In [1]:

```
import numpy as np
import dolfin as dl
from ufl import lhs, rhs, replace
import scipy.sparse as sps
import scipy.sparse.linalg as spla
import scipy.linalg as sla
import matplotlib.pyplot as plt
from pathlib import Path
from nalger_helper_functions import *
import meshio
import sys
from scipy.spatial import KDTree as scipy_KDTree
from scipy.optimize import root_scalar
from localpsf import localpsf_root
from localpsf.newtoncg import newtoncg_ls, cgsteihaug
from nalger_helper_functions import *

import hlibpro_python_wrapper as hpro
from localpsf.product_convolution_kernel import ProductConvolutionKernel
from localpsf.product_convolution_hmatrix import make_hmatrix_from_kernel
from localpsf.derivatives_at_point import StokesDerivativesAtPoint
from localpsf.bilaplacian_regularization import BiLaplacianRegularization
```

Options

In [2]:

```
# mesh_type = 'coarse'
# mesh_type = 'medium'
mesh_type = 'fine'
noise_level=1e-2
relative_prior_correlation_length=0.1
save_plots=True
gamma=1e4 # regularization gamma
m0_constant_value = 1.5 * 7.
rel_correlation_length = 0.1
reg_robin_bc=True
# mtrue_type = 'aces_building'
mtrue_type = 'angel_peak'
# mtrue_type = 'sinusoid'
# mtrue_string = 'm0 - (m0 / 7.)*std::cos((x[0]*x[0]+x[1]*x[1])*pi/(Radius*Radiu
mtrue_string = 'm0 - (m0 / 7.)*std::cos(2.*x[0]*pi/Radius)'
solver_type = 'mumps'
# solver_type = 'default'
# solver_type = 'petsc'
# solver_type = 'umfpack'
# solver_type = 'superlu'
outflow_constant = 1.0e6

num_batches = 5 # number of batches used for Newton solves
tau = 3.0
num_neighbors = 10
hmatrix_tol = 1e-6
newton_rtol = 1e-8
# gamma_morozov = 15428.012899090285 # medium mesh, noise_level=1e-2
# gamma_morozov= 6932.714344119822 # fine mesh, noise_level=1e-2
# gamma_morozov= 6975.784285975133 # fine mesh, noise_level=1e-2
gamma_morozov = None
```

```
forcing_sequence_power = 0.5 # 1.0

run_finite_difference_checks = False
check_gauss_newton_hessian = False

all_num_batches = [1, 5, 25] # number of batches used for spectral comparisons
```

In [3]:

```
save_dir = localpsf_root / 'numerical_examples' / 'stokes' / 'data'
save_dir.mkdir(parents=True, exist_ok=True)
save_dir_str = str(save_dir)
```

Mesh

Load unmodified mesh

In [4]:

```
if mesh_type == 'coarse':
    mfile_name = str(localpsf_root) + "/numerical_examples/stokes/meshes/cylinde
    lam = 1e10
elif mesh_type == 'medium':
    mfile_name = str(localpsf_root) + "/numerical_examples/stokes/meshes/cylinde
    lam = 1e11
elif mesh_type == 'fine':
    mfile_name = str(localpsf_root) + "/numerical_examples/stokes/meshes/cylinde
    lam = 1e12
else:
    raise RuntimeError('invalid mesh type '+mesh_type+', valid types are coarse,

mesh = dl.Mesh(mfile_name+".xml")

mesh
```

Out[4]:

Options

Menu Options

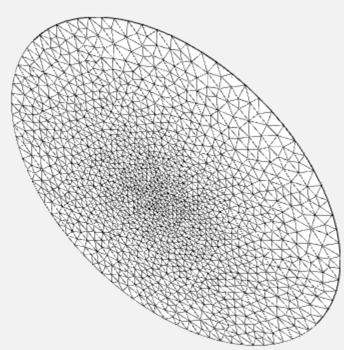
Summary

Boundary submeshes and subdomains

In [5]:

```
class BasalBoundary(dl.SubDomain):
    def inside(me, x, on_boundary):
        return dl.near(x[2], 0.) and on_boundary

class BasalBoundarySub(dl.SubDomain):
    def inside(me, x, on_boundary):
        return dl.near(x[2], 0.)

class TopBoundary(dl.SubDomain):
    def __init__(me, Height):
        me.Height = Height
        dl.SubDomain.__init__(me)
    def inside(me, x, on_boundary):
        return dl.near(x[1], me.Height) and on_boundary

boundary_markers = dl.MeshFunction("size_t", mesh, mfile_name+"_facet_region.xml
boundary_mesh = dl.BoundaryMesh(mesh, "exterior", True)
basal_mesh3D = dl.SubMesh(boundary_mesh, BasalBoundarySub())

basal_mesh3D
```

Out[5]:



Options

Menu Options

Summary

Modify mesh

In [6]:

```
r0 = 0.05
sig = 0.4
valleys = 4
valley_depth = 0.35
bump_height = 0.2
```

```
min_thickness = 0.08 / 8.
avg_thickness = 0.2 / 8.
theta = -np.pi / 2.
max_thickness = avg_thickness + (avg_thickness - min_thickness)
A_thickness = max_thickness - avg_thickness

dilitation = 1.e4
Length = 1.
Width = 1.
Length *= 2 * dilitation
Width *= 2 * dilitation
Radius = dilitation

prior_correlation_length = relative_prior_correlation_length * Radius

coords = mesh.coordinates()
bcoords = boundary_mesh.coordinates()
subbcoords = basal_mesh3D.coordinates()
coord_sets = [coords, bcoords, subbcoords]

def topography(r, t):
    zero = np.zeros(r.shape)
    R0 = r0 * np.ones(r.shape)
    return bump_height * np.exp(-(r / sig) ** 2) * (
            1. + valley_depth * np.sin(valleys * t - theta) * np.fmax(zero,

def depth(r, t):
    zero = np.zeros(r.shape)
    R0 = r0 * np.ones(r.shape)
    return min_thickness - A_thickness * np.sin(valleys * t - theta) * np.exp(
        -(r / sig) ** 2) * np.fmax(zero, (r - R0) / sig)

for k in range(len(coord_sets)):
    for i in range(len(coord_sets[k])):
        x, y, z = coord_sets[k][i]
        r = np.sqrt(x ** 2 + y ** 2)
        t = np.arctan2(y, x)
        coord_sets[k][i, 2] = depth(r, t) * z + topography(r, t)
        coord_sets[k][i] *= dilitation

mesh
```

Out[6]:

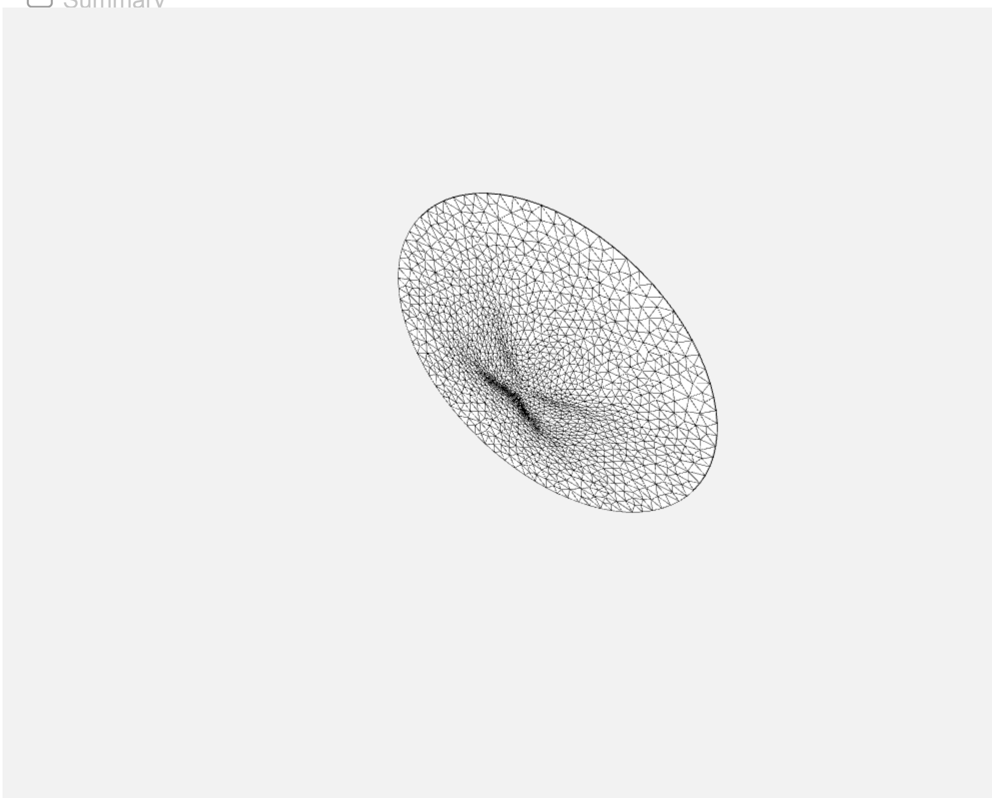
Options

In [7]:

```
basal_mesh3D
```

Summary

Out[7]:



Options

2D Basal mesh

Menu Options

Summary

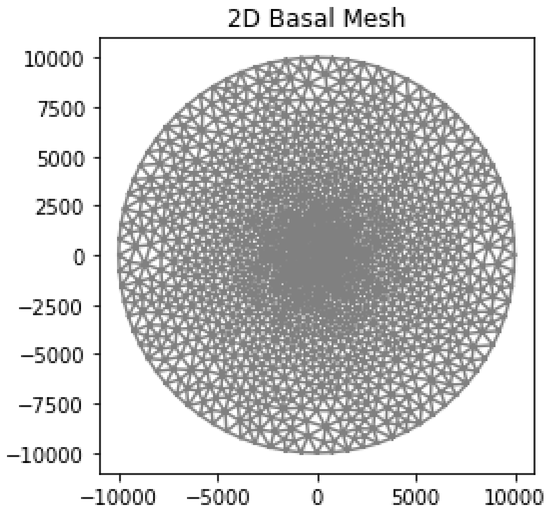
In [8]:

```
# ------ generate 2D mesh from 3D boundary subset mesh
coords = basal_mesh3D.coordinates()[:, :2]
cells = [("triangle", basal_mesh3D.cells())]
mesh2D = meshio.Mesh(coords, cells)
mesh2D.write("mesh2D.xml")
basal_mesh2D = dl.Mesh("mesh2D.xml")

dl.plot(basal_mesh2D)
plt.title('2D Basal Mesh')
```

```
WARNING:root:DOLFIN XML is a legacy format. Consider using XDMF instead.
```

Out[8]:

```
Text(0.5, 1.0, '2D Basal Mesh')
```



Plots to make (1):

- 3D mesh, colors, light mesh edges
- 2D mesh top-view, edges only, black and white

In [9]:

```
# Save mesh
dl.File(save_dir_str + "/ice_mountain_mesh3D.xml") << mesh
dl.File(save_dir_str + "/ice_mountain_mesh_base3D.xml") << basal_mesh3D
dl.File(save_dir_str + "/ice_mountain_mesh_base2D.xml") << basal_mesh2D
```

Function spaces

In [10]:

```
P1 = dl.FiniteElement("Lagrange", mesh.ufl_cell(), 1)
P2 = dl.VectorElement("Lagrange", mesh.ufl_cell(), 2)
TH = P2 * P1

Zh = dl.FunctionSpace(mesh, TH)                          # Zh:  state, also adjoint
Wh = dl.FunctionSpace(mesh, 'Lagrange', 1)               # Wh:  parameter, full 3D do
Vh3 = dl.FunctionSpace(basal_mesh3D, 'Lagrange', 1) # Vh3: parameter, 2d basal m
Vh2 = dl.FunctionSpace(basal_mesh2D, 'Lagrange', 1) # Vh2: parameter, 2d basal m
Xh = [Zh, Wh, Zh]                                        # Xh:  (state, parameter ful
```

Transfer operators between function spaces

In [11]:

```
def function_space_prolongate_numpy(x_numpy, dim_Yh, inds_Xh_in_Yh):
    y_numpy = np.zeros(dim_Yh)
    y_numpy[inds_Xh_in_Yh] = x_numpy
    return y_numpy

def function_space_restrict_numpy(y_numpy, inds_Xh_in_Yh):
    return y_numpy[inds_Xh_in_Yh].copy()

def function_space_prolongate_petsc(x_petsc, Yh, inds_Xh_in_Yh):
    y_petsc = dl.Function(Yh).vector()
    y_petsc[:] = function_space_prolongate_numpy(x_petsc[:], Yh.dim(), inds_Xh_i
```

```
    return y_petsc

def function_space_restrict_petsc(y_petsc, Xh, inds_Xh_in_Yh):
    x_petsc = dl.Function(Xh).vector()
    x_petsc[:] = function_space_restrict_numpy(y_petsc[:], inds_Xh_in_Yh)
    return x_petsc

def make_prolongation_and_restriction_operators(Vsmall, Vbig, inds_Vsmall_in_Vbi
    Vbig_to_Vsmall_numpy = lambda vb: function_space_restrict_numpy(vb, inds_Vsm
    Vsmall_to_Vbig_numpy = lambda vs: function_space_prolongate_numpy(vs, Vbig.c

    Vbig_to_Vsmall_petsc = lambda vb: function_space_restrict_petsc(vb, Vsmall,
    Vsmall_to_Vbig_petsc = lambda vs: function_space_prolongate_petsc(vs, Vbig,

    return Vbig_to_Vsmall_numpy, Vsmall_to_Vbig_numpy, Vbig_to_Vsmall_petsc, Vsm
```

In [12]:

```
pp_Vh3 = Vh3.tabulate_dof_coordinates()
pp_Vh2 = Vh2.tabulate_dof_coordinates()
pp_Wh = Wh.tabulate_dof_coordinates()

KDT_Wh = scipy_KDTree(pp_Wh)
inds_Vh3_in_Wh = KDT_Wh.query(pp_Vh3)[1]
if np.linalg.norm(pp_Vh3 - pp_Wh[inds_Vh3_in_Wh, :]) / np.linalg.norm(pp_Vh3) >
    warnings.warn('problem with basal function space inclusion')

pp_Vh3_2D = pp_Vh3[:,:2]

KDT_Vh3_2D = scipy_KDTree(pp_Vh3_2D)
inds_Vh2_in_Vh3 = KDT_Vh3_2D.query(pp_Vh2)[1]
if np.linalg.norm(pp_Vh2 - pp_Vh3[inds_Vh2_in_Vh3, :2]) / np.linalg.norm(pp_Vh2)
    warnings.warn('inconsistency between manifold basal mesh and flat basal mesh

inds_Vh2_in_Wh = inds_Vh3_in_Wh[inds_Vh2_in_Vh3]

Wh_to_Vh3_numpy, Vh3_to_Wh_numpy, Wh_to_Vh3_petsc, Vh3_to_Wh_petsc = \
    make_prolongation_and_restriction_operators(Vh3, Wh, inds_Vh3_in_Wh)

Vh3_to_Vh2_numpy, Vh2_to_Vh3_numpy, Vh3_to_Vh2_petsc, Vh2_to_Vh3_petsc = \
    make_prolongation_and_restriction_operators(Vh2, Vh3, inds_Vh2_in_Vh3)

Wh_to_Vh2_numpy, Vh2_to_Wh_numpy, Wh_to_Vh2_petsc, Vh2_to_Wh_petsc = \
    make_prolongation_and_restriction_operators(Vh2, Wh, inds_Vh2_in_Wh)
```

Forward Stokes forms

In [13]:

```
m_Wh = dl.Function(Wh) # Basal sliding friction
u = dl.Function(Zh) # Stokes state: (velocity, pressure)
p = dl.Function(Zh) # Stokes adjoint variable

mtrue_Wh = dl.Function(Wh) # True parameter
utrue = dl.Function(Zh) # True state
uobs = dl.Function(Zh) # observed state (true state plus noise)

grav  = 9.81           # acceleration due to gravity
rho   = 910.0          # volumetric mass density of ice
stokes_forcing = dl.Constant( (0., 0., -rho*grav) )
```

```
ds = dl.Measure("ds", domain=mesh, subdomain_data=boundary_markers)
ds = ds
ds_base = ds(1)
ds_top = ds(2)
ds_lateral = ds(3)
normal = dl.FacetNormal(mesh)
# Strongly enforced Dirichlet conditions. The no outflow condition will be enfo
bcs  = []
bcs0 = []

# Define the Nonlinear Stokes varfs
# rheology
stokes_n = 1.0
stokes_A = dl.Constant(2.140373e-7) # dl.Constant(1.e-16)
smooth_strain = dl.Constant(1e-6)

velocity, pressure = dl.split(u)
strain_rate = dl.sym(dl.grad(velocity))
normEu12 = 0.5 * dl.inner(strain_rate, strain_rate) + smooth_strain

tangent_velocity = (velocity - dl.outer(normal, normal)*velocity)

energy_t1 = stokes_A ** (-1.) * normEu12 * dl.dx
energy_t2 = -dl.inner(stokes_forcing, velocity) * dl.dx
energy_t3 = dl.Constant(.5) * dl.inner(dl.exp(m_Wh) * tangent_velocity, tangent_
energy_t4 = lam * dl.inner(velocity, normal) ** 2 * ds_base

energy_t5 = dl.Constant(outflow_constant) * dl.inner(velocity, normal)**2 * ds_l

energy = energy_t1 + energy_t2 + energy_t3 + energy_t4 + energy_t5
energy_gradient = dl.derivative(energy, u, p)

adjoint_velocity, adjoint_pressure = dl.split(p)
div_constraint           = dl.inner(-dl.div(velocity),         adjoint_pressure)
div_constraint_transpose = dl.inner(-dl.div(adjoint_velocity), pressure)

forward_form_ff = energy_gradient + div_constraint + div_constraint_transpose #
```

Misfit forms

In [14]:

```
def Tang(vel, n):
    return vel - dl.outer(n, n)*vel

dummy_u1 = dl.Function(Zh)
dummy_u2 = dl.Function(Zh)

dummy_velocity1, dummy_pressure1 = dl.split(dummy_u1)
dummy_velocity2, dummy_pressure2 = dl.split(dummy_u2)

data_inner_product_form = dl.inner(Tang(dummy_velocity1, normal),
                                   Tang(dummy_velocity2, normal))*ds_top

def data_inner_product(u_petsc, v_petsc):
    dummy_u1.vector()[:] = u_petsc[:]
    dummy_u2.vector()[:] = v_petsc[:]
    return dl.assemble(data_inner_product_form)
```

```
true_velocity, true_pressure = dl.split(utrue)
observed_velocity, observed_pressure = dl.split(uobs)

velocity_discrepancy = observed_velocity - velocity

misfit_form = 0.5*dl.inner(Tang(velocity_discrepancy, normal),
                           Tang(velocity_discrepancy, normal))*ds_top
```

Lagrangian, adjoint, gradient, hessian

In [15]:

```
def input_vector_transformation(m_Vh2_numpy):
    m_Vh2_petsc = dl.Function(Vh2).vector()
    m_Vh2_petsc[:] = m_Vh2_numpy
    return Vh2_to_Wh_petsc(m_Vh2_petsc)

def output_vector_transformation(m_Wh_petsc):
    m_Vh2_numpy = Wh_to_Vh2_petsc(m_Wh_petsc)[:]
    return m_Vh2_numpy

SDAP = StokesDerivativesAtPoint(misfit_form, forward_form_ff, bcs,
                                m_Wh, u, p,
                                input_vector_transformation,
                                output_vector_transformation,
                                solver_type=solver_type)
```

Solve for true observations

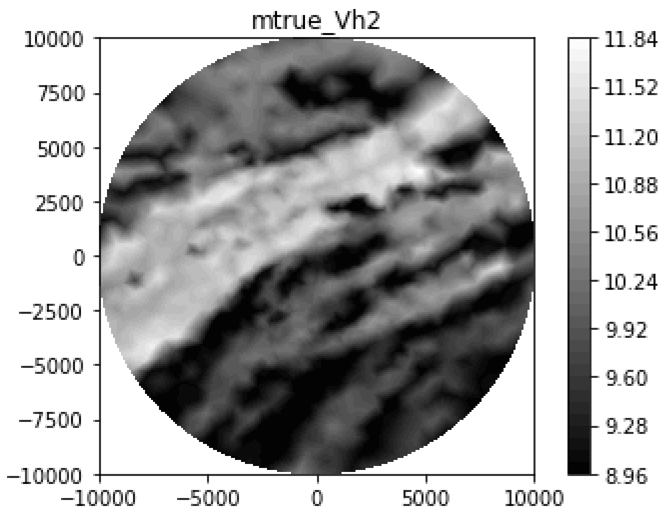
In [16]:

```
if mtrue_type == 'angel_peak' or mtrue_type == 'aces_building':
    image_dir = localpsf_root / 'localpsf'

    if mtrue_type == 'angel_peak':
#         image_file = image_dir / 'angel_peak_badlands_cropped9.png' # good
        image_file = image_dir / 'angel_peak_badlands_cropped7.png'
    else:
        image_file = image_dir / 'aces_building.png'

    mtrue_Vh2 = load_image_into_fenics(Vh2, image_file)
    minc = 9
    maxc = 12
    mtrue_Vh2_vec = mtrue_Vh2.vector()[:]
    mtrue_Vh2_vec *= 3
    mtrue_Vh2_vec += 9
    mtrue_Vh2.vector()[:] = mtrue_Vh2_vec
elif mtrue_type == 'sinusoid':
    mtrue_expr = dl.Expression(mtrue_string, element=Vh2.ufl_element(), m0=m0_cc
    mtrue_Vh2 = dl.interpolate(mtrue_expr, Vh2)
else:
    raise RuntimeError('BAD mtrue_type')


mtrue_Vh3 = dl.Function(Vh3)
mtrue_Vh3.vector()[:] = Vh2_to_Vh3_numpy(mtrue_Vh2.vector()[:])

cm = dl.plot(mtrue_Vh2, cmap='gray')
```

```
plt.colorbar(cm)
plt.title('mtrue_Vh2')
```

Out[16]:

```
Text(0.5, 1.0, 'mtrue_Vh2')
```



In [17]:

```
SDAP.update_m(mtrue_Vh2.vector()[:])
SDAP.update_forward()
utrue.vector()[:] = SDAP.u.vector()[:].copy()
```

Plots to make (2):

- True velocity (arrows or not, up to you)
- mtrue (basal sliding friction field)

In [18]:

```
dl.File(save_dir_str + "/mtrue_Vh2.pvd") << mtrue_Vh2
dl.File(save_dir_str + "/mtrue_Vh3.pvd") << mtrue_Vh3

dl.File(save_dir_str + "/true_velocity" + str(outflow_constant)+".pvd") << utrue
dl.File(save_dir_str + "/true_pressure" + str(outflow_constant)+".pvd") << utrue
```

Add noise

In [19]:

```
utrue_vec = utrue.vector()[:]
noise_vec = noise_level * np.random.randn(Zh.dim()) * np.abs(utrue_vec)
uobs.vector()[:] = utrue_vec + noise_vec
```

In [20]:

```
def compute_misfit_datanorm():
    return np.sqrt(2.0 * SDAP.misfit())

noise_vec_petsc = dl.Function(Zh).vector()
noise_vec_petsc[:] = noise_vec
noise_datanorm = np.sqrt(data_inner_product(noise_vec_petsc, noise_vec_petsc))
print('noise_datanorm=', noise_datanorm)
```

```
noise_datanorm= 863.5355316705534
```

Regularization

In [21]:

```
m_prior_mean_Vh2 = dl.Function(Vh2)
m_prior_mean_Vh2.vector()[:] = m0_constant_value * np.ones(Vh2.dim())
m_Vh2 = dl.Function(Vh2)
m_Vh2.vector()[:] = Wh_to_Vh2_numpy(m_Wh.vector()[:])

REG = BiLaplacianRegularization(gamma, prior_correlation_length, m_Vh2, m_prior_
```

```
/home/nick/anaconda3/envs/fenics4/lib/python3.9/site-packages/scipy/sparse/linal
g/dsolve/linsolve.py:318: SparseEfficiencyWarning: splu requires CSC matrix form
at
  warn('splu requires CSC matrix format', SparseEfficiencyWarning)
/home/nick/anaconda3/envs/fenics4/lib/python3.9/site-packages/scipy/sparse/coo.p
y:431: SparseEfficiencyWarning: Constructing a DIA matrix with 1416 diagonals is
inefficient
  warn("Constructing a DIA matrix with %d diagonals "
```

In [22]:

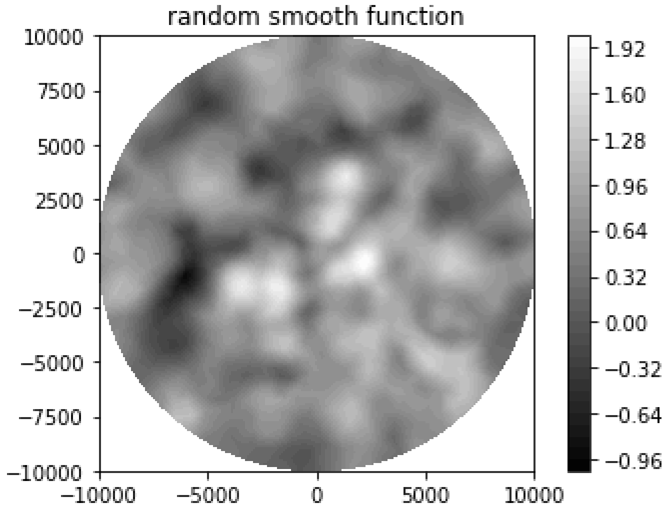
```
REG.update_gamma(1e5)
REG.update_correlation_length(prior_correlation_length)
```

In [23]:

```
def random_smooth_Vh2_function(min_f, max_f):
    f_vec = REG.solve_R_numpy(np.random.randn(Vh2.dim()))
    f_vec = f_vec - np.min(f_vec)
    f_vec = (max_f - min_f) * f_vec / np.max(f_vec)
    f_vec = f_vec + min_f
    f = dl.Function(Vh2)
    f.vector()[:] = f_vec
    return f

random_smooth = random_smooth_Vh2_function(-1.0, 2.0)

cm = dl.plot(random_smooth, cmap='gray')
plt.colorbar(cm)
plt.title('random smooth function')
```

Out[23]:

```
Text(0.5, 1.0, 'random smooth function')
```



In [24]:

```
def update_m(new_m_Vh2_numpy):
    SDAP.update_m(new_m_Vh2_numpy)
    REG.parameter.vector()[:] = new_m_Vh2_numpy

def cost():
    return SDAP.misfit() + REG.cost()

def gradient():
    return SDAP.gradient() + REG.gradient_numpy()

def apply_hessian(z_Vh2_numpy):
    return SDAP.apply_hessian(z_Vh2_numpy) + REG.apply_hessian_numpy(z_Vh2_numpy

def apply_gauss_newton_hessian(z_Vh2_numpy):
    return SDAP.apply_gauss_newton_hessian(z_Vh2_numpy) + REG.apply_hessian_nump
```

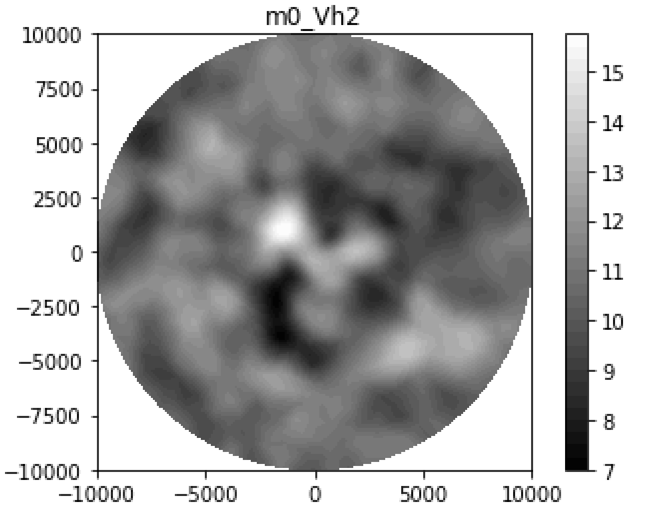
Finite difference check

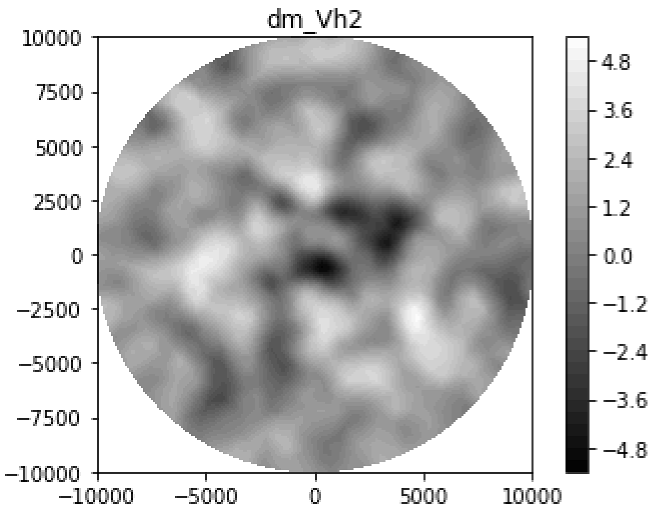
In [25]:

```
m0_Vh2 = random_smooth_Vh2_function(m0_constant_value/1.5, m0_constant_value*1.5
plt.figure()
cm = dl.plot(m0_Vh2, cmap='gray')
plt.colorbar(cm)
plt.title('m0_Vh2')

plt.figure()
dm_Vh2 = random_smooth_Vh2_function(-m0_constant_value/2.0, m0_constant_value/2.
cm = dl.plot(dm_Vh2, cmap='gray')
plt.colorbar(cm)
plt.title('dm_Vh2')
```

