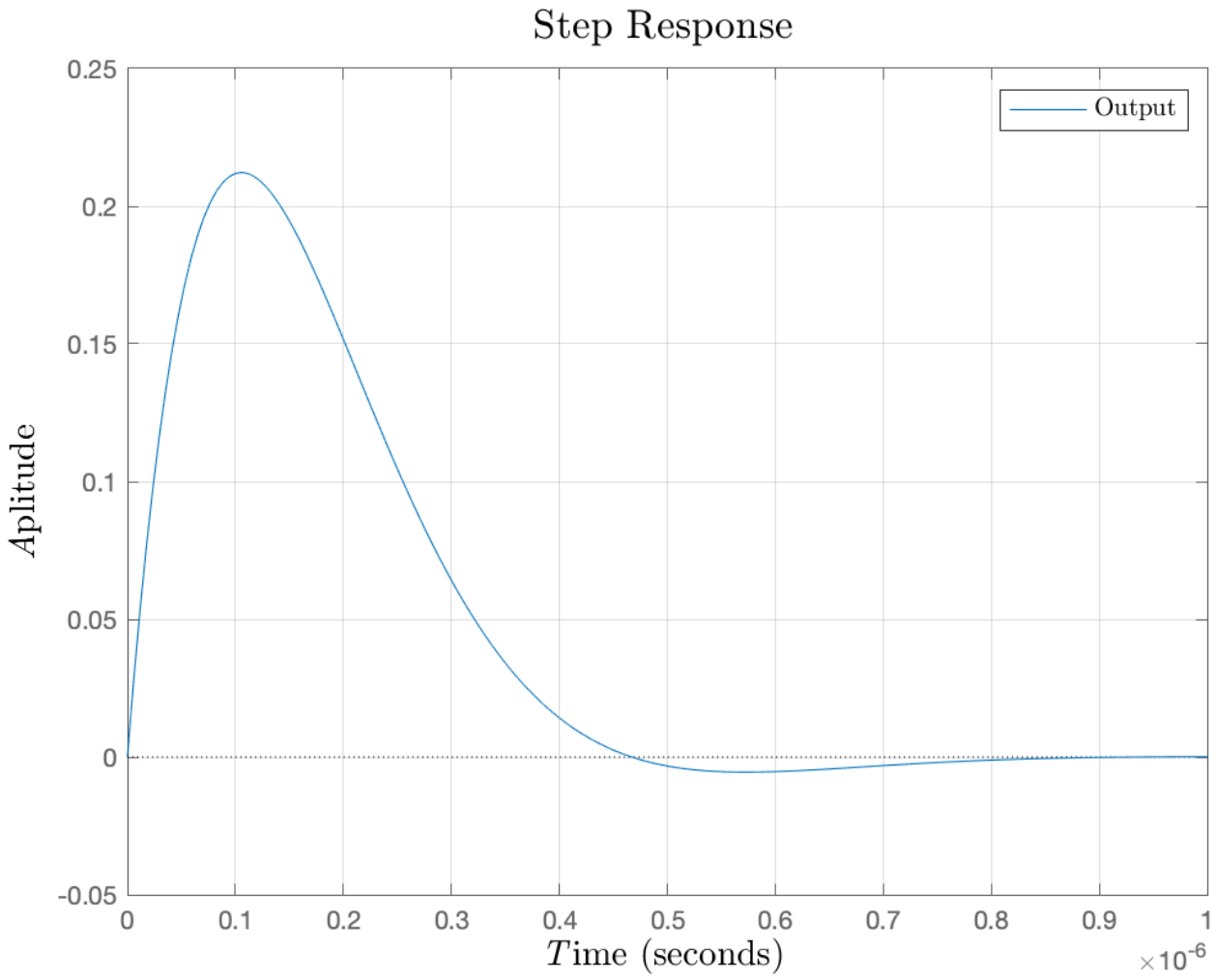
Answer 1A

Output $v_c(t)$ to an input step response $v(t)$:



MATLAB Code:

```
close all; clear; clc;
```

Given Parameters

```
% Resistance 1 [ohms]
R1 = 100000;
% Resistance 2 [Ohms]
R2 = 100000;

% Inductance [Henrys]
L = 4.7e-3;

% Capacitance [Farads]
C = .001e-9;
```

```
% Conductace [1 / Ohms]
% G = 1 / R
G1 = 1 / R1;
G2 = 1 / R2;
```

Transfer Function

```
% as^2 + bs + c = 0
% Normalizing the polynomial: s^2 + (b/a)s + (c/a) = 0
a = (G1 + G2);
b = ((G1 * G2 * L) + C) / (L * C);
c = G2 / (L * C);

% Numerator of tranfer function
numerator = [(G1 * G2 ) / C, 0];

% Denominator of transfer function
denominator = [a, b, c];

% Constructing transfer function
Gs = tf(numerator, denominator);

% Creating new figure
figure(1)

% Ploting the step responce of the system
step(Gs);

% Plot parameters
hold on
grid on
grid minor

% Plot descriptors
xlabel('{\emph Time}','fontsize',14,'Interpreter','latex');
ylabel('{\emph Aplitude }','fontsize',14,'Interpreter','latex');
title('{Step Response}','fontsize',16,'Interpreter','latex');
legend('Output', 'fontsize', 10, 'Interpreter', 'latex');
```

Answer 1B

Characteristic parameters of the system:

Natural frequency:

$$\omega_n = \sqrt{\frac{\frac{G_2}{LC}}{G_1 + G_2}}$$

$$\omega_n = 1.0314 \times 10^7\ Hz$$

Damping ratio:

$$\zeta = \frac{x}{2 * \omega_n}$$

Where: $x = \frac{\frac{(G1*G2*L)+C}{L*C}}{G_1+G_2}$

$$\zeta = 0.7581$$

Since $\zeta < 1$, the system is underdamped.

