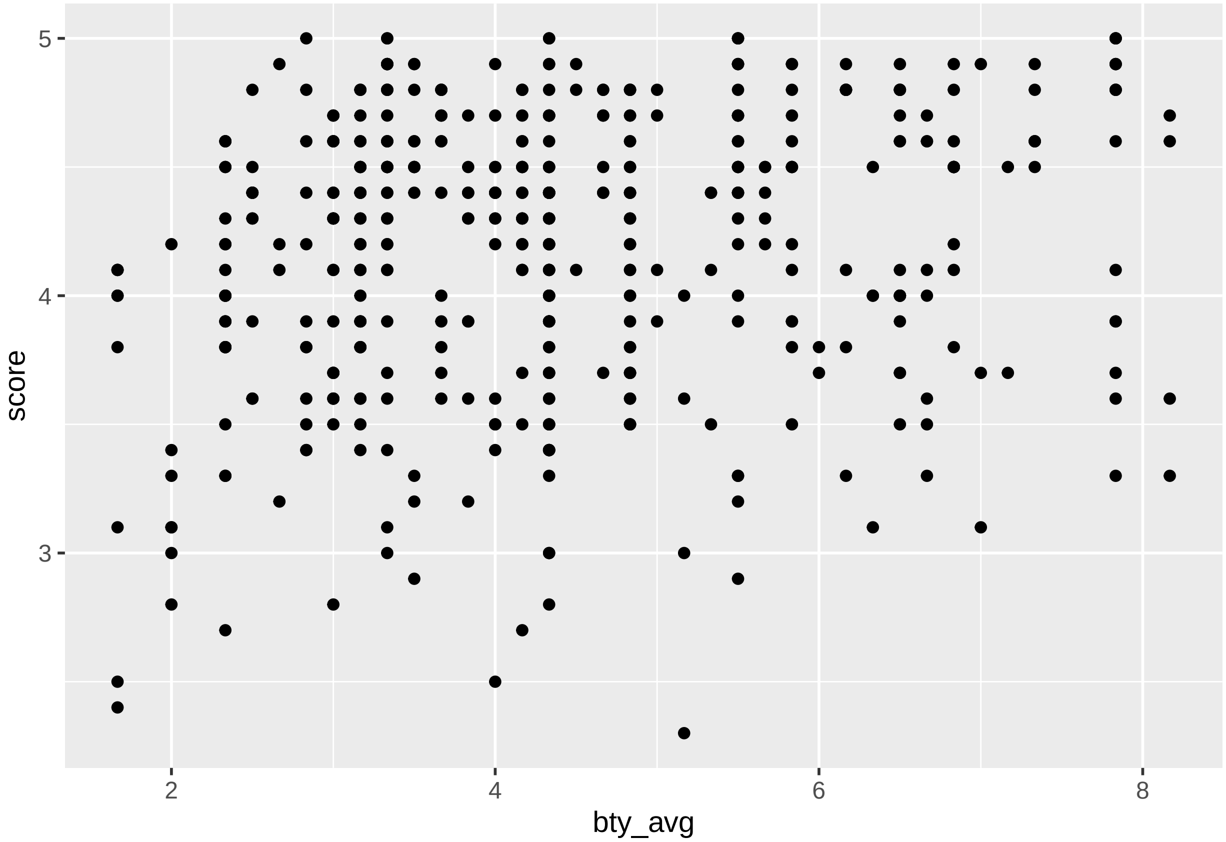
```
ggplot(data = evals, aes(x = cls_students, y = cls_did_eval)) + geom_point()
```



Simple linear regression

The fundamental phenomenon suggested by the study is that better looking teachers are evaluated more favorably. Let's create a scatterplot to see if this appears to be the case:

```
ggplot(data = evals, aes(x = bty_avg, y = score)) + geom_point()
```

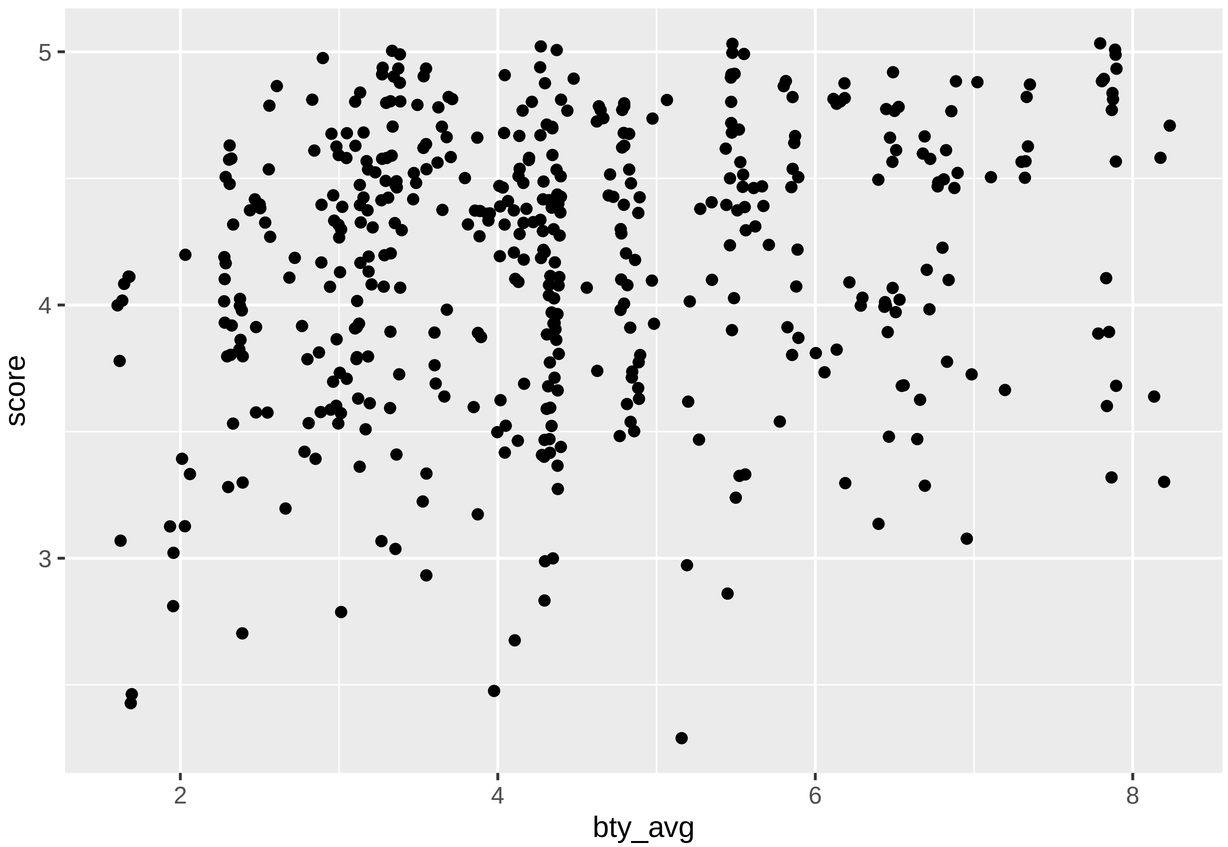


Before you draw conclusions about the trend, compare the number of observations in the data frame with the approximate number of points on the scatterplot. Is anything awry?