Out[25]:

```
Text(0.5, 1.0, 'dm_Vh2')
```





In [26]:

```
if run_finite_difference_checks:
    m0_Vh2_numpy = m0_Vh2.vector()[:]
    dm_Vh2_numpy = dm_Vh2.vector()[:]

    update_m(m0_Vh2_numpy)
    J0_d = SDAP.misfit()
    g0_d = SDAP.gradient()
    H0dm_d = SDAP.apply_hessian(dm_Vh2_numpy)

    J0_r = REG.cost()
    g0_r = REG.gradient_numpy()
    H0dm_r = REG.apply_hessian_numpy(dm_Vh2_numpy)

    J0 = cost()
    g0 = gradient()
    H0dm = apply_hessian(dm_Vh2_numpy)

    # ss = [1e-6]
    ss = [1e0, 1e-1, 1e-2, 1e-3, 1e-4, 1e-5, 1e-6, 1e-7, 1e-8, 1e-9, 1e-10, 1e-1
    errs_grad_d = []
    errs_hess_d = []
    errs_grad_r = []
    errs_hess_r = []
    errs_grad = []
    errs_hess = []
    for s in ss:
```

```
        m1_Vh2_numpy = m0_Vh2_numpy + s * dm_Vh2_numpy
        update_m(m1_Vh2_numpy)
        J1_d = SDAP.misfit()
        g1_d = SDAP.gradient()
#         H1dm_d = SDAP.apply_hessian(dm_Vh2_numpy)

        J1_r = REG.cost()
        g1_r = REG.gradient_numpy()
#         H1dm_r = REG.apply_hessian_numpy(dm_Vh2_numpy)

        J1 = cost()
        g1 = gradient()
#         H1dm = apply_hessian(dm_Vh2_numpy)

        # Gradient

        dJ_diff_d = (J1_d - J0_d) / s
        dJ_d = np.dot(g0_d, dm_Vh2_numpy)
        err_grad_d = np.abs(dJ_diff_d - dJ_d) / np.abs(dJ_diff_d)
        errs_grad_d.append(err_grad_d)

        dJ_diff_r = (J1_r - J0_r) / s
        dJ_r = np.dot(g0_r, dm_Vh2_numpy)
        err_grad_r = np.abs(dJ_diff_r - dJ_r) / np.abs(dJ_diff_r)
        errs_grad_r.append(err_grad_r)

        dJ_diff = (J1 - J0) / s
        dJ = np.dot(g0, dm_Vh2_numpy)
        err_grad = np.abs(dJ_diff - dJ) / np.abs(dJ_diff)
        errs_grad.append(err_grad)

        # Hessian

        dg_diff_d = (g1_d - g0_d) / s
        err_hess_d = np.linalg.norm(dg_diff_d - H0dm_d) / np.linalg.norm(dg_dif
        errs_hess_d.append(err_hess_d)

        dg_diff_r = (g1_r - g0_r) / s
        err_hess_r = np.linalg.norm(dg_diff_r - H0dm_r) / np.linalg.norm(dg_dif
        errs_hess_r.append(err_hess_r)

        dg_diff = (g1 - g0) / s
        err_hess = np.linalg.norm(dg_diff - H0dm) / np.linalg.norm(dg_diff)
        errs_hess.append(err_hess)

        print('s=', s)
        print('err_grad_d=', err_grad_d, ', err_grad_r=', err_grad_r, ', err_gra
        print('err_hess_d=', err_hess_d, ', err_hess_r=', err_hess_r, ', err_hes

    ss = np.array(ss)
    errs_grad_d = np.array(errs_grad_d)
    errs_hess_d = np.array(errs_hess_d)
    errs_grad_r = np.array(errs_grad_r)
    errs_hess_r = np.array(errs_hess_r)
    errs_grad = np.array(errs_grad)
    errs_hess = np.array(errs_hess)

    plt.figure()
    plt.loglog(ss, errs_grad_d)
    plt.loglog(ss, errs_hess_d)
    plt.title('finite difference check for misfit')
```

```
    plt.xlabel('step size s')
    plt.ylabel('error')
    plt.legend(['misfit gradient', 'misfit hessian'])

    plt.figure()
    plt.loglog(ss, errs_grad_r)
    plt.loglog(ss, errs_hess_r)
    plt.title('finite difference check for regularization')
    plt.xlabel('step size s')
    plt.ylabel('error')
    plt.legend(['regularization gradient', 'regularization hessian'])

    plt.figure()
    plt.loglog(ss, errs_grad)
    plt.loglog(ss, errs_hess)
    plt.title('finite difference check for overall cost')
    plt.xlabel('step size s')
    plt.ylabel('error')
    plt.legend(['gradient', 'hessian'])
```

Check Gauss-Newton Hessian

In [27]:

```
if check_gauss_newton_hessian:
    m0_Vh2_numpy = m0_Vh2.vector()[:]

    uobs_vec_backup = uobs.vector()[:].copy()
    uobs.vector()[:] = utrue.vector()[:].copy()

    update_m(mtrue_Vh2.vector()[:].copy())

    z_Vh2_numpy = random_smooth_Vh2_function(-m0_constant_value/2.0, m0_constant

    Hz = SDAP.apply_hessian(z_Vh2_numpy)
    HGNz = SDAP.apply_gauss_newton_hessian(z_Vh2_numpy)

    GN_discrepancy_at_truth_nonoise = np.linalg.norm(Hz - HGNz) / np.linalg.norm
    print('GN_discrepancy_at_truth_nonoise=', GN_discrepancy_at_truth_nonoise)

    update_m(m0_Vh2_numpy)

    Hz = SDAP.apply_hessian(z_Vh2_numpy)
    HGNz = SDAP.apply_gauss_newton_hessian(z_Vh2_numpy)

    GN_discrepancy_elsewhere_nonoise = np.linalg.norm(Hz - HGNz) / np.linalg.no
    print('GN_discrepancy_elsewhere_nonoise=', GN_discrepancy_elsewhere_nonoise)


    uobs.vector()[:] = uobs_vec_backup

    update_m(mtrue_Vh2.vector()[:].copy())
    Hz = SDAP.apply_hessian(z_Vh2_numpy)
    HGNz = SDAP.apply_gauss_newton_hessian(z_Vh2_numpy)

    GN_discrepancy_at_truth_yesnoise = np.linalg.norm(Hz - HGNz) / np.linalg.no
    print('GN_discrepancy_at_truth_yesnoise=', GN_discrepancy_at_truth_yesnoise)

    update_m(m0_Vh2_numpy)
    Hz = SDAP.apply_hessian(z_Vh2_numpy)
```

```
    HGNz = SDAP.apply_gauss_newton_hessian(z_Vh2_numpy)

    GN_discrepancy_elsewhere_yesnoise = np.linalg.norm(Hz - HGNz) / np.linalg.nc
    print('GN_discrepancy_elsewhere_yesnoise=', GN_discrepancy_elsewhere_yesnois

    #

    SDAP.update_m(m0_Vh2_numpy)
    SDAP.update_forward()

    p_Vh2_numpy = random_smooth_Vh2_function(-m0_constant_value/2.0, m0_constant
    q_Vh2_numpy = random_smooth_Vh2_function(-m0_constant_value/2.0, m0_constant

    SDAP.update_z(p_Vh2_numpy)
    SDAP.update_incremental_forward()
    du_dm_p = dl.Function(Zh)
    du_dm_p.vector()[:] = SDAP.du_dm_z.vector()[:].copy()

    SDAP.update_z(q_Vh2_numpy)
    SDAP.update_incremental_forward()
    du_dm_q = dl.Function(Zh)
    du_dm_q.vector()[:] = SDAP.du_dm_z.vector()[:].copy()

    C1 = data_inner_product(du_dm_p.vector(), du_dm_q.vector())

    Hgn_p = SDAP.apply_gauss_newton_hessian(p_Vh2_numpy)
    C2 = np.dot(q_Vh2_numpy, Hgn_p)

    gauss_newton_error = np.abs(C1 - C2) / np.abs(C1)

    print('gauss_newton_error=', gauss_newton_error)
```

In [28]:

```
def apply_misfit_gauss_newton_hessian_petsc(u_petsc):
    v = dl.Function(Vh2).vector()
    v[:] = SDAP.apply_gauss_newton_hessian(u_petsc[:])
    return v

class PCHWrapper:
    def __init__(me, do_updates=False):
        me.do_updates = do_updates

        me.PCK = None
        me.Hd_pch = None
        me.R_hmatrix = None
        me.H_pch = None
        me.preconditioner = None

    def build(me):
        me._make_Hd_hmatrix()
        me._make_reg_hmatrix()
        me._make_pre_hmatrix()

    def _make_pre_hmatrix(me):
        me.H_pch = me.Hd_pch + me.R_hmatrix
        me.preconditioner = me.H_pch.factorized_inverse()

    def _make_Hd_hmatrix(me):
        print('building PCH preconditioner')
        me.PCK = ProductConvolutionKernel(Vh2, Vh2,
```

```
                                                  apply_misfit_gauss_newton_hessian_pets
                                                  apply_misfit_gauss_newton_hessian_pets
                                                  num_batches, num_batches,
                                                  tau_rows=tau, tau_cols=tau,
                                                  num_neighbors_rows=num_neighbors,
                                                  num_neighbors_cols=num_neighbors)
        Hd_pch_nonsym, extras = make_hmatrix_from_kernel(me.PCK, hmatrix_tol=hma
        me.Hd_pch = Hd_pch_nonsym.spd()

    def _make_reg_hmatrix(me):
        print('Building Regularization H-Matrix')
        me.R_hmatrix = REG.make_R_hmatrix(me.Hd_pch.bct, rtol=1e-6)

    def update_reg(me):
        if me.Hd_pch is None:
            me._make_Hd_hmatrix()
        me._make_reg_hmatrix()
        me._make_pre_hmatrix()

    def update_Hd(me):
        me._make_Hd_hmatrix()
        if me.R_hmatrix is None:
            me._make_reg_hmatrix()
        me._make_pre_hmatrix()


    def low_rank_update(me, xx, yy):
        if me.do_updates and (me.preconditioner_hmatrix is not None):
            pass
            #             me.preconditioner_hmatrix = me.preconditioner_hmatrix.

    def solve(me, u_Vh2_numpy):
        if me.preconditioner is None:
            return REG.solve_hessian_numpy(u_Vh2_numpy)
        else:
            return me.preconditioner.matvec(u_Vh2_numpy)

    def reset(me):
        me.PCK = None
        me.Hd_pch = None
        me.R_hmatrix = None
        me.H_pch = None
        me.preconditioner = None
```

In [29]:

```
REG.update_gamma(1e4)

get_optimization_variable = lambda : Wh_to_Vh2_numpy(m_Wh.vector()[:])
set_optimization_variable = update_m
cost_triple = lambda : [cost(), SDAP.misfit(), REG.cost()]
build_nothing = lambda : None
update_nothing = lambda X, Y: None
```