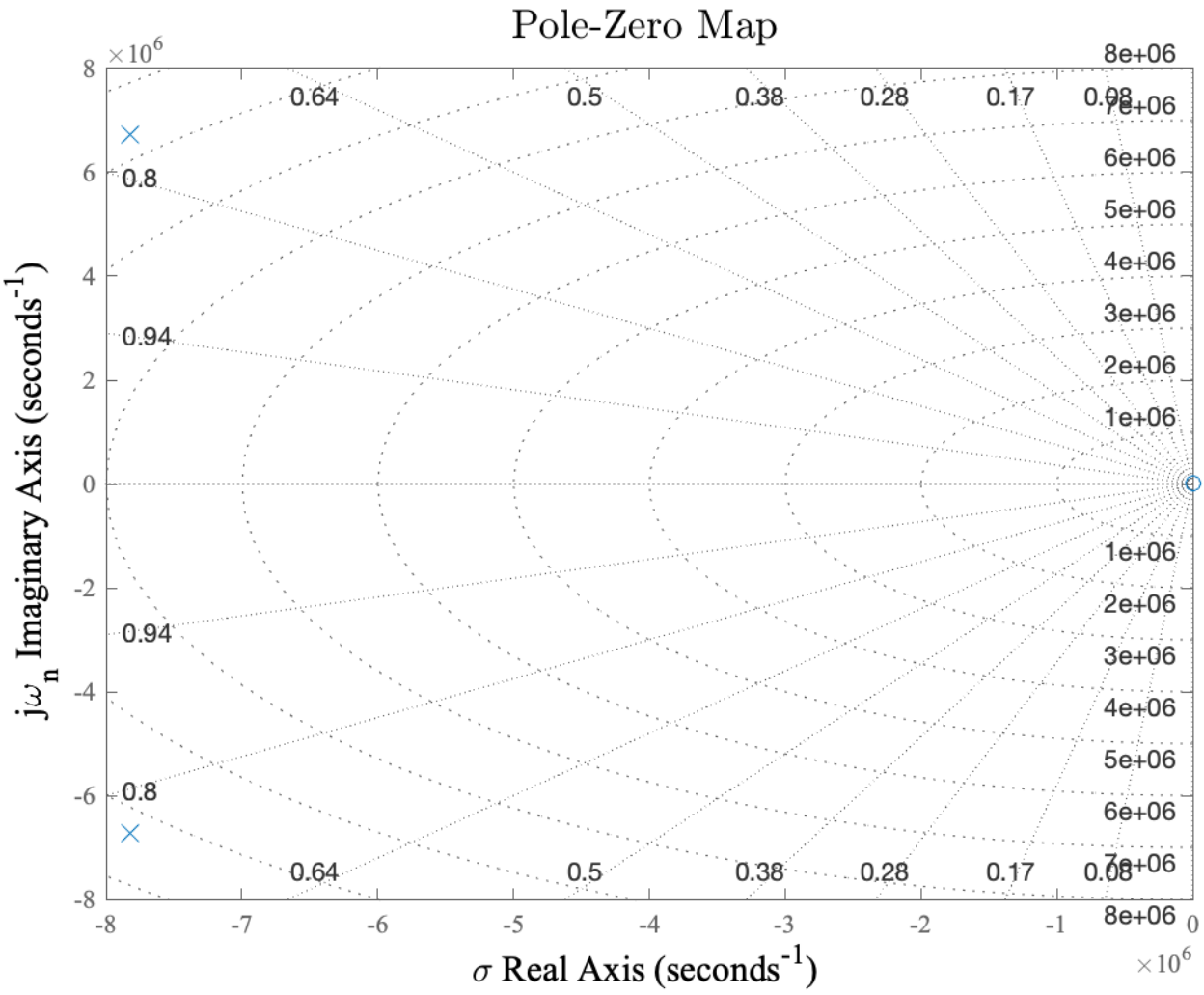
MATLAB Code:

Characteristic Parameters

```
% Natural frequency [Hz]
wn = sqrt(c/a);

% x = 2 * zeta * wn
x = b / a;

% Daming ratio
zeta = x / (2 * wn);
```

Note that this section of code is dependent on the previous section of code.

Answer 1C



System Response:

$s_1 = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$

$s_1 = [-7.8191 + 6.7264i] \times 10^6$

$s_2 = -\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}$

$s_2 = [-7.8191 - 6.7264i] \times 10^6$

The poles are complex, with a negative real portion. There is a fast rise time, but it oscillates and is slow to converge. Despite this, it eventually converges and is therefore stable.