4. Replot the scatterplot, but this time use `geom_jitter` as your layer. What was misleading about the initial scatterplot?

With geom_jitter(), we can better see the density and distribution of the data.

```
ggplot(data = evals, aes(x = bty_avg, y = score)) + geom_jitter()
```



5. Let's see if the apparent trend in the plot is something more than natural variation. Fit a linear model called `m_bty` to predict average professor score by average beauty rating. Write out the equation for the linear model and interpret the slope. Is average beauty score a statistically significant predictor? Does it appear to be a practically significant predictor?

Add the line of the bet fit model to your plot using the following:

```
m_bty <- lm(score ~ bty_avg, data = evals)
summary(m_bty)
```

```
##
## Call:
## lm(formula = score ~ bty_avg, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.9246 -0.3690  0.1420  0.3977  0.9309
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.88034    0.07614   50.96  < 2e-16 ***
## bty_avg      0.06664    0.01629    4.09 5.08e-05 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
```

```
## Residual standard error: 0.5348 on 461 degrees of freedom
## Multiple R-squared:  0.03502,    Adjusted R-squared:  0.03293
## F-statistic: 16.73 on 1 and 461 DF,  p-value: 5.083e-05
```

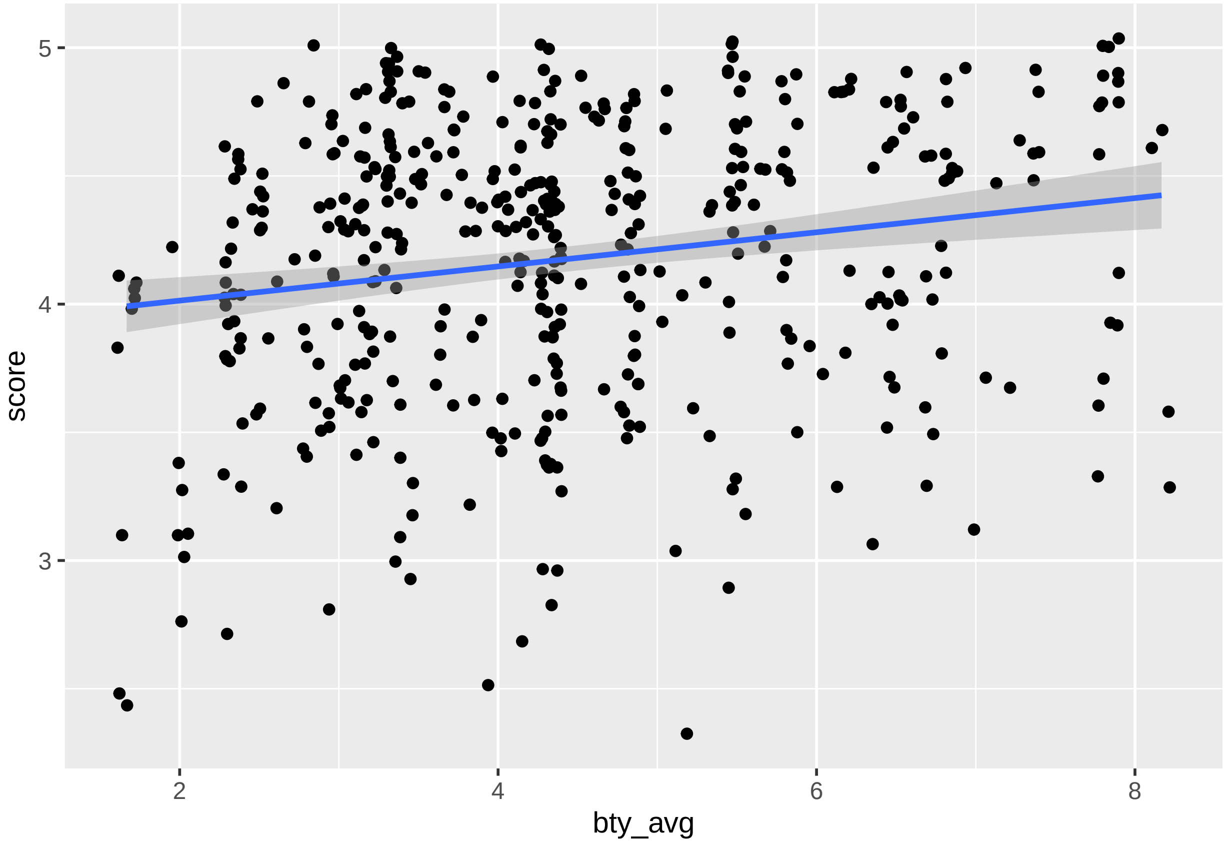
$$\widehat{score} = 3.88 + 0.067 avg\_beauty$$

The beta coefficient of 0.067 of avg beauty means that, on average, the predicted score increases 0.067 per point in beauty.

With a p-value less than 0.05, suggests that average beauty is a significant predictor.