In [30]:

```
PCH1 = PCHWrapper(do_updates=False)

def morozov_helper():
    misfit_datanorm = np.sqrt(2.0 * SDAP.misfit())
    print('gamma=', REG.gamma, ', noise_datanorm=', noise_datanorm, ', misfit_da
```

```
    return misfit_datanorm

morozov_gammas = list()
morozov_discrepancies = list()

def log_morozov_function(log_gamma):
    gamma = np.exp(log_gamma)
    print()
    print('-------- gamma=', gamma, ' --------')
    REG.update_gamma(gamma)

    preconditioner_build_iters = (3,)
    if PCH1.preconditioner is None:
        preconditioner_build_iters = (3,)
    else:
        PCH1.update_reg()
        preconditioner_build_iters = tuple([])

    update_m(m0_constant_value * np.ones(Vh2.dim()))
    newtoncg_ls(get_optimization_variable,
                set_optimization_variable,
                cost_triple,
                gradient,
                apply_hessian,
                apply_gauss_newton_hessian,
                PCH1.build,
                update_nothing,
                PCH1.solve,
                preconditioner_build_iters = preconditioner_build_iters,
                rtol=newton_rtol,
                forcing_sequence_power=forcing_sequence_power)

    misfit_datanorm = morozov_helper()
    morozov_gammas.append(gamma)
    morozov_discrepancies.append(misfit_datanorm)

    log_mismatch = np.log(noise_datanorm) - np.log(misfit_datanorm)
    return log_mismatch

if gamma_morozov is None:
    sol_morozov = root_scalar(log_morozov_function, x0=np.log(5e3), x1=np.log(5e
    print(sol_morozov)
    gamma_morozov = np.exp(sol_morozov.root)
    print('gamma_morozov=', gamma_morozov)

REG.update_gamma(gamma_morozov)
```

```
-------- gamma= 5000.000000000004  --------
using Gauss-Newton Hessian
it= 0 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.951e+07
||M r0|| = 2.528e+05
(M r0, r0) = 2.037e+06
 Iteration :  0  (r, r) =  380758959655851.75
 Iteration :  1  (r, r) =  60800772279799.06
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  7797484.9970871415
```

```
===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 1
cumulative cost evaluations : 1
cumulative gradient evaluations : 1
cumulative Hessian vector products (excluding preconditioner builds) : 1
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 1 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 7.906e+06
||M r0|| = 5.409e+04
(M r0, r0) = 5.845e+05
 Iteration :  0  (r, r) =  62500719723087.62
 Iteration :  1  (r, r) =  25868298506353.766
 Iteration :  2  (r, r) =  15319779174433.328
Relative/Absolute residual less than tol
Converged in  2  iterations with final norm  3914048.948906149

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
```

```
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.080028e+07 6.1e+07 1.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 3
cumulative cost evaluations : 2
cumulative gradient evaluations : 2
cumulative Hessian vector products (excluding preconditioner builds) : 3
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 2 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 3.876e+06
||M r0|| = 2.271e+04
(M r0, r0) = 2.391e+05
 Iteration :  0  (r, r) =  15021920577626.777
 Iteration :  1  (r, r) =  9845330889144.463
 Iteration :  2  (r, r) =  7737089442423.863
 Iteration :  3  (r, r) =  4611240518703.695
 Iteration :  4  (r, r) =  3497989937372.867
 Iteration :  5  (r, r) =  2796040040126.543
Relative/Absolute residual less than tol
Converged in  5  iterations with final norm  1672136.3700746847

====================== Begin Newton CG convergence information =================
```

```
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.080028e+07 6.1e+07 1.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.178416e+07 2.2e+07 2.6e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 8
cumulative cost evaluations : 3
cumulative gradient evaluations : 3
cumulative Hessian vector products (excluding preconditioner builds) : 8
===================== End Newton CG convergence information =================
=====

using Gauss-Newton Hessian
it= 3 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
building preconditioner
building PCH preconditioner
Computing impulse response moments
getting spatially varying volume
getting spatially varying mean
getting spatially varying covariance
Preparing sample point batch stuff
Preparing c++ object
Building initial sample point batches

/home/nick/repos/localpsf/localpsf/sample_point_batches.py:94: NumbaPerformanceW
```

```
arning: np.dot() is faster on contiguous arrays, called on (array(float64, 2d,
A), array(float64, 2d, C))
  if ellipsoids_intersect(Sigma_perm[x_ind, :, :], Sigma_perm[p_ind, :, :], mu_p
erm[x_ind, :],
Making row and column cluster treesUsing ProductConvolutionKernelRBFColsOnly!
Making block cluster trees

Building A kernel hmatrix
━━ building H-matrix ( tol = Making input and output mass matrix hmatrices
Computing A_hmatrix = M_out_hmatrix * A_kernel_hmatrix * M_in_hmatrix
making hmatrix spd1e-06 )
 ==-                                     ]   7% ETA 11 s (5.12 GB)
symmetrizing
                                                               getting smallest
eigenvalue with Lanczos
    done in 12.97s
    size of H-matrix = 4.42 MB
lambda_min= -76580.83327169948
scaling_at_lambda_min= 0.9878048780487805
scaling_at_zero= 0.5
Setting up operator T = (2*A - (b+a) I) / (b-a)
computing T^(2^k)
computing T^(2^1) = T^(2^0) * T^(2^0)

computing T^(2^2) = T^(2^1) * T^(2^1)
computing negative spectral projector Pi_minus = I / (I + T^(2^k))━━ H-matrix mu
ltiplication C=A*B
    done in 33.53s
    size of C = 5.27 MB

━━ H-matrix multiplication C=A*B
    done in 34.77s
    size of C = 7.70 MB

━━ H-matrix inverse ( rtol =  1e-07 , atol =  1e-12 , overwrite= False  )
                                                                  97397613525
    size of inverse =  14640401  bytes

computing A_plus = Pi * A

━━ H-matrix multiplication C=A*B
    done in 37.30s
    size of C = 17.53 MB
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is            real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 806144
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 806144
 |S-A|_2 = 5.4232e-09
  matrix has dimension ━━ H-matrix inverse ( rtol =  1e-06 , atol =  1e-14 , ove
rwrite= False  )
1416 x 1416
    no of non-zeroes    = 9746
    matrix is            real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
```

```
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
[==                                                                 done in  1
5.418750524520874
    size of inverse =  3361874  bytes

━━ H-matrix multiplication C=A*B
    done in 24.67s
]   4% ETA 27 s (5.20 GB)
    size of C = 3.86 MB

━━ H-matrix multiplication C=A*B
    done in 33.28s
    size of C = 8.53 MB

Residual: r=b-Ax
━━ LU factorisation ( rtol = 1e-07 )
[==-                               Preconditioner: M =approx= A^-1
||r0|| = 1.627e+06
||M r0|| = 3.933e+01
(M r0, r0) = 3.905e+03
 Iteration :  0  (r, r) =  2646144442184.8745
[====  (r, r) =  366414301859.5593
[============ 2  (r, r) =  57802220637.35317
Relative/Absolute residual less than tol
Converged in  2  iterations with final norm  240420.9238759247
                       =============-    ]  89% ETA 3 s (5.21 GB)
===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
```

```
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.080028e+07 6.1e+07 1.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.178416e+07 2.2e+07 2.6e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01
 3   2  1  1   2  T  T 6.165938e+06 6.1e+06 6.1e+04 -1.1e+07 1.6e+06 1.0e+00 2.9
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 10
cumulative cost evaluations : 4
cumulative gradient evaluations : 4
cumulative Hessian vector products (excluding preconditioner builds) : 10
======================= End Newton CG convergence information ==================
=====

    done in 31.24s
    size of LU factor = 18.69 MB
using Gauss-Newton Hessian
it= 4 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 2.617e+05
||M r0|| = 1.288e+01
(M r0, r0) = 1.027e+03
 Iteration :  0  (r, r) =  68483623043.65376
 Iteration :  1  (r, r) =  31981559678.06558
 Iteration :  2  (r, r) =  10761652533.239012
 Iteration :  3  (r, r) =  3809862148.703622
 Iteration :  4  (r, r) =  1112038312.778342
 Iteration :  5  (r, r) =  571265944.5628774
Relative/Absolute residual less than tol
Converged in  5  iterations with final norm  23901.17035968903

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
```

```
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.080028e+07 6.1e+07 1.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.178416e+07 2.2e+07 2.6e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01
 3   2  1  1   2  T  T 6.165938e+06 6.1e+06 6.1e+04 -1.1e+07 1.6e+06 1.0e+00 2.9
e-01
 4   5  1  1   5  T  F 8.218143e+05 5.4e+05 2.8e+05 -5.1e+05 2.6e+05 1.0e+00 1.2
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 15
cumulative cost evaluations : 5
cumulative gradient evaluations : 5
cumulative Hessian vector products (excluding preconditioner builds) : 15
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 5 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 3.487e+04
||M r0|| = 1.403e+00
(M r0, r0) = 8.386e+01
 Iteration :  0  (r, r) =  1215958116.5525784
 Iteration :  1  (r, r) =  417884304.99576837
 Iteration :  2  (r, r) =  128318492.13023134
 Iteration :  3  (r, r) =  151795373.92534086
 Iteration :  4  (r, r) =  104411095.17273203
 Iteration :  5  (r, r) =  30524202.764465027
 Iteration :  6  (r, r) =  8843132.773494504
 Iteration :  7  (r, r) =  5275252.058082572
 Iteration :  8  (r, r) =  3148887.3993596765
 Iteration :  9  (r, r) =  1756242.544646705
Relative/Absolute residual less than tol
Converged in  9  iterations with final norm  1325.233015226645

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
```

```
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.080028e+07 6.1e+07 1.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.178416e+07 2.2e+07 2.6e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01
 3   2  1  1   2  T  T 6.165938e+06 6.1e+06 6.1e+04 -1.1e+07 1.6e+06 1.0e+00 2.9
e-01
 4   5  1  1   5  T  F 8.218143e+05 5.4e+05 2.8e+05 -5.1e+05 2.6e+05 1.0e+00 1.2
e-01
 5   9  1  1   9  T  F 5.696227e+05 3.3e+05 2.4e+05 -7.1e+03 3.5e+04 1.0e+00 4.2
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 24
cumulative cost evaluations : 6
cumulative gradient evaluations : 6
cumulative Hessian vector products (excluding preconditioner builds) : 24
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 6 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.832e+03
||M r0|| = 9.013e-02
(M r0, r0) = 4.920e+00
 Iteration :  0  (r, r) =  3356716.5020248247
 Iteration :  1  (r, r) =  1446937.6878977364
 Iteration :  2  (r, r) =  420612.97650490917
 Iteration :  3  (r, r) =  321700.54030757665
 Iteration :  4  (r, r) =  277535.755474856
 Iteration :  5  (r, r) =  148023.123180567
 Iteration :  6  (r, r) =  58260.53671776735
 Iteration :  7  (r, r) =  29460.732394096784
 Iteration :  8  (r, r) =  24558.39352351096
 Iteration :  9  (r, r) =  18189.053385198655
```

```
 Iteration :  10  (r, r) =  3128.9248763057767
 Iteration :  11  (r, r) =  1285.0892650385576
 Iteration :  12  (r, r) =  486.70348065888885
 Iteration :  13  (r, r) =  212.35924359665927
Relative/Absolute residual less than tol
Converged in  13  iterations with final norm  14.572551032563217

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.080028e+07 6.1e+07 1.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.178416e+07 2.2e+07 2.6e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01
 3   2  1  1   2  T  T 6.165938e+06 6.1e+06 6.1e+04 -1.1e+07 1.6e+06 1.0e+00 2.9
e-01
 4   5  1  1   5  T  F 8.218143e+05 5.4e+05 2.8e+05 -5.1e+05 2.6e+05 1.0e+00 1.2
e-01
 5   9  1  1   9  T  F 5.696227e+05 3.3e+05 2.4e+05 -7.1e+03 3.5e+04 1.0e+00 4.2
e-02
 6  13  1  1  13  T  F 5.661010e+05 3.3e+05 2.4e+05 -2.5e+01 1.8e+03 1.0e+00 9.7
e-03

converged : False
reason    : unknown reason
cumulative CG iterations : 37
cumulative cost evaluations : 7
cumulative gradient evaluations : 7
cumulative Hessian vector products (excluding preconditioner builds) : 37
====================== End Newton CG convergence information =================
```

```
=====

using Hessian
it= 7 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
False
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.749e+01
||M r0|| = 1.862e-03
(M r0, r0) = 7.587e-02
 Iteration :  0  (r, r) =  305.8281629576305
 Iteration :  1  (r, r) =  178.476651349541
 Iteration :  2  (r, r) =  96.83062502098187
 Iteration :  3  (r, r) =  52.936712304136634
 Iteration :  4  (r, r) =  40.71405985354616
 Iteration :  5  (r, r) =  19.06338122616416
 Iteration :  6  (r, r) =  6.647026458057597
 Iteration :  7  (r, r) =  2.078940355305485
 Iteration :  8  (r, r) =  1.090015705544459
 Iteration :  9  (r, r) =  0.5818700576444459
 Iteration :  10  (r, r) =  0.2905726315017592
 Iteration :  11  (r, r) =  0.1570359409728928
 Iteration :  12  (r, r) =  0.07972613501548531
 Iteration :  13  (r, r) =  0.04541808865839962
 Iteration :  14  (r, r) =  0.01923156224586578
 Iteration :  15  (r, r) =  0.00998209894438901
 Iteration :  16  (r, r) =  0.004508057294932381
 Iteration :  17  (r, r) =  0.001772705308016505
 Iteration :  18  (r, r) =  0.0007139075328989931
 Iteration :  19  (r, r) =  0.0003153627074175871
 Iteration :  20  (r, r) =  0.00013280439322403394
Relative/Absolute residual less than tol
Converged in  20  iterations with final norm  0.011524078844924393

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
```

```
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.080028e+07 6.1e+07 1.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.178416e+07 2.2e+07 2.6e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01
 3   2  1  1   2  T  T 6.165938e+06 6.1e+06 6.1e+04 -1.1e+07 1.6e+06 1.0e+00 2.9
e-01
 4   5  1  1   5  T  F 8.218143e+05 5.4e+05 2.8e+05 -5.1e+05 2.6e+05 1.0e+00 1.2
e-01
 5   9  1  1   9  T  F 5.696227e+05 3.3e+05 2.4e+05 -7.1e+03 3.5e+04 1.0e+00 4.2
e-02
 6  13  1  1  13  T  F 5.661010e+05 3.3e+05 2.4e+05 -2.5e+01 1.8e+03 1.0e+00 9.7
e-03
 7  20  1  1  20  F  F 5.660884e+05 3.3e+05 2.4e+05 -5.4e-03 1.7e+01 1.0e+00 9.5
e-04

converged : False
reason    : unknown reason
cumulative CG iterations : 57
cumulative cost evaluations : 8
cumulative gradient evaluations : 8
cumulative Hessian vector products (excluding preconditioner builds) : 57
======================= End Newton CG convergence information ==================
=====

using Hessian
it= 8 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
False

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
```

```
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.080028e+07 6.1e+07 1.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.178416e+07 2.2e+07 2.6e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01
 3   2  1  1   2  T  T 6.165938e+06 6.1e+06 6.1e+04 -1.1e+07 1.6e+06 1.0e+00 2.9
e-01
 4   5  1  1   5  T  F 8.218143e+05 5.4e+05 2.8e+05 -5.1e+05 2.6e+05 1.0e+00 1.2
e-01
 5   9  1  1   9  T  F 5.696227e+05 3.3e+05 2.4e+05 -7.1e+03 3.5e+04 1.0e+00 4.2
e-02
 6  13  1  1  13  T  F 5.661010e+05 3.3e+05 2.4e+05 -2.5e+01 1.8e+03 1.0e+00 9.7
e-03
 7  20  1  1  20  F  F 5.660884e+05 3.3e+05 2.4e+05 -5.4e-03 1.7e+01 1.0e+00 9.5
e-04
 8   0  1  1   0  F  F 5.660884e+05 3.3e+05 2.4e+05     ---- 5.2e-02 1.0e+00
----

converged : True
reason    : Norm of the gradient less than tolerance
cumulative CG iterations : 57
cumulative cost evaluations : 9
cumulative gradient evaluations : 9
cumulative Hessian vector products (excluding preconditioner builds) : 57
======================= End Newton CG convergence information ==================
=====

gamma= 5000.000000000004 , noise_datanorm= 863.5355316705534 , misfit_datanorm=
812.6169880110655

-------- gamma= 50000.00000000001  --------
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 8.06144e+06
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 8.06144e+06
 |S-A|_2 = 6.76813e-09
  matrix has dimension ━━ H-matrix inverse ( rtol =  1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
1e-06 , atol =  1e-14 , overwrite= False  )
```

```
/home/nick/anaconda3/envs/fenics4/lib/python3.9/site-packages/scipy/sparse/linal
g/dsolve/linsolve.py:318: SparseEfficiencyWarning: splu requires CSC matrix form
at
  warn('splu requires CSC matrix format', SparseEfficiencyWarning)
    done in  14.279305696487427
    size of inverse =  3361874  bytes

━━ H-matrix multiplication C=A*B
    done in 24.03s
    size of C = 3.95 MB

━━ H-matrix multiplication C=A*B
                                                              done in 33.39s
    size of C = 8.64 MB

━━ LU factorisation ( rtol = 1e-07 )
[================-                                  n
it= 0 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
[==================================-     ]  Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.951e+07
||M r0|| = 3.595e+01
(M r0, r0) = 2.169e+04
 Iteration :  0  (r, r) =  380758959655851.75
    done in 30.63s
    size of LU factor = 17.94 MB
 Iteration :  1  (r, r) =  22458528040128.277
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  4739042.945588094

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)
```

```
it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 1
cumulative cost evaluations : 1
cumulative gradient evaluations : 1
cumulative Hessian vector products (excluding preconditioner builds) : 1
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 1 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 3.923e+06
||M r0|| = 1.007e+01
(M r0, r0) = 4.632e+03
 Iteration :  0  (r, r) =  15392549306050.43
 Iteration :  1  (r, r) =  352544684504.33435
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  593754.7343005649

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
```

```
 1   1  1  1   1  T  F 2.047000e+07 1.4e+07 6.9e+06 -2.1e+07 3.9e+06 1.0e+00 4.5
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 2
cumulative cost evaluations : 2
cumulative gradient evaluations : 2
cumulative Hessian vector products (excluding preconditioner builds) : 2
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 2 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 5.329e+05
||M r0|| = 3.243e+00
(M r0, r0) = 8.008e+02
 Iteration :  0  (r, r) =  283941708606.2915
 Iteration :  1  (r, r) =  17216423679.299385
 Iteration :  2  (r, r) =  999462153.3520812
Relative/Absolute residual less than tol
Converged in  2  iterations with final norm  31614.271355703917

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 2.047000e+07 1.4e+07 6.9e+06 -2.1e+07 3.9e+06 1.0e+00 4.5
```

```
e-01
 2   2  1  1   2  T  F 1.002705e+07 3.5e+06 6.5e+06 -5.2e+05 5.3e+05 1.0e+00 1.7
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 4
cumulative cost evaluations : 3
cumulative gradient evaluations : 3
cumulative Hessian vector products (excluding preconditioner builds) : 4
====================== End Newton CG convergence information =================
=====

using Gauss-Newton Hessian
it= 3 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 3.363e+04
||M r0|| = 2.028e-01
(M r0, r0) = 5.799e+01
 Iteration :  0  (r, r) =  1130766339.588983
 Iteration :  1  (r, r) =  59144820.533889666
 Iteration :  2  (r, r) =  7040928.623888069
 Iteration :  3  (r, r) =  203541.26394750192
Relative/Absolute residual less than tol
Converged in  3  iterations with final norm  451.1554764684808

====================== Begin Newton CG convergence information ===============
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
```

```
e-01
 1   1  1  1   1  T  F 2.047000e+07 1.4e+07 6.9e+06 -2.1e+07 3.9e+06 1.0e+00 4.5
e-01
 2   2  1  1   2  T  F 1.002705e+07 3.5e+06 6.5e+06 -5.2e+05 5.3e+05 1.0e+00 1.7
e-01
 3   3  1  1   3  T  F 9.770928e+06 3.3e+06 6.4e+06 -2.7e+03 3.4e+04 1.0e+00 4.2
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 7
cumulative cost evaluations : 4
cumulative gradient evaluations : 4
cumulative Hessian vector products (excluding preconditioner builds) : 7
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 4 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 9.132e+02
||M r0|| = 1.103e-02
(M r0, r0) = 2.542e+00
 Iteration :  0  (r, r) =  833849.4027094174
 Iteration :  1  (r, r) =  146794.71532708744
 Iteration :  2  (r, r) =  9749.073084496502
 Iteration :  3  (r, r) =  317.4747197346938
 Iteration :  4  (r, r) =  17.518234131799126
Relative/Absolute residual less than tol
Converged in  4  iterations with final norm  4.185478960859692

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
```

```
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 2.047000e+07 1.4e+07 6.9e+06 -2.1e+07 3.9e+06 1.0e+00 4.5
e-01
 2   2  1  1   2  T  F 1.002705e+07 3.5e+06 6.5e+06 -5.2e+05 5.3e+05 1.0e+00 1.7
e-01
 3   3  1  1   3  T  F 9.770928e+06 3.3e+06 6.4e+06 -2.7e+03 3.4e+04 1.0e+00 4.2
e-02
 4   4  1  1   4  T  F 9.769621e+06 3.3e+06 6.4e+06 -5.1e+00 9.1e+02 1.0e+00 6.8
e-03

converged : False
reason    : unknown reason
cumulative CG iterations : 11
cumulative cost evaluations : 5
cumulative gradient evaluations : 5
cumulative Hessian vector products (excluding preconditioner builds) : 11
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 5 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 4.873e+01
||M r0|| = 7.853e-04
(M r0, r0) = 1.626e-01
 Iteration :  0  (r, r) =  2374.1513963100197
 Iteration :  1  (r, r) =  594.3755554841518
 Iteration :  2  (r, r) =  30.536713225260378
 Iteration :  3  (r, r) =  0.9773040458769024
 Iteration :  4  (r, r) =  0.04648430875090048
 Iteration :  5  (r, r) =  0.003952444565034312
Relative/Absolute residual less than tol
Converged in  5  iterations with final norm  0.06286847035704235

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
```

```
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 2.047000e+07 1.4e+07 6.9e+06 -2.1e+07 3.9e+06 1.0e+00 4.5
e-01
 2   2  1  1   2  T  F 1.002705e+07 3.5e+06 6.5e+06 -5.2e+05 5.3e+05 1.0e+00 1.7
e-01
 3   3  1  1   3  T  F 9.770928e+06 3.3e+06 6.4e+06 -2.7e+03 3.4e+04 1.0e+00 4.2
e-02
 4   4  1  1   4  T  F 9.769621e+06 3.3e+06 6.4e+06 -5.1e+00 9.1e+02 1.0e+00 6.8
e-03
 5   5  1  1   5  T  F 9.769619e+06 3.3e+06 6.4e+06 -2.0e-02 4.9e+01 1.0e+00 1.6
e-03

converged : False
reason    : unknown reason
cumulative CG iterations : 16
cumulative cost evaluations : 6
cumulative gradient evaluations : 6
cumulative Hessian vector products (excluding preconditioner builds) : 16
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 6 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 3.128e+00
||M r0|| = 5.626e-05
(M r0, r0) = 1.099e-02
 Iteration :  0  (r, r) =  9.785201657116076
 Iteration :  1  (r, r) =  3.008032134304214
 Iteration :  2  (r, r) =  0.12813413814545815
 Iteration :  3  (r, r) =  0.005516259996165365
 Iteration :  4  (r, r) =  0.0002661537357763828
 Iteration :  5  (r, r) =  1.9430069563418738e-05
 Iteration :  6  (r, r) =  1.3714377913856614e-06
Relative/Absolute residual less than tol
Converged in  6  iterations with final norm  0.0011710840240502223

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
```

```
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 2.047000e+07 1.4e+07 6.9e+06 -2.1e+07 3.9e+06 1.0e+00 4.5
e-01
 2   2  1  1   2  T  F 1.002705e+07 3.5e+06 6.5e+06 -5.2e+05 5.3e+05 1.0e+00 1.7
e-01
 3   3  1  1   3  T  F 9.770928e+06 3.3e+06 6.4e+06 -2.7e+03 3.4e+04 1.0e+00 4.2
e-02
 4   4  1  1   4  T  F 9.769621e+06 3.3e+06 6.4e+06 -5.1e+00 9.1e+02 1.0e+00 6.8
e-03
 5   5  1  1   5  T  F 9.769619e+06 3.3e+06 6.4e+06 -2.0e-02 4.9e+01 1.0e+00 1.6
e-03
 6   6  1  1   6  T  F 9.769619e+06 3.3e+06 6.4e+06 -9.0e-05 3.1e+00 1.0e+00 4.0
e-04

converged : False
reason    : unknown reason
cumulative CG iterations : 22
cumulative cost evaluations : 7
cumulative gradient evaluations : 7
cumulative Hessian vector products (excluding preconditioner builds) : 22
====================== End Newton CG convergence information =================
=====


===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.
```

```
it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 2.047000e+07 1.4e+07 6.9e+06 -2.1e+07 3.9e+06 1.0e+00 4.5
e-01
 2   2  1  1   2  T  F 1.002705e+07 3.5e+06 6.5e+06 -5.2e+05 5.3e+05 1.0e+00 1.7
e-01
 3   3  1  1   3  T  F 9.770928e+06 3.3e+06 6.4e+06 -2.7e+03 3.4e+04 1.0e+00 4.2
e-02
 4   4  1  1   4  T  F 9.769621e+06 3.3e+06 6.4e+06 -5.1e+00 9.1e+02 1.0e+00 6.8
e-03
 5   5  1  1   5  T  F 9.769619e+06 3.3e+06 6.4e+06 -2.0e-02 4.9e+01 1.0e+00 1.6
e-03
 6   6  1  1   6  T  F 9.769619e+06 3.3e+06 6.4e+06 -9.0e-05 3.1e+00 1.0e+00 4.0
e-04
 7   0 10  1   0  T  F 9.769619e+06 3.3e+06 6.4e+06     ---- 3.1e+00 9.8e-04
----

converged : False
reason    : Maximum number of backtracking reached
cumulative CG iterations : 22
cumulative cost evaluations : 17
cumulative gradient evaluations : 8
cumulative Hessian vector products (excluding preconditioner builds) : 22
======================= End Newton CG convergence information ==================
=====

gamma= 50000.00000000001 , noise_datanorm= 863.5355316705534 , misfit_datanorm=
2585.9744874650937

-------- gamma= 5642.493956627751  --------
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 909732
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 909732
 |S-A|_2 = 2.77941e-09
  matrix has dimension ━━ H-matrix inverse ( rtol = 1416 x 1416
```

```
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
 1e-06 , atol =  1e-14 , overwrite= False  )
                                                                  0868759155
(5.21 GB)
    size of inverse =  3361874  bytes

━━ H-matrix multiplication C=A*B
    done in 24.08s
    size of C = 3.87 MB

━━ H-matrix multiplication C=A*B
    done in 32.96s
    size of C = 8.53 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 30.75s
    size of LU factor = 18.71 MB
using Gauss-Newton Hessian
it= 0 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.951e+07
||M r0|| = 6.356e+01
(M r0, r0) = 2.263e+04
 Iteration :  0  (r, r) =  380758959655851.75
 Iteration :  1  (r, r) =  27959776538156.125
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  5287700.4962607445

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
```

```
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.5e+08 2.0e+07    ---- 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 1
cumulative cost evaluations : 1
cumulative gradient evaluations : 1
cumulative Hessian vector products (excluding preconditioner builds) : 1
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 1 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 5.224e+06
||M r0|| = 6.381e+01
(M r0, r0) = 8.469e+03
 Iteration :  0  (r, r) =  27289142408423.758
 Iteration :  1  (r, r) =  7701060835297.035
 Iteration :  2  (r, r) =  1311442969153.9053
Relative/Absolute residual less than tol
Converged in  2  iterations with final norm  1145182.5047362125

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
```

```
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.5e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 2.132606e+07 2.0e+07 1.5e+06 -3.7e+07 5.2e+06 1.0e+00 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 3
cumulative cost evaluations : 2
cumulative gradient evaluations : 2
cumulative Hessian vector products (excluding preconditioner builds) : 3
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 2 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.530e+06
||M r0|| = 3.820e+01
(M r0, r0) = 3.525e+03
 Iteration :  0  (r, r) =  2340536424140.0137
 Iteration :  1  (r, r) =  712914200013.5758
 Iteration :  2  (r, r) =  175316183704.7832
Relative/Absolute residual less than tol
Converged in  2  iterations with final norm  418707.754531467

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
```

```
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.5e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 2.132606e+07 2.0e+07 1.5e+06 -3.7e+07 5.2e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 3.338753e+06 2.8e+06 4.9e+05 -4.3e+06 1.5e+06 1.0e+00 2.8
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 5
cumulative cost evaluations : 3
cumulative gradient evaluations : 3
cumulative Hessian vector products (excluding preconditioner builds) : 5
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 3 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 4.986e+05
||M r0|| = 1.761e+01
(M r0, r0) = 1.161e+03
 Iteration :  0  (r, r) =  248568164917.9255
 Iteration :  1  (r, r) =  106842720955.19986
 Iteration :  2  (r, r) =  20930111230.756157
 Iteration :  3  (r, r) =  17866906665.744617
 Iteration :  4  (r, r) =  14926724379.708942
 Iteration :  5  (r, r) =  4978380900.0606365
Relative/Absolute residual less than tol
Converged in  5  iterations with final norm  70557.64239301535

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
```

```
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.5e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 2.132606e+07 2.0e+07 1.5e+06 -3.7e+07 5.2e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 3.338753e+06 2.8e+06 4.9e+05 -4.3e+06 1.5e+06 1.0e+00 2.8
e-01
 3   5  1  1   5  T  F 1.195733e+06 8.2e+05 3.7e+05 -1.0e+06 5.0e+05 1.0e+00 1.6
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 10
cumulative cost evaluations : 4
cumulative gradient evaluations : 4
cumulative Hessian vector products (excluding preconditioner builds) : 10
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 4 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 8.543e+04
||M r0|| = 5.409e+00
(M r0, r0) = 3.824e+02
 Iteration :  0  (r, r) =  7297845207.598768
 Iteration :  1  (r, r) =  2280685646.174207
 Iteration :  2  (r, r) =  659728017.0973521
 Iteration :  3  (r, r) =  225314734.4025255
 Iteration :  4  (r, r) =  89762691.02489269
 Iteration :  5  (r, r) =  57860846.27469475
 Iteration :  6  (r, r) =  48513613.62454468
 Iteration :  7  (r, r) =  21170138.144938618
Relative/Absolute residual less than tol
Converged in  7  iterations with final norm  4601.101840313754

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
```

```
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.5e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 2.132606e+07 2.0e+07 1.5e+06 -3.7e+07 5.2e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 3.338753e+06 2.8e+06 4.9e+05 -4.3e+06 1.5e+06 1.0e+00 2.8
e-01
 3   5  1  1   5  T  F 1.195733e+06 8.2e+05 3.7e+05 -1.0e+06 5.0e+05 1.0e+00 1.6
e-01
 4   7  1  1   7  T  F 6.670863e+05 3.7e+05 3.0e+05 -7.6e+04 8.5e+04 1.0e+00 6.6
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 17
cumulative cost evaluations : 5
cumulative gradient evaluations : 5
cumulative Hessian vector products (excluding preconditioner builds) : 17
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 5 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 7.056e+03
||M r0|| = 2.762e-01
(M r0, r0) = 2.327e+01
 Iteration :  0  (r, r) =  49792409.53401345
 Iteration :  1  (r, r) =  22010573.92844159
 Iteration :  2  (r, r) =  6471232.010882744
 Iteration :  3  (r, r) =  3532920.94038814
 Iteration :  4  (r, r) =  2141096.200956701
 Iteration :  5  (r, r) =  1358342.6692035976
 Iteration :  6  (r, r) =  778075.9056996037
 Iteration :  7  (r, r) =  281847.7694706188
 Iteration :  8  (r, r) =  100866.0569116978
 Iteration :  9  (r, r) =  30838.7852635023
 Iteration :  10  (r, r) =  9049.563496632909
Relative/Absolute residual less than tol
Converged in  10  iterations with final norm  95.1291937137749

===================== Begin Newton CG convergence information =================
=====
```

```
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.5e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 2.132606e+07 2.0e+07 1.5e+06 -3.7e+07 5.2e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 3.338753e+06 2.8e+06 4.9e+05 -4.3e+06 1.5e+06 1.0e+00 2.8
e-01
 3   5  1  1   5  T  F 1.195733e+06 8.2e+05 3.7e+05 -1.0e+06 5.0e+05 1.0e+00 1.6
e-01
 4   7  1  1   7  T  F 6.670863e+05 3.7e+05 3.0e+05 -7.6e+04 8.5e+04 1.0e+00 6.6
e-02
 5  10  1  1  10  T  F 6.292693e+05 3.4e+05 2.9e+05 -4.5e+02 7.1e+03 1.0e+00 1.9
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 27
cumulative cost evaluations : 6
cumulative gradient evaluations : 6
cumulative Hessian vector products (excluding preconditioner builds) : 27
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 6 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.340e+02
||M r0|| = 6.322e-03
(M r0, r0) = 4.462e-01
 Iteration :  0  (r, r) =  17958.22980205646
```

```
 Iteration :  1  (r, r) =  6858.528240221176
 Iteration :  2  (r, r) =  1876.9757529167366
 Iteration :  3  (r, r) =  1065.5891962126195
 Iteration :  4  (r, r) =  733.3525130555047
 Iteration :  5  (r, r) =  344.57629869435925
 Iteration :  6  (r, r) =  138.15005937735427
 Iteration :  7  (r, r) =  76.76910479247397
 Iteration :  8  (r, r) =  35.07656150156684
 Iteration :  9  (r, r) =  14.76832484573847
 Iteration :  10  (r, r) =  4.795889229333161
 Iteration :  11  (r, r) =  1.9263074200520975
 Iteration :  12  (r, r) =  0.8235434270191263
 Iteration :  13  (r, r) =  0.4654588938497404
 Iteration :  14  (r, r) =  0.254923188605036
 Iteration :  15  (r, r) =  0.11286904328390368
Relative/Absolute residual less than tol
Converged in  15  iterations with final norm  0.3359598834442941

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.5e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 2.132606e+07 2.0e+07 1.5e+06 -3.7e+07 5.2e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 3.338753e+06 2.8e+06 4.9e+05 -4.3e+06 1.5e+06 1.0e+00 2.8
e-01
 3   5  1  1   5  T  F 1.195733e+06 8.2e+05 3.7e+05 -1.0e+06 5.0e+05 1.0e+00 1.6
e-01
 4   7  1  1   7  T  F 6.670863e+05 3.7e+05 3.0e+05 -7.6e+04 8.5e+04 1.0e+00 6.6
e-02
 5  10  1  1  10  T  F 6.292693e+05 3.4e+05 2.9e+05 -4.5e+02 7.1e+03 1.0e+00 1.9
```

```
e-02
 6  15  1  1  15  T  F 6.290439e+05 3.4e+05 2.9e+05 -1.6e-01 1.3e+02 1.0e+00 2.6
e-03

converged : False
reason    : unknown reason
cumulative CG iterations : 42
cumulative cost evaluations : 7
cumulative gradient evaluations : 7
cumulative Hessian vector products (excluding preconditioner builds) : 42
======================= End Newton CG convergence information ==================
=====

using Hessian
it= 7 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Fa
lse
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.173e+00
||M r0|| = 2.675e-04
(M r0, r0) = 9.478e-03
 Iteration :  0  (r, r) =  1.3755405355551833
 Iteration :  1  (r, r) =  0.6227369434083823
 Iteration :  2  (r, r) =  0.22928756030759018
 Iteration :  3  (r, r) =  0.10367138361380868
 Iteration :  4  (r, r) =  0.06324886955755232
 Iteration :  5  (r, r) =  0.03711992955355433
 Iteration :  6  (r, r) =  0.018845951386799842
 Iteration :  7  (r, r) =  0.0077388123714759015
 Iteration :  8  (r, r) =  0.0036279920549297604
 Iteration :  9  (r, r) =  0.0013675002671183008
 Iteration :  10  (r, r) =  0.0005571189427543016
 Iteration :  11  (r, r) =  0.0002814548589007438
 Iteration :  12  (r, r) =  0.00013737385872023458
 Iteration :  13  (r, r) =  6.896216035485181e-05
 Iteration :  14  (r, r) =  3.024945284928301e-05
 Iteration :  15  (r, r) =  1.4963198146291305e-05
 Iteration :  16  (r, r) =  6.051372710972423e-06
 Iteration :  17  (r, r) =  2.7617118528609315e-06
 Iteration :  18  (r, r) =  1.4594959022473484e-06
 Iteration :  19  (r, r) =  7.161965143108112e-07
 Iteration :  20  (r, r) =  2.2493266982281057e-07
 Iteration :  21  (r, r) =  9.439702462987222e-08
 Iteration :  22  (r, r) =  4.069067152507951e-08
Relative/Absolute residual less than tol
Converged in  22  iterations with final norm  0.00020171928892666654

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
```

```
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.5e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 2.132606e+07 2.0e+07 1.5e+06 -3.7e+07 5.2e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 3.338753e+06 2.8e+06 4.9e+05 -4.3e+06 1.5e+06 1.0e+00 2.8
e-01
 3   5  1  1   5  T  F 1.195733e+06 8.2e+05 3.7e+05 -1.0e+06 5.0e+05 1.0e+00 1.6
e-01
 4   7  1  1   7  T  F 6.670863e+05 3.7e+05 3.0e+05 -7.6e+04 8.5e+04 1.0e+00 6.6
e-02
 5  10  1  1  10  T  F 6.292693e+05 3.4e+05 2.9e+05 -4.5e+02 7.1e+03 1.0e+00 1.9
e-02
 6  15  1  1  15  T  F 6.290439e+05 3.4e+05 2.9e+05 -1.6e-01 1.3e+02 1.0e+00 2.6
e-03
 7  22  1  1  22  F  F 6.290438e+05 3.4e+05 2.9e+05 -5.1e-05 1.2e+00 1.0e+00 2.5
e-04

converged : False
reason    : unknown reason
cumulative CG iterations : 64
cumulative cost evaluations : 8
cumulative gradient evaluations : 8
cumulative Hessian vector products (excluding preconditioner builds) : 64
====================== End Newton CG convergence information ==================
=====

using Hessian
it= 8 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Fa
lse
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.029e+00
||M r0|| = 2.340e-04
(M r0, r0) = 8.291e-03
 Iteration :  0  (r, r) =  1.0582034675173972
 Iteration :  1  (r, r) =  0.47877975463098
 Iteration :  2  (r, r) =  0.1758227178656292
 Iteration :  3  (r, r) =  0.07950096551186883
 Iteration :  4  (r, r) =  0.04860318715260439
 Iteration :  5  (r, r) =  0.02846569764644217
 Iteration :  6  (r, r) =  0.01445260068055998
 Iteration :  7  (r, r) =  0.005926764608446447
 Iteration :  8  (r, r) =  0.0027776855731647095
```

```
 Iteration :  9  (r, r) =  0.0010475326373439976
 Iteration :  10  (r, r) =  0.00042735154789226256
 Iteration :  11  (r, r) =  0.00021617944665006133
 Iteration :  12  (r, r) =  0.00010554465227443668
 Iteration :  13  (r, r) =  5.296695753497628e-05
 Iteration :  14  (r, r) =  2.3154808605604264e-05
 Iteration :  15  (r, r) =  1.1445675389251117e-05
 Iteration :  16  (r, r) =  4.639358406142161e-06
 Iteration :  17  (r, r) =  2.119967894102724e-06
 Iteration :  18  (r, r) =  1.1188220520013279e-06
 Iteration :  19  (r, r) =  5.482384253147205e-07
 Iteration :  20  (r, r) =  1.7232321902324986e-07
 Iteration :  21  (r, r) =  7.238109400925625e-08
 Iteration :  22  (r, r) =  3.111892231841758e-08
Relative/Absolute residual less than tol
Converged in  22  iterations with final norm  0.00017640556203934608

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.5e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 2.132606e+07 2.0e+07 1.5e+06 -3.7e+07 5.2e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 3.338753e+06 2.8e+06 4.9e+05 -4.3e+06 1.5e+06 1.0e+00 2.8
e-01
 3   5  1  1   5  T  F 1.195733e+06 8.2e+05 3.7e+05 -1.0e+06 5.0e+05 1.0e+00 1.6
e-01
 4   7  1  1   7  T  F 6.670863e+05 3.7e+05 3.0e+05 -7.6e+04 8.5e+04 1.0e+00 6.6
e-02
 5  10  1  1  10  T  F 6.292693e+05 3.4e+05 2.9e+05 -4.5e+02 7.1e+03 1.0e+00 1.9
e-02
```

```
 6  15  1  1  15  T  F 6.290439e+05 3.4e+05 2.9e+05 -1.6e-01 1.3e+02 1.0e+00 2.6
e-03
 7  22  1  1  22  F  F 6.290438e+05 3.4e+05 2.9e+05 -5.1e-05 1.2e+00 1.0e+00 2.5
e-04
 8  22  4  1  22  F  F 6.290438e+05 3.4e+05 2.9e+05 -3.9e-05 1.0e+00 1.2e-01 2.3
e-04

converged : False
reason    : unknown reason
cumulative CG iterations : 86
cumulative cost evaluations : 12
cumulative gradient evaluations : 9
cumulative Hessian vector products (excluding preconditioner builds) : 86
====================== End Newton CG convergence information ==================
=====


===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.5e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 2.132606e+07 2.0e+07 1.5e+06 -3.7e+07 5.2e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 3.338753e+06 2.8e+06 4.9e+05 -4.3e+06 1.5e+06 1.0e+00 2.8
e-01
 3   5  1  1   5  T  F 1.195733e+06 8.2e+05 3.7e+05 -1.0e+06 5.0e+05 1.0e+00 1.6
e-01
 4   7  1  1   7  T  F 6.670863e+05 3.7e+05 3.0e+05 -7.6e+04 8.5e+04 1.0e+00 6.6
e-02
 5  10  1  1  10  T  F 6.292693e+05 3.4e+05 2.9e+05 -4.5e+02 7.1e+03 1.0e+00 1.9
e-02
```

```
 6  15  1  1  15  T  F 6.290439e+05 3.4e+05 2.9e+05 -1.6e-01 1.3e+02 1.0e+00 2.6
e-03
 7  22  1  1  22  F  F 6.290438e+05 3.4e+05 2.9e+05 -5.1e-05 1.2e+00 1.0e+00 2.5
e-04
 8  22  4  1  22  F  F 6.290438e+05 3.4e+05 2.9e+05 -3.9e-05 1.0e+00 1.2e-01 2.3
e-04
 9   0 10  1   0  T  F 6.290438e+05 3.4e+05 2.9e+05     ---- 1.0e+00 9.8e-04
----

converged : False
reason    : Maximum number of backtracking reached
cumulative CG iterations : 86
cumulative cost evaluations : 22
cumulative gradient evaluations : 10
cumulative Hessian vector products (excluding preconditioner builds) : 86
======================= End Newton CG convergence information ==================
=====

gamma= 5642.493956627751 , noise_datanorm= 863.5355316705534 , misfit_datanorm=
827.8100435380474

-------- gamma= 6118.097561746672  --------
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 986413
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 986413
 |S-A|_2 = 4.10017e-09
  matrix has dimension 1416━━ H-matrix inverse ( rtol =  1e-06 , atol =  1e-14 ,
overwrite= False  )
 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
    done in  14.772184371948242
    size of inverse =  3361874  bytes

━━ H-matrix multiplication C=A*B
    done in 24.07s
    size of C = 3.87 MB

━━ H-matrix multiplication C=A*B
    done in 32.68s
    size of C = 8.54 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 30.89s
    size of LU factor = 18.69 MB
using Gauss-Newton Hessian
it= 0 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
```

```
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.951e+07
||M r0|| = 6.138e+01
(M r0, r0) = 2.260e+04
 Iteration :  0  (r, r) =  380758959655851.75
 Iteration :  1  (r, r) =  27552264456638.92
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  5249025.095828646

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 1
cumulative cost evaluations : 1
cumulative gradient evaluations : 1
cumulative Hessian vector products (excluding preconditioner builds) : 1
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 1 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 5.104e+06
```

```
||M r0|| = 5.441e+01
(M r0, r0) = 7.913e+03
 Iteration :  0  (r, r) =  26054966978283.848
 Iteration :  1  (r, r) =  6473242690748.515
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  2544256.805188603

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 2.035642e+07 1.9e+07 1.5e+06 -2.6e+07 5.1e+06 1.0e+00 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 2
cumulative cost evaluations : 2
cumulative gradient evaluations : 2
cumulative Hessian vector products (excluding preconditioner builds) : 2
====================== End Newton CG convergence information =================
=====

using Gauss-Newton Hessian
it= 2 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 3.058e+06
||M r0|| = 4.495e+01
(M r0, r0) = 5.731e+03
```

```
 Iteration :  0  (r, r) =  9349885539661.662
 Iteration :  1  (r, r) =  3913714972610.4346
 Iteration :  2  (r, r) =  380462945413.51056
Relative/Absolute residual less than tol
Converged in  2  iterations with final norm  616816.7843156593

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 2.035642e+07 1.9e+07 1.5e+06 -2.6e+07 5.1e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 8.191928e+06 7.5e+06 7.3e+05 -1.3e+07 3.1e+06 1.0e+00 4.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 4
cumulative cost evaluations : 3
cumulative gradient evaluations : 3
cumulative Hessian vector products (excluding preconditioner builds) : 4
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 3 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 7.263e+05
||M r0|| = 2.282e+01
```

```
(M r0, r0) = 1.935e+03
 Iteration :  0  (r, r) =  527449808817.5456
 Iteration :  1  (r, r) =  160687550941.08383
 Iteration :  2  (r, r) =  33790535932.055767
 Iteration :  3  (r, r) =  18894419458.856133
Relative/Absolute residual less than tol
Converged in  3  iterations with final norm  137456.9731183403

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 2.035642e+07 1.9e+07 1.5e+06 -2.6e+07 5.1e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 8.191928e+06 7.5e+06 7.3e+05 -1.3e+07 3.1e+06 1.0e+00 4.0
e-01
 3   3  1  1   3  T  F 1.727353e+06 1.2e+06 4.8e+05 -1.8e+06 7.3e+05 1.0e+00 1.9
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 7
cumulative cost evaluations : 4
cumulative gradient evaluations : 4
cumulative Hessian vector products (excluding preconditioner builds) : 7
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 4 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
```

```
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.390e+05
||M r0|| = 4.718e+00
(M r0, r0) = 4.313e+02
 Iteration :  0  (r, r) =  19334059092.520103
 Iteration :  1  (r, r) =  7247875720.422212
 Iteration :  2  (r, r) =  4475951574.946624
 Iteration :  3  (r, r) =  3452168534.8928995
 Iteration :  4  (r, r) =  922591237.8618386
 Iteration :  5  (r, r) =  215262624.86511877
 Iteration :  6  (r, r) =  86070662.6460247
Relative/Absolute residual less than tol
Converged in  6  iterations with final norm  9277.427587754308

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 2.035642e+07 1.9e+07 1.5e+06 -2.6e+07 5.1e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 8.191928e+06 7.5e+06 7.3e+05 -1.3e+07 3.1e+06 1.0e+00 4.0
e-01
 3   3  1  1   3  T  F 1.727353e+06 1.2e+06 4.8e+05 -1.8e+06 7.3e+05 1.0e+00 1.9
e-01
 4   6  1  1   6  T  F 7.894271e+05 4.4e+05 3.5e+05 -2.1e+05 1.4e+05 1.0e+00 8.4
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 13
```

```
cumulative cost evaluations : 5
cumulative gradient evaluations : 5
cumulative Hessian vector products (excluding preconditioner builds) : 13
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 5 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 4.731e+04
||M r0|| = 4.250e+00
(M r0, r0) = 3.134e+02
 Iteration :  0  (r, r) =  2238172375.0731626
 Iteration :  1  (r, r) =  618700128.6462255
 Iteration :  2  (r, r) =  87700054.27766621
 Iteration :  3  (r, r) =  33430072.922515627
 Iteration :  4  (r, r) =  19504510.562068425
 Iteration :  5  (r, r) =  14327744.380251084
 Iteration :  6  (r, r) =  7508999.932956887
 Iteration :  7  (r, r) =  2390789.343373282
Relative/Absolute residual less than tol
Converged in  7  iterations with final norm  1546.2177541902959

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 2.035642e+07 1.9e+07 1.5e+06 -2.6e+07 5.1e+06 1.0e+00 5.0
e-01
```

```
 2   2  1  1   2  T  F 8.191928e+06 7.5e+06 7.3e+05 -1.3e+07 3.1e+06 1.0e+00 4.0
e-01
 3   3  1  1   3  T  F 1.727353e+06 1.2e+06 4.8e+05 -1.8e+06 7.3e+05 1.0e+00 1.9
e-01
 4   6  1  1   6  T  F 7.894271e+05 4.4e+05 3.5e+05 -2.1e+05 1.4e+05 1.0e+00 8.4
e-02
 5   7  1  1   7  T  F 6.975534e+05 3.7e+05 3.3e+05 -3.8e+04 4.7e+04 1.0e+00 4.9
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 20
cumulative cost evaluations : 6
cumulative gradient evaluations : 6
cumulative Hessian vector products (excluding preconditioner builds) : 20
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 6 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 3.452e+03
||M r0|| = 3.327e-01
(M r0, r0) = 2.394e+01
 Iteration :  0  (r, r) =  11917813.161069373
 Iteration :  1  (r, r) =  4606513.007703627
 Iteration :  2  (r, r) =  1420040.9352825829
 Iteration :  3  (r, r) =  291530.37694936426
 Iteration :  4  (r, r) =  146419.0735492895
 Iteration :  5  (r, r) =  97358.04409870662
 Iteration :  6  (r, r) =  48598.18128093131
 Iteration :  7  (r, r) =  26698.38451811125
 Iteration :  8  (r, r) =  6044.951726411782
 Iteration :  9  (r, r) =  2235.356100178333
 Iteration :  10  (r, r) =  1092.0475576731521
Relative/Absolute residual less than tol
Converged in  10  iterations with final norm  33.04614285621171

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
```

```
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 2.035642e+07 1.9e+07 1.5e+06 -2.6e+07 5.1e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 8.191928e+06 7.5e+06 7.3e+05 -1.3e+07 3.1e+06 1.0e+00 4.0
e-01
 3   3  1  1   3  T  F 1.727353e+06 1.2e+06 4.8e+05 -1.8e+06 7.3e+05 1.0e+00 1.9
e-01
 4   6  1  1   6  T  F 7.894271e+05 4.4e+05 3.5e+05 -2.1e+05 1.4e+05 1.0e+00 8.4
e-02
 5   7  1  1   7  T  F 6.975534e+05 3.7e+05 3.3e+05 -3.8e+04 4.7e+04 1.0e+00 4.9
e-02
 6  10  1  1  10  T  F 6.786696e+05 3.5e+05 3.3e+05 -3.0e+02 3.5e+03 1.0e+00 1.3
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 30
cumulative cost evaluations : 7
cumulative gradient evaluations : 7
cumulative Hessian vector products (excluding preconditioner builds) : 30
======================= End Newton CG convergence information ==================
=====

using Hessian
it= 7 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Fa
lse
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 6.775e+01
||M r0|| = 9.697e-03
(M r0, r0) = 6.108e-01
 Iteration :  0  (r, r) =  4590.15483639251
 Iteration :  1  (r, r) =  1664.681519007714
 Iteration :  2  (r, r) =  443.28130339939287
 Iteration :  3  (r, r) =  229.65525611157375
 Iteration :  4  (r, r) =  90.54086617396655
 Iteration :  5  (r, r) =  52.5888956772388
 Iteration :  6  (r, r) =  19.89682549041862
 Iteration :  7  (r, r) =  10.082156636912782
 Iteration :  8  (r, r) =  4.29242688564112
 Iteration :  9  (r, r) =  1.6595071118831572
 Iteration :  10  (r, r) =  0.9519281920629445
 Iteration :  11  (r, r) =  0.3710331930139767
 Iteration :  12  (r, r) =  0.17407273758540226
 Iteration :  13  (r, r) =  0.07348842838545781
 Iteration :  14  (r, r) =  0.03208038019063672
 Iteration :  15  (r, r) =  0.011814915820827995
Relative/Absolute residual less than tol
Converged in  15  iterations with final norm  0.10869643885991848
```

```
===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 2.035642e+07 1.9e+07 1.5e+06 -2.6e+07 5.1e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 8.191928e+06 7.5e+06 7.3e+05 -1.3e+07 3.1e+06 1.0e+00 4.0
e-01
 3   3  1  1   3  T  F 1.727353e+06 1.2e+06 4.8e+05 -1.8e+06 7.3e+05 1.0e+00 1.9
e-01
 4   6  1  1   6  T  F 7.894271e+05 4.4e+05 3.5e+05 -2.1e+05 1.4e+05 1.0e+00 8.4
e-02
 5   7  1  1   7  T  F 6.975534e+05 3.7e+05 3.3e+05 -3.8e+04 4.7e+04 1.0e+00 4.9
e-02
 6  10  1  1  10  T  F 6.786696e+05 3.5e+05 3.3e+05 -3.0e+02 3.5e+03 1.0e+00 1.3
e-02
 7  15  1  1  15  F  F 6.785206e+05 3.5e+05 3.3e+05 -1.9e-01 6.8e+01 1.0e+00 1.9
e-03

converged : False
reason    : unknown reason
cumulative CG iterations : 45
cumulative cost evaluations : 8
cumulative gradient evaluations : 8
cumulative Hessian vector products (excluding preconditioner builds) : 45
====================== End Newton CG convergence information ==================
=====

using Hessian
it= 8 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Fa
```

```
lse

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 2.035642e+07 1.9e+07 1.5e+06 -2.6e+07 5.1e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 8.191928e+06 7.5e+06 7.3e+05 -1.3e+07 3.1e+06 1.0e+00 4.0
e-01
 3   3  1  1   3  T  F 1.727353e+06 1.2e+06 4.8e+05 -1.8e+06 7.3e+05 1.0e+00 1.9
e-01
 4   6  1  1   6  T  F 7.894271e+05 4.4e+05 3.5e+05 -2.1e+05 1.4e+05 1.0e+00 8.4
e-02
 5   7  1  1   7  T  F 6.975534e+05 3.7e+05 3.3e+05 -3.8e+04 4.7e+04 1.0e+00 4.9
e-02
 6  10  1  1  10  T  F 6.786696e+05 3.5e+05 3.3e+05 -3.0e+02 3.5e+03 1.0e+00 1.3
e-02
 7  15  1  1  15  F  F 6.785206e+05 3.5e+05 3.3e+05 -1.9e-01 6.8e+01 1.0e+00 1.9
e-03
 8   0  1  1   0  F  F 6.785205e+05 3.5e+05 3.3e+05     ---- 1.3e-01 1.0e+00
 ----

converged : True
reason    : Norm of the gradient less than tolerance
cumulative CG iterations : 45
cumulative cost evaluations : 9
cumulative gradient evaluations : 9
cumulative Hessian vector products (excluding preconditioner builds) : 45
====================== End Newton CG convergence information ==================
=====
```

```
gamma= 6118.097561746672 , noise_datanorm= 863.5355316705534 , misfit_datanorm=
 839.9312773370468

-------- gamma= 7138.784456052863  --------
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 1.15098e+06
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 1.15098e+06
 |S-A|_2 = 6.6538e-09
  matrix has dimension ━━ H-matrix inverse ( rtol =  1e-06 , atol =  1e-14 , ove
rwrite= False  )
    done in 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
 14.250972747802734
    size of inverse =  3361874  bytes

━━ H-matrix multiplication C=A*B
    done in 23.86s
    size of C = 3.88 MB

━━ H-matrix multiplication C=A*B
    done in 32.78s                                                (5.27 GB)
    size of C = 8.54 MB

━━ LU factorisation ( rtol = 1e-07 )
[===================================using Gauss-Newton Hessian           ]   7% E
TA 27 s (5.28 GB)                 ]  22% ETA 23 s (5.28 GB)
it= 0 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
    done in 30.47s
    size of LU factor = 18.68 MB
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.951e+07
||M r0|| = 5.778e+01
(M r0, r0) = 2.256e+04
 Iteration :  0  (r, r) =  380758959655851.75
 Iteration :  1  (r, r) =  26896234250609.027
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  5186157.94694001

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
```

```
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 1
cumulative cost evaluations : 1
cumulative gradient evaluations : 1
cumulative Hessian vector products (excluding preconditioner builds) : 1
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 1 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 4.915e+06
||M r0|| = 4.050e+01
(M r0, r0) = 7.108e+03
 Iteration :  0  (r, r) =  24160913725032.47
 Iteration :  1  (r, r) =  4453064542156.394
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  2110228.5521138213

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
```

```
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.889649e+07 1.7e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 2
cumulative cost evaluations : 2
cumulative gradient evaluations : 2
cumulative Hessian vector products (excluding preconditioner builds) : 2
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 2 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 2.509e+06
||M r0|| = 3.539e+01
(M r0, r0) = 4.837e+03
 Iteration :  0  (r, r) =  6296777088407.215
 Iteration :  1  (r, r) =  2481593417039.141
 Iteration :  2  (r, r) =  249372931029.08386
Relative/Absolute residual less than tol
Converged in  2  iterations with final norm  499372.5373196686

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
```

```
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.889649e+07 1.7e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.437744e+06 5.7e+06 7.8e+05 -9.9e+06 2.5e+06 1.0e+00 3.6
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 4
cumulative cost evaluations : 3
cumulative gradient evaluations : 3
cumulative Hessian vector products (excluding preconditioner builds) : 4
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 3 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 5.512e+05
||M r0|| = 1.881e+01
(M r0, r0) = 1.492e+03
 Iteration :  0  (r, r) =  303848102337.3596
 Iteration :  1  (r, r) =  81081111335.34378
 Iteration :  2  (r, r) =  15445353572.752089
 Iteration :  3  (r, r) =  18080127981.030334
 Iteration :  4  (r, r) =  8238387134.2983265