MATLAB Code:

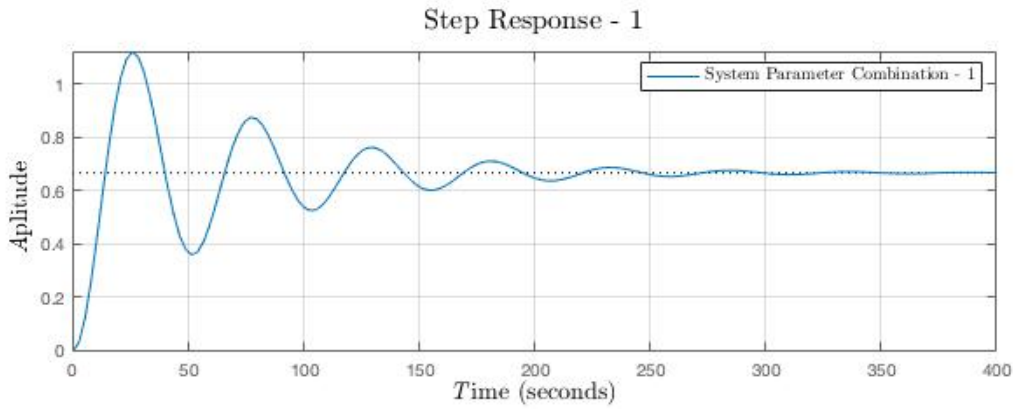
System Response

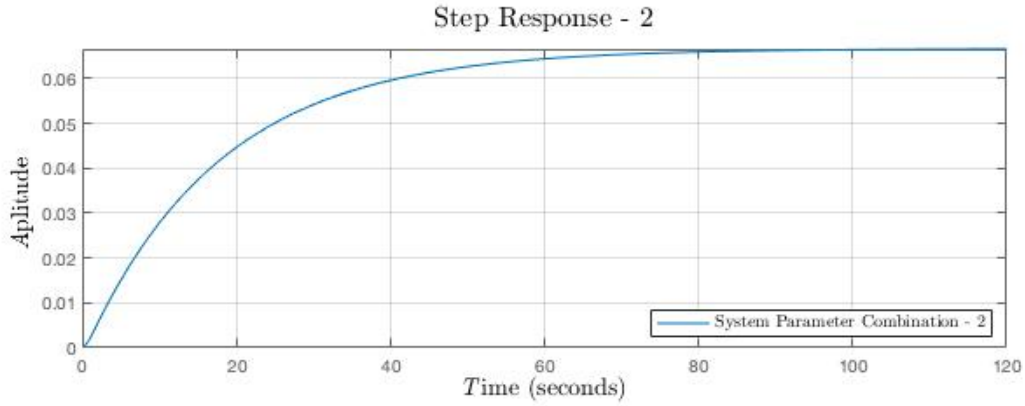
```
% Solving for roots of characteristic polynomial
characteristicRoots = roots(denominator);

% Creating new figure
figure(2)

pzplot(Gs)
hold on
grid on

% Plot descriptors
set(gca, 'fontName', 'Times');
xlabel('\sigma Real Axis', 'fontName', 'Times', 'fontSize', 14);
ylabel('j\omega_{n} Imaginary Axis', 'fontName', 'Times', 'fontSize', 14);
title('{Pole-Zero Map}','fontsize',16,'Interpreter','latex');
```

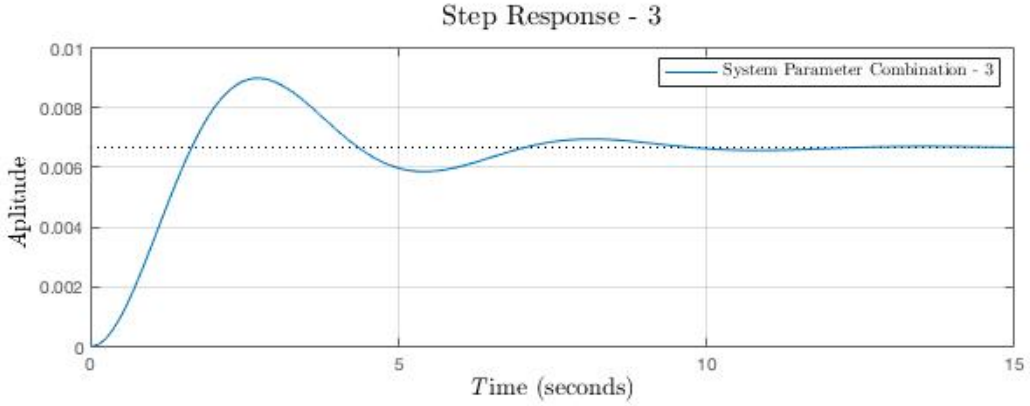
Note that this section of code is dependent on the previous section of code.