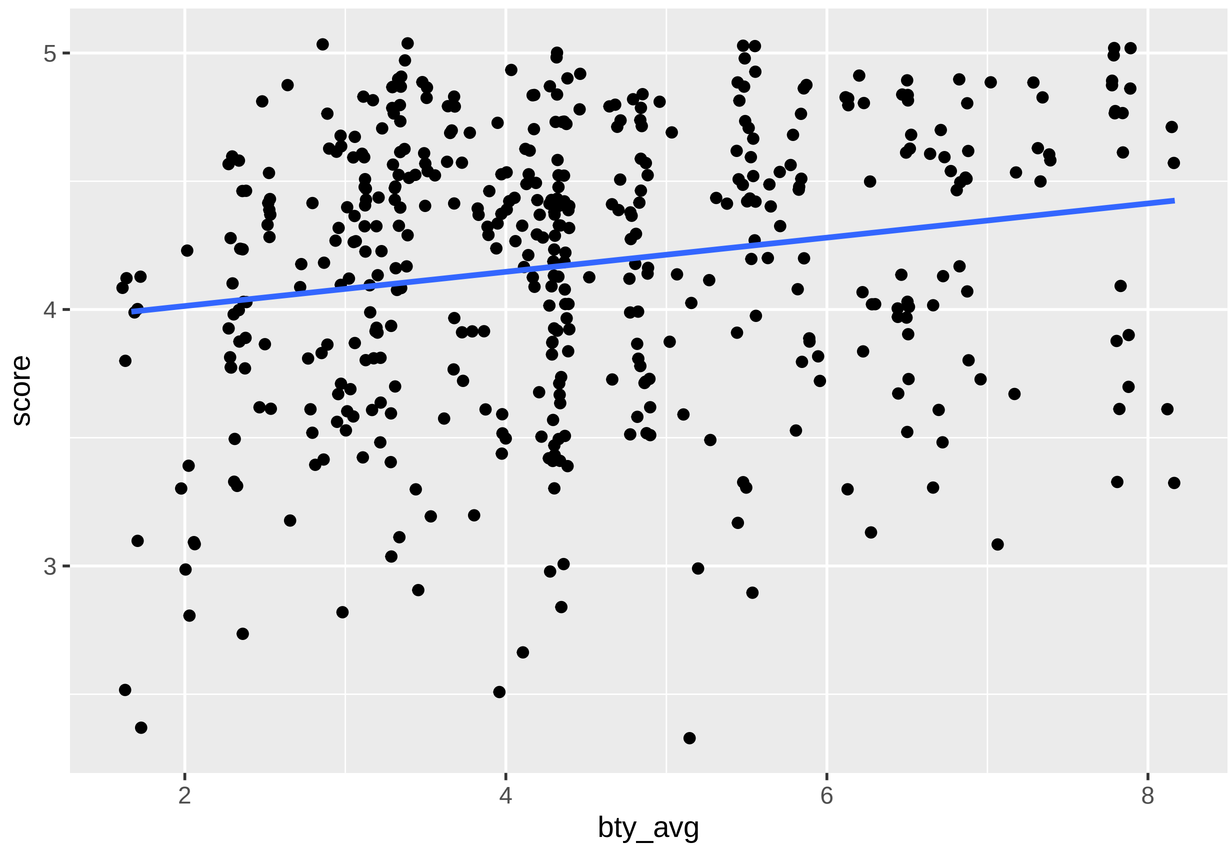
But the R squared values indicate that this model is a not a fit since it only explains 3% of the variability of the data.

```
ggplot(data = evals, aes(x = bty_avg, y = score)) + geom_jitter() + geom_smooth(method = "lm")
```



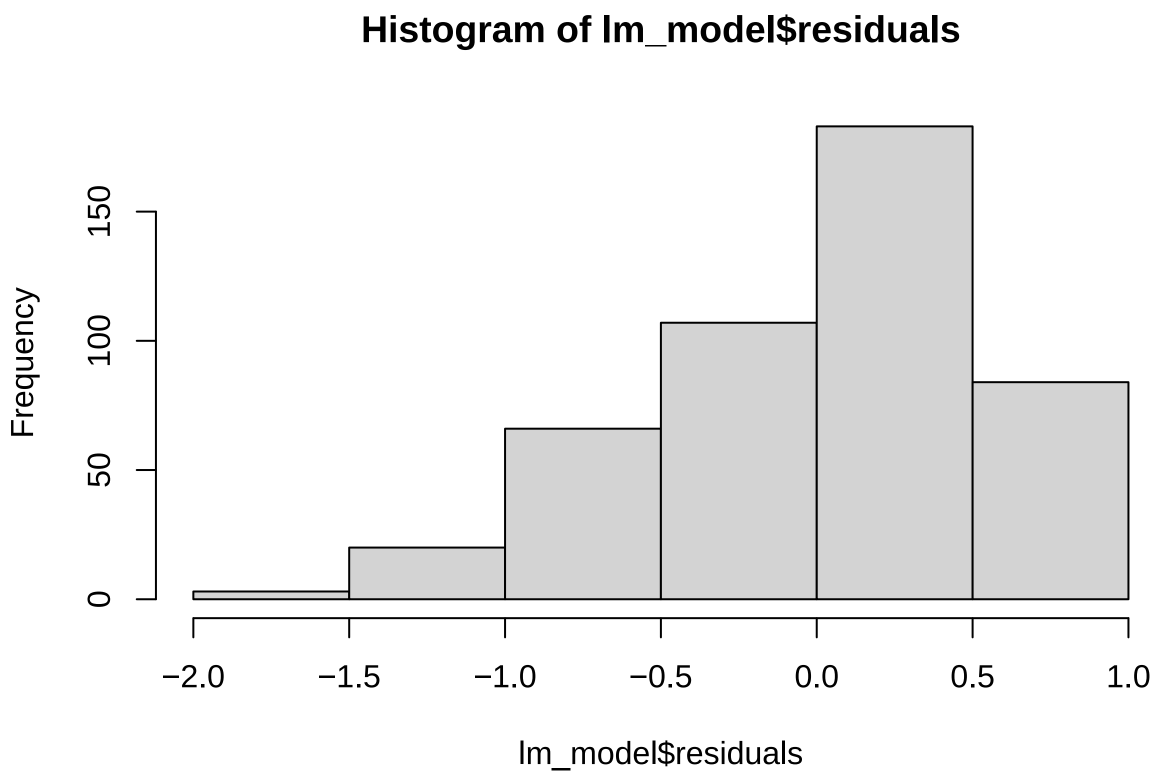
The blue line is the model. The shaded gray area around the line tells you about the variability you might expect in your predictions. To turn that off, use `se = FALSE`.

```
ggplot(data = evals, aes(x = bty_avg, y = score)) + geom_jitter() + geom_smooth(method = "lm",
    se = FALSE)
```