Relative/Absolute residual less than tol
Converged in  4  iterations with final norm  90765.56138920932

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion
```

```
it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.889649e+07 1.7e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.437744e+06 5.7e+06 7.8e+05 -9.9e+06 2.5e+06 1.0e+00 3.6
e-01
 3   4  1  1   4  T  F 1.512410e+06 9.8e+05 5.4e+05 -1.3e+06 5.5e+05 1.0e+00 1.7
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 8
cumulative cost evaluations : 4
cumulative gradient evaluations : 4
cumulative Hessian vector products (excluding preconditioner builds) : 8
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 4 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 9.127e+04
||M r0|| = 4.140e+00
(M r0, r0) = 3.319e+02
 Iteration :  0  (r, r) =  8329932130.716492
 Iteration :  1  (r, r) =  2496202487.435622
 Iteration :  2  (r, r) =  532603679.47196174
 Iteration :  3  (r, r) =  155225476.16986752
 Iteration :  4  (r, r) =  92899021.41982985
 Iteration :  5  (r, r) =  62839648.63868518
 Iteration :  6  (r, r) =  36067250.26972498
Relative/Absolute residual less than tol
Converged in  6  iterations with final norm  6005.601574340823
```

```
===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.889649e+07 1.7e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.437744e+06 5.7e+06 7.8e+05 -9.9e+06 2.5e+06 1.0e+00 3.6
e-01
 3   4  1  1   4  T  F 1.512410e+06 9.8e+05 5.4e+05 -1.3e+06 5.5e+05 1.0e+00 1.7
e-01
 4   6  1  1   6  T  F 8.248326e+05 4.0e+05 4.2e+05 -6.5e+04 9.1e+04 1.0e+00 6.8
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 14
cumulative cost evaluations : 5
cumulative gradient evaluations : 5
cumulative Hessian vector products (excluding preconditioner builds) : 14
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 5 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 7.603e+03
||M r0|| = 1.964e-01
(M r0, r0) = 2.055e+01
 Iteration :  0  (r, r) =  57799774.423620336
```

```
 Iteration :  1  (r, r) =  20459278.332493618
 Iteration :  2  (r, r) =  5850681.833356286
 Iteration :  3  (r, r) =  3141111.5110887783
 Iteration :  4  (r, r) =  1860590.8383449935
 Iteration :  5  (r, r) =  790604.9220607233
 Iteration :  6  (r, r) =  226270.55212328077
 Iteration :  7  (r, r) =  57373.93873669866
 Iteration :  8  (r, r) =  13054.16419750683
Relative/Absolute residual less than tol
Converged in  8  iterations with final norm  114.25482133156058

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.889649e+07 1.7e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.437744e+06 5.7e+06 7.8e+05 -9.9e+06 2.5e+06 1.0e+00 3.6
e-01
 3   4  1  1   4  T  F 1.512410e+06 9.8e+05 5.4e+05 -1.3e+06 5.5e+05 1.0e+00 1.7
e-01
 4   6  1  1   6  T  F 8.248326e+05 4.0e+05 4.2e+05 -6.5e+04 9.1e+04 1.0e+00 6.8
e-02
 5   8  1  1   8  T  F 7.925317e+05 3.8e+05 4.1e+05 -3.6e+02 7.6e+03 1.0e+00 2.0
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 22
cumulative cost evaluations : 6
cumulative gradient evaluations : 6
```

```
cumulative Hessian vector products (excluding preconditioner builds) : 22
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 6 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.401e+02
||M r0|| = 8.849e-03
(M r0, r0) = 5.157e-01
 Iteration :  0  (r, r) =  19638.404328884742
 Iteration :  1  (r, r) =  8285.306905766436
 Iteration :  2  (r, r) =  1890.3477045625775
 Iteration :  3  (r, r) =  966.9571175218595
 Iteration :  4  (r, r) =  401.7769648490362
 Iteration :  5  (r, r) =  150.4335585114488
 Iteration :  6  (r, r) =  83.94608334727351
 Iteration :  7  (r, r) =  38.066598193353585
 Iteration :  8  (r, r) =  13.989544974915455
 Iteration :  9  (r, r) =  2.9651534590621953
 Iteration :  10  (r, r) =  0.9213248930296841
 Iteration :  11  (r, r) =  0.4743181490770158
 Iteration :  12  (r, r) =  0.2159035316330078
 Iteration :  13  (r, r) =  0.07901412561683538
Relative/Absolute residual less than tol
Converged in  13  iterations with final norm  0.28109451367259974

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
```

```
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.889649e+07 1.7e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.437744e+06 5.7e+06 7.8e+05 -9.9e+06 2.5e+06 1.0e+00 3.6
e-01
 3   4  1  1   4  T  F 1.512410e+06 9.8e+05 5.4e+05 -1.3e+06 5.5e+05 1.0e+00 1.7
e-01
 4   6  1  1   6  T  F 8.248326e+05 4.0e+05 4.2e+05 -6.5e+04 9.1e+04 1.0e+00 6.8
e-02
 5   8  1  1   8  T  F 7.925317e+05 3.8e+05 4.1e+05 -3.6e+02 7.6e+03 1.0e+00 2.0
e-02
 6  13  1  1  13  T  F 7.923532e+05 3.8e+05 4.2e+05 -1.9e-01 1.4e+02 1.0e+00 2.7
e-03

converged : False
reason    : unknown reason
cumulative CG iterations : 35
cumulative cost evaluations : 7
cumulative gradient evaluations : 7
cumulative Hessian vector products (excluding preconditioner builds) : 35
======================= End Newton CG convergence information ==================
=====

using Hessian
it= 7 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Fa
lse
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.265e+00
||M r0|| = 1.335e-04
(M r0, r0) = 7.977e-03
 Iteration :  0  (r, r) =  1.6008585910543833
 Iteration :  1  (r, r) =  0.6159330822101328
 Iteration :  2  (r, r) =  0.21619970253233936
 Iteration :  3  (r, r) =  0.09148577901167894
 Iteration :  4  (r, r) =  0.03977000800932326
 Iteration :  5  (r, r) =  0.017194087226798146
 Iteration :  6  (r, r) =  0.006674669191000224
 Iteration :  7  (r, r) =  0.0021554660938514787
 Iteration :  8  (r, r) =  0.0006252031774234368
 Iteration :  9  (r, r) =  0.00022292934827211635
 Iteration :  10  (r, r) =  9.29293999476769e-05
 Iteration :  11  (r, r) =  3.998573440339819e-05
 Iteration :  12  (r, r) =  1.718379362944741e-05
 Iteration :  13  (r, r) =  7.231998152333773e-06
 Iteration :  14  (r, r) =  2.429794614487861e-06
 Iteration :  15  (r, r) =  8.638154780095389e-07
 Iteration :  16  (r, r) =  4.104322110962867e-07
 Iteration :  17  (r, r) =  1.7858321238271204e-07
 Iteration :  18  (r, r) =  6.387157959502488e-08
Relative/Absolute residual less than tol
Converged in  18  iterations with final norm  0.00025272827225109756

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion
```

```
it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.889649e+07 1.7e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.437744e+06 5.7e+06 7.8e+05 -9.9e+06 2.5e+06 1.0e+00 3.6
e-01
 3   4  1  1   4  T  F 1.512410e+06 9.8e+05 5.4e+05 -1.3e+06 5.5e+05 1.0e+00 1.7
e-01
 4   6  1  1   6  T  F 8.248326e+05 4.0e+05 4.2e+05 -6.5e+04 9.1e+04 1.0e+00 6.8
e-02
 5   8  1  1   8  T  F 7.925317e+05 3.8e+05 4.1e+05 -3.6e+02 7.6e+03 1.0e+00 2.0
e-02
 6  13  1  1  13  T  F 7.923532e+05 3.8e+05 4.2e+05 -1.9e-01 1.4e+02 1.0e+00 2.7
e-03
 7  18  1  1  18  F  F 7.923531e+05 3.8e+05 4.2e+05 -4.3e-05 1.3e+00 1.0e+00 2.5
e-04

converged : False
reason    : unknown reason
cumulative CG iterations : 53
cumulative cost evaluations : 8
cumulative gradient evaluations : 8
cumulative Hessian vector products (excluding preconditioner builds) : 53
======================= End Newton CG convergence information ==================
=====

using Hessian
it= 8 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Fa
lse

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
```

```
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.889649e+07 1.7e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.437744e+06 5.7e+06 7.8e+05 -9.9e+06 2.5e+06 1.0e+00 3.6
e-01
 3   4  1  1   4  T  F 1.512410e+06 9.8e+05 5.4e+05 -1.3e+06 5.5e+05 1.0e+00 1.7
e-01
 4   6  1  1   6  T  F 8.248326e+05 4.0e+05 4.2e+05 -6.5e+04 9.1e+04 1.0e+00 6.8
e-02
 5   8  1  1   8  T  F 7.925317e+05 3.8e+05 4.1e+05 -3.6e+02 7.6e+03 1.0e+00 2.0
e-02
 6  13  1  1  13  T  F 7.923532e+05 3.8e+05 4.2e+05 -1.9e-01 1.4e+02 1.0e+00 2.7
e-03
 7  18  1  1  18  F  F 7.923531e+05 3.8e+05 4.2e+05 -4.3e-05 1.3e+00 1.0e+00 2.5
e-04
 8   0  1  1   0  F  F 7.923531e+05 3.8e+05 4.2e+05     ---- 3.8e-02 1.0e+00
 ----

converged : True
reason    : Norm of the gradient less than tolerance
cumulative CG iterations : 53
cumulative cost evaluations : 9
cumulative gradient evaluations : 9
cumulative Hessian vector products (excluding preconditioner builds) : 53
======================= End Newton CG convergence information ==================
=====

gamma= 7138.784456052863 , noise_datanorm= 863.5355316705534 , misfit_datanorm=
 868.260089302643

-------- gamma= 6959.888561201022  --------
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
```

```
    matrix is            real valued
    format             = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                = 1.12213e+06
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F            = 1.12213e+06
 |S-A|_2 = 1.01053e-09
  matrix has dimension ━━ H-matrix inverse ( rtol =  1e-06 , atol =  1e-14 , ove
rwrite= False  )
1416 x 1416
    no of non-zeroes    = 9746
    matrix is            real valued
    format             = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F            = 5.18561e+06
 |S-A|_2 = 4.66653e-09
          14.947875022888184             ]   7% ETA 14 s (5.28 GB)
    size of inverse =  3361874  bytes

━━ H-matrix multiplication C=A*B
                           using Gauss-Newton Hessian s (5.28 GB)
[=================================     () , num_gn_iter= 7 , using_gauss_newton=
 True
    done in 24.44s
    size of C = 3.88 MB

━━ H-matrix multiplication C=A*B
[==========             Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.951e+07
||M r0|| = 5.832e+01
(M r0, r0) = 2.257e+04
 Iteration :  0  (r, r) =  380758959655851.75
                                 1  (r, r) =  26994998919279.85B)4 s (5.28 GB)
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  5195671.171203952

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
```

```
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 1
cumulative cost evaluations : 1
cumulative gradient evaluations : 1
cumulative Hessian vector products (excluding preconditioner builds) : 1
====================== End Newton CG convergence information =================
=====

    done in 33.03s                                                  100% ETA 0 s
 (5.28 GB)
    size of C = 8.54 MB

━━ LU factorisation ( rtol = 1e-07 )
                                                  using Gauss-Newton Hessian
it= 1 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
    done in 30.79s
    size of LU factor = 18.69 MB
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 4.943e+06
||M r0|| = 4.248e+01
(M r0, r0) = 7.221e+03
 Iteration :  0  (r, r) =  24437921378935.86
 Iteration :  1  (r, r) =  4751109067522.971
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  2179703.894459743

===================== Begin Newton CG convergence information ================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
```

```
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.910665e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 2
cumulative cost evaluations : 2
cumulative gradient evaluations : 2
cumulative Hessian vector products (excluding preconditioner builds) : 2
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 2 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 2.598e+06
||M r0|| = 3.683e+01
(M r0, r0) = 4.981e+03
 Iteration :  0  (r, r) =  6751410678051.522
 Iteration :  1  (r, r) =  2681652363909.4595
 Iteration :  2  (r, r) =  263710772450.128
Relative/Absolute residual less than tol
Converged in  2  iterations with final norm  513527.7718391947

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
```

```
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.910665e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.700183e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 4
cumulative cost evaluations : 3
cumulative gradient evaluations : 3
cumulative Hessian vector products (excluding preconditioner builds) : 4
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 3 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 5.728e+05
||M r0|| = 1.955e+01
(M r0, r0) = 1.548e+03
 Iteration :  0  (r, r) =  328139769359.62915
 Iteration :  1  (r, r) =  89739741084.64735
 Iteration :  2  (r, r) =  16224672247.429895
 Iteration :  3  (r, r) =  18226735419.339443
 Iteration :  4  (r, r) =  9990812893.898542
 Iteration :  5  (r, r) =  2721734252.176254
Relative/Absolute residual less than tol
Converged in  5  iterations with final norm  52170.24297601319

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
```

```
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.910665e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.700183e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.534167e+06 1.0e+06 5.2e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 9
cumulative cost evaluations : 4
cumulative gradient evaluations : 4
cumulative Hessian vector products (excluding preconditioner builds) : 9
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 4 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.209e+05
||M r0|| = 3.436e+00
(M r0, r0) = 4.296e+02
 Iteration :  0  (r, r) =  14618524977.755209
 Iteration :  1  (r, r) =  2824849143.357995
 Iteration :  2  (r, r) =  596817292.8396115
 Iteration :  3  (r, r) =  150264785.5796662
 Iteration :  4  (r, r) =  47829032.384854436
Relative/Absolute residual less than tol
Converged in  4  iterations with final norm  6915.853698919204

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
```

```
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.910665e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.700183e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.534167e+06 1.0e+06 5.2e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.230069e+05 4.2e+05 4.0e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 13
cumulative cost evaluations : 5
cumulative gradient evaluations : 5
cumulative Hessian vector products (excluding preconditioner builds) : 13
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 5 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 7.751e+03
||M r0|| = 2.715e-01
(M r0, r0) = 2.877e+01
 Iteration :  0  (r, r) =  60084635.63848385
 Iteration :  1  (r, r) =  29032064.085037418
 Iteration :  2  (r, r) =  16385361.19183522
 Iteration :  3  (r, r) =  9546030.446397679
 Iteration :  4  (r, r) =  5178463.116675905
 Iteration :  5  (r, r) =  1921669.6559056407
 Iteration :  6  (r, r) =  700002.845715906
 Iteration :  7  (r, r) =  224592.21862906194
 Iteration :  8  (r, r) =  49460.683178396255
 Iteration :  9  (r, r) =  15957.502524407959
```

```
Relative/Absolute residual less than tol
Converged in  9  iterations with final norm  126.32300868966017

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.910665e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.700183e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.534167e+06 1.0e+06 5.2e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.230069e+05 4.2e+05 4.0e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02
 5   9  1  1   9  T  F 7.721033e+05 3.7e+05 4.0e+05 -8.4e+02 7.8e+03 1.0e+00 2.0
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 22
cumulative cost evaluations : 6
cumulative gradient evaluations : 6
cumulative Hessian vector products (excluding preconditioner builds) : 22
====================== End Newton CG convergence information ===================
=====

using Gauss-Newton Hessian
it= 6 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
```

```
Preconditioner: M =approx= A^-1
||r0|| = 2.523e+02
||M r0|| = 8.286e-03
(M r0, r0) = 6.153e-01
 Iteration :  0  (r, r) =  63644.77410180435
 Iteration :  1  (r, r) =  18220.1013835282
 Iteration :  2  (r, r) =  3418.748949646263
 Iteration :  3  (r, r) =  2461.226163052246
 Iteration :  4  (r, r) =  1117.9012560910874
 Iteration :  5  (r, r) =  263.68204340349564
 Iteration :  6  (r, r) =  111.33970591014665
 Iteration :  7  (r, r) =  52.9728069177894
 Iteration :  8  (r, r) =  15.931270925174996
 Iteration :  9  (r, r) =  7.743578792046668
 Iteration :  10  (r, r) =  2.612351129862421
 Iteration :  11  (r, r) =  0.9782850927934768
 Iteration :  12  (r, r) =  0.44378291462218056
Relative/Absolute residual less than tol
Converged in  12  iterations with final norm  0.6661703345407843

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.910665e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.700183e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.534167e+06 1.0e+06 5.2e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.230069e+05 4.2e+05 4.0e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
```

```
e-02
 5   9  1  1   9  T  F 7.721033e+05 3.7e+05 4.0e+05 -8.4e+02 7.8e+03 1.0e+00 2.0
e-02
 6  12  1  1  12  T  F 7.716808e+05 3.7e+05 4.0e+05 -3.3e-01 2.5e+02 1.0e+00 3.6
e-03

converged : False
reason    : unknown reason
cumulative CG iterations : 34
cumulative cost evaluations : 7
cumulative gradient evaluations : 7
cumulative Hessian vector products (excluding preconditioner builds) : 34
====================== End Newton CG convergence information ==================
=====

using Hessian
it= 7 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Fa
lse
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.660e+00
||M r0|| = 1.342e-04
(M r0, r0) = 8.641e-03
 Iteration :  0  (r, r) =  2.7544941060569483
 Iteration :  1  (r, r) =  1.176639982063275
 Iteration :  2  (r, r) =  0.4726279462324404
 Iteration :  3  (r, r) =  0.2303162865384366
 Iteration :  4  (r, r) =  0.13953013709331114
 Iteration :  5  (r, r) =  0.05438198365811223
 Iteration :  6  (r, r) =  0.019923023823623057
 Iteration :  7  (r, r) =  0.00717110597089139
 Iteration :  8  (r, r) =  0.0028992477866755053
 Iteration :  9  (r, r) =  0.001446070443569261
 Iteration :  10  (r, r) =  0.00072446710874602 57
 Iteration :  11  (r, r) =  0.0002932677316112949
 Iteration :  12  (r, r) =  0.00010507512280767351
 Iteration :  13  (r, r) =  3.72073125707393e-05
 Iteration :  14  (r, r) =  1.1765484101651073e-05
 Iteration :  15  (r, r) =  4.322235664123169e-06
 Iteration :  16  (r, r) =  1.4039536226039814e-06
 Iteration :  17  (r, r) =  5.527179294902568e-07
 Iteration :  18  (r, r) =  2.5110672677278734e-07
 Iteration :  19  (r, r) =  7.873527934973784e-08
Relative/Absolute residual less than tol
Converged in  19  iterations with final norm  0.000280598074387081

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
```

```
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.910665e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.700183e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.534167e+06 1.0e+06 5.2e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.230069e+05 4.2e+05 4.0e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02
 5   9  1  1   9  T  F 7.721033e+05 3.7e+05 4.0e+05 -8.4e+02 7.8e+03 1.0e+00 2.0
e-02
 6  12  1  1  12  T  F 7.716808e+05 3.7e+05 4.0e+05 -3.3e-01 2.5e+02 1.0e+00 3.6
e-03
 7  19  1  1  19  F  F 7.716807e+05 3.7e+05 4.0e+05 -5.9e-05 1.7e+00 1.0e+00 2.9
e-04

converged : False
reason    : unknown reason
cumulative CG iterations : 53
cumulative cost evaluations : 8
cumulative gradient evaluations : 8
cumulative Hessian vector products (excluding preconditioner builds) : 53
======================= End Newton CG convergence information ==================
=====

using Hessian
it= 8 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Fa
lse

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
```

```
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.910665e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.700183e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.534167e+06 1.0e+06 5.2e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.230069e+05 4.2e+05 4.0e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02
 5   9  1  1   9  T  F 7.721033e+05 3.7e+05 4.0e+05 -8.4e+02 7.8e+03 1.0e+00 2.0
e-02
 6  12  1  1  12  T  F 7.716808e+05 3.7e+05 4.0e+05 -3.3e-01 2.5e+02 1.0e+00 3.6
e-03
 7  19  1  1  19  F  F 7.716807e+05 3.7e+05 4.0e+05 -5.9e-05 1.7e+00 1.0e+00 2.9
e-04
 8   0  1  1   0  F  F 7.716807e+05 3.7e+05 4.0e+05     ---- 3.9e-02 1.0e+00
 ----

converged : True
reason    : Norm of the gradient less than tolerance
cumulative CG iterations : 53
cumulative cost evaluations : 9
cumulative gradient evaluations : 9
cumulative Hessian vector products (excluding preconditioner builds) : 53
===================== End Newton CG convergence information ==================
=====

gamma= 6959.888561201022 , noise_datanorm= 863.5355316705534 , misfit_datanorm=
 863.0787236500014

-------- gamma= 6975.521475652478  --------
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 1.12465e+06
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 1.12465e+06
 |S-A|_2 = 3.50572e-09
```

```
  matrix has dimension ━━ H-matrix inverse ( rtol =  1e-06 , atol =  1e-14 , ove
rwrite= False  )
1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
    done in  14.838630676269531
    size of inverse =  3361874  bytes

━━ H-matrix multiplication C=A*B
    done in 23.15s
    size of C = 3.88 MB

━━ H-matrix multiplication C=A*B
    done in 32.63s
    size of C = 8.54 MB

━━ LU factorisation ( rtol = 1e-07 )
[====================-                   ]  52using Gauss-Newton Hessian
it= 0 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
[================================-       ]  82% ETA 5 s Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.951e+07
||M r0|| = 5.828e+01
(M r0, r0) = 2.257e+04
 Iteration :  0  (r, r) =  380758959655851.75
    done in 30.54s
    size of LU factor = 18.68 MB
 Iteration :  1  (r, r) =  26986137540882.38
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  5194818.335695906

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
```

```
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 1
cumulative cost evaluations : 1
cumulative gradient evaluations : 1
cumulative Hessian vector products (excluding preconditioner builds) : 1
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 1 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 4.941e+06
||M r0|| = 4.230e+01
(M r0, r0) = 7.211e+03
 Iteration :  0  (r, r) =  24412943490066.926
 Iteration :  1  (r, r) =  4724179467642.846
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  2173517.7633603197

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
```

```
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908764e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 2
cumulative cost evaluations : 2
cumulative gradient evaluations : 2
cumulative Hessian vector products (excluding preconditioner builds) : 2
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 2 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 2.590e+06
||M r0|| = 3.670e+01
(M r0, r0) = 4.968e+03
 Iteration :  0  (r, r) =  6710410662649.094
 Iteration :  1  (r, r) =  2663424519425.3853
 Iteration :  2  (r, r) =  262383989939.66464
Relative/Absolute residual less than tol
Converged in  2  iterations with final norm  512234.3115603099

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
```

```
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908764e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676546e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 4
cumulative cost evaluations : 3
cumulative gradient evaluations : 3
cumulative Hessian vector products (excluding preconditioner builds) : 4
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 3 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 5.708e+05
||M r0|| = 1.948e+01
(M r0, r0) = 1.543e+03
 Iteration :  0  (r, r) =  325862045174.4467
 Iteration :  1  (r, r) =  88927921774.37445
 Iteration :  2  (r, r) =  16139933250.665852
 Iteration :  3  (r, r) =  18219006083.39456
 Iteration :  4  (r, r) =  9807126242.911156
 Iteration :  5  (r, r) =  2640484583.114731
Relative/Absolute residual less than tol
Converged in  5  iterations with final norm  51385.64569132834

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
```

```
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908764e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676546e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532065e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 9
cumulative cost evaluations : 4
cumulative gradient evaluations : 4
cumulative Hessian vector products (excluding preconditioner builds) : 9
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 4 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.207e+05
||M r0|| = 3.452e+00
(M r0, r0) = 4.292e+02
 Iteration :  0  (r, r) =  14560597816.619476
 Iteration :  1  (r, r) =  2744621290.539221
 Iteration :  2  (r, r) =  590918825.7678332
 Iteration :  3  (r, r) =  146934718.8114643
 Iteration :  4  (r, r) =  46961452.457745984
Relative/Absolute residual less than tol
Converged in  4  iterations with final norm  6852.842655259639

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
```

```
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908764e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676546e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532065e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.249513e+05 4.2e+05 4.1e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 13
cumulative cost evaluations : 5
cumulative gradient evaluations : 5
cumulative Hessian vector products (excluding preconditioner builds) : 13
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 5 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 7.731e+03
||M r0|| = 2.759e-01
(M r0, r0) = 2.884e+01
 Iteration :  0  (r, r) =  59764721.66030115
 Iteration :  1  (r, r) =  28736506.922672182
 Iteration :  2  (r, r) =  16147166.295022048
 Iteration :  3  (r, r) =  9345878.847465008
 Iteration :  4  (r, r) =  5078625.805474104
 Iteration :  5  (r, r) =  1900877.6928434072
 Iteration :  6  (r, r) =  686175.4882842426
 Iteration :  7  (r, r) =  222592.27080003245
 Iteration :  8  (r, r) =  49276.8067354832
 Iteration :  9  (r, r) =  16046.743004559743
Relative/Absolute residual less than tol
Converged in  9  iterations with final norm  126.67573960533936

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
```

```
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908764e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676546e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532065e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.249513e+05 4.2e+05 4.1e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02
 5   9  1  1   9  T  F 7.738958e+05 3.7e+05 4.0e+05 -8.4e+02 7.7e+03 1.0e+00 2.0
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 22
cumulative cost evaluations : 6
cumulative gradient evaluations : 6
cumulative Hessian vector products (excluding preconditioner builds) : 22
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 6 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 2.492e+02
||M r0|| = 8.120e-03
(M r0, r0) = 6.144e-01
 Iteration :  0  (r, r) =  62079.25217709316
 Iteration :  1  (r, r) =  17820.525276786488
 Iteration :  2  (r, r) =  3329.24037129121
 Iteration :  3  (r, r) =  2346.838581740703
 Iteration :  4  (r, r) =  1071.5360812891247
```

```
 Iteration :  5  (r, r) =  250.77167391711976
 Iteration :  6  (r, r) =  107.58866544384756
 Iteration :  7  (r, r) =  51.31551518144478
 Iteration :  8  (r, r) =  15.597486739681184
 Iteration :  9  (r, r) =  7.515118900809812
 Iteration :  10  (r, r) =  2.498509297210107
 Iteration :  11  (r, r) =  0.9220808618184567
 Iteration :  12  (r, r) =  0.41492071630487126
Relative/Absolute residual less than tol
Converged in  12  iterations with final norm  0.6441433973152805

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908764e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676546e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532065e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.249513e+05 4.2e+05 4.1e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02
 5   9  1  1   9  T  F 7.738958e+05 3.7e+05 4.0e+05 -8.4e+02 7.7e+03 1.0e+00 2.0
e-02
 6  12  1  1  12  T  F 7.734754e+05 3.7e+05 4.0e+05 -3.3e-01 2.5e+02 1.0e+00 3.6
e-03

converged : False
reason    : unknown reason
cumulative CG iterations : 34
```

```
cumulative cost evaluations : 7
cumulative gradient evaluations : 7
cumulative Hessian vector products (excluding preconditioner builds) : 34
====================== End Newton CG convergence information ==================
=====

using Hessian
it= 7 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Fa
lse
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.626e+00
||M r0|| = 1.314e-04
(M r0, r0) = 8.542e-03
 Iteration :  0  (r, r) =  2.6447626949387555
 Iteration :  1  (r, r) =  1.1047201600059233
 Iteration :  2  (r, r) =  0.4386994619299106
 Iteration :  3  (r, r) =  0.21167325627094197
 Iteration :  4  (r, r) =  0.1293118812986188
 Iteration :  5  (r, r) =  0.05214425783584359
 Iteration :  6  (r, r) =  0.019147589746999066
 Iteration :  7  (r, r) =  0.006863200589848971
 Iteration :  8  (r, r) =  0.002727831677560976
 Iteration :  9  (r, r) =  0.001331467839776729
 Iteration :  10  (r, r) =  0.0006641766947584329
 Iteration :  11  (r, r) =  0.00027318130329950975
 Iteration :  12  (r, r) =  9.61178228052334e-05
 Iteration :  13  (r, r) =  3.439319736998566e-05
 Iteration :  14  (r, r) =  1.0598821216691e-05
 Iteration :  15  (r, r) =  3.9056143787202344e-06
 Iteration :  16  (r, r) =  1.2832964185919377e-06
 Iteration :  17  (r, r) =  5.026065464888799e-07
 Iteration :  18  (r, r) =  2.2586032906766434e-07
 Iteration :  19  (r, r) =  7.20061079874386e-08
Relative/Absolute residual less than tol
Converged in  19  iterations with final norm  0.00026833953862120025

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
```

```
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908764e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676546e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532065e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.249513e+05 4.2e+05 4.1e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02
 5   9  1  1   9  T  F 7.738958e+05 3.7e+05 4.0e+05 -8.4e+02 7.7e+03 1.0e+00 2.0
e-02
 6  12  1  1  12  T  F 7.734754e+05 3.7e+05 4.0e+05 -3.3e-01 2.5e+02 1.0e+00 3.6
e-03
 7  19  1  1  19  F  F 7.734753e+05 3.7e+05 4.0e+05 -5.7e-05 1.6e+00 1.0e+00 2.9
e-04

converged : False
reason    : unknown reason
cumulative CG iterations : 53
cumulative cost evaluations : 8
cumulative gradient evaluations : 8
cumulative Hessian vector products (excluding preconditioner builds) : 53
======================= End Newton CG convergence information ==================
=====


====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
```

```
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908764e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676546e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532065e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.249513e+05 4.2e+05 4.1e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02
 5   9  1  1   9  T  F 7.738958e+05 3.7e+05 4.0e+05 -8.4e+02 7.7e+03 1.0e+00 2.0
e-02
 6  12  1  1  12  T  F 7.734754e+05 3.7e+05 4.0e+05 -3.3e-01 2.5e+02 1.0e+00 3.6
e-03
 7  19  1  1  19  F  F 7.734753e+05 3.7e+05 4.0e+05 -5.7e-05 1.6e+00 1.0e+00 2.9
e-04
 8   0 10  1   0  T  F 7.734753e+05 3.7e+05 4.0e+05     ---- 1.6e+00 9.8e-04
 ----

converged : False
reason    : Maximum number of backtracking reached
cumulative CG iterations : 53
cumulative cost evaluations : 18
cumulative gradient evaluations : 9
cumulative Hessian vector products (excluding preconditioner builds) : 53
====================== End Newton CG convergence information ==================
=====

gamma= 6975.521475652478 , noise_datanorm= 863.5355316705534 , misfit_datanorm=
 863.5279664560159

-------- gamma= 6975.784963434012  --------
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 1.1247e+06
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 1.1247e+06
 |S-A|_2 = 6.53779e-09
  matrix has dimension ━━ H-matrix inverse ( rtol = 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
 1e-06 , atol =  1e-14 , overwrite= False  )
    done in  14.96683955192566
```

```
    size of inverse =  3361874  bytes

━━ H-matrix multiplication C=A*B
    done in 24.33s
    size of C = 3.88 MB

━━ H-matrix multiplication C=A*B
    done in 33.59s
    size of C = 8.54 MB

━━ LU factorisation ( rtol = 1e-07 )
[================================
]  37% ETA 19 s (5.28 GB)================-                   ]  52% ETA 14 s (5.
28 GB)
it= 0 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
    done in 30.85s
    size of LU factor = 18.68 MB
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.951e+07
||M r0|| = 5.827e+01
(M r0, r0) = 2.257e+04
 Iteration :  0  (r, r) =  380758959655851.75
 Iteration :  1  (r, r) =  26985988575434.09
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  5194803.997787991

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
```

```
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 1
cumulative cost evaluations : 1
cumulative gradient evaluations : 1
cumulative Hessian vector products (excluding preconditioner builds) : 1
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 1 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 4.941e+06
||M r0|| = 4.230e+01
(M r0, r0) = 7.211e+03
 Iteration :  0  (r, r) =  24412523831736.195
 Iteration :  1  (r, r) =  4723727061156.914
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  2173413.6884534694

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP        cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908732e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
```

```
converged : False
reason    : unknown reason
cumulative CG iterations : 2
cumulative cost evaluations : 2
cumulative gradient evaluations : 2
cumulative Hessian vector products (excluding preconditioner builds) : 2
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 2 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 2.590e+06
||M r0|| = 3.670e+01
(M r0, r0) = 4.968e+03
 Iteration :  0  (r, r) =  6709721660493.758
 Iteration :  1  (r, r) =  2663118516506.841
 Iteration :  2  (r, r) =  262361761555.70374
Relative/Absolute residual less than tol
Converged in  2  iterations with final norm  512212.6136241705

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908732e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676149e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
```

```
converged : False
reason    : unknown reason
cumulative CG iterations : 4
cumulative cost evaluations : 3
cumulative gradient evaluations : 3
cumulative Hessian vector products (excluding preconditioner builds) : 4
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 3 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 5.708e+05
||M r0|| = 1.948e+01
(M r0, r0) = 1.543e+03
 Iteration :  0  (r, r) =  325823942410.9524
 Iteration :  1  (r, r) =  88914339428.99207
 Iteration :  2  (r, r) =  16138539026.315361
 Iteration :  3  (r, r) =  18218870484.611763
 Iteration :  4  (r, r) =  9804070321.366941
 Iteration :  5  (r, r) =  2639133370.625786
Relative/Absolute residual less than tol
Converged in  5  iterations with final norm  51372.49624678351

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
```

```
 1   1  1  1   1  T  F 1.908732e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676149e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532030e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 9
cumulative cost evaluations : 4
cumulative gradient evaluations : 4
cumulative Hessian vector products (excluding preconditioner builds) : 9
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 4 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.207e+05
||M r0|| = 3.452e+00
(M r0, r0) = 4.292e+02
 Iteration :  0  (r, r) =  14559585462.508945
 Iteration :  1  (r, r) =  2743233587.129353
 Iteration :  2  (r, r) =  590817306.181736
 Iteration :  3  (r, r) =  146878827.28704348
 Iteration :  4  (r, r) =  46947078.0784325
Relative/Absolute residual less than tol
Converged in  4  iterations with final norm  6851.793785457389

===================== Begin Newton CG convergence information ==================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)
```

```
it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908732e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676149e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532030e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.249840e+05 4.2e+05 4.1e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 13
cumulative cost evaluations : 5
cumulative gradient evaluations : 5
cumulative Hessian vector products (excluding preconditioner builds) : 13
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 5 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 7.730e+03
||M r0|| = 2.760e-01
(M r0, r0) = 2.884e+01
 Iteration :  0  (r, r) =  59759317.696027935
 Iteration :  1  (r, r) =  28731720.293041974
 Iteration :  2  (r, r) =  16143262.141422078
 Iteration :  3  (r, r) =  9342633.197015144
 Iteration :  4  (r, r) =  5076984.054706373
 Iteration :  5  (r, r) =  1900521.2619439233
 Iteration :  6  (r, r) =  685941.8592473419
 Iteration :  7  (r, r) =  222556.75456658046
 Iteration :  8  (r, r) =  49272.61862664743
 Iteration :  9  (r, r) =  16047.785900776398
Relative/Absolute residual less than tol
Converged in  9  iterations with final norm  126.67985593919974

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
```

```
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908732e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676149e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532030e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.249840e+05 4.2e+05 4.1e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02
 5   9  1  1   9  T  F 7.739261e+05 3.7e+05 4.0e+05 -8.4e+02 7.7e+03 1.0e+00 2.0
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 22
cumulative cost evaluations : 6
cumulative gradient evaluations : 6
cumulative Hessian vector products (excluding preconditioner builds) : 22
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 6 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 2.491e+02
||M r0|| = 8.117e-03
(M r0, r0) = 6.144e-01
 Iteration :  0  (r, r) =  62058.439250244424
 Iteration :  1  (r, r) =  17814.407700301672
 Iteration :  2  (r, r) =  3327.902969464403
 Iteration :  3  (r, r) =  2344.9836647559982
 Iteration :  4  (r, r) =  1070.8140537124389
 Iteration :  5  (r, r) =  250.56244329211685
 Iteration :  6  (r, r) =  107.52763895864862
 Iteration :  7  (r, r) =  51.28823507945161
 Iteration :  8  (r, r) =  15.591701323935272
 Iteration :  9  (r, r) =  7.511493688747701
 Iteration :  10  (r, r) =  2.496720934047522
 Iteration :  11  (r, r) =  0.921224661109163
 Iteration :  12  (r, r) =  0.4144705377550145
Relative/Absolute residual less than tol
Converged in  12  iterations with final norm  0.6437938627814143
```

```
===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908732e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676149e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532030e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.249840e+05 4.2e+05 4.1e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02
 5   9  1  1   9  T  F 7.739261e+05 3.7e+05 4.0e+05 -8.4e+02 7.7e+03 1.0e+00 2.0
e-02
 6  12  1  1  12  T  F 7.735057e+05 3.7e+05 4.0e+05 -3.3e-01 2.5e+02 1.0e+00 3.6
e-03

converged : False
reason    : unknown reason
cumulative CG iterations : 34
cumulative cost evaluations : 7
cumulative gradient evaluations : 7
cumulative Hessian vector products (excluding preconditioner builds) : 34
====================== End Newton CG convergence information ==================
=====

using Hessian
it= 7 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Fa
lse
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
```

```
||r0|| = 1.623e+00
||M r0|| = 1.314e-04
(M r0, r0) = 8.544e-03
 Iteration :  0  (r, r) =  2.633967188440769
 Iteration :  1  (r, r) =  1.1024529077736012
 Iteration :  2  (r, r) =  0.4386488845931381
 Iteration :  3  (r, r) =  0.21238073485041603
 Iteration :  4  (r, r) =  0.12948340181010617
 Iteration :  5  (r, r) =  0.052230074546741716
 Iteration :  6  (r, r) =  0.019163275521390938
 Iteration :  7  (r, r) =  0.006856352729135722
 Iteration :  8  (r, r) =  0.00272804091874754
 Iteration :  9  (r, r) =  0.001331541590609222
 Iteration :  10  (r, r) =  0.0006644383964490574
 Iteration :  11  (r, r) =  0.00027234016006245855
 Iteration :  12  (r, r) =  9.599314296330871e-05
 Iteration :  13  (r, r) =  3.4389019181241365e-05
 Iteration :  14  (r, r) =  1.0581667883611036e-05
 Iteration :  15  (r, r) =  3.892591692362938e-06
 Iteration :  16  (r, r) =  1.2833389844066836e-06
 Iteration :  17  (r, r) =  5.028329845578269e-07
 Iteration :  18  (r, r) =  2.2534349144204616e-07
 Iteration :  19  (r, r) =  7.206965224998767e-08
Relative/Absolute residual less than tol
Converged in  19  iterations with final norm  0.0002684579152306515

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908732e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
```

```
e-01
 2   2  1  1   2  T  F 6.676149e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532030e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.249840e+05 4.2e+05 4.1e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02
 5   9  1  1   9  T  F 7.739261e+05 3.7e+05 4.0e+05 -8.4e+02 7.7e+03 1.0e+00 2.0
e-02
 6  12  1  1  12  T  F 7.735057e+05 3.7e+05 4.0e+05 -3.3e-01 2.5e+02 1.0e+00 3.6
e-03
 7  19  1  1  19  F  F 7.735055e+05 3.7e+05 4.0e+05 -5.7e-05 1.6e+00 1.0e+00 2.9
e-04

converged : False
reason    : unknown reason
cumulative CG iterations : 53
cumulative cost evaluations : 8
cumulative gradient evaluations : 8
cumulative Hessian vector products (excluding preconditioner builds) : 53
======================= End Newton CG convergence information ==================
=====

using Hessian
it= 8 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Fa
lse

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
```

```
 1   1  1  1   1  T  F 1.908732e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676149e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532030e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.249840e+05 4.2e+05 4.1e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02
 5   9  1  1   9  T  F 7.739261e+05 3.7e+05 4.0e+05 -8.4e+02 7.7e+03 1.0e+00 2.0
e-02
 6  12  1  1  12  T  F 7.735057e+05 3.7e+05 4.0e+05 -3.3e-01 2.5e+02 1.0e+00 3.6
e-03
 7  19  1  1  19  F  F 7.735055e+05 3.7e+05 4.0e+05 -5.7e-05 1.6e+00 1.0e+00 2.9
e-04
 8   0  1  1   0  F  F 7.735055e+05 3.7e+05 4.0e+05     ---- 3.6e-02 1.0e+00
 ----

converged : True
reason    : Norm of the gradient less than tolerance
cumulative CG iterations : 53
cumulative cost evaluations : 9
cumulative gradient evaluations : 9
cumulative Hessian vector products (excluding preconditioner builds) : 53
====================== End Newton CG convergence information ==================
=====

gamma= 6975.784963434012 , noise_datanorm= 863.5355316705534 , misfit_datanorm=
 863.5355515346085

-------- gamma= 6975.784273393516  --------
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 1.1247e+06
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 1.1247e+06
 |S-A|_2 = 7.02535e-10
  matrix has dimension ━━ H-matrix inverse ( rtol =  1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
1e-06 , atol =  1e-14 , overwrite= False  )
    done in  14.302937030792236
    size of inverse =  3361874  bytes

━━ H-matrix multiplication C=A*B
    done in 23.91s
    size of C = 3.88 MB

━━ H-matrix multiplication C=A*B
    done in 32.89s
```

```
    size of C = 8.54 MB

── LU factorisation ( rtol = 1e-07 )
    done in 30.35s
    size of LU factor = 18.69 MB
using Gauss-Newton Hessian
it= 0 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.951e+07
||M r0|| = 5.827e+01
(M r0, r0) = 2.257e+04
 Iteration :  0  (r, r) =  380758959655851.75
 Iteration :  1  (r, r) =  26985988965503.54
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  5194804.035332183

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 1
cumulative cost evaluations : 1
cumulative gradient evaluations : 1
cumulative Hessian vector products (excluding preconditioner builds) : 1
====================== End Newton CG convergence information ==================
=====
```

```
using Gauss-Newton Hessian
it= 1 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 4.941e+06
||M r0|| = 4.230e+01
(M r0, r0) = 7.211e+03
 Iteration :  0  (r, r) =  24412524966420.227
 Iteration :  1  (r, r) =  4723728264318.135
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  2173413.965244112

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908732e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 2
cumulative cost evaluations : 2
cumulative gradient evaluations : 2
cumulative Hessian vector products (excluding preconditioner builds) : 2
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
```

```
it= 2 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 2.590e+06
||M r0|| = 3.670e+01
(M r0, r0) = 4.968e+03
 Iteration :  0  (r, r) =  6709723502914.271
 Iteration :  1  (r, r) =  2663119337060.9033
 Iteration :  2  (r, r) =  262361819223.133
Relative/Absolute residual less than tol
Converged in  2  iterations with final norm  512212.6699166402

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908732e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676150e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 4
cumulative cost evaluations : 3
cumulative gradient evaluations : 3
cumulative Hessian vector products (excluding preconditioner builds) : 4
====================== End Newton CG convergence information ==================
=====
```

```
using Gauss-Newton Hessian
it= 3 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 5.708e+05
||M r0|| = 1.948e+01
(M r0, r0) = 1.543e+03
 Iteration :  0  (r, r) =  325824031054.1389
 Iteration :  1  (r, r) =  88914373127.82126
 Iteration :  2  (r, r) =  16138542638.227646
 Iteration :  3  (r, r) =  18218870739.925636
 Iteration :  4  (r, r) =  9804078273.521988
 Iteration :  5  (r, r) =  2639136942.956028
Relative/Absolute residual less than tol
Converged in  5  iterations with final norm  51372.5310156705

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908732e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676150e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532030e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 9
```

```
cumulative cost evaluations : 4
cumulative gradient evaluations : 4
cumulative Hessian vector products (excluding preconditioner builds) : 9
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 4 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.207e+05
||M r0|| = 3.452e+00
(M r0, r0) = 4.292e+02
 Iteration :  0  (r, r) =  14559587774.812725
 Iteration :  1  (r, r) =  2743237233.9661226
 Iteration :  2  (r, r) =  590817485.3079911
 Iteration :  3  (r, r) =  146878956.0452265
 Iteration :  4  (r, r) =  46947112.831418544
Relative/Absolute residual less than tol
Converged in  4  iterations with final norm  6851.796321507123

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:       Newton iteration number
nCG:      number of CG iterations in Newton iteration
nJ:       number of cost function evaluations in Newton iteration
nG:       number of gradient evaluations in Newton iteration
nHp:      number of Hessian-vector products in Newton iteration
GN:       True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:       True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:     cost, J = Jd + Jr
misfit:   misfit cost, Jd
reg:      regularization cost, Jr
(g,p):    inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:    step size
tolcg:    relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908732e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676150e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532030e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
```

```
e-01
 4   4  1  1   4  T  F 8.249839e+05 4.2e+05 4.1e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 13
cumulative cost evaluations : 5
cumulative gradient evaluations : 5
cumulative Hessian vector products (excluding preconditioner builds) : 13
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 5 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 7.730e+03
||M r0|| = 2.760e-01
(M r0, r0) = 2.884e+01
 Iteration :  0  (r, r) =  59759315.870615825
 Iteration :  1  (r, r) =  28731736.060458593
 Iteration :  2  (r, r) =  16143277.103502933
 Iteration :  3  (r, r) =  9342645.571804605
 Iteration :  4  (r, r) =  5076988.515400544
 Iteration :  5  (r, r) =  1900522.3241589246
 Iteration :  6  (r, r) =  685942.5012781622
 Iteration :  7  (r, r) =  222556.85340325884
 Iteration :  8  (r, r) =  49272.65866337853
 Iteration :  9  (r, r) =  16047.785113285427
Relative/Absolute residual less than tol
Converged in  9  iterations with final norm  126.67985283100634

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
```

```
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908732e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676150e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532030e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.249839e+05 4.2e+05 4.1e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02
 5   9  1  1   9  T  F 7.739260e+05 3.7e+05 4.0e+05 -8.4e+02 7.7e+03 1.0e+00 2.0
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 22
cumulative cost evaluations : 6
cumulative gradient evaluations : 6
cumulative Hessian vector products (excluding preconditioner builds) : 22
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 6 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Tr
ue
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 2.491e+02
||M r0|| = 8.117e-03
(M r0, r0) = 6.144e-01
 Iteration :  0  (r, r) =  62055.25627862993
 Iteration :  1  (r, r) =  17813.99086948527
 Iteration :  2  (r, r) =  3327.8855390116096
 Iteration :  3  (r, r) =  2344.986878741371
 Iteration :  4  (r, r) =  1070.796146913166
 Iteration :  5  (r, r) =  250.5671246548092
 Iteration :  6  (r, r) =  107.52551605927837
 Iteration :  7  (r, r) =  51.287253784420585
 Iteration :  8  (r, r) =  15.591087623569367
 Iteration :  9  (r, r) =  7.511362217603848
 Iteration :  10  (r, r) =  2.496644433043406
 Iteration :  11  (r, r) =  0.9211951070843012
 Iteration :  12  (r, r) =  0.4144659637366318
Relative/Absolute residual less than tol
Converged in  12  iterations with final norm  0.6437903103780235

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
```

```
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908732e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676150e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532030e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.249839e+05 4.2e+05 4.1e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02
 5   9  1  1   9  T  F 7.739260e+05 3.7e+05 4.0e+05 -8.4e+02 7.7e+03 1.0e+00 2.0
e-02
 6  12  1  1  12  T  F 7.735056e+05 3.7e+05 4.0e+05 -3.3e-01 2.5e+02 1.0e+00 3.6
e-03

converged : False
reason    : unknown reason
cumulative CG iterations : 34
cumulative cost evaluations : 7
cumulative gradient evaluations : 7
cumulative Hessian vector products (excluding preconditioner builds) : 34
====================== End Newton CG convergence information ==================
=====

using Hessian
it= 7 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Fa
lse
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.622e+00
||M r0|| = 1.314e-04
(M r0, r0) = 8.544e-03
 Iteration :  0  (r, r) =  2.632324150102312
 Iteration :  1  (r, r) =  1.1004878090835133
 Iteration :  2  (r, r) =  0.4381865251812377
 Iteration :  3  (r, r) =  0.21169953907339312
 Iteration :  4  (r, r) =  0.12901175715537366
 Iteration :  5  (r, r) =  0.05202813129889891
 Iteration :  6  (r, r) =  0.019110488023099646
```

```
 Iteration :  7  (r, r) =  0.006849034953224545
 Iteration :  8  (r, r) =  0.0027241778745961545
 Iteration :  9  (r, r) =  0.0013305864089768202
 Iteration :  10  (r, r) =  0.0006622521046680118
 Iteration :  11  (r, r) =  0.0002728674586366191
 Iteration :  12  (r, r) =  9.594554176196423e-05
 Iteration :  13  (r, r) =  3.446989298960136e-05
 Iteration :  14  (r, r) =  1.0641595807109882e-05
 Iteration :  15  (r, r) =  3.917750475282861e-06
 Iteration :  16  (r, r) =  1.2817278787248201e-06
 Iteration :  17  (r, r) =  5.022689663570074e-07
 Iteration :  18  (r, r) =  2.2573702896450665e-07
 Iteration :  19  (r, r) =  7.213822657979337e-08
Relative/Absolute residual less than tol
Converged in  19  iterations with final norm  0.00026858560382081795

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908732e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676150e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532030e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.249839e+05 4.2e+05 4.1e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02
 5   9  1  1   9  T  F 7.739260e+05 3.7e+05 4.0e+05 -8.4e+02 7.7e+03 1.0e+00 2.0
e-02
 6  12  1  1  12  T  F 7.735056e+05 3.7e+05 4.0e+05 -3.3e-01 2.5e+02 1.0e+00 3.6
```

```
e-03
 7  19  1  1  19  F  F 7.735055e+05 3.7e+05 4.0e+05 -5.7e-05 1.6e+00 1.0e+00 2.9
e-04

converged : False
reason    : unknown reason
cumulative CG iterations : 53
cumulative cost evaluations : 8
cumulative gradient evaluations : 8
cumulative Hessian vector products (excluding preconditioner builds) : 53
====================== End Newton CG convergence information =================
=====

using Hessian
it= 8 , preconditioner_build_iters= () , num_gn_iter= 7 , using_gauss_newton= Fa
lse

===================== Begin Newton CG convergence information ================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -3.6e+08 2.0e+07    ---- 5.0
e-01
 1   1  1  1   1  T  F 1.908732e+07 1.8e+07 1.5e+06 -2.6e+07 4.9e+06 1.0e+00 5.0
e-01
 2   2  1  1   2  T  F 6.676150e+06 5.9e+06 7.7e+05 -1.0e+07 2.6e+06 1.0e+00 3.6
e-01
 3   5  1  1   5  T  F 1.532030e+06 1.0e+06 5.3e+05 -1.5e+06 5.7e+05 1.0e+00 1.7
e-01
 4   4  1  1   4  T  F 8.249839e+05 4.2e+05 4.1e+05 -1.0e+05 1.2e+05 1.0e+00 7.9
e-02
 5   9  1  1   9  T  F 7.739260e+05 3.7e+05 4.0e+05 -8.4e+02 7.7e+03 1.0e+00 2.0
e-02
```

```
 6  12  1  1  12  T  F 7.735056e+05 3.7e+05 4.0e+05 -3.3e-01 2.5e+02 1.0e+00 3.6
e-03
 7  19  1  1  19  F  F 7.735055e+05 3.7e+05 4.0e+05 -5.7e-05 1.6e+00 1.0e+00 2.9
e-04
 8   0  1  1   0  F  F 7.735055e+05 3.7e+05 4.0e+05     ---- 3.6e-02 1.0e+00
 ----

converged : True
reason    : Norm of the gradient less than tolerance
cumulative CG iterations : 53
cumulative cost evaluations : 9
cumulative gradient evaluations : 9
cumulative Hessian vector products (excluding preconditioner builds) : 53
======================= End Newton CG convergence information ==================
=====

gamma= 6975.784273393516 , noise_datanorm= 863.5355316705534 , misfit_datanorm=
 863.5355313016433
      converged: True
           flag: 'converged'
 function_calls: 9
     iterations: 8
           root: 8.8502000428021
gamma_morozov= 6975.784285975133
```