Answer 2A

Output $v_c(t)$ to an input step response $v(t)$ for three parameter combinations:







MATLAB Code:

```
close all; clear; clc;
```

Given Parameters

```
% Mass [kg]
M = 100;

% K [N/m]
K = [1.5 15 150];

% fv [kg/s]
fv = [3 270 78];
```

Constructing Transfer Function and Plotting Output

```
% Creating new figure
figure(1)

% Numerator of transfer function
% Constant for all cases
numerator = 1/M;


for ii = 1:length(K)

    % Denominator of transfer function for all 3 cases
    denominator = [1 fv(ii)/M K(ii)/M];

    % Generating transfer function
    Gs = tf(numerator, denominator);

    % Creating subplot
    subplot(3, 1, ii)

    % Plotting the step response of the system
    step(Gs)
    hold on

    % Plot parameters
    grid on
    grid minor

    % Plot descriptors
    xlabel('{\emph Time}','fontsize',14,'Interpreter','latex');
    ylabel('{\emph Aplitude }','fontsize',14,'Interpreter','latex');
    titleText = sprintf('Step Response - %d', ii);
    title(titleText,'fontsize',16,'Interpreter','latex');
    leg = sprintf('System Parameter Combination - %d', ii);

    % Placing legend in southeast corner for cobination 2
    if ii == 2

    legend(leg, 'fontsize', 10, 'location', 'southeast','Interpreter', ...
        'latex')

    % Placing legend in northeast corner for combination 1 & 3
    else
```

```
        legend(leg, 'fontsize', 10, 'location', 'northeast','Interpreter', ...
        'latex')

    end

end
```

Answer 2B

Characteristic parameters of the system:

$$\omega_n = \sqrt{\frac{K}{M}}$$

$$\zeta = \frac{x}{2 * \omega_n}$$

Where: $x = 2 * \zeta * \omega_n$

Case 1:

$\omega_n = .1225\ Hz$

$\zeta = .1225$

Since $\zeta < 1$, the system is underdamped.

Case 2:

$\omega_n = .3873\ Hz$

$\zeta = 3.4857$

Since $\zeta > 1$, the system is overdamped.

Case 3:

$\omega_n = 1.2247\ Hz$

$\zeta = .3184$

Since $\zeta < 1$, the system is underdamped.

MATLAB Code:

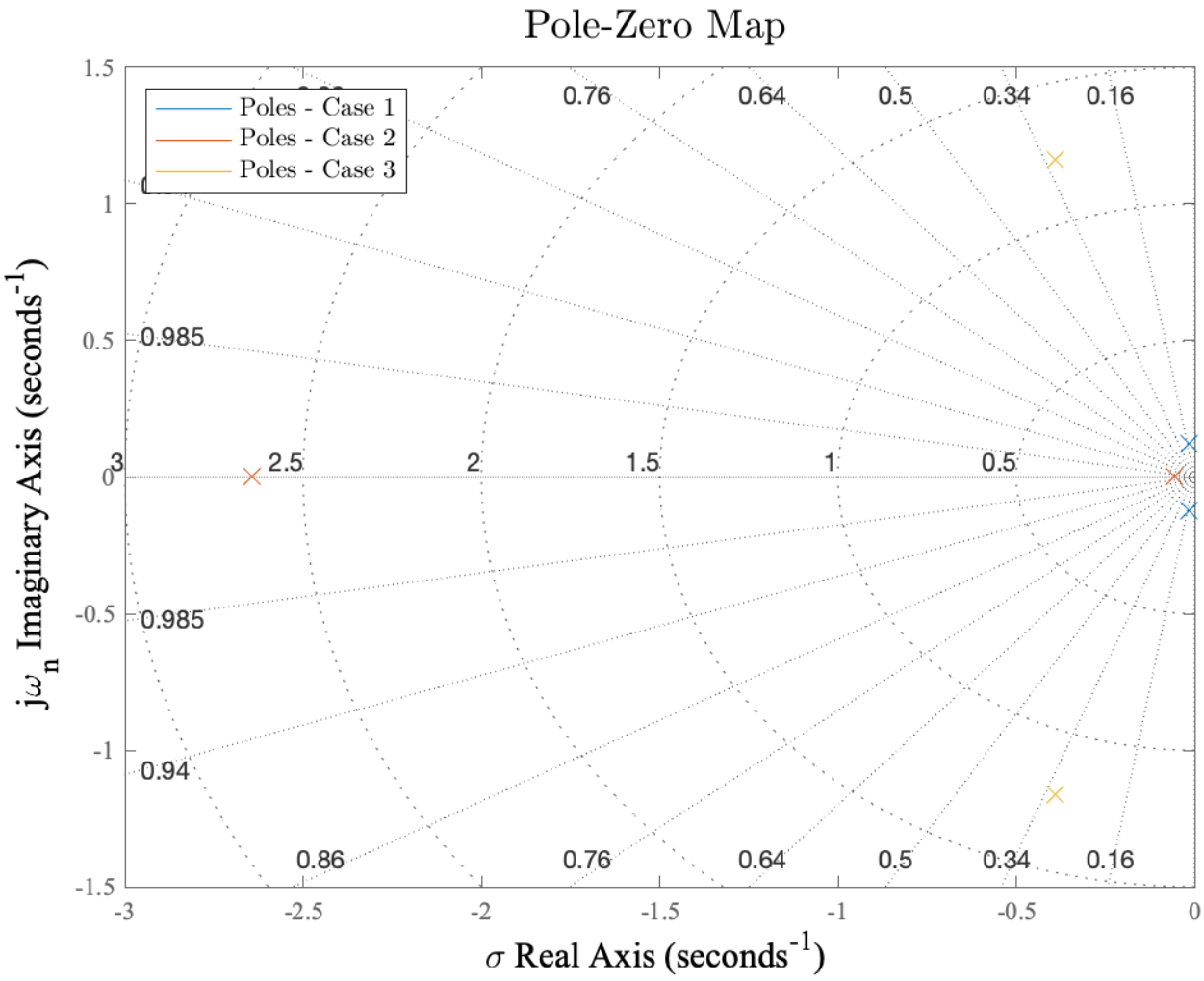
Characteristic Parameters

```
% Initializing vectors
wn = zeros(1, length(K));
x = zeros(1, length(K));
zeta = zeros(1, length(K));

for ii = 1: length(K)

    % Natural frequency
    wn(ii) = sqrt(K(ii) / M);

    % x = 2 * zeta * wn
    x(ii) = fv(ii) / M;
```

```
    zeta(ii) = x(ii) / (2 * wn(ii));
end
```