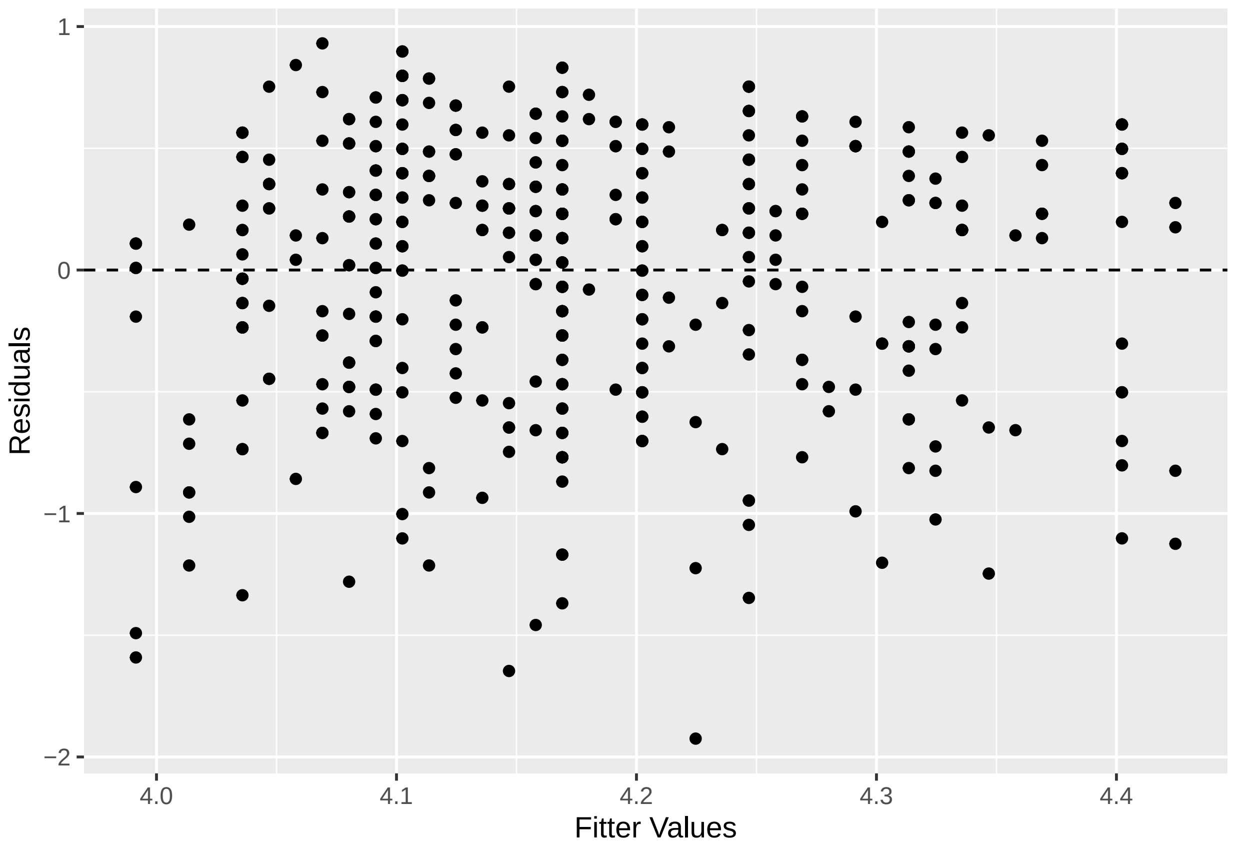


6. Use residual plots to evaluate whether the conditions of least squares regression are reasonable. Provide plots and comments for each one (see the Simple Regression Lab for a reminder of how to make these).

```
lm_model <- lm(score ~ bty_avg, data = evals)
hist(lm_model$residuals)
```



```
ggplot(data = m_bty, aes(x = .fitted, y = .resid)) + geom_point() + geom_hline(yintercept = 0,
    linetype = "dashed") + xlab("Fitter Values") + ylab("Residuals")
```

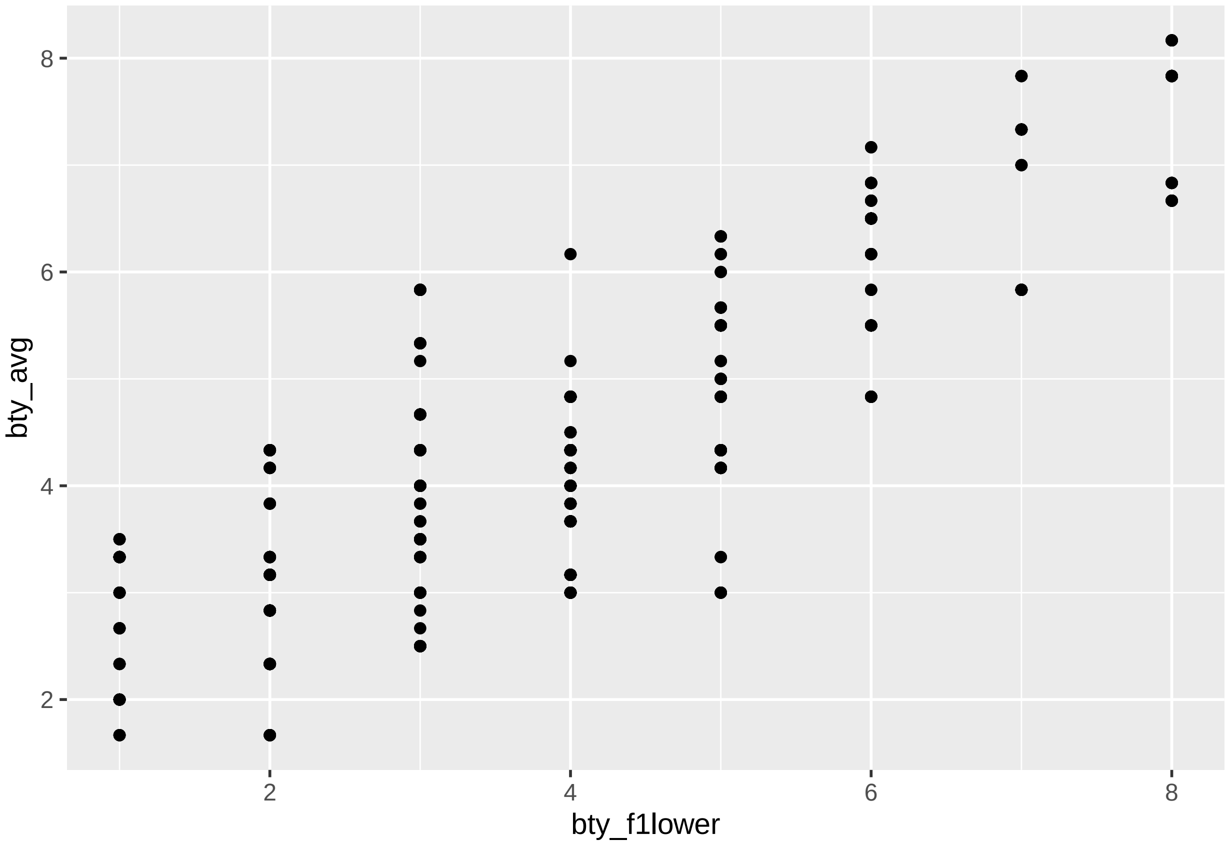


It seems that most of the residuals are between -1 and 1 which implies that the residuals do not greatly devaite from the line.