Plots to make (3):

- Morozov discrepancy log-log plot

In [64]:

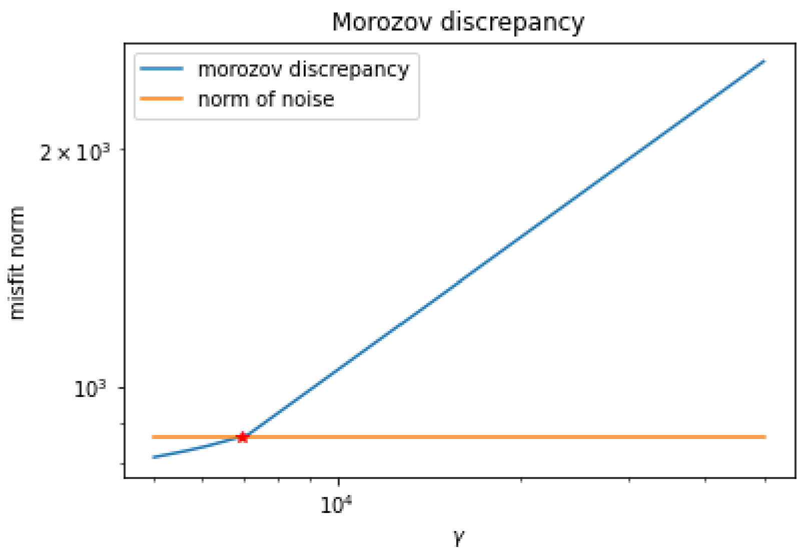
```
morozov_gammas = np.array(morozov_gammas)
morozov_discrepancies = np.array(morozov_discrepancies)

sort_inds = np.argsort(morozov_gammas)
morozov_gammas = morozov_gammas[sort_inds]
morozov_discrepancies = morozov_discrepancies[sort_inds]

print('morozov_gammas=', morozov_gammas)
print('morozov_discrepancies=', morozov_discrepancies)
print('noise_datanorm=', noise_datanorm)

np.savetxt(save_dir_str + '/morozov_gammas.txt', morozov_gammas)
np.savetxt(save_dir_str + '/morozov_discrepancies.txt', morozov_discrepancies)
np.savetxt(save_dir_str + '/noise_datanorm.txt', np.array([noise_datanorm]))

plt.figure()
plt.loglog(morozov_gammas, morozov_discrepancies)
plt.loglog(morozov_gammas, noise_datanorm*np.ones(len(morozov_gammas)))
plt.plot(gamma_morozov, noise_datanorm, '*r')
plt.title('Morozov discrepancy')
plt.xlabel(r'$\gamma$')
plt.ylabel('misfit norm')
plt.legend(['morozov discrepancy', 'norm of noise'])
```

```
morozov_gammas= [ 5000.          5642.49395663  6118.09756175  6959.8885612
  6975.52147565  6975.78427339  6975.78496343  7138.78445605
 50000.        ]
morozov_discrepancies= [ 812.61698801  827.81004354  839.93127734  863.07872365
```

```
863.52796646
  863.5355313   863.53555153  868.2600893  2585.97448747]
noise_datanorm= 863.5355316705534
```

Out[64]:

```
<matplotlib.legend.Legend at 0x7f3a932ce4c0>
```



Newton Solve with PCH preconditioning

In [32]:

```
PCH1 = PCHWrapper(do_updates=False)
update_m(m0_constant_value * np.ones(Vh2.dim()))

iter_PCH = list()
m_iter_PCH = list()
def PCH_callback(it, m_Vh2_numpy):
    iter_PCH.append(it)
    m_iter_PCH.append(m_Vh2_numpy)
    m_Vh2 = dl.Function(Vh2)
    m_Vh2.vector()[:] = m_Vh2_numpy
    plt.figure()
    cm = dl.plot(m_Vh2, cmap='gray')
    plt.colorbar(cm)
    plt.title('m_'+str(it))

PCH_convergence_info = newtoncg_ls(get_optimization_variable,
                                   set_optimization_variable,
                                   cost_triple,
                                   gradient,
                                   apply_hessian,
                                   apply_gauss_newton_hessian,
                                   PCH1.build,
                                   update_nothing,
                                   PCH1.solve,
                                   callback=PCH_callback,
                                   preconditioner_build_iters = (3,),
                                   rtol=newton_rtol,
                                   forcing_sequence_power=forcing_sequence_powe

misfit_datanorm = morozov_helper()
```

```
using Gauss-Newton Hessian
it= 0 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
```

```
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.951e+07
||M r0|| = 1.299e+05
(M r0, r0) = 1.460e+06
 Iteration :  0  (r, r) =  380758959655851.75
 Iteration :  1  (r, r) =  60792616517657.08
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  7796962.005656888

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 1
cumulative cost evaluations : 1
cumulative gradient evaluations : 1
cumulative Hessian vector products (excluding preconditioner builds) : 1
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 1 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 7.905e+06
```

```
||M r0|| = 2.779e+04
(M r0, r0) = 4.189e+05
 Iteration :  0  (r, r) =  62494263002660.72
 Iteration :  1  (r, r) =  25858348443099.72
 Iteration :  2  (r, r) =  15307513353856.215
Relative/Absolute residual less than tol
Converged in  2  iterations with final norm  3912481.7384693585

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.081021e+07 6.1e+07 2.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 3
cumulative cost evaluations : 2
cumulative gradient evaluations : 2
cumulative Hessian vector products (excluding preconditioner builds) : 3
====================== End Newton CG convergence information =================
=====

using Gauss-Newton Hessian
it= 2 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 3.874e+06
||M r0|| = 1.166e+04
```

```
(M r0, r0) = 1.713e+05
 Iteration :  0  (r, r) =  15010672486864.512
 Iteration :  1  (r, r) =  9835012971501.21
 Iteration :  2  (r, r) =  7723910396975.338
 Iteration :  3  (r, r) =  4596707853392.447
 Iteration :  4  (r, r) =  3484908890885.7627
 Iteration :  5  (r, r) =  2782124709218.0215
Relative/Absolute residual less than tol
Converged in  5  iterations with final norm  1667970.2363105947

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.081021e+07 6.1e+07 2.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.180904e+07 2.2e+07 5.1e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 8
cumulative cost evaluations : 3
cumulative gradient evaluations : 3
cumulative Hessian vector products (excluding preconditioner builds) : 8
====================== End Newton CG convergence information =================
=====

using Gauss-Newton Hessian
it= 3 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
```

```
building preconditioner
building PCH preconditioner
Computing impulse response moments
getting spatially varying volume
getting spatially varying mean
getting spatially varying covariance
Preparing sample point batch stuff
Preparing c++ object
Building initial sample point batches

Making row and column cluster treesUsing ProductConvolutionKernelRBFColsOnly!

Making block cluster trees
Building A kernel hmatrix
Making input and output mass matrix hmatrices━━ building H-matrix ( tol =
Computing A_hmatrix = M_out_hmatrix * A_kernel_hmatrix * M_in_hmatrix
making hmatrix spd
symmetrizing
getting smallest eigenvalue with Lanczos
1e-06 )
    done in 13.44s
    size of H-matrix = 4.42 MB
lambda_min= -76590.67133677886
scaling_at_lambda_min= 0.9878048780487805
scaling_at_zero= 0.5
Setting up operator T = (2*A - (b+a) I) / (b-a)
computing T^(2^k)
computing T^(2^1) = T^(2^0) * T^(2^0)

━━ H-matrix multiplication C=A*B
    done in 32.54s
    size of C = 5.27 MB


computing negative spectral projector Pi_minus = I / (I + T^(2^k))
━━ H-matrix inverse ( rtol =  1e-07 , atol =  1e-12 , overwrite= False  )
━━ H-matrix multiplication C=A*B
[==-                                    done in  23.251529932022095
    size of inverse =  14645377  bytes
computing absolute value projector Pi = I - 2*Pi_minus
                              computing A_plus = Pi * A (5.27 GB)
    done in 33.83s
    size of C = 7.70 MB
                                                                  )
━━ H-matrix multiplication C=A*B
    done in 36.50s
    size of C = 17.55 MB
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 1.1247e+06
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 1.1247e+06
 |S-A|_2 = 3.13924e-09
  matrix has dimension ━━ H-matrix inverse ( rtol =  1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
```

```
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
1e-06 , atol =  1e-14 , overwrite= False  )
    done in  14.54411268234253
    size of inverse =  3361874  bytes

━━ H-matrix multiplication C=A*B
    done in 24.28s
    size of C = 3.88 MB

━━ H-matrix multiplication C=A*B
    done in 33.17s
    size of C = 8.54 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 30.85s
    size of LU factor = 18.69 MB
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.623e+06
||M r0|| = 3.030e+01
(M r0, r0) = 3.755e+03
 Iteration :  0  (r, r) =  2633546763856.491
 Iteration :  1  (r, r) =  250580861251.00507
 Iteration :  2  (r, r) =  33496680698.47232
Relative/Absolute residual less than tol
Converged in  2  iterations with final norm  183020.98431183328

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
```

```
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.081021e+07 6.1e+07 2.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.180904e+07 2.2e+07 5.1e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01
 3   2  1  1   2  T  T 6.223754e+06 6.1e+06 1.2e+05 -1.1e+07 1.6e+06 1.0e+00 2.9
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 10
cumulative cost evaluations : 4
cumulative gradient evaluations : 4
cumulative Hessian vector products (excluding preconditioner builds) : 10
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 4 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 2.050e+05
||M r0|| = 6.264e+00
(M r0, r0) = 6.686e+02
 Iteration :  0  (r, r) =  42042735743.235
 Iteration :  1  (r, r) =  15671755066.120493
 Iteration :  2  (r, r) =  3499198012.00104
 Iteration :  3  (r, r) =  1337266702.9177585
 Iteration :  4  (r, r) =  446370599.19143844
 Iteration :  5  (r, r) =  168805258.69329643
Relative/Absolute residual less than tol
Converged in  5  iterations with final norm  12992.507790772974

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
```

```
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.081021e+07 6.1e+07 2.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.180904e+07 2.2e+07 5.1e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01
 3   2  1  1   2  T  T 6.223754e+06 6.1e+06 1.2e+05 -1.1e+07 1.6e+06 1.0e+00 2.9
e-01
 4   5  1  1   5  T  F 9.138152e+05 4.8e+05 4.3e+05 -2.8e+05 2.1e+05 1.0e+00 1.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 15
cumulative cost evaluations : 5
cumulative gradient evaluations : 5
cumulative Hessian vector products (excluding preconditioner builds) : 15
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 5 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.958e+04
||M r0|| = 5.044e-01
(M r0, r0) = 4.470e+01
 Iteration :  0  (r, r) =  383345197.19962656
 Iteration :  1  (r, r) =  104864321.5827917
 Iteration :  2  (r, r) =  46136303.28401123
 Iteration :  3  (r, r) =  39817683.35112052
 Iteration :  4  (r, r) =  17026279.5081695
 Iteration :  5  (r, r) =  3342774.9987876136
 Iteration :  6  (r, r) =  1254356.5052999544
 Iteration :  7  (r, r) =  564533.0684810837
 Iteration :  8  (r, r) =  209884.7463130094
Relative/Absolute residual less than tol
Converged in  8  iterations with final norm  458.13180015472557

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
```