Note that this section of code is dependent on the previous section of code.

Answer 2C



Case 1:

$s_1 = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$

$s_1 = -.015 + .1216i$

$s_2 = -\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}$

$s_2 = -.015 - .1216i$

The poles are complex, with a negative real portion. There is a fast rise time, but it oscillates and is slow to converge. Despite this, it eventually converges and is therefore stable.

Case 2:

$s_1 = -\zeta\omega_n + \omega_n\sqrt{1-\zeta^2}$

$s_1 = -2.6433$

$s_2 = -\zeta\omega_n - \omega_n\sqrt{1-\zeta^2}$

$s_2 = -.0567$

The poles are negative on the real axis and have no imaginary part. This response is slow to converge but is still considered steady.

Case 3:

$s_1 = -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$

$s_1 = -.39 + 1.161i$

$s_2 = -\zeta\omega_n - j\omega_n\sqrt{1-\zeta^2}$

$s_2 = -.39 - 1.161i$

The poles are complex, with a negative real portion. There is a fast rise time, but it oscillates and is slow to converge. Despite this, it eventually converges and is therefore stable.

MATLAB Code:

System Response

```
% Creating new figure
figure(2)

% Initializing arrays
characteristicRoots = zeros(length(K), 2);

% Plotting poles
for ii = 1:length(K)

    % Denominator of transfer function for all 3 cases
    denominator = [1 fv(ii)/M K(ii)/M];

    % Generating transfer function
    Gs = tf(numerator, denominator);

    % Solving for roots of characteristic polynomial
    polynomialRoots = roots(denominator);

    % Storing roots
    characteristicRoots(ii, 1) = polynomialRoots(1, 1);
    characteristicRoots(ii, 2) = polynomialRoots(2, 1);

    % Creating pole - zero plot
    pzplot(Gs);

    hold on

end
```

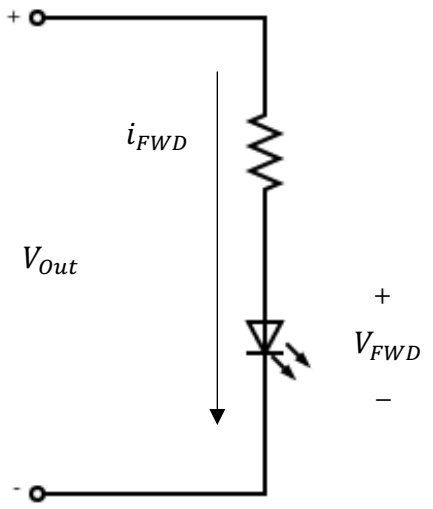
```
% Turning grid on
grid on

% Plot descriptors
set(gca, 'fontName', 'Times');
xlabel('\sigma Real Axis', 'fontName', 'Times', 'fontSize', 14);
ylabel('j\omega_{n} Imaginary Axis', 'fontName', 'Times', 'fontSize', 14);
title('{Pole-Zero Map}','fontsize',16,'Interpreter','latex');
legend('Poles - Case 1', 'Poles - Case 2', 'Poles - Case 3', ...
       'fontsize', 10, 'Interpreter', 'latex', 'location', 'northwest');
```

Note that this section of code is dependent on the previous section of code.

Answer 3A

Resistors R_2 & R_3 are current limiting resistors for the LEDs in the circuit. Their purpose is to limit the forward current into the diodes since they have a certain operating region. The forward current (typical and surge) as well as the forward voltage (typical and maximum) for 5mm red and green diodes can be seen in the table below.