Multiple linear regression

The data set contains several variables on the beauty score of the professor: individual ratings from each of the six students who were asked to score the physical appearance of the professors and the average of these six scores. Let's take a look at the relationship between one of these scores and the average beauty score.

```
ggplot(data = evals, aes(x = bty_f1lower, y = bty_avg)) + geom_point()
```

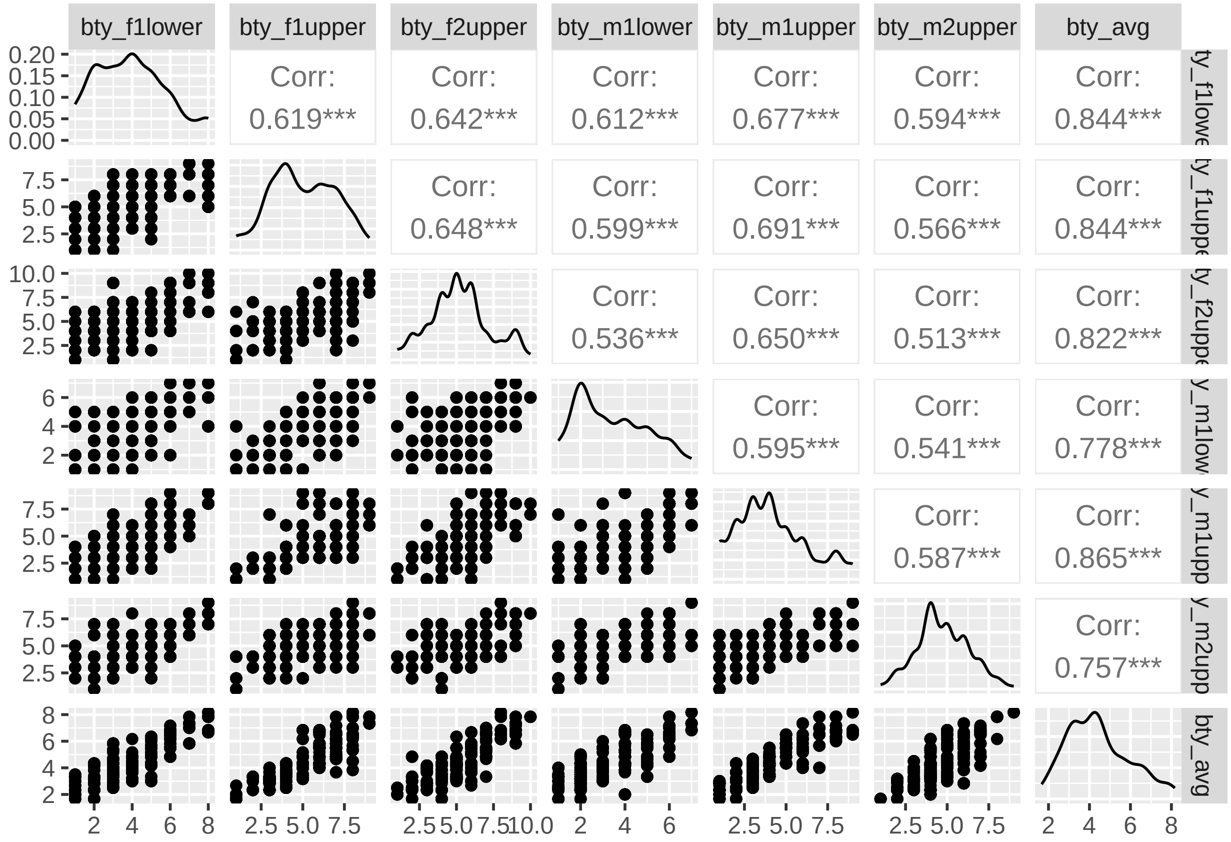


```
evals %>%
    summarise(cor(bty_avg, bty_f1lower))
```

```
## # A tibble: 1 x 1
##   `cor(bty_avg, bty_f1lower)`
##                         <dbl>
## 1                       0.844
```

As expected, the relationship is quite strong—after all, the average score is calculated using the individual scores. You can actually look at the relationships between all beauty variables (columns 13 through 19) using the following command:

```
evals %>%
    select(contains("bty")) %>%
    ggpairs()
```



These variables are collinear (correlated), and adding more than one of these variables to the model would not add much value to the model. In this application and with these highly-correlated predictors, it is reasonable to use the average beauty score as the single representative of these variables.

In order to see if beauty is still a significant predictor of professor score after you've accounted for the professor's gender, you can add the gender term into the model.

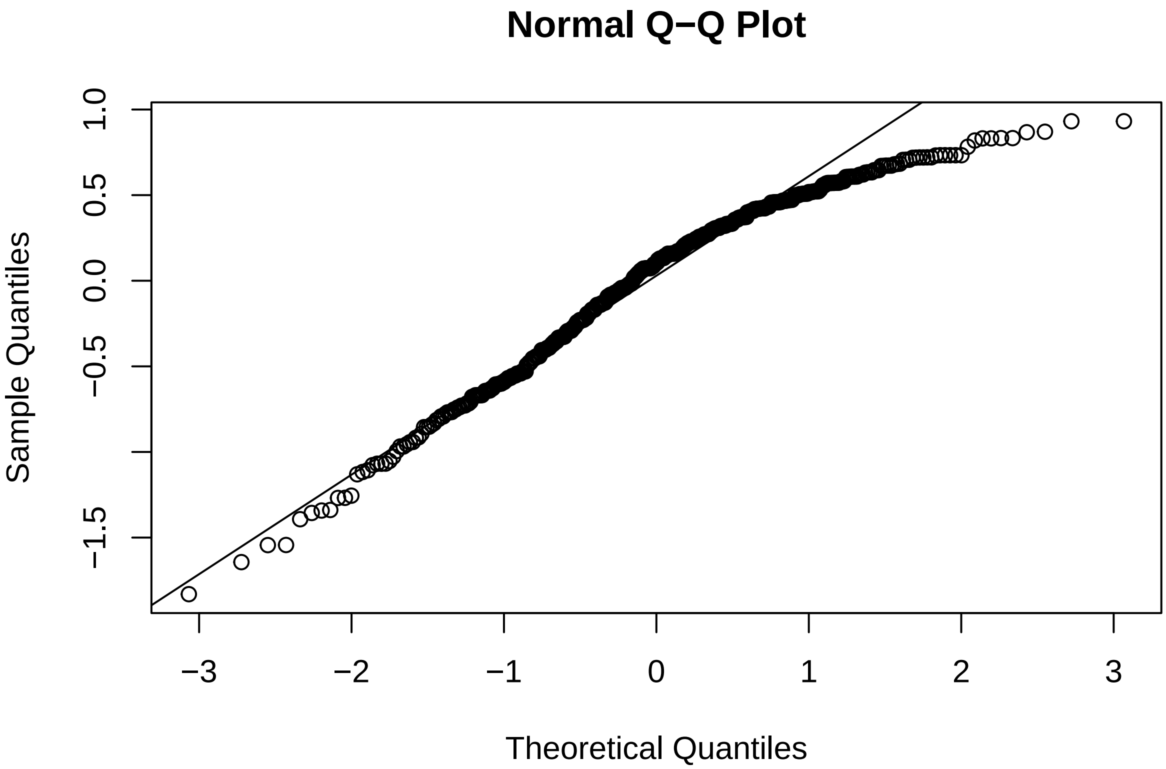
```r
m_bty_gen <- lm(score ~ bty_avg + gender, data = evals)
summary(m_bty_gen)
```

```
##
## Call:
## lm(formula = score ~ bty_avg + gender, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.8305 -0.3625  0.1055  0.4213  0.9314
##
## Coefficients:
##             Estimate Std. Error t value Pr(>|t|)
## (Intercept)  3.74734    0.08466  44.266  < 2e-16 ***
## bty_avg      0.07416    0.01625   4.563 6.48e-06 ***
## gendermale   0.17239    0.05022   3.433 0.000652 ***
## ---
## Signif. codes:  0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 ’.’ 0.1 ’ ’ 1
##
```

```
## Residual standard error: 0.5287 on 460 degrees of freedom
## Multiple R-squared:  0.05912,    Adjusted R-squared:  0.05503
## F-statistic: 14.45 on 2 and 460 DF,  p-value: 8.177e-07
```