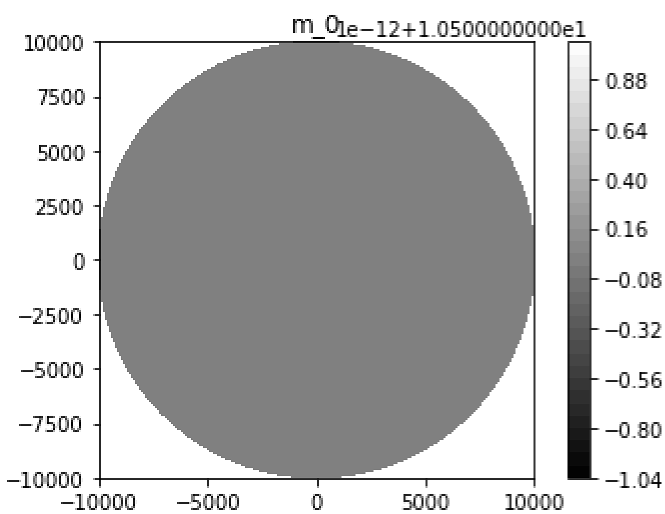
```
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.081021e+07 6.1e+07 2.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.180904e+07 2.2e+07 5.1e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01
 3   2  1  1   2  T  T 6.223754e+06 6.1e+06 1.2e+05 -1.1e+07 1.6e+06 1.0e+00 2.9
e-01
 4   5  1  1   5  T  F 9.138152e+05 4.8e+05 4.3e+05 -2.8e+05 2.1e+05 1.0e+00 1.0
e-01
 5   8  1  1   8  T  F 7.745874e+05 3.7e+05 4.0e+05 -2.2e+03 2.0e+04 1.0e+00 3.2
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 23
cumulative cost evaluations : 6
cumulative gradient evaluations : 6
cumulative Hessian vector products (excluding preconditioner builds) : 23
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 6 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 6.239e+02
||M r0|| = 2.045e-02
(M r0, r0) = 1.620e+00
 Iteration :  0  (r, r) =  389309.8150192415
 Iteration :  1  (r, r) =  149321.32328557732
 Iteration :  2  (r, r) =  48628.06275758954
 Iteration :  3  (r, r) =  40624.77118927481
 Iteration :  4  (r, r) =  16846.476248559346
 Iteration :  5  (r, r) =  4726.28974340554
 Iteration :  6  (r, r) =  2304.610238560923
 Iteration :  7  (r, r) =  1509.5834879965591
 Iteration :  8  (r, r) =  825.8301378265722
 Iteration :  9  (r, r) =  146.22076271265655
```

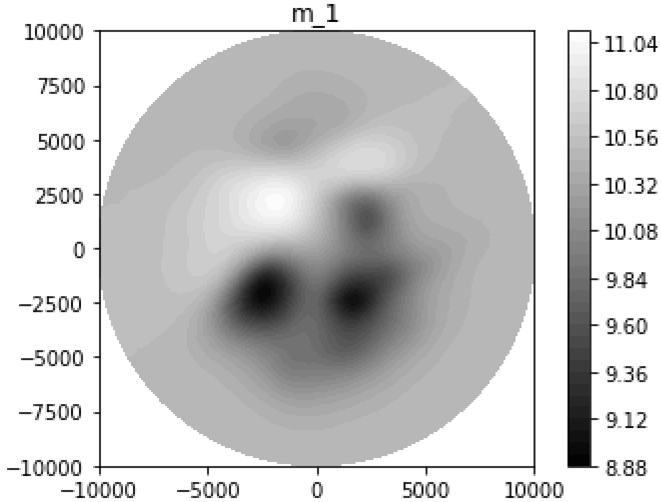
```
 Iteration :  10  (r, r) =  57.289019345246544
 Iteration :  11  (r, r) =  19.72065406458009
 Iteration :  12  (r, r) =  6.631543741303982
Relative/Absolute residual less than tol
Converged in  12  iterations with final norm  2.5751783901904703

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.081021e+07 6.1e+07 2.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.180904e+07 2.2e+07 5.1e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01
 3   2  1  1   2  T  T 6.223754e+06 6.1e+06 1.2e+05 -1.1e+07 1.6e+06 1.0e+00 2.9
e-01
 4   5  1  1   5  T  F 9.138152e+05 4.8e+05 4.3e+05 -2.8e+05 2.1e+05 1.0e+00 1.0
e-01
 5   8  1  1   8  T  F 7.745874e+05 3.7e+05 4.0e+05 -2.2e+03 2.0e+04 1.0e+00 3.2
e-02
 6  12  1  1  12  T  F 7.735069e+05 3.7e+05 4.0e+05 -3.0e+00 6.2e+02 1.0e+00 5.7
e-03

converged : False
reason    : unknown reason
cumulative CG iterations : 35
cumulative cost evaluations : 7
cumulative gradient evaluations : 7
cumulative Hessian vector products (excluding preconditioner builds) : 35
====================== End Newton CG convergence information ==================
=====
```

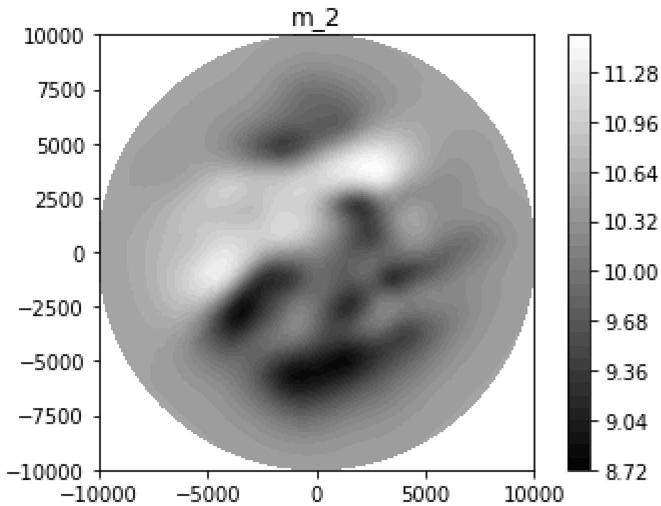
```
using Hessian
it= 7 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
False
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 5.815e+00
||M r0|| = 4.991e-04
(M r0, r0) = 2.603e-02
 Iteration :  0  (r, r) =  33.818990213008064
 Iteration :  1  (r, r) =  15.463723098182285
 Iteration :  2  (r, r) =  6.28924230001418
 Iteration :  3  (r, r) =  3.8049837307050565
 Iteration :  4  (r, r) =  2.1644727022801864
 Iteration :  5  (r, r) =  0.5541918233703111
 Iteration :  6  (r, r) =  0.1656993563436443
 Iteration :  7  (r, r) =  0.04751182038890159
 Iteration :  8  (r, r) =  0.01601287735337807
 Iteration :  9  (r, r) =  0.006514407673800337
 Iteration :  10  (r, r) =  0.002937606184435253
 Iteration :  11  (r, r) =  0.0009972596591672413
 Iteration :  12  (r, r) =  0.00039270536270223123
 Iteration :  13  (r, r) =  0.00016607290233615143
 Iteration :  14  (r, r) =  5.7395285788918e-05
 Iteration :  15  (r, r) =  1.9187311896355508e-05
 Iteration :  16  (r, r) =  7.5771151067822606e-06
Relative/Absolute residual less than tol
Converged in  16  iterations with final norm  0.002752656009526483

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
```

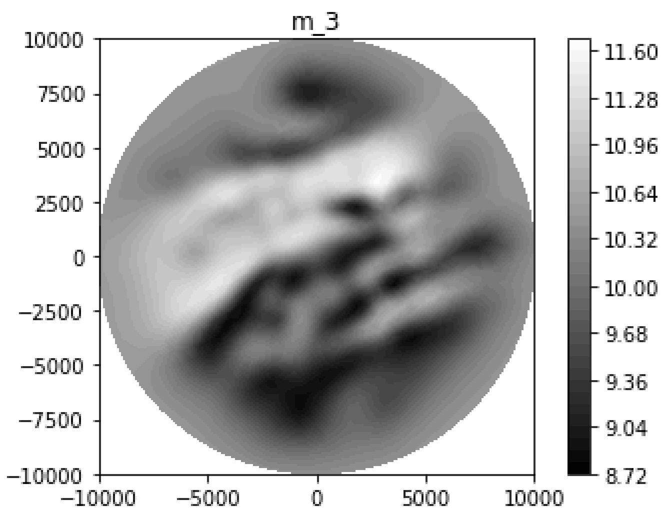
```
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.081021e+07 6.1e+07 2.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.180904e+07 2.2e+07 5.1e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01
 3   2  1  1   2  T  T 6.223754e+06 6.1e+06 1.2e+05 -1.1e+07 1.6e+06 1.0e+00 2.9
e-01
 4   5  1  1   5  T  F 9.138152e+05 4.8e+05 4.3e+05 -2.8e+05 2.1e+05 1.0e+00 1.0
e-01
 5   8  1  1   8  T  F 7.745874e+05 3.7e+05 4.0e+05 -2.2e+03 2.0e+04 1.0e+00 3.2
e-02
 6  12  1  1  12  T  F 7.735069e+05 3.7e+05 4.0e+05 -3.0e+00 6.2e+02 1.0e+00 5.7
e-03
 7  16  1  1  16  F  F 7.735055e+05 3.7e+05 4.0e+05 -6.7e-04 5.8e+00 1.0e+00 5.5
e-04

converged : False
reason    : unknown reason
cumulative CG iterations : 51
cumulative cost evaluations : 8
cumulative gradient evaluations : 8
cumulative Hessian vector products (excluding preconditioner builds) : 51
======================= End Newton CG convergence information ==================
=====

using Hessian
it= 8 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
False

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
```

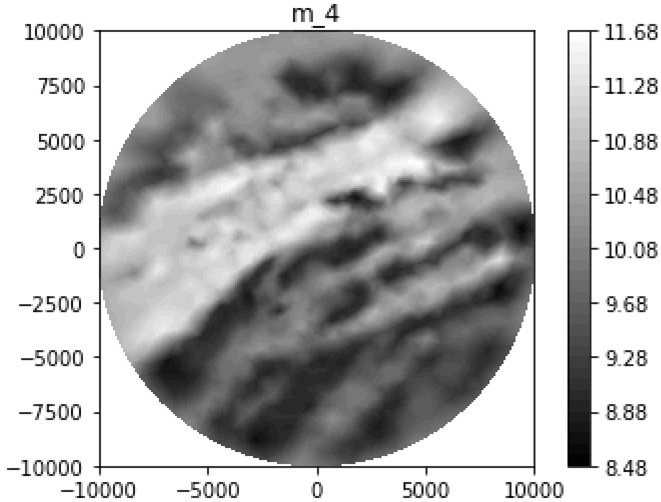
```
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.081021e+07 6.1e+07 2.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.180904e+07 2.2e+07 5.1e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01
 3   2  1  1   2  T  T 6.223754e+06 6.1e+06 1.2e+05 -1.1e+07 1.6e+06 1.0e+00 2.9
e-01
 4   5  1  1   5  T  F 9.138152e+05 4.8e+05 4.3e+05 -2.8e+05 2.1e+05 1.0e+00 1.0
e-01
 5   8  1  1   8  T  F 7.745874e+05 3.7e+05 4.0e+05 -2.2e+03 2.0e+04 1.0e+00 3.2
e-02
 6  12  1  1  12  T  F 7.735069e+05 3.7e+05 4.0e+05 -3.0e+00 6.2e+02 1.0e+00 5.7
e-03
 7  16  1  1  16  F  F 7.735055e+05 3.7e+05 4.0e+05 -6.7e-04 5.8e+00 1.0e+00 5.5
e-04
 8   0  1  1   0  F  F 7.735055e+05 3.7e+05 4.0e+05     ---- 4.5e-02 1.0e+00
----

converged : True
reason    : Norm of the gradient less than tolerance
cumulative CG iterations : 51
cumulative cost evaluations : 9
cumulative gradient evaluations : 9
cumulative Hessian vector products (excluding preconditioner builds) : 51
===================== End Newton CG convergence information =================
=====

gamma= 6975.784285975133 , noise_datanorm= 863.5355316705534 , misfit_datanorm=
863.5355308077618
```

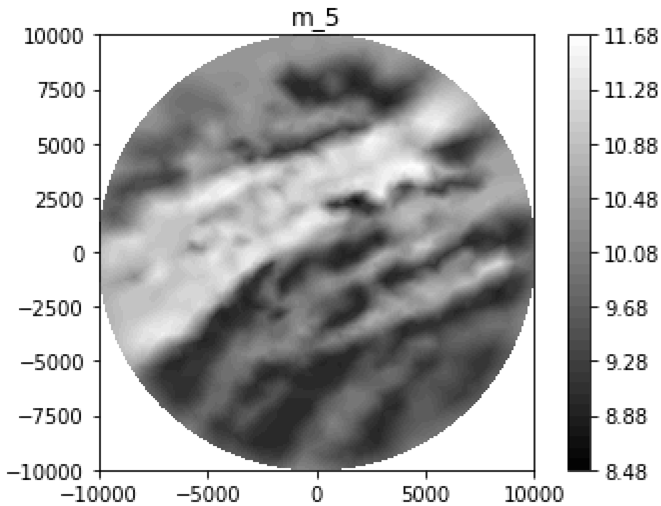


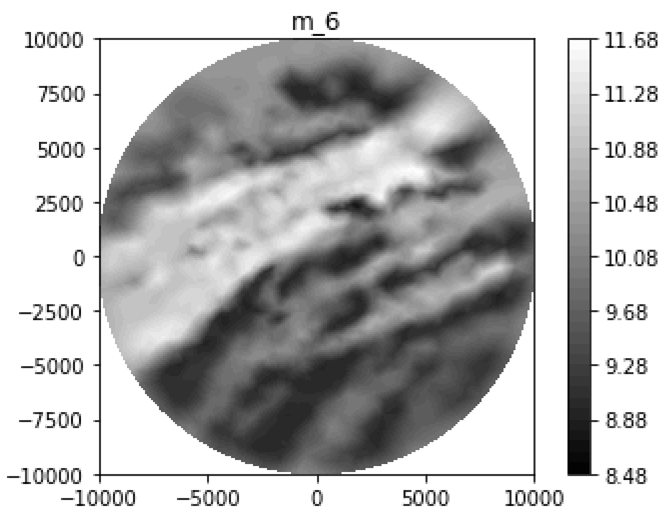

m_1
10000
7500
5000
2500
0
−2500
−5000
−7500
−10000
−10000 −5000 0 5000 10000
11.04
10.80
10.56
10.32
10.08
9.84
9.60
9.36
9.12
8.88

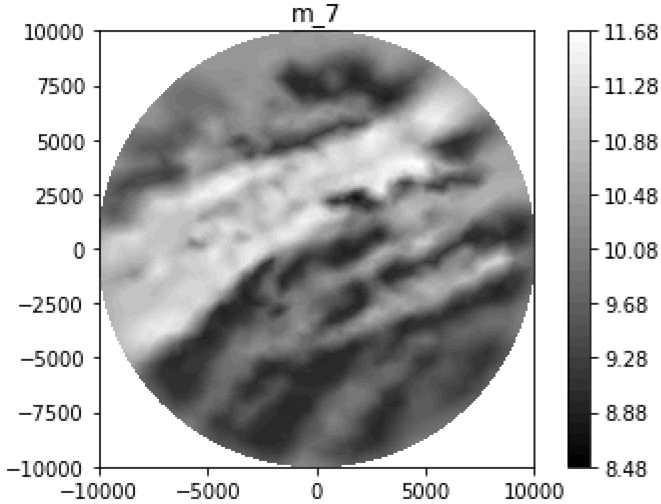

m_2
10000
7500
5000
2500
0
−2500
−5000
−7500
−10000
−10000 −5000 0 5000 10000
11.28
10.96
10.64
10.32
10.00
9.68
9.36
9.04
8.72


m_3
10000
7500
5000
2500
0
−2500
−5000
−7500
−10000
−10000 −5000 0 5000 10000
11.60
11.28
10.96
10.64
10.32
10.00
9.68
9.36
9.04
8.72


m_4
10000
7500
5000
2500
0
−2500
−5000
−7500
−10000
−10000
−5000
0
5000
10000
11.68
11.28
10.88
10.48
10.08
9.68
9.28
8.88
8.48


m_5
10000
7500
5000
2500
0
−2500
−5000
−7500
−10000
−10000
−5000
0
5000
10000
11.68
11.28
10.88
10.48
10.08
9.68
9.28
8.88
8.48


m_6
10000
7500
5000
2500
0
−2500
−5000
−7500
−10000
−10000
−5000
0
5000
10000
11.68
11.28
10.88
10.48
10.08
9.68
9.28
8.88
8.48



In [33]:

```
misfit_datanorm = np.sqrt(2.0 * SDAP.misfit())
print('noise_datanorm=', noise_datanorm, ', misfit_datanorm=', misfit_datanorm)
```

```
noise_datanorm= 863.5355316705534 , misfit_datanorm= 863.5355308077618
```