	Red LED	**Green LED**
$\boldsymbol{i_{FWD}}$	$20mA$	$30mA$
$\boldsymbol{i_{FWD-Surge}}$	$1A$	$1A$
$\boldsymbol{V_{FWD}}$	$2V$	$2V$
$\boldsymbol{V_{FWD-MAX}}$	$3V$	$3V$



Using KVL, the output voltage is represented by the following equation:

$$V_{Out} = V_R + V_{FWD}$$

Rearranging this and solving for the voltage across the resistor, the equation becomes:

$$V_R = V_{Out} - V_{FWD}$$

From Ohm's law, the voltage across the resistor is represented by:

$$V_R = Ri_{FWD}$$

Substituting the equations and solving for R, the equation becomes:

$$R = \frac{V_{Out} - V_{FWD}}{I_{FWD}}$$

The values for R_2 & R_3 are as follows:

Resistance	
Red LED	**Green LED**
$2\Omega < R_2 < 150\Omega$	$2\Omega < R_3 < 100\Omega$

Resistors R_4& R_5 serve as pull down resistors. They ensure that when the switch is open, despite any leakage current from the Arduino micro controller, the voltage will be zero. The leakage current is assumed to be 30% of the supply voltage. Ohm's law on these resistors is as follows:

$$.3V_C = i_{Leak}R_{Pull-down}$$

Rearranging and solving for the pull-down resistance, the equation becomes:

$$R_{Pull-down} = \frac{.3V_C}{i_{Leak}}$$

With $V_C = 5V$ and $i_{Leak} = 1mA$, the pull-down resistance is:

$$R_{Pull-down} = 1.5k\Omega$$

This resistance is the maximum allowable resistance for the desired voltage. Because of this, a resistance value less that $R_{Pull-down}$ should be used in the circuit.

MATLAB Code:

```
close all; clear; clc

% LED Values
LED(1).color = 'RED';
LED(1).FWDcurrent = [20 * 10 ^ -3, 1];
LED(1).FWDvoltage = [2 3];

LED(2).color = 'GREEN';
LED(2).FWDcurrent = [30 * 10 ^ -3, 1];
LED(2).FWDvoltage = [2 3];

Vout = 5;
Vc = 5;
Ileak = 1 * 10 ^ -3;
```

Solving for resistances R2 & R3

```
% Initializing arry
R = zeros(length(LED(1).FWDcurrent), ...
    length(LED(1).FWDvoltage), ...
    length(LED));
% R(:,:,1) = R2
% R(:,:,2) = R3

for ii = 1:length(LED)
    for jj = 1:length(LED(1).FWDcurrent)
        for kk = 1:length(LED(1).FWDvoltage)

            R(jj, kk, ii) = (Vout - LED(ii).FWDvoltage(kk)) / ...
                LED(ii).FWDcurrent(jj);

        end
    end
end
```

Solving for pull-down resistance (R4 & R5)

```
RpullDown = (.3 * Vc) / Ileak;
```

Answer 3B

IO:

Component	Classification	Signal Type	Pin
Switch 1	Input	Digital	7
Switch 2	Input	Digital	8
Motor	Output	Analog	6
Red LED	Output	Digital	2
Green LED	Output	Digital	3

Both switches are digital inputs that are either HIGH (depressed) or LOW (not depressed). They dictate the operation of the DC motor.

The DC motor is an analog output. In this case since no duty cycle is specified, the motor is off with a PWM of zero and on with a PWM of 255.

Both LEDs are digital outputs. An LED that is on receives a HIGH signal, and an LED that is off receives a LOW signal.