7. P-values and parameter estimates should only be trusted if the conditions for the regression are reasonable. Verify that the conditions for this model are reasonable using diagnostic plots.

```
qqnorm(m_bty_gen$residuals)
qqline(m_bty_gen$residuals)
```



8. Is `bty_avg` still a significant predictor of `score`? Has the addition of `gender` to the model changed the parameter estimate for `bty_avg`?

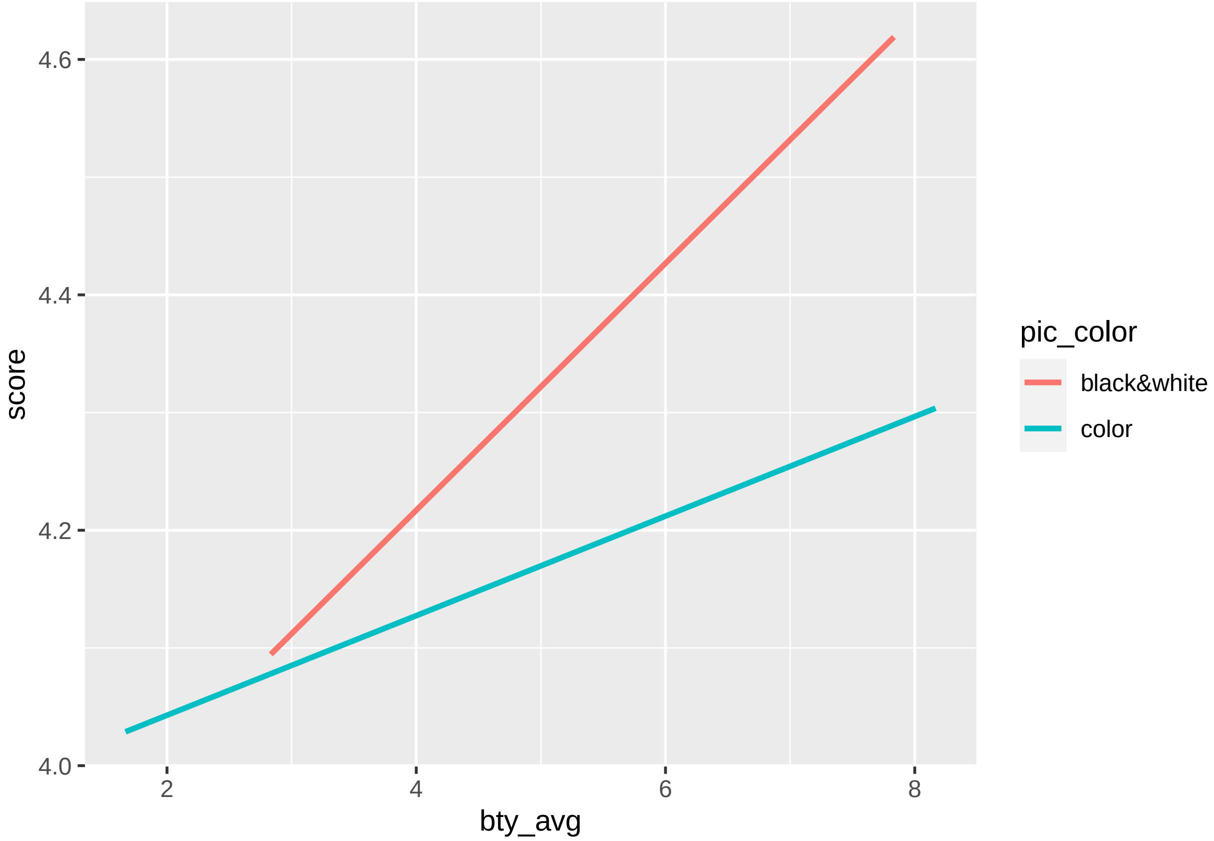
bty_avg is still a significant predictor of score. The addition of gender to the model descrease the coefficient for bty_avg and lowered the R-squared value as well

Note that the estimate for `gender` is now called `gendermale`. You'll see this name change whenever you introduce a categorical variable. The reason is that R recodes `gender` from having the values of `male` and `female` to being an indicator variable called `gendermale` that takes a value of 0 for female professors and a value of 1 for male professors. (Such variables are often referred to as "dummy" variables.)

As a result, for female professors, the parameter estimate is multiplied by zero, leaving the intercept and slope form familiar from simple regression.

$$\begin{aligned}\widehat{score} &= \hat{\beta}_0 + \hat{\beta}_1 \times bty\_avg + \hat{\beta}_2 \times (0) \\ &= \hat{\beta}_0 + \hat{\beta}_1 \times bty\_avg\end{aligned}$$

```
ggplot(data = evals, aes(x = bty_avg, y = score, color = pic_color)) + geom_smooth(method = "lm",
    formula = y ~ x, se = FALSE)
```



9. What is the equation of the line corresponding to those with color pictures? (*Hint:* For those with color pictures, the parameter estimate is multiplied by 1.) For two professors who received the same beauty rating, which color picture tends to have the higher course evaluation score?

   The equation of the color line is

$$\widehat{score} = 3.47 + 0.07bty\_avg + 0.17(0)$$

For two professors who received the same beauty rating white&black professors tends to score higher than color.