Newton solve with regularization preconditioning

In [34]:

```
update_m(m0_constant_value * np.ones(Vh2.dim()))

iter_REG = list()
m_iter_REG = list()
def REG_callback(it, m_Vh2_numpy):
    iter_REG.append(it)
    m_iter_REG.append(m_Vh2_numpy)
    m_Vh2 = dl.Function(Vh2)
    m_Vh2.vector()[:] = m_Vh2_numpy
    plt.figure()
    cm = dl.plot(m_Vh2, cmap='gray')
    plt.colorbar(cm)
    plt.title('m_'+str(it))

REG_convergence_info = newtoncg_ls(get_optimization_variable,
                                   set_optimization_variable,
                                   cost_triple,
                                   gradient,
                                   apply_hessian,
                                   apply_gauss_newton_hessian,
                                   build_nothing,
                                   update_nothing,
                                   REG.solve_hessian_numpy,
                                   callback=REG_callback,
                                   rtol=newton_rtol,
                                   forcing_sequence_power=forcing_sequence_powe

misfit_datanorm = morozov_helper()
```

```
using Gauss-Newton Hessian
it= 0 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
```

```
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.951e+07
||M r0|| = 1.299e+05
(M r0, r0) = 1.460e+06
 Iteration :  0  (r, r) =  380758959655851.75
 Iteration :  1  (r, r) =  60792616517657.08
Relative/Absolute residual less than tol
Converged in  1  iterations with final norm  7796962.005656888

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 1
cumulative cost evaluations : 1
cumulative gradient evaluations : 1
cumulative Hessian vector products (excluding preconditioner builds) : 1
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 1 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 7.905e+06
```

```
||M r0|| = 2.779e+04
(M r0, r0) = 4.189e+05
 Iteration :  0  (r, r) =  62494263002660.72
 Iteration :  1  (r, r) =  25858348443099.72
 Iteration :  2  (r, r) =  15307513353856.215
Relative/Absolute residual less than tol
Converged in  2  iterations with final norm  3912481.7384693585

======================= Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.081021e+07 6.1e+07 2.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 3
cumulative cost evaluations : 2
cumulative gradient evaluations : 2
cumulative Hessian vector products (excluding preconditioner builds) : 3
======================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 2 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 3.874e+06
||M r0|| = 1.166e+04
```

```
(M r0, r0) = 1.713e+05
 Iteration :  0  (r, r) =  15010672486864.512
 Iteration :  1  (r, r) =  9835012971501.21
 Iteration :  2  (r, r) =  7723910396975.338
 Iteration :  3  (r, r) =  4596707853392.447
 Iteration :  4  (r, r) =  3484908890885.7627
 Iteration :  5  (r, r) =  2782124709218.0215
Relative/Absolute residual less than tol
Converged in  5  iterations with final norm  1667970.2363105947

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.081021e+07 6.1e+07 2.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.180904e+07 2.2e+07 5.1e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 8
cumulative cost evaluations : 3
cumulative gradient evaluations : 3
cumulative Hessian vector products (excluding preconditioner builds) : 8
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 3 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
```

```
building preconditioner
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 1.623e+06
||M r0|| = 4.183e+03
(M r0, r0) = 6.505e+04
 Iteration :  0  (r, r) =  2633546763856.491
 Iteration :  1  (r, r) =  1797914212244.619
 Iteration :  2  (r, r) =  1660712510306.205
 Iteration :  3  (r, r) =  1288347515817.5286
 Iteration :  4  (r, r) =  1078521385205.6877
 Iteration :  5  (r, r) =  1093309960593.429
 Iteration :  6  (r, r) =  867617429843.6278
 Iteration :  7  (r, r) =  621255636446.1287
 Iteration :  8  (r, r) =  441183332097.97125
 Iteration :  9  (r, r) =  386169031113.4435
 Iteration :  10  (r, r) =  342336435763.02625
 Iteration :  11  (r, r) =  300238243125.9479
 Iteration :  12  (r, r) =  290555871850.3236
 Iteration :  13  (r, r) =  227244801699.8081
 Iteration :  14  (r, r) =  204198054022.70337
Relative/Absolute residual less than tol
Converged in  14  iterations with final norm  451882.787924815

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.081021e+07 6.1e+07 2.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.180904e+07 2.2e+07 5.1e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
```

```
e-01
 3  14  1  1  14  T  T 6.223754e+06 6.1e+06 1.2e+05 -9.3e+06 1.6e+06 1.0e+00 2.9
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 22
cumulative cost evaluations : 4
cumulative gradient evaluations : 4
cumulative Hessian vector products (excluding preconditioner builds) : 22
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 4 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 4.570e+05
||M r0|| = 9.034e+02
(M r0, r0) = 1.480e+04
 Iteration :  0  (r, r) =  208814699704.0423
 Iteration :  1  (r, r) =  156274225178.69598
 Iteration :  2  (r, r) =  151558142671.16113
 Iteration :  3  (r, r) =  139091094339.03308
 Iteration :  4  (r, r) =  124827891646.63843
 Iteration :  5  (r, r) =  118888187106.44522
 Iteration :  6  (r, r) =  120794486505.15369
 Iteration :  7  (r, r) =  118097898625.76532
 Iteration :  8  (r, r) =  111036746788.02646
 Iteration :  9  (r, r) =  109337941880.37997
 Iteration :  10  (r, r) =  92489603628.72995
 Iteration :  11  (r, r) =  93401237923.22665
 Iteration :  12  (r, r) =  91017574745.29724
 Iteration :  13  (r, r) =  76587142830.50244
 Iteration :  14  (r, r) =  77144013743.98473
 Iteration :  15  (r, r) =  61378015445.73114
 Iteration :  16  (r, r) =  49186435048.63666
 Iteration :  17  (r, r) =  40794446122.67066
 Iteration :  18  (r, r) =  41165358626.87737
 Iteration :  19  (r, r) =  32366270257.165955
 Iteration :  20  (r, r) =  24490363937.322876
 Iteration :  21  (r, r) =  22375551821.195145
 Iteration :  22  (r, r) =  20607845095.096878
 Iteration :  23  (r, r) =  19832684577.885567
 Iteration :  24  (r, r) =  19033706200.83288
 Iteration :  25  (r, r) =  20062166698.41462
 Iteration :  26  (r, r) =  18504125522.7228
 Iteration :  27  (r, r) =  16579008330.08011
 Iteration :  28  (r, r) =  15450347892.541042
 Iteration :  29  (r, r) =  14179588778.35804
 Iteration :  30  (r, r) =  13444878476.572296
 Iteration :  31  (r, r) =  12803163454.573095
 Iteration :  32  (r, r) =  13062730201.124844
 Iteration :  33  (r, r) =  10815349608.264198
 Iteration :  34  (r, r) =  10751956994.9739
 Iteration :  35  (r, r) =  9245388494.787855
 Iteration :  36  (r, r) =  7906997921.438799
 Iteration :  37  (r, r) =  6714593399.253725
 Iteration :  38  (r, r) =  6177051751.096422
 Iteration :  39  (r, r) =  5296056953.159194
```

```
 Iteration :  40  (r, r) =  5233834882.77063
 Iteration :  41  (r, r) =  5619340684.244795
 Iteration :  42  (r, r) =  4974313791.958543
 Iteration :  43  (r, r) =  4536892864.879036
Relative/Absolute residual less than tol
Converged in  43  iterations with final norm  67356.4611962285

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.081021e+07 6.1e+07 2.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.180904e+07 2.2e+07 5.1e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01
 3  14  1  1  14  T  T 6.223754e+06 6.1e+06 1.2e+05 -9.3e+06 1.6e+06 1.0e+00 2.9
e-01
 4  43  1  1  43  T  F 1.578787e+06 1.3e+06 2.3e+05 -1.6e+06 4.6e+05 1.0e+00 1.5
e-01

converged : False
reason    : unknown reason
cumulative CG iterations : 65
cumulative cost evaluations : 5
cumulative gradient evaluations : 5
cumulative Hessian vector products (excluding preconditioner builds) : 65
====================== End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 5 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
```

```
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 7.082e+04
||M r0|| = 1.549e+02
(M r0, r0) = 2.476e+03
 Iteration :  0  (r, r) =  5015446977.454357
 Iteration :  1  (r, r) =  4226926584.7344627
 Iteration :  2  (r, r) =  3363265424.767089
 Iteration :  3  (r, r) =  3065205625.257473
 Iteration :  4  (r, r) =  2895892680.1961317
 Iteration :  5  (r, r) =  2694295995.4240704
 Iteration :  6  (r, r) =  2663749270.0654273
 Iteration :  7  (r, r) =  2674392231.483264
 Iteration :  8  (r, r) =  2474937662.3631306
 Iteration :  9  (r, r) =  2471182155.1995716
 Iteration :  10  (r, r) =  2439106743.897315
 Iteration :  11  (r, r) =  2267162619.562894
 Iteration :  12  (r, r) =  2249240345.345931
 Iteration :  13  (r, r) =  2201342871.9013405
 Iteration :  14  (r, r) =  2075682297.12861
 Iteration :  15  (r, r) =  2019005094.7968073
 Iteration :  16  (r, r) =  2039068322.4237216
 Iteration :  17  (r, r) =  1864430412.6118493
 Iteration :  18  (r, r) =  1899908662.222554
 Iteration :  19  (r, r) =  1754068401.5958602
 Iteration :  20  (r, r) =  1627886921.4528928
 Iteration :  21  (r, r) =  1503799574.1301122
 Iteration :  22  (r, r) =  1362655895.2547865
 Iteration :  23  (r, r) =  1388231238.116437
 Iteration :  24  (r, r) =  1248423948.0803251
 Iteration :  25  (r, r) =  1265456393.6449885
 Iteration :  26  (r, r) =  1216503675.778665
 Iteration :  27  (r, r) =  1246855640.0427613
 Iteration :  28  (r, r) =  1219735217.0996556
 Iteration :  29  (r, r) =  1156914082.6075768
 Iteration :  30  (r, r) =  1114715491.1751218
 Iteration :  31  (r, r) =  1171956936.1756299
 Iteration :  32  (r, r) =  1187055069.3202627
 Iteration :  33  (r, r) =  999755685.0054071
 Iteration :  34  (r, r) =  809843541.626469
 Iteration :  35  (r, r) =  729466418.4460568
 Iteration :  36  (r, r) =  658076548.5206707
 Iteration :  37  (r, r) =  602093840.7458813
 Iteration :  38  (r, r) =  587351585.8145802
 Iteration :  39  (r, r) =  520714485.45405227
 Iteration :  40  (r, r) =  481512325.3770869
 Iteration :  41  (r, r) =  464423675.5660394
 Iteration :  42  (r, r) =  460557702.58099496
 Iteration :  43  (r, r) =  466164878.4468198
 Iteration :  44  (r, r) =  477464220.8458015
 Iteration :  45  (r, r) =  505762582.2837423
 Iteration :  46  (r, r) =  397796527.03082037
 Iteration :  47  (r, r) =  347173334.34753907
 Iteration :  48  (r, r) =  296656305.8055004
 Iteration :  49  (r, r) =  297043853.5139904
 Iteration :  50  (r, r) =  225731696.27324462
 Iteration :  51  (r, r) =  211507996.48067746
 Iteration :  52  (r, r) =  166827986.70790637
 Iteration :  53  (r, r) =  149332495.55657285
 Iteration :  54  (r, r) =  140640265.82695824
```

```
 Iteration :  55  (r, r) =  129092345.70772202
 Iteration :  56  (r, r) =  111639175.09074448
 Iteration :  57  (r, r) =  119095368.81640287
 Iteration :  58  (r, r) =  108796530.38956907
 Iteration :  59  (r, r) =  106722362.2696543
 Iteration :  60  (r, r) =  94766493.93004248
 Iteration :  61  (r, r) =  90098656.83230785
 Iteration :  62  (r, r) =  87562834.06547919
 Iteration :  63  (r, r) =  78407669.47752306
 Iteration :  64  (r, r) =  78513738.26139826
 Iteration :  65  (r, r) =  71323276.4488349
 Iteration :  66  (r, r) =  67457491.63288079
 Iteration :  67  (r, r) =  69746637.77956004
 Iteration :  68  (r, r) =  62105544.51546043
 Iteration :  69  (r, r) =  66286379.850912064
 Iteration :  70  (r, r) =  54802885.828488916
 Iteration :  71  (r, r) =  65354458.55278109
 Iteration :  72  (r, r) =  54401053.31802486
 Iteration :  73  (r, r) =  59142553.55050546
 Iteration :  74  (r, r) =  56409531.57357912
 Iteration :  75  (r, r) =  57345472.46728028
 Iteration :  76  (r, r) =  50227049.91947527
 Iteration :  77  (r, r) =  45988631.34193316
 Iteration :  78  (r, r) =  47356475.87091111
 Iteration :  79  (r, r) =  39119412.330537036
 Iteration :  80  (r, r) =  36366548.49069657
 Iteration :  81  (r, r) =  30454649.533442654
 Iteration :  82  (r, r) =  30886333.70319294
 Iteration :  83  (r, r) =  26557315.929344974
 Iteration :  84  (r, r) =  23989731.761429753
 Iteration :  85  (r, r) =  24191600.979336746
 Iteration :  86  (r, r) =  21760403.168978468
 Iteration :  87  (r, r) =  22696557.77637533
 Iteration :  88  (r, r) =  19470611.01371662
 Iteration :  89  (r, r) =  15711254.562275952
Relative/Absolute residual less than tol
Converged in  89  iterations with final norm  3963.742494445868

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
```

```
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.081021e+07 6.1e+07 2.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.180904e+07 2.2e+07 5.1e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01
 3  14  1  1  14  T  T 6.223754e+06 6.1e+06 1.2e+05 -9.3e+06 1.6e+06 1.0e+00 2.9
e-01
 4  43  1  1  43  T  F 1.578787e+06 1.3e+06 2.3e+05 -1.6e+06 4.6e+05 1.0e+00 1.5
e-01
 5  89  1  1  89  T  F 7.995442e+05 4.3e+05 3.7e+05 -5.2e+04 7.1e+04 1.0e+00 6.0
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 154
cumulative cost evaluations : 6
cumulative gradient evaluations : 6
cumulative Hessian vector products (excluding preconditioner builds) : 154
======================= End Newton CG convergence information ==================
=====

using Gauss-Newton Hessian
it= 6 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
True
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 4.116e+03
||M r0|| = 7.644e+00
(M r0, r0) = 1.156e+02
 Iteration :  0  (r, r) =  16939358.21579752
 Iteration :  1  (r, r) =  15829940.842864025
 Iteration :  2  (r, r) =  13676674.18744998
 Iteration :  3  (r, r) =  13000319.677204274
 Iteration :  4  (r, r) =  12839903.085058603
 Iteration :  5  (r, r) =  11252931.106234163
 Iteration :  6  (r, r) =  11117532.353566494
 Iteration :  7  (r, r) =  10454889.809724286
 Iteration :  8  (r, r) =  10473481.648212474
 Iteration :  9  (r, r) =  9995063.508553853
 Iteration :  10  (r, r) =  10032096.937647901
 Iteration :  11  (r, r) =  9773935.1979649
 Iteration :  12  (r, r) =  10054966.654117588
 Iteration :  13  (r, r) =  9698673.34881819
 Iteration :  14  (r, r) =  9245175.130086863
 Iteration :  15  (r, r) =  8425372.838980898
 Iteration :  16  (r, r) =  8095330.444068493
 Iteration :  17  (r, r) =  7218567.876135841
 Iteration :  18  (r, r) =  6627809.069332445
 Iteration :  19  (r, r) =  6419969.876993366
 Iteration :  20  (r, r) =  5726033.871260938
 Iteration :  21  (r, r) =  5370349.602071915
```

```
Iteration :  22  (r, r) =  5336818.750085354
Iteration :  23  (r, r) =  5112306.267622681
Iteration :  24  (r, r) =  5249001.404878503
Iteration :  25  (r, r) =  5100659.549067544
Iteration :  26  (r, r) =  4951474.685012238
Iteration :  27  (r, r) =  4740931.757715403
Iteration :  28  (r, r) =  4084323.088254988
Iteration :  29  (r, r) =  4120431.0633110167
Iteration :  30  (r, r) =  3925581.033911786
Iteration :  31  (r, r) =  3619150.5510794306
Iteration :  32  (r, r) =  3893175.7999986187
Iteration :  33  (r, r) =  3938525.778453321
Iteration :  34  (r, r) =  3632118.344556088
Iteration :  35  (r, r) =  3611091.2217316125
Iteration :  36  (r, r) =  3581500.8359455946
Iteration :  37  (r, r) =  3134781.4435195243
Iteration :  38  (r, r) =  2945147.370118011
Iteration :  39  (r, r) =  2768632.5140945353
Iteration :  40  (r, r) =  2643943.6410198957
Iteration :  41  (r, r) =  2509313.4685574668
Iteration :  42  (r, r) =  2525362.050482229
Iteration :  43  (r, r) =  2503614.002094605
Iteration :  44  (r, r) =  2433353.3767861007
Iteration :  45  (r, r) =  2279107.903945623
Iteration :  46  (r, r) =  2436987.664032352
Iteration :  47  (r, r) =  2057757.6094985926
Iteration :  48  (r, r) =  1867418.2079785536
Iteration :  49  (r, r) =  1602104.3682036689
Iteration :  50  (r, r) =  1401867.1044460777
Iteration :  51  (r, r) =  1583244.829775154
Iteration :  52  (r, r) =  1211240.1496423692
Iteration :  53  (r, r) =  1099416.808403374
Iteration :  54  (r, r) =  1144208.1836736002
Iteration :  55  (r, r) =  959710.3778465224
Iteration :  56  (r, r) =  929986.8833463367
Iteration :  57  (r, r) =  956057.8332374836
Iteration :  58  (r, r) =  848423.3046664663
Iteration :  59  (r, r) =  852778.2829900356
Iteration :  60  (r, r) =  803417.9227235925
Iteration :  61  (r, r) =  786807.3034546932
Iteration :  62  (r, r) =  688890.398518729
Iteration :  63  (r, r) =  640988.2662193326
Iteration :  64  (r, r) =  572724.8220247928
Iteration :  65  (r, r) =  522808.34792641096
Iteration :  66  (r, r) =  462477.1346515586
Iteration :  67  (r, r) =  449178.817166253
Iteration :  68  (r, r) =  402003.2518144209
Iteration :  69  (r, r) =  363891.0248270657
Iteration :  70  (r, r) =  386721.07399884146
Iteration :  71  (r, r) =  394908.4263579035
Iteration :  72  (r, r) =  320555.25108559534
Iteration :  73  (r, r) =  289994.12139775674
Iteration :  74  (r, r) =  298829.38327594043
Iteration :  75  (r, r) =  295065.2895523781
Iteration :  76  (r, r) =  272769.4997038597
Iteration :  77  (r, r) =  239906.0208355223
Iteration :  78  (r, r) =  244573.04901723412
Iteration :  79  (r, r) =  226412.17988788214
Iteration :  80  (r, r) =  224791.61556675954
Iteration :  81  (r, r) =  190442.29768792976
Iteration :  82  (r, r) =  223710.99710221085
```

```
 Iteration :  83  (r, r) =  188888.5667581929
 Iteration :  84  (r, r) =  171893.79806178517
 Iteration :  85  (r, r) =  156179.34690351546
 Iteration :  86  (r, r) =  156974.41108843716
 Iteration :  87  (r, r) =  145485.56109250695
 Iteration :  88  (r, r) =  146058.1677343908
 Iteration :  89  (r, r) =  166649.29226406774
 Iteration :  90  (r, r) =  165773.34212792426
 Iteration :  91  (r, r) =  167302.90164516587
 Iteration :  92  (r, r) =  132887.5852534274
 Iteration :  93  (r, r) =  117611.97769440032
 Iteration :  94  (r, r) =  97165.91166148696
 Iteration :  95  (r, r) =  101226.10529205346
 Iteration :  96  (r, r) =  102333.03931087881
 Iteration :  97  (r, r) =  68459.04460998529
 Iteration :  98  (r, r) =  67136.11753525406
 Iteration :  99  (r, r) =  53694.62540588234
 Iteration :  100  (r, r) =  52037.305645079075
 Iteration :  101  (r, r) =  51502.65928632441
 Iteration :  102  (r, r) =  40210.428838687556
 Iteration :  103  (r, r) =  43180.85507375995
 Iteration :  104  (r, r) =  34839.62640147778
 Iteration :  105  (r, r) =  34799.50712406801
 Iteration :  106  (r, r) =  32060.363863718914
 Iteration :  107  (r, r) =  26668.910943214672
 Iteration :  108  (r, r) =  28209.541591361587
 Iteration :  109  (r, r) =  24351.569981106713
 Iteration :  110  (r, r) =  24618.85152375905
 Iteration :  111  (r, r) =  22574.1255187585
 Iteration :  112  (r, r) =  24083.233419933353
 Iteration :  113  (r, r) =  19471.090558898683
 Iteration :  114  (r, r) =  17581.983650198323
 Iteration :  115  (r, r) =  14966.4278973316
 Iteration :  116  (r, r) =  16176.492725193664
 Iteration :  117  (r, r) =  14274.473062932364
 Iteration :  118  (r, r) =  13842.43236500987
 Iteration :  119  (r, r) =  11565.902571608436
 Iteration :  120  (r, r) =  10105.482899867682
 Iteration :  121  (r, r) =  9851.00982699451
 Iteration :  122  (r, r) =  9669.446252100868
 Iteration :  123  (r, r) =  8491.918126617493
 Iteration :  124  (r, r) =  9159.148313408537
 Iteration :  125  (r, r) =  7409.885204931529
 Iteration :  126  (r, r) =  6873.122725768691
 Iteration :  127  (r, r) =  6788.865461705542
 Iteration :  128  (r, r) =  5760.6773020573555
 Iteration :  129  (r, r) =  5991.673137903212
 Iteration :  130  (r, r) =  5259.797881030383
 Iteration :  131  (r, r) =  4750.881974249822
 Iteration :  132  (r, r) =  4497.623487228953
 Iteration :  133  (r, r) =  4228.438088141882
 Iteration :  134  (r, r) =  4440.442729312833
 Iteration :  135  (r, r) =  3699.046457289701
 Iteration :  136  (r, r) =  4055.216109672579
 Iteration :  137  (r, r) =  3449.8565814041594
Relative/Absolute residual less than tol
Converged in  137  iterations with final norm  58.735479749502

===================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
```

```
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.081021e+07 6.1e+07 2.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.180904e+07 2.2e+07 5.1e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01
 3  14  1  1  14  T  T 6.223754e+06 6.1e+06 1.2e+05 -9.3e+06 1.6e+06 1.0e+00 2.9
e-01
 4  43  1  1  43  T  F 1.578787e+06 1.3e+06 2.3e+05 -1.6e+06 4.6e+05 1.0e+00 1.5
e-01
 5  89  1  1  89  T  F 7.995442e+05 4.3e+05 3.7e+05 -5.2e+04 7.1e+04 1.0e+00 6.0
e-02
 6 137  1  1 137  T  F 7.736097e+05 3.7e+05 4.0e+05 -2.1e+02 4.1e+03 1.0e+00 1.5
e-02

converged : False
reason    : unknown reason
cumulative CG iterations : 291
cumulative cost evaluations : 7
cumulative gradient evaluations : 7
cumulative Hessian vector products (excluding preconditioner builds) : 291
====================== End Newton CG convergence information ==================
=====

using Hessian
it= 7 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
False
Residual: r=b-Ax
Preconditioner: M =approx= A^-1
||r0|| = 6.841e+01
||M r0|| = 1.358e-01
(M r0, r0) = 2.103e+00
```

```
Iteration :  0  (r, r) =  4680.392646908169
Iteration :  1  (r, r) =  4125.295051961384
Iteration :  2  (r, r) =  3570.7867989924634
Iteration :  3  (r, r) =  3771.976830214595
Iteration :  4  (r, r) =  3671.394206207255
Iteration :  5  (r, r) =  3312.7577119459775
Iteration :  6  (r, r) =  3721.5064242710087
Iteration :  7  (r, r) =  3213.1669008171484
Iteration :  8  (r, r) =  3309.287436353742
Iteration :  9  (r, r) =  3420.335177030822
Iteration :  10  (r, r) =  3247.873061773187
Iteration :  11  (r, r) =  3155.130485673222
Iteration :  12  (r, r) =  3286.953993505009
Iteration :  13  (r, r) =  3084.6154189378385
Iteration :  14  (r, r) =  2821.258128727842
Iteration :  15  (r, r) =  2804.1359708451337
Iteration :  16  (r, r) =  2651.600236407752
Iteration :  17  (r, r) =  2575.1569320326835
Iteration :  18  (r, r) =  2421.8591473017564
Iteration :  19  (r, r) =  2379.252628879637
Iteration :  20  (r, r) =  2221.3606985140564
Iteration :  21  (r, r) =  2053.137391125238
Iteration :  22  (r, r) =  2101.5220486625676
Iteration :  23  (r, r) =  2132.2451257003145
Iteration :  24  (r, r) =  2065.1842201354225
Iteration :  25  (r, r) =  1974.6594957525162
Iteration :  26  (r, r) =  1836.1499633367102
Iteration :  27  (r, r) =  1749.5913182016552
Iteration :  28  (r, r) =  1686.6753582815124
Iteration :  29  (r, r) =  1652.822124245469
Iteration :  30  (r, r) =  1477.0821745084106
Iteration :  31  (r, r) =  1475.4020794142273
Iteration :  32  (r, r) =  1482.3647251149264
Iteration :  33  (r, r) =  1319.051816323662
Iteration :  34  (r, r) =  1231.384509488952
Iteration :  35  (r, r) =  1152.715529036428
Iteration :  36  (r, r) =  1036.5747273685474
Iteration :  37  (r, r) =  1003.747247756638
Iteration :  38  (r, r) =  951.5576342408037
Iteration :  39  (r, r) =  909.3720215978352
Iteration :  40  (r, r) =  855.3747459810445
Iteration :  41  (r, r) =  910.453435778993
Iteration :  42  (r, r) =  855.3709843228849
Iteration :  43  (r, r) =  813.1830992632001
Iteration :  44  (r, r) =  823.1334508962059
Iteration :  45  (r, r) =  755.3497859962672
Iteration :  46  (r, r) =  702.6525528831562
Iteration :  47  (r, r) =  709.9724640457166
Iteration :  48  (r, r) =  677.5598368649179
Iteration :  49  (r, r) =  601.4270860250836
Iteration :  50  (r, r) =  642.7149247447705
Iteration :  51  (r, r) =  553.714303259302
Iteration :  52  (r, r) =  506.22036055772423
Iteration :  53  (r, r) =  523.7209960183188
Iteration :  54  (r, r) =  433.1275655734473
Iteration :  55  (r, r) =  364.6176476482824
Iteration :  56  (r, r) =  354.98926303217485
Iteration :  57  (r, r) =  369.6713523861059
Iteration :  58  (r, r) =  300.9088481998485
Iteration :  59  (r, r) =  273.1113300877132
Iteration :  60  (r, r) =  281.5484238499139
```

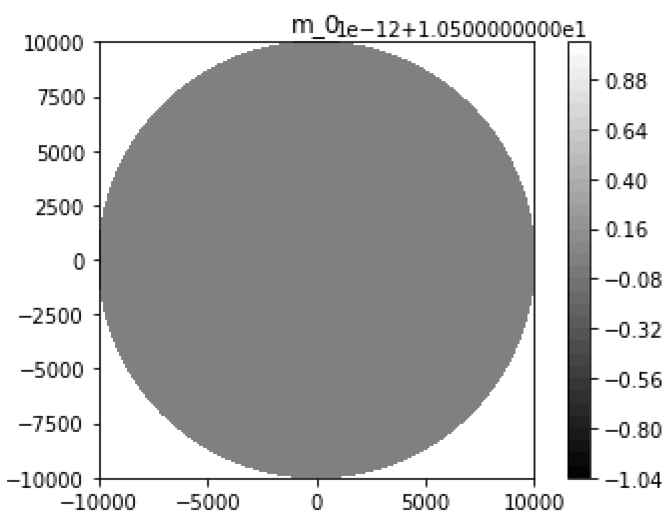
```
Iteration :  61  (r, r) =  264.8181460079386
Iteration :  62  (r, r) =  285.36321038634503
Iteration :  63  (r, r) =  234.0417146377423
Iteration :  64  (r, r) =  203.0694059439502
Iteration :  65  (r, r) =  207.55719934638637
Iteration :  66  (r, r) =  170.48283365546456
Iteration :  67  (r, r) =  152.65372043551918
Iteration :  68  (r, r) =  149.14819114917975
Iteration :  69  (r, r) =  129.8359957193319
Iteration :  70  (r, r) =  128.11965976720126
Iteration :  71  (r, r) =  116.81292916576874
Iteration :  72  (r, r) =  104.76204432428133
Iteration :  73  (r, r) =  104.42263356681264
Iteration :  74  (r, r) =  99.69168960650548
Iteration :  75  (r, r) =  106.99407276562034
Iteration :  76  (r, r) =  84.87805819638604
Iteration :  77  (r, r) =  78.99305963043359
Iteration :  78  (r, r) =  77.42621601533313
Iteration :  79  (r, r) =  81.72445706828888
Iteration :  80  (r, r) =  79.95118184009556
Iteration :  81  (r, r) =  83.08786826210033
Iteration :  82  (r, r) =  69.66739338385287
Iteration :  83  (r, r) =  67.37027089055397
Iteration :  84  (r, r) =  63.52627698908434
Iteration :  85  (r, r) =  59.51788386615817
Iteration :  86  (r, r) =  54.27958679415797
Iteration :  87  (r, r) =  53.975569114629884
Iteration :  88  (r, r) =  51.38871441161929
Iteration :  89  (r, r) =  43.458274808963694
Iteration :  90  (r, r) =  43.26307153252954
Iteration :  91  (r, r) =  41.158981676196106
Iteration :  92  (r, r) =  42.04407302511191
Iteration :  93  (r, r) =  38.07955602885774
Iteration :  94  (r, r) =  34.49310133442029
Iteration :  95  (r, r) =  35.66331608526443
Iteration :  96  (r, r) =  36.18038852104269
Iteration :  97  (r, r) =  29.552178153534854
Iteration :  98  (r, r) =  34.00489136798697
Iteration :  99  (r, r) =  27.055125492969132
Iteration :  100  (r, r) =  23.990591290819037
Iteration :  101  (r, r) =  23.8595822107147
Iteration :  102  (r, r) =  18.12485894144455
Iteration :  103  (r, r) =  19.22384510369739
Iteration :  104  (r, r) =  16.519220985581242
Iteration :  105  (r, r) =  16.62171487780605
Iteration :  106  (r, r) =  14.473290521789258
Iteration :  107  (r, r) =  14.674954409377849
Iteration :  108  (r, r) =  12.514701746631474
Iteration :  109  (r, r) =  13.70061409240968
Iteration :  110  (r, r) =  11.6703426684449
Iteration :  111  (r, r) =  11.211845731277151
Iteration :  112  (r, r) =  11.774597389386162
Iteration :  113  (r, r) =  10.384368278004377
Iteration :  114  (r, r) =  10.63791774735525
Iteration :  115  (r, r) =  9.87316965739577
Iteration :  116  (r, r) =  8.918127340856994
Iteration :  117  (r, r) =  10.683883609570572
Iteration :  118  (r, r) =  8.157487133311948
Iteration :  119  (r, r) =  7.934014640171668
Iteration :  120  (r, r) =  6.971172330559067
Iteration :  121  (r, r) =  6.5091676141562385
```

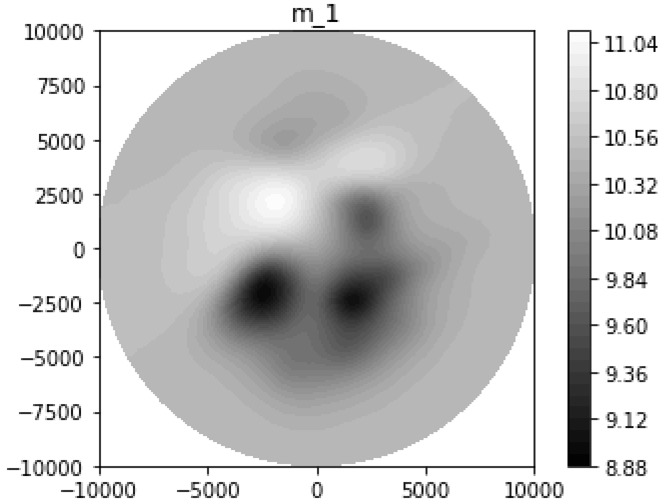
```
Iteration :  122  (r, r) =  6.27194490888004
Iteration :  123  (r, r) =  5.270275741557111
Iteration :  124  (r, r) =  4.835730272439543
Iteration :  125  (r, r) =  5.086948679305122
Iteration :  126  (r, r) =  4.899524752774311
Iteration :  127  (r, r) =  4.544025022882121
Iteration :  128  (r, r) =  3.8372105499989053
Iteration :  129  (r, r) =  3.9869242522528827
Iteration :  130  (r, r) =  3.118803422519643
Iteration :  131  (r, r) =  2.8485665986646778
Iteration :  132  (r, r) =  2.785138110628126
Iteration :  133  (r, r) =  2.673457280689694
Iteration :  134  (r, r) =  2.5496742076636583
Iteration :  135  (r, r) =  2.383969666569546
Iteration :  136  (r, r) =  2.187770115319438
Iteration :  137  (r, r) =  2.6327844581605735
Iteration :  138  (r, r) =  2.244977017001406
Iteration :  139  (r, r) =  1.8441373815371702
Iteration :  140  (r, r) =  1.8059530694850436
Iteration :  141  (r, r) =  1.7441619001221698
Iteration :  142  (r, r) =  1.5368610930642599
Iteration :  143  (r, r) =  1.6158540668184316
Iteration :  144  (r, r) =  1.223951089857826
Iteration :  145  (r, r) =  1.3467021454426398
Iteration :  146  (r, r) =  1.0617517053766816
Iteration :  147  (r, r) =  1.0239121633656532
Iteration :  148  (r, r) =  1.0084510086331964
Iteration :  149  (r, r) =  0.9635221060606705
Iteration :  150  (r, r) =  0.893076935501322
Iteration :  151  (r, r) =  0.8559293176223289
Iteration :  152  (r, r) =  0.7353157010827055
Iteration :  153  (r, r) =  0.8267628835571275
Iteration :  154  (r, r) =  0.6507583938446451
Iteration :  155  (r, r) =  0.6463162311819997
Iteration :  156  (r, r) =  0.5421571528223947
Iteration :  157  (r, r) =  0.49750459010952125
Iteration :  158  (r, r) =  0.5279000398746043
Iteration :  159  (r, r) =  0.4754291369378048
Iteration :  160  (r, r) =  0.47039733637700587
Iteration :  161  (r, r) =  0.3874828851864443
Iteration :  162  (r, r) =  0.36691537469590774
Iteration :  163  (r, r) =  0.4123569000570968
Iteration :  164  (r, r) =  0.38441890363422004
Iteration :  165  (r, r) =  0.3333579298331476
Iteration :  166  (r, r) =  0.31156491571040745
Iteration :  167  (r, r) =  0.27219968812199025
Iteration :  168  (r, r) =  0.27578480208396117
Iteration :  169  (r, r) =  0.2398043637198655
Iteration :  170  (r, r) =  0.23456731584957974
Iteration :  171  (r, r) =  0.22223224772953254
Iteration :  172  (r, r) =  0.23865907114845222
Iteration :  173  (r, r) =  0.1983149049734706
Iteration :  174  (r, r) =  0.23990648985795715
Iteration :  175  (r, r) =  0.1819098298466856
Iteration :  176  (r, r) =  0.1701360872768624
Iteration :  177  (r, r) =  0.1567644614047008
Iteration :  178  (r, r) =  0.14819074540635124
Iteration :  179  (r, r) =  0.14118284746866672
Iteration :  180  (r, r) =  0.150794916653695
Iteration :  181  (r, r) =  0.13427128403133148
Iteration :  182  (r, r) =  0.11579318556919715
```

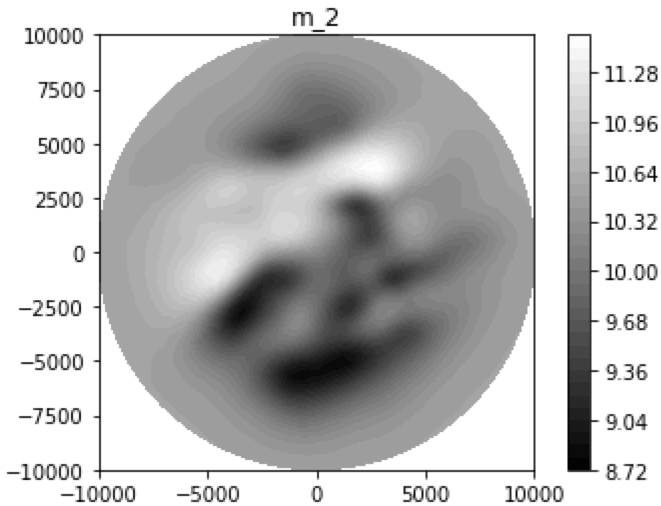
```
 Iteration :  183  (r, r) =  0.10650846048683599
 Iteration :  184  (r, r) =  0.09956408986540763
 Iteration :  185  (r, r) =  0.10673307335008737
 Iteration :  186  (r, r) =  0.08935995206142411
 Iteration :  187  (r, r) =  0.08466192639984654
 Iteration :  188  (r, r) =  0.08508403091309337
 Iteration :  189  (r, r) =  0.06717343504233254
 Iteration :  190  (r, r) =  0.0710844153338924
 Iteration :  191  (r, r) =  0.06838014894964176
 Iteration :  192  (r, r) =  0.053089076681112735
 Iteration :  193  (r, r) =  0.05794948996029318
 Iteration :  194  (r, r) =  0.05335007518443348
 Iteration :  195  (r, r) =  0.05277685192086817
 Iteration :  196  (r, r) =  0.04492426513354285
 Iteration :  197  (r, r) =  0.04585274923738247
 Iteration :  198  (r, r) =  0.03586349698197262
 Iteration :  199  (r, r) =  0.04099082911719869
 Iteration :  200  (r, r) =  0.035647617821953075
 Iteration :  201  (r, r) =  0.03625441777487742
 Iteration :  202  (r, r) =  0.03276052532271956
 Iteration :  203  (r, r) =  0.03234473137515734
 Iteration :  204  (r, r) =  0.028902904819872836
 Iteration :  205  (r, r) =  0.023775574665537418
 Iteration :  206  (r, r) =  0.02604100631198865
 Iteration :  207  (r, r) =  0.021139723632776185
 Iteration :  208  (r, r) =  0.020506606500749473
 Iteration :  209  (r, r) =  0.019229319034283807
 Iteration :  210  (r, r) =  0.019362233407578807
 Iteration :  211  (r, r) =  0.017265726108519425
 Iteration :  212  (r, r) =  0.014864424413508052
Relative/Absolute residual less than tol
Converged in  212  iterations with final norm  0.12191974579004031

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
```

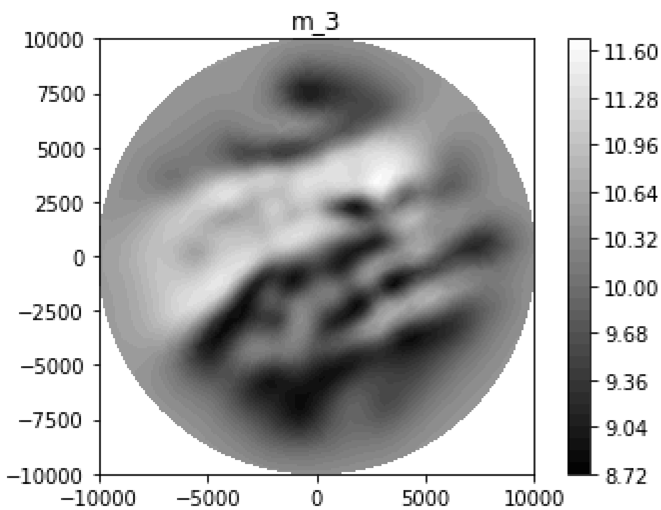
```
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.081021e+07 6.1e+07 2.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.180904e+07 2.2e+07 5.1e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01
 3  14  1  1  14  T  T 6.223754e+06 6.1e+06 1.2e+05 -9.3e+06 1.6e+06 1.0e+00 2.9
e-01
 4  43  1  1  43  T  F 1.578787e+06 1.3e+06 2.3e+05 -1.6e+06 4.6e+05 1.0e+00 1.5
e-01
 5  89  1  1  89  T  F 7.995442e+05 4.3e+05 3.7e+05 -5.2e+04 7.1e+04 1.0e+00 6.0
e-02
 6 137  1  1 137  T  F 7.736097e+05 3.7e+05 4.0e+05 -2.1e+02 4.1e+03 1.0e+00 1.5
e-02
 7 212  1  1 212  F  F 7.735055e+05 3.7e+05 4.0e+05 -8.1e-02 6.8e+01 1.0e+00 1.9
e-03

converged : False
reason    : unknown reason
cumulative CG iterations : 503
cumulative cost evaluations : 8
cumulative gradient evaluations : 8
cumulative Hessian vector products (excluding preconditioner builds) : 503
======================= End Newton CG convergence information ==================
=====

using Hessian
it= 8 , preconditioner_build_iters= (3,) , num_gn_iter= 7 , using_gauss_newton=
False

====================== Begin Newton CG convergence information =================
=====
Preconditioned inexact Newton-CG with line search
Hp=-g
u <- u + alpha * p
u: parameter, J: cost, g: gradient, H: Hessian, alpha=step size, p=search direct
ion

it=0    : u=u0         -> J -> g -> build precond (optional) -> cgsolve Hp=-g
it=1    : linesearch u -> J -> g -> build precond (optional) -> cgsolve Hp=-g
...
it=last : linesearch u -> J -> g -> Done.

it:      Newton iteration number
nCG:     number of CG iterations in Newton iteration
nJ:      number of cost function evaluations in Newton iteration
nG:      number of gradient evaluations in Newton iteration
nHp:     number of Hessian-vector products in Newton iteration
GN:      True (T) if Gauss-Newton Hessian is used, False (F) if Hessian is used
BP:      True (T) if we built or rebuilt the preconditioner, False (F) otherwis
e.
cost:    cost, J = Jd + Jr
misfit:  misfit cost, Jd
reg:     regularization cost, Jr
(g,p):   inner product between gradient, g, and Newton search direction, p
||g||L2: l2 norm of gradient
alpha:   step size
```