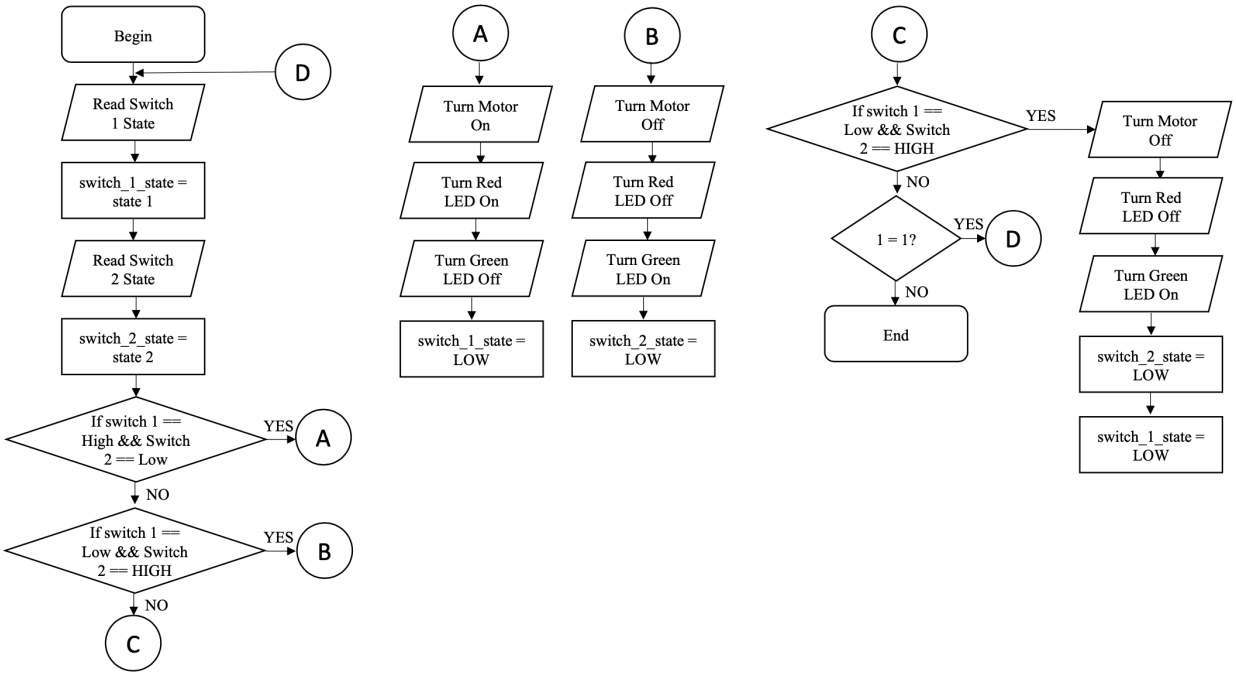
Answer 3C

Arduino Sketch:

```
# define switch_1 7
# define switch_2 8
# define RED_LED 2
# define GREEN_LED 3
# define MOTOR 6

// Initializing both pushbottons in the off position
int switch_1_state = LOW;
int switch_2_state = LOW;

void setup() {

  // Defining IO
  pinMode(switch_1, INPUT);
  pinMode(switch_2, INPUT);
  pinMode(RED_LED, OUTPUT);
  pinMode(GREEN_LED, OUTPUT);
  pinMode(MOTOR, OUTPUT);

}

void loop() {

  // Determining the state of both swithces
  switch_1_state = digitalRead(switch_1);
  switch_2_state = digitalRead(switch_2);

  // IF the first swithc is pressed
  if (switch_1_state == HIGH && switch_2_state == LOW) {

    // Turn on the motor
    digitalWrite(MOTOR, HIGH);

    // Turn red LED on and green LED off
    digitalWrite(RED_LED, HIGH);
    digitalWrite(GREEN_LED, LOW);

    // Set state of swich 1 to off
    switch_1_state == LOW;

  }

  // If swith 2 is pressed
  else if (switch_2_state == HIGH && switch_1_state == LOW) {

    // Turn the motor off
    analogWrite(MOTOR, 0);

    // Turn the red LED off and green LED on
    digitalWrite(RED_LED, LOW);
    digitalWrite(GREEN_LED, HIGH);
```

```
    // Set the state of switch 2 back to 0
    switch_2_state == LOW;

  }

  // If both buttons are pressed simultaneously
  else{

    // Turn the motor off
    analogWrite(MOTOR, 255);

    // Turn the red LED off and the green LED on
    digitalWrite(RED_LED, LOW);
    digitalWrite(GREEN_LED, HIGH);

    // Set the state of both swithces to 0
    switch_1_state == LOW;
    switch_2_state == LOW;

  }

}
```

Flow Diagram:



Answer 3D

In this system, there are two pushbuttons, two pull-down resistors, two LEDs (red and green), two current limiting resistors, a 5V voltage source, an Arduino Uno microcontroller, and a DC motor. The Arduino input pin has a leakage current of around 1mA causing a voltage in circuit (around 30% of the supply voltage); including these two pull-down resistors ensures that when the switch is open, there is zero voltage in the circuit. The current limiting resistors, ensure that the LEDs are within viable operating conditions.

In terms of the operation of the system, the two pushbuttons act as switches for the motor. When the first switch is pressed, the motor starts. When the second pushbutton is pressed, the motor stops. In the case that both pushbuttons are simultaneously pressed, the motor will not start. Whenever the motor is operational, the red LED is on, and the green LED is off; this is reversed when the motor is not operational. The code is based on the state of the switches with conditional statements included to satisfy the execution of the tasks.

Answer 4A

IO:

Component	Classification	Signal Type	Pin
Potentiometer	Input	Analog	A1
Thermistor	Input	Analog	A0
Heater	Output	Analog	3
Red LED	Output	Digital	2
Green LED	Output	Digital	4

The potentiometer is an analog input that has a range of 0 – 1023. In the code, these values are mapped to a range of 0 – 100 allowing for a comparison with the temperature received from the thermistor.

The thermistor is also an analog input. This contains the temperature in $°C$, which is then converted to $°F$. This value is also mapped from 50 – 100 to 0 – 100.

The heater is a digital output. Based on the if statement, the heater will either receive a HIGH or LOW command, dictating whether it is on or off.

The red and green LEDs also operate in the same manner. Based on certain conditions, the red and green LEDs will receive a HIGH command for on or a LOW command for off as they are digital outputs.