The decision to call the indicator variable `gendermale` instead of `genderfemale` has no deeper meaning. R simply codes the category that comes first alphabetically as a 0. (You can change the reference level of a categorical variable, which is the level that is coded as a 0, using the`relevel()` function. Use `?relevel` to learn more.)

10. Create a new model called `m_bty_rank` with `gender` removed and `rank` added in. How does R appear to handle categorical variables that have more than two levels? Note that the rank variable has three levels: `teaching`, `tenure track`, `tenured`.

```
m_bty_rank <- lm(score ~ bty_avg + rank, data = evals)
summary(m_bty_rank)
```

```
##
## Call:
## lm(formula = score ~ bty_avg + rank, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.8713 -0.3642  0.1489  0.4103  0.9525
##
## Coefficients:
##                  Estimate Std. Error t value Pr(>|t|)
## (Intercept)       3.98155    0.09078  43.860  < 2e-16 ***
## bty_avg           0.06783    0.01655   4.098 4.92e-05 ***
## ranktenure track -0.16070    0.07395  -2.173   0.0303 *
## ranktenured      -0.12623    0.06266  -2.014   0.0445 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5328 on 459 degrees of freedom
## Multiple R-squared:  0.04652,    Adjusted R-squared:  0.04029
## F-statistic: 7.465 on 3 and 459 DF,  p-value: 6.88e-05
```

The interpretation of the coefficients in multiple regression is slightly different from that of simple regression. The estimate for `bty_avg` reflects how much higher a group of professors is expected to score if they have a beauty rating that is one point higher *while holding all other variables constant*. In this case, that translates into considering only professors of the same rank with `bty_avg` scores that are one point apart.

The search for the best model

We will start with a full model that predicts professor score based on rank, gender, ethnicity, language of the university where they got their degree, age, proportion of students that filled out evaluations, class size, course level, number of professors, number of credits, average beauty rating, outfit, and picture color.

11. Which variable would you expect to have the highest p-value in this model? Why? *Hint:* Think about which variable would you expect to not have any association with the professor score.

The variable with highest p value is cls_prof because the number of professors teaching would not impact model since we are taking look at individual professor rating.

Let's run the model…

```
m_full <- lm(score ~ rank + gender + ethnicity + language + age + cls_perc_eval
             + cls_students + cls_level + cls_profs + cls_credits + bty_avg
             + pic_outfit + pic_color, data = evals)
summary(m_full)
```

```
##
## Call:
## lm(formula = score ~ rank + gender + ethnicity + language + age +
##     cls_perc_eval + cls_students + cls_level + cls_profs + cls_credits +
##     bty_avg + pic_outfit + pic_color, data = evals)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.77397 -0.32432  0.09067  0.35183  0.95036
##
## Coefficients:
##                         Estimate Std. Error t value Pr(>|t|)
## (Intercept)            4.0952141  0.2905277  14.096  < 2e-16 ***
## ranktenure track      -0.1475932  0.0820671  -1.798  0.07278 .
## ranktenured           -0.0973378  0.0663296  -1.467  0.14295
## gendermale             0.2109481  0.0518230   4.071 5.54e-05 ***
## ethnicitynot minority  0.1234929  0.0786273   1.571  0.11698
## languagenon-english   -0.2298112  0.1113754  -2.063  0.03965 *
## age                   -0.0090072  0.0031359  -2.872  0.00427 **
## cls_perc_eval          0.0053272  0.0015393   3.461  0.00059 ***
## cls_students           0.0004546  0.0003774   1.205  0.22896
## cls_levelupper         0.0605140  0.0575617   1.051  0.29369
## cls_profssingle       -0.0146619  0.0519885  -0.282  0.77806
## cls_creditsone credit  0.5020432  0.1159388   4.330 1.84e-05 ***
## bty_avg                0.0400333  0.0175064   2.287  0.02267 *
## pic_outfitnot formal  -0.1126817  0.0738800  -1.525  0.12792
## pic_colorcolor        -0.2172630  0.0715021  -3.039  0.00252 **
## ---
## Signif. codes:  0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 ’.’ 0.1 ’ ’ 1
##
## Residual standard error: 0.498 on 448 degrees of freedom
## Multiple R-squared:  0.1871, Adjusted R-squared:  0.1617
## F-statistic: 7.366 on 14 and 448 DF,  p-value: 6.552e-14
```

12. Check your suspicions from the previous exercise. Include the model output in your response.

```
sus_lm <- lm(score ~ cls_profs, data = evals)
summary(sus_lm)
```

```
##
## Call:
## lm(formula = score ~ cls_profs, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.8554 -0.3846  0.1154  0.4154  0.8446
##
## Coefficients:
##                 Estimate Std. Error t value Pr(>|t|)
## (Intercept)      4.18464    0.03111 134.493   <2e-16 ***
## cls_profssingle -0.02923    0.05343  -0.547    0.585
## ---
## Signif. codes:  0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 ’.’ 0.1 ’ ’ 1
##
## Residual standard error: 0.5443 on 461 degrees of freedom
## Multiple R-squared:  0.0006486,  Adjusted R-squared:  -0.001519
## F-statistic: 0.2992 on 1 and 461 DF,  p-value: 0.5847
```

We see that cls_profs is not a significant predictor of score and very low R squared indicates that the model is not a good fit. In addition, a small F statistic and high p value further indicates that this variable is not significant

13. Interpret the coefficient associated with the ethnicity variable.

The coefficient of ethnictiy in the model is 0.123 with p-value of 0.1169 which suggests that this varaible is nto a significable indicator for score

14. Drop the variable with the highest p-value and re-fit the model. Did the coefficients and significance of the other explanatory variables change? (One of the things that makes multiple regression interesting is that coefficient estimates depend on the other variables that are included in the model.) If not, what does this say about whether or not the dropped variable was collinear with the other explanatory variables?

```
another_lm <- lm(score ~ rank + gender + ethnicity + language + age + cls_perc_eval +
    cls_students + cls_level + cls_credits + bty_avg + pic_outfit + pic_color, data = evals)
summary(another_lm)
```

```
##
## Call:
## lm(formula = score ~ rank + gender + ethnicity + language + age +
##     cls_perc_eval + cls_students + cls_level + cls_credits +
##     bty_avg + pic_outfit + pic_color, data = evals)
##
## Residuals:
##     Min      1Q  Median      3Q     Max
## -1.7836 -0.3257  0.0859  0.3513  0.9551
##
## Coefficients:
##                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)           4.0872523  0.2888562  14.150  < 2e-16 ***
## ranktenure track     -0.1476746  0.0819824  -1.801 0.072327 .
## ranktenured          -0.0973829  0.0662614  -1.470 0.142349
## gendermale            0.2101231  0.0516873   4.065 5.66e-05 ***
## ethnicitynot minority  0.1274458  0.0772887   1.649 0.099856 .
## languagenon-english  -0.2282894  0.1111305  -2.054 0.040530 *
## age                  -0.0089992  0.0031326  -2.873 0.004262 **
## cls_perc_eval         0.0052888  0.0015317   3.453 0.000607 ***
## cls_students          0.0004687  0.0003737   1.254 0.210384
## cls_levelupper        0.0606374  0.0575010   1.055 0.292200
## cls_creditsone credit  0.5061196  0.1149163   4.404 1.33e-05 ***
## bty_avg               0.0398629  0.0174780   2.281 0.023032 *
## pic_outfitnot formal -0.1083227  0.0721711  -1.501 0.134080
## pic_colorcolor       -0.2190527  0.0711469  -3.079 0.002205 **
## ---
## Signif. codes:  0 ’***’ 0.001 ’**’ 0.01 ’*’ 0.05 ’.’ 0.1 ’ ’ 1
##
## Residual standard error: 0.4974 on 449 degrees of freedom
## Multiple R-squared:  0.187,  Adjusted R-squared:  0.1634
## F-statistic: 7.943 on 13 and 449 DF,  p-value: 2.336e-14
```

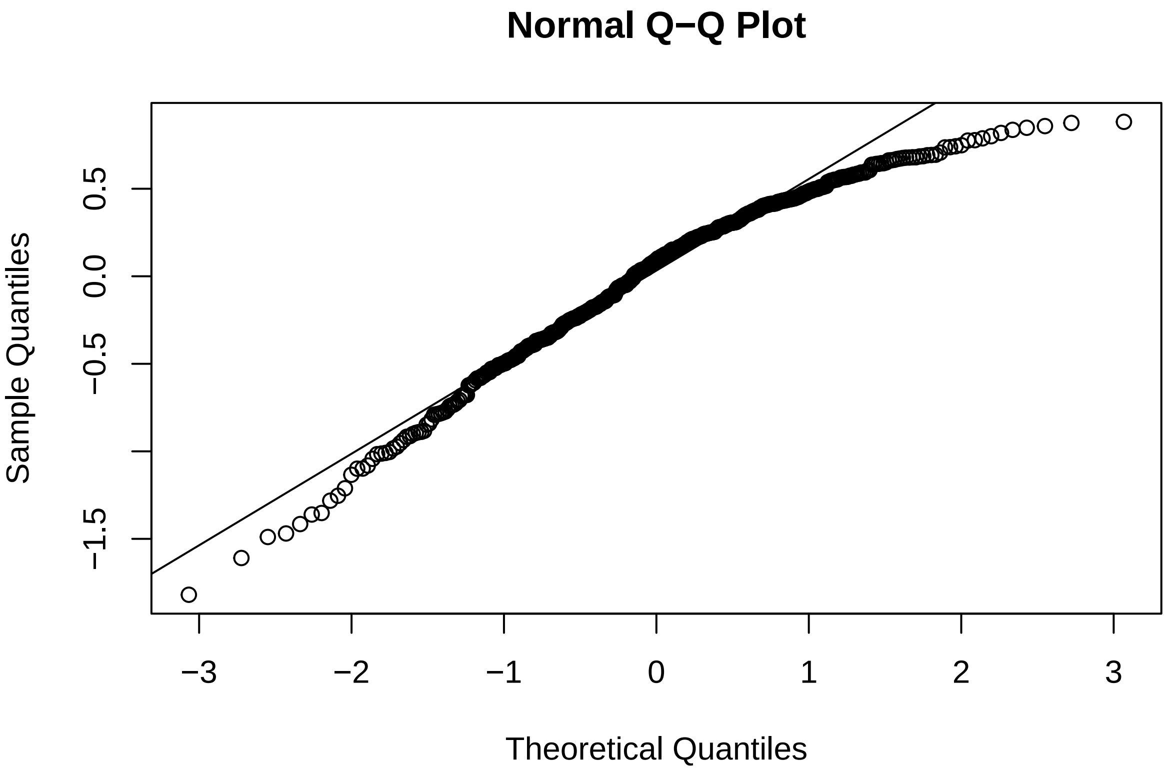
15. Using backward-selection and p-value as the selection criterion, determine the best model. You do not need to show all steps in your answer, just the output for the final model. Also, write out the linear model for predicting score based on the final model you settle on.

```
best_lm <- lm(score ~ gender + language + age + cls_perc_eval + cls_credits + bty_avg +
    pic_color, data = evals)
summary(best_lm)
```

```
##
## Call:
## lm(formula = score ~ gender + language + age + cls_perc_eval +
##     cls_credits + bty_avg + pic_color, data = evals)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.81919 -0.32035  0.09272  0.38526  0.88213
##
## Coefficients:
##                        Estimate Std. Error t value Pr(>|t|)
## (Intercept)            3.967255   0.215824  18.382  < 2e-16 ***
## gendermale             0.221457   0.049937   4.435 1.16e-05 ***
## languagenon-english   -0.281933   0.098341  -2.867  0.00434 **
## age                   -0.005877   0.002622  -2.241  0.02551 *
## cls_perc_eval          0.004295   0.001432   2.999  0.00286 **
## cls_creditsone credit  0.444392   0.100910   4.404 1.33e-05 ***
## bty_avg                0.048679   0.016974   2.868  0.00432 **
## pic_colorcolor        -0.216556   0.066625  -3.250  0.00124 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.5014 on 455 degrees of freedom
## Multiple R-squared:  0.1631, Adjusted R-squared:  0.1502
## F-statistic: 12.67 on 7 and 455 DF,  p-value: 6.996e-15
```

16. Verify that the conditions for this model are reasonable using diagnostic plots.

```
qqnorm(best_lm$residuals)
qqline(best_lm$residuals)
```



17. The original paper describes how these data were gathered by taking a sample of professors from the University of Texas at Austin and including all courses that they have taught. Considering that each row represents a course, could this new information have an impact on any of the conditions of linear regression?

It might have an impact on the regression becuase some courses will be taught by the same professors which may introduce bias in the analysis.

18. Based on your final model, describe the characteristics of a professor and course at University of Texas at Austin that would be associated with a high evaluation score.

The most significant characteritics of a professor to receive a high evaluation score are if they are black&white english speaking male teaching a multi credit course

19. Would you be comfortable generalizing your conclusions to apply to professors generally (at any university)? Why or why not?

It would be not be right to generalize conculsion infered from a sample from one universtiy and apply it to any universities without first taking samples from every other university. So no, I would not be comfortable generalizing this conclusions.