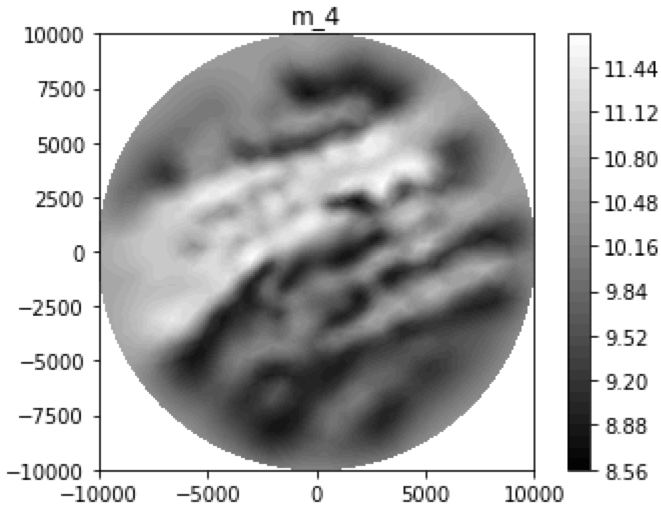
```
tolcg:   relative tolerance for Hp=-g CG solve (unpreconditioned residual decrea
se)

it nCG nJ nG nHp GN BP         cost  misfit     reg    (g,p) ||g||L2   alpha   t
olcg
 0   1  1  1   1  T  F 2.093839e+08 2.1e+08 0.0e+00 -2.9e+08 2.0e+07    ---- 5.0
e-01
 1   2  1  1   2  T  F 6.081021e+07 6.1e+07 2.0e+04 -7.7e+07 7.9e+06 1.0e+00 5.0
e-01
 2   5  1  1   5  T  F 2.180904e+07 2.2e+07 5.1e+04 -3.1e+07 3.9e+06 1.0e+00 4.5
e-01
 3  14  1  1  14  T  T 6.223754e+06 6.1e+06 1.2e+05 -9.3e+06 1.6e+06 1.0e+00 2.9
e-01
 4  43  1  1  43  T  F 1.578787e+06 1.3e+06 2.3e+05 -1.6e+06 4.6e+05 1.0e+00 1.5
e-01
 5  89  1  1  89  T  F 7.995442e+05 4.3e+05 3.7e+05 -5.2e+04 7.1e+04 1.0e+00 6.0
e-02
 6 137  1  1 137  T  F 7.736097e+05 3.7e+05 4.0e+05 -2.1e+02 4.1e+03 1.0e+00 1.5
e-02
 7 212  1  1 212  F  F 7.735055e+05 3.7e+05 4.0e+05 -8.1e-02 6.8e+01 1.0e+00 1.9
e-03
 8   0  1  1   0  F  F 7.735055e+05 3.7e+05 4.0e+05     ---- 1.4e-01 1.0e+00
----

converged : True
reason    : Norm of the gradient less than tolerance
cumulative CG iterations : 503
cumulative cost evaluations : 9
cumulative gradient evaluations : 9
cumulative Hessian vector products (excluding preconditioner builds) : 503
====================== End Newton CG convergence information =================
=====

gamma= 6975.784285975133 , noise_datanorm= 863.5355316705534 , misfit_datanorm=
863.5355639023904
```

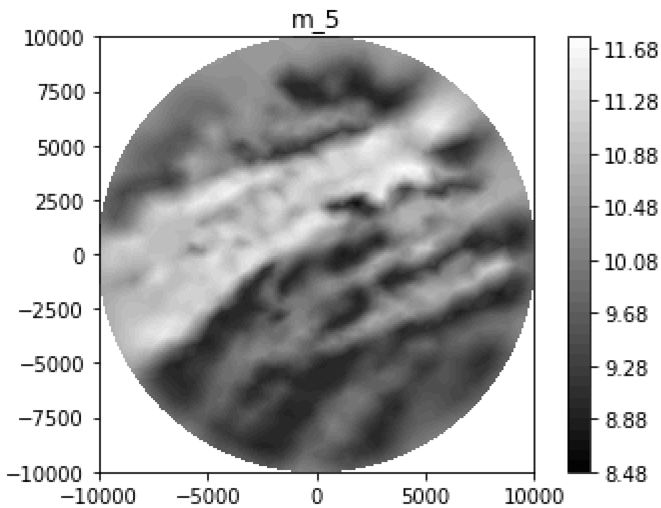


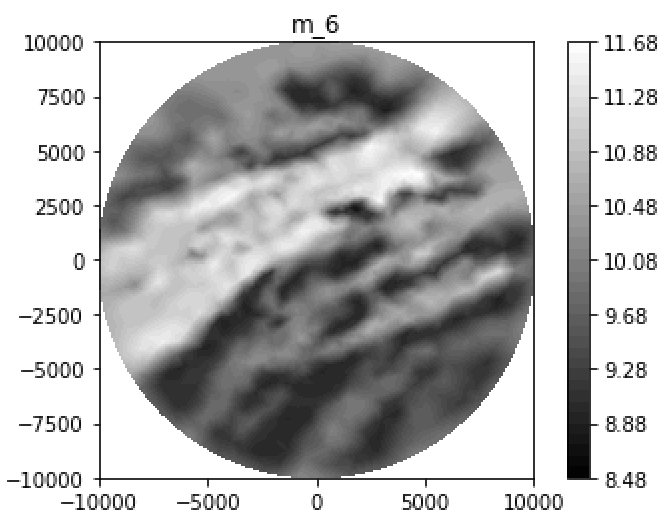

m_1
10000
7500
5000
2500
0
−2500
−5000
−7500
−10000
−10000 −5000 0 5000 10000
11.04
10.80
10.56
10.32
10.08
9.84
9.60
9.36
9.12
8.88

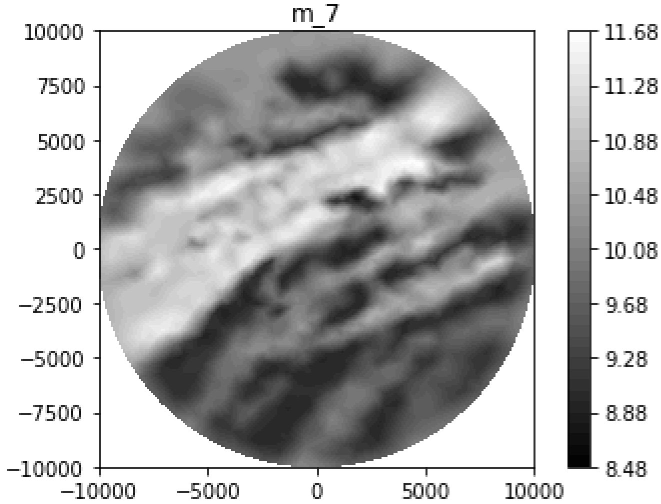

m_2
10000
7500
5000
2500
0
−2500
−5000
−7500
−10000
−10000 −5000 0 5000 10000
11.28
10.96
10.64
10.32
10.00
9.68
9.36
9.04
8.72


m_3
10000
7500
5000
2500
0
−2500
−5000
−7500
−10000
−10000 −5000 0 5000 10000
11.60
11.28
10.96
10.64
10.32
10.00
9.68
9.36
9.04
8.72


m_4
10000
7500
5000
2500
0
−2500
−5000
−7500
−10000
−10000
−5000
0
5000
10000
11.44
11.12
10.80
10.48
10.16
9.84
9.52
9.20
8.88
8.56


m_5
10000
7500
5000
2500
0
−2500
−5000
−7500
−10000
−10000
−5000
0
5000
10000
11.68
11.28
10.88
10.48
10.08
9.68
9.28
8.88
8.48


m_6
10000
7500
5000
2500
0
−2500
−5000
−7500
−10000
−10000
−5000
0
5000
10000
11.68
11.28
10.88
10.48
10.08
9.68
9.28
8.88
8.48



In [35]:

```
misfit_datanorm = morozov_helper()
```

```
gamma= 6975.784285975133 , noise_datanorm= 863.5355316705534 , misfit_datanorm=
863.5355639023904
```

Plots to make (4):

- Newton CG iterations and gradient table (psf and reg)
- Newton iterates (reg and psf)
- u recovered (with arrows)

In [63]:

```
norm_grad_newton_pch = np.array(PCH_convergence_info.gradnorm)
norm_grad_newton_reg = np.array(REG_convergence_info.gradnorm)

print('norm_grad_newton_pch=', norm_grad_newton_pch)
print('norm_grad_newton_reg=', norm_grad_newton_reg)
print('norm_grad_newton_pch/norm_grad_newton_pch[0]=', norm_grad_newton_pch/norm
print('norm_grad_newton_reg/norm_grad_newton_reg[0]=', norm_grad_newton_reg/norm
```

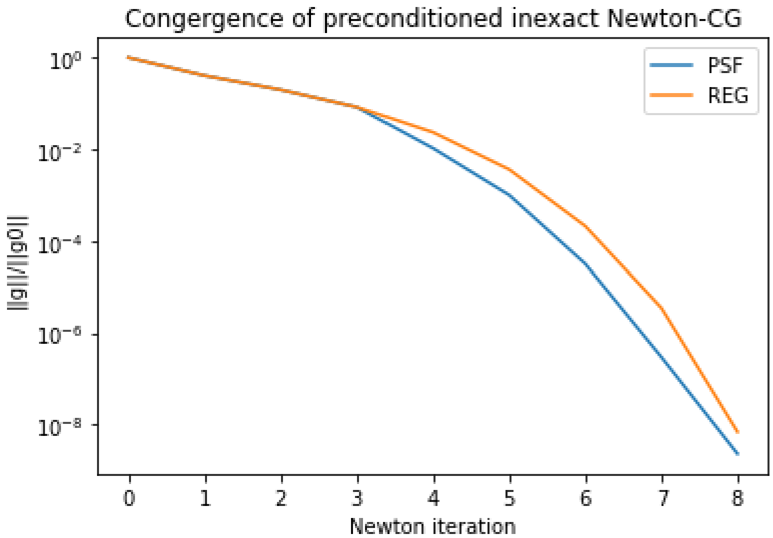
```
norm_grad_newton_pch= [1.95130459e+07 7.90533130e+06 3.87436091e+06 1.62282062e+
06
 2.05043253e+05 1.95792032e+04 6.23946965e+02 5.81540972e+00
 4.54048560e-02]
norm_grad_newton_reg= [1.95130459e+07 7.90533130e+06 3.87436091e+06 1.62282062e+
06
 4.56962471e+05 7.08198205e+04 4.11574516e+03 6.84133952e+01
 1.36654632e-01]
norm_grad_newton_pch/norm_grad_newton_pch[0]= [1.00000000e+00 4.05130565e-01 1.9
8552340e-01 8.31659306e-02
 1.05080086e-02 1.00339041e-03 3.19758878e-05 2.98026754e-07
 2.32689741e-09]
norm_grad_newton_reg/norm_grad_newton_reg[0]= [1.00000000e+00 4.05130565e-01 1.9
8552340e-01 8.31659306e-02
 2.34183056e-02 3.62935756e-03 2.10922743e-04 3.50603364e-06
 7.00324453e-09]
```

In [36]:

```
plt.figure()
plt.semilogy(norm_grad_newton_pch/norm_grad_newton_pch[0])
```

```
plt.semilogy(norm_grad_newton_reg/norm_grad_newton_reg[0])
plt.legend(['PSF', 'REG'])
plt.xlabel('Newton iteration')
plt.ylabel('||g||/||g0||')
plt.title('Congergence of preconditioned inexact Newton-CG')
```

Out[36]: Text(0.5, 1.0, 'Congergence of preconditioned inexact Newton-CG')

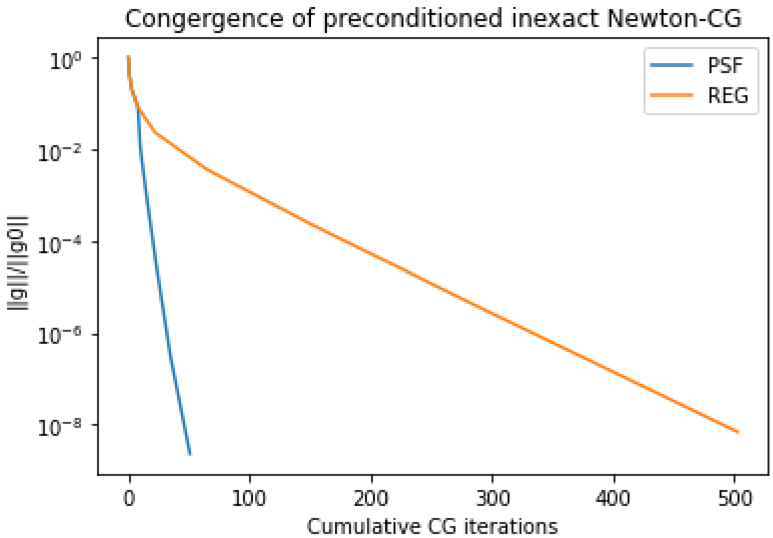


In [37]:

```
cumulative_cg_pch = np.concatenate([np.array([0.0]), np.cumsum(PCH_convergence_i
cumulative_cg_reg = np.concatenate([np.array([0.0]), np.cumsum(REG_convergence_i

print('cumulative_cg_pch=', cumulative_cg_pch)
print('cumulative_cg_reg=', cumulative_cg_reg)

plt.figure()
plt.semilogy(cumulative_cg_pch, norm_grad_newton_pch/norm_grad_newton_pch[0])
plt.semilogy(cumulative_cg_reg, norm_grad_newton_reg/norm_grad_newton_reg[0])
plt.legend(['PSF', 'REG'])
plt.xlabel('Cumulative CG iterations')
plt.ylabel('||g||/||g0||')
plt.title('Congergence of preconditioned inexact Newton-CG')
```

```
cumulative_cg_pch= [ 0.   1.   3.   8. 10. 15. 23. 35. 51.]
cumulative_cg_reg= [  0.    1.    3.    8.   22.   65. 154. 291. 503.]
```

Out[37]: Text(0.5, 1.0, 'Congergence of preconditioned inexact Newton-CG')



In [38]:

```
print(PCH_convergence_info.hess_matvecs)
print(PCH_convergence_info.cg_iter)
```

```
[1, 2, 5, 2, 5, 8, 12, 16, 0]
[1, 2, 5, 2, 5, 8, 12, 16, 0]
```

In [39]:

```
spatialdim = 2
num_moment_matvecs = 1 + spatialdim + spatialdim * (spatialdim + 1) / 2.0
num_preconditioner_build_matvecs = num_moment_matvecs + num_batches

print('num_moment_matvecs=', num_moment_matvecs)
print('num_preconditioner_build_matvecs=', num_preconditioner_build_matvecs)
```

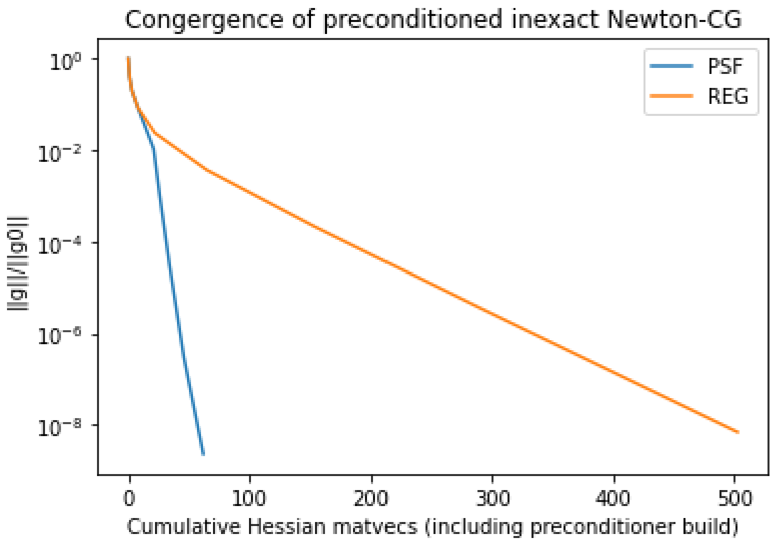
```
num_moment_matvecs= 6.0
num_preconditioner_build_matvecs= 11.0
```

In [40]:

```
cumulative_hessian_matvecs_psf = [0.0]
for nh, build in zip(PCH_convergence_info.hess_matvecs[:-1], PCH_convergence_in
    prev_matvecs = cumulative_hessian_matvecs_psf[-1]
    if build:
        new_matvecs = nh + num_preconditioner_build_matvecs
    else:
        new_matvecs = nh
    cumulative_hessian_matvecs_psf.append(prev_matvecs + new_matvecs)

cumulative_hessian_matvecs_psf = np.array(cumulative_hessian_matvecs_psf)
cumulative_hessian_matvecs_reg = cumulative_cg_reg
print('cumulative_hessian_matvecs_psf=', cumulative_hessian_matvecs_psf)
print('cumulative_hessian_matvecs_reg=', cumulative_hessian_matvecs_reg)

plt.figure()
plt.semilogy(cumulative_hessian_matvecs_psf, norm_grad_newton_pch/norm_grad_newt
plt.semilogy(cumulative_hessian_matvecs_reg, norm_grad_newton_reg/norm_grad_newt
plt.legend(['PSF', 'REG'])
plt.xlabel('Cumulative Hessian matvecs (including preconditioner build)')
plt.ylabel('||g||/||g0||')
plt.title('Congergence of preconditioned inexact Newton-CG')
```

```
cumulative_hessian_matvecs_psf= [ 0.   1.   3.   8. 21. 26. 34. 46. 62.]
cumulative_hessian_matvecs_reg= [  0.   1.   3.   8.  22.  65. 154. 291. 503.]
```

Out[40]: Text(0.5, 1.0, 'Congergence of preconditioned inexact Newton-CG')



In [65]:

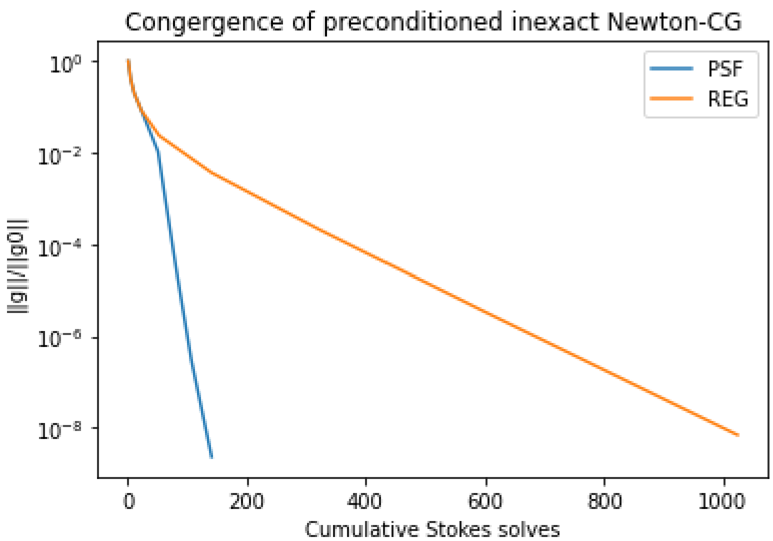
```
cumulative_stokes_psf = [PCH_convergence_info.cost_calls[0] + PCH_convergence_in
for k in range(1,len(PCH_convergence_info.newton_iter)):
    cs = cumulative_stokes_psf[-1]
    cs += PCH_convergence_info.cost_calls[k]
    cs += PCH_convergence_info.grad_calls[k]
    cs += 2*PCH_convergence_info.hess_matvecs[k-1] # incremental forward and inc
    if PCH_convergence_info.build_precond[k-1]:
        cs += 2*num_preconditioner_build_matvecs
    cumulative_stokes_psf.append(cs)

cumulative_stokes_psf = np.array(cumulative_stokes_psf, dtype=int)
print('cumulative_stokes_psf=', cumulative_stokes_psf)

cumulative_stokes_reg = [REG_convergence_info.cost_calls[0] + REG_convergence_in
for k in range(1,len(REG_convergence_info.newton_iter)):
    cs = cumulative_stokes_reg[-1]
    cs += REG_convergence_info.cost_calls[k]
    cs += REG_convergence_info.grad_calls[k]
    cs += 2*REG_convergence_info.hess_matvecs[k-1] # incremental forward and inc
    cumulative_stokes_reg.append(cs)

cumulative_stokes_reg = np.array(cumulative_stokes_reg)
print('cumulative_stokes_reg=', cumulative_stokes_reg)

relative_grad_psf = norm_grad_newton_pch/norm_grad_newton_pch[0]
relative_grad_reg = norm_grad_newton_reg/norm_grad_newton_reg[0]

print('relative_grad_psf=', relative_grad_psf)
print('relative_grad_reg=', relative_grad_reg)

plt.figure()
plt.semilogy(cumulative_stokes_psf, relative_grad_psf)
plt.semilogy(cumulative_stokes_reg, relative_grad_reg)
plt.legend(['PSF', 'REG'])
plt.xlabel('Cumulative Stokes solves')
plt.ylabel('||g||/||g0||')
plt.title('Congergence of preconditioned inexact Newton-CG')
```

```
cumulative_stokes_psf= [  2   6  12  24  52  64  82 108 142]
cumulative_stokes_reg= [   2    6   12   24   54  142  322  598 1024]
```

```
relative_grad_psf= [1.00000000e+00 4.05130565e-01 1.98552340e-01 8.31659306e-02
 1.05080086e-02 1.00339041e-03 3.19758878e-05 2.98026754e-07
 2.32689741e-09]
relative_grad_reg= [1.00000000e+00 4.05130565e-01 1.98552340e-01 8.31659306e-02
 2.34183056e-02 3.62935756e-03 2.10922743e-04 3.50603364e-06
 7.00324453e-09]
```

Out[65]:

```
Text(0.5, 1.0, 'Congergence of preconditioned inexact Newton-CG')
```



In [68]:

```
np.savetxt(save_dir_str + '/cumulative_cg_pch.txt', cumulative_cg_pch)
np.savetxt(save_dir_str + '/cumulative_cg_reg.txt', cumulative_cg_reg)
np.savetxt(save_dir_str + '/cumulative_hessian_matvecs_psf.txt', cumulative_hess
np.savetxt(save_dir_str + '/cumulative_hessian_matvecs_reg.txt', cumulative_hess
np.savetxt(save_dir_str + '/cumulative_stokes_psf.txt', cumulative_stokes_psf)
np.savetxt(save_dir_str + '/cumulative_stokes_reg.txt', cumulative_stokes_reg)
np.savetxt(save_dir_str + '/relative_grad_psf.txt', relative_grad_psf)
np.savetxt(save_dir_str + '/relative_grad_reg.txt', relative_grad_reg)
```

In [43]:

```
dl.File(save_dir_str + "/recovered_pressure" + str(outflow_constant)+".pvd") <<
dl.File(save_dir_str + "/recovered_velocity" + str(outflow_constant)+".pvd") <<

num_cg_newton_reg = np.array(REG_convergence_info.hess_matvecs)
num_stokes_newton_reg = (np.array(REG_convergence_info.cost_calls) +
                         np.array(REG_convergence_info.grad_calls) +
                         2*np.array(REG_convergence_info.hess_matvecs))

num_cg_newton_psf = np.array(PCH_convergence_info.hess_matvecs)
num_stokes_newton_psf = (np.array(PCH_convergence_info.cost_calls) +
                         np.array(PCH_convergence_info.grad_calls) +
                         2*np.array(PCH_convergence_info.hess_matvecs))

num_stokes_build_preconditioner = 2*num_preconditioner_build_matvecs*np.array(PC
num_stokes_build_preconditioner = np.array(num_stokes_build_preconditioner, dtyp


np.savetxt(save_dir_str + '/norm_grad_newton_reg.txt', norm_grad_newton_reg)
np.savetxt(save_dir_str + '/norm_grad_newton_psf.txt', norm_grad_newton_pch)
np.savetxt(save_dir_str + '/num_cg_newton_reg.txt', num_cg_newton_reg)
np.savetxt(save_dir_str + '/num_cg_newton_psf.txt', num_cg_newton_psf)
np.savetxt(save_dir_str + '/num_stokes_newton_reg.txt', num_stokes_newton_reg)
np.savetxt(save_dir_str + '/num_stokes_newton_psf.txt', num_stokes_newton_psf)
```

```
np.savetxt(save_dir_str + '/num_stokes_build_preconditioner.txt', num_stokes_bui

ps = " "*5 + "|" + " "*10 + "PSF" + " "*10 + "|" + " "*10 + "REG" + " "*10 + "\n
ps += "-"*5 + "|" + "-"*23 + "|" + "-"*23 + "\n"
ps += "{0:>4} | {1:<3}  {2:<7} {3:>9} | {4:<3}  {5:<7} {6:>9}\n".format(
            "Iter", "#CG", "#Stokes", "||g||", "#CG", "#Stokes", "||g||")
for k in range(len(norm_grad_newton_reg)):
    g_psf = norm_grad_newton_pch[k]
    g_reg = norm_grad_newton_reg[k]
    ncg_psf = num_cg_newton_psf[k]
    ncg_reg = num_cg_newton_reg[k]
    ns_psf = num_stokes_newton_psf[k]
    ns_reg = num_stokes_newton_reg[k]
    nbuild = num_stokes_build_preconditioner[k]

    ps += "{:>4}".format(k)
    ps += " | "
    ps += "{:<3d}".format(ncg_psf)
    ps += "  "
    if nbuild > 0:
        ps += "{:<7}".format(str(ns_psf) + " + " + str(int(nbuild)))
    else:
        ps += "{:<7d}".format(ns_psf)
    ps += " "
    ps += "{:>9}".format(np.format_float_scientific(g_psf, precision=2, trim='k'
    ps += " | "
    ps += "{:<3d}".format(ncg_reg)
    ps += "  "
    ps += "{:<7d}".format(ns_reg)
    ps += " "
    ps += "{:>9}".format(np.format_float_scientific(g_reg, precision=2, trim='k'
    ps += "\n"

print(ps)

num_cg_total_psf = np.sum(num_cg_newton_psf)
print('num_cg_total_psf=', num_cg_total_psf)

num_cg_total_reg = np.sum(num_cg_newton_reg)
print('num_cg_total_reg=', num_cg_total_reg)

num_stokes_total_psf = np.sum(num_stokes_newton_psf)
print('num_stokes_total_psf=', num_stokes_total_psf)

num_stokes_total_reg = np.sum(num_stokes_newton_reg)
print('num_stokes_total_reg=', num_stokes_total_reg)

# Iter  |  #CG (PSF)   #Stokes (PSF)    ||g|| (PSF) |  #CG (REG)   #Stokes (REG)
# 1     |  1           1234             1.95e+07    |  1           2345
# 2     |  2           1234             7.88e+06    |  2           2345
# 3     |  5           1234             3.86e+06    |  5           2345
# 4     |  2           1234 + 24        1.62e+06    |  14          2345
# ...   |  ...         ...              ...         |  ...         ...
# 8     |  15          1234             3.74e+01    |  210         2345
# 9     |  ---         2                1.26e-01    |  ---         2
```

```
     |          PSF          |          REG
-----|-----------------------|-----------------------
Iter | #CG  #Stokes     ||g|| | #CG  #Stokes     ||g||
   0 | 1    4        1.95e+07 | 1    4        1.95e+07
   1 | 2    6        7.91e+06 | 2    6        7.91e+06
```

```
  2 | 5    12       3.87e+06 | 5    12       3.87e+06
  3 | 2    6 + 22   1.62e+06 | 14   30       1.62e+06
  4 | 5    12       2.05e+05 | 43   88       4.57e+05
  5 | 8    18       1.96e+04 | 89   180      7.08e+04
  6 | 12   26       6.24e+02 | 137  276      4.12e+03
  7 | 16   34       5.82e+00 | 212  426      6.84e+01
  8 | 0    2        4.54e-02 | 0    2        1.37e-01

num_cg_total_psf= 51
num_cg_total_reg= 503
num_stokes_total_psf= 120
num_stokes_total_reg= 1024
```

In [44]:

```python
for k in range(len(m_iter_PCH)):
    mk_pch = dl.Function(Vh2)
    mk_pch.vector()[:] = m_iter_PCH[k]
    dl.File(save_dir_str + "/m" + str(k) + "_psf.pvd") << mk_pch

for k in range(len(m_iter_REG)):
    mk_reg = dl.Function(Vh2)
    mk_reg.vector()[:] = m_iter_REG[k]
    dl.File(save_dir_str + "/m" + str(k) + "_reg.pvd") << mk_reg
```

CG solve comparing PCH to REG to None preconditioning

In [45]:

```python
all_PCK = list()
all_Hd_pch = list()
for nb in all_num_batches:
    print('building PCH' + str(nb) + ' preconditioner')
    PCKx = ProductConvolutionKernel(Vh2, Vh2,
                                    apply_misfit_gauss_newton_hessian_petsc,
                                    apply_misfit_gauss_newton_hessian_petsc,
                                    nb, nb,
                                    tau_rows=tau, tau_cols=tau,
                                    num_neighbors_rows=num_neighbors,
                                    num_neighbors_cols=num_neighbors)
    Hd_pch_nonsymx, extrasx = make_hmatrix_from_kernel(PCKx, hmatrix_tol=hmatrix

    # ----- build spd approximation of Hd
    Hd_pchx = Hd_pch_nonsymx.spd()
    all_PCK.append(PCKx)
    all_Hd_pch.append(Hd_pchx)
```

```
building PCH1 preconditioner
Computing impulse response moments
getting spatially varying volume
getting spatially varying mean
getting spatially varying covariance
Preparing sample point batch stuff
Preparing c++ object
Building initial sample point batches

Making row and column cluster treesUsing ProductConvolutionKernelRBFColsOnly!

Making block cluster trees
```

```
Building A kernel hmatrix
━━ building H-matrix ( tol = Making input and output mass matrix hmatrices
Computing A_hmatrix = M_out_hmatrix * A_kernel_hmatrix * M_in_hmatrix
1e-06 )
 making hmatrix spd                      ]   7% ETA 13 s (5.30 GB)
symmetrizing
getting smallest eigenvalue with Lanczos
    done in 13.55s
    size of H-matrix = 4.42 MB
lambda_min= -21939.888935957897
scaling_at_lambda_min= 0.9878048780487805
scaling_at_zero= 0.5
Setting up operator T = (2*A - (b+a) I) / (b-a)
computing T^(2^k)
computing T^(2^1) = T^(2^0) * T^(2^0)

computing T^(2^2) = T^(2^1) * T^(2^1)
computing negative spectral projector Pi_minus = I / (I + T^(2^k))
━━ H-matrix multiplication C=A*B
[====================== )
    done in  24.047887325286865
    done in 32.00s                                              5.30 GB)
    size of C = 5.37 MB

━━ H-matrix multiplication C=A*B
    done in 33.39s
    size of C = 8.05 MB
 bytes

computing A_plus = Pi * A

━━ H-matrix multiplication C=A*B
[==-                                  building PCH5 preconditioner
                                                                Computing impul
se response moments
getting spatially varying volume
    done in 38.49s
    size of C = 21.30 MB
getting spatially varying mean
getting spatially varying covariance
Preparing sample point batch stuff
Preparing c++ object
Building initial sample point batches
```

```
Making row and column cluster treesUsing ProductConvolutionKernelRBFColsOnly!
Making block cluster trees
Building A kernel hmatrix
Making input and output mass matrix hmatrices
━━ building H-matrix ( tol = 1e-06 )
[==-
Computing A_hmatrix = M_out_hmatrix * A_kernel_hmatrix * M_in_hmatrix
                                         ]  14% ETA 12 s (5.35 GB)
symmetrizing
[========-
    done in 14.31s
    size of H-matrix = 4.42 MB
lambda_min= -31571.25915903734
scaling_at_lambda_min= 0.9878048780487805
scaling_at_zero= 0.5
Setting up operator T = (2*A - (b+a) I) / (b-a)
```

```
computing T^(2^k)
computing T^(2^1) = T^(2^0) * T^(2^0)

━━ H-matrix multiplication C=A*B
                                         ]   7% ETA 22 s (5.35 GB)computing T^(2
^2) = T^