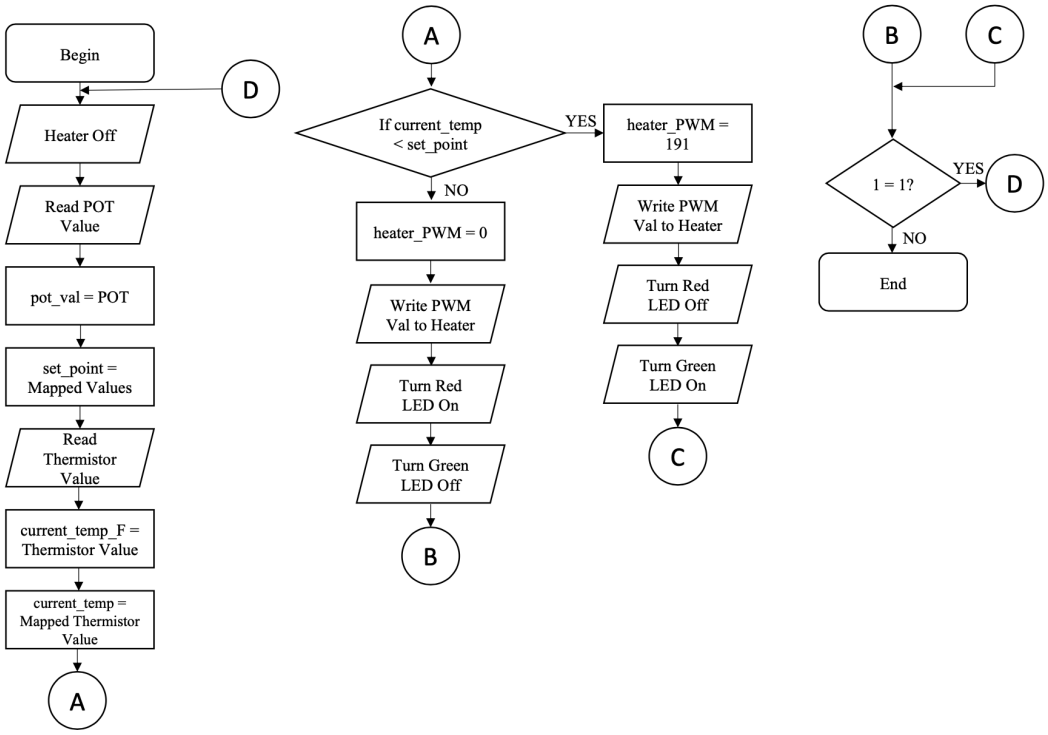
Answer 4B

Arduino Sketch:

```
#define POTENTIOMETER A1  // Defining pin A1 as the potentioketer
#define THERMISTOR A0     // Defining A0 as the thermistor
#define HEATER 3          // Defining pin 3 as the heater
#define RED_LED 2         // Defining pin 2 as the red LED
#define GREEN_LED 4       // Defining pin 4 as the green LED

// Defining integers
int pot_val;
int set_point;

// Defining floats
float current_temp;
float current_temp_F;

// Initializing the heater PWM
int heater_PWM = 0;

void setup() {

  // Defining IO
  pinMode(POTENTIOMETER, INPUT);
  pinMode(THERMISTOR, INPUT);
  pinMode(HEATER, OUTPUT);
  pinMode(RED_LED, OUTPUT);
  pinMode(GREEN_LED, OUTPUT);

 // Initialiing the heater as off
  analogWrite(HEATER, heater_PWM);


}

void loop() {


  // Reading the potentiometer value
  pot_val = analogRead(POTENTIOMETER);

  // Mapping the pot values
  set_point = map(pot_val, 0, 1023, 0, 100);

  // Reading the thermistor and coverting to Farenheit
  current_temp_F = (analogRead(THERMISTOR) * (9 / 5)) + 32;

  // Mapping the thermistor values
  current_temp = map(current_temp_F, 50, 100, 0, 100);

  // If the current temperature is lower than the setpoint
  if (current_temp < set_point) {

    // Heater PWM is set to a duty cycle of 75%
```

```
    // Max PWM value is 255
    heater_PWM = 191;

    // Heater is written the PWM value
    analogWrite(HEATER, heater_PWM);

    // Red LED is turned on
    digitalWrite(RED_LED, HIGH);

    // Green LED is turned off
    digitalWrite(GREEN_LED, LOW);
  }

  // If the current temperature is greater than or equal to the setpoint
  else if (current_temp >= set_point) {

    // The heater PWM is set to 0
    heater_PWM = 0;

    // The heater is turned off (PWM = 0)
    analogWrite(HEATER, heater_PWM);

    // Red LED is turned off
    digitalWrite(RED_LED, LOW);

    // Green LED is turned on
    digitalWrite(GREEN_LED, HIGH);
  }

}
```

Flow Diagram:


Begin
D
Heater Off
Read POT Value
pot_val = POT
set_point = Mapped Values
Read Thermistor Value
current_temp_F = Thermistor Value
current_temp = Mapped Thermistor Value
A
A
If current_temp < set_point
YES
heater_PWM = 191
Write PWM Val to Heater
Turn Red LED Off
Turn Green LED On
C
NO
heater_PWM = 0
Write PWM Val to Heater
Turn Red LED On
Turn Green LED Off
B
B
C
1 = 1?
YES
D
NO
End

Answer 4C

The components involved in this system consisted of a potentiometer, thermistor, two LEDs (red and green), two current limiting resistors, a 5V voltage source, and an Arduino Uno microcontroller. The potentiometer is used to control the temperature setpoint. Since the Arduino reads analog data on a scale from 0 – 1023, these values must be mapped to 0 – 100. This value is compared with a temperature reading from a thermistor. The thermistor reads the temperature in degrees Celsius; this value is converted to degrees Fahrenheit and mapped from an input range of 50 - 100° F to 0 – 100. This allows for an equal comparison of the temperature read from the thermistor and the set temperature.

When the temperature is less than the set temperature, the heater operates with a duty cycle of 75%; under this condition, the red LED is on, and the green LED is off. When the temperature is greater than or equal to the set temperature, the heater stops, and the red LED turns off while the green LED turns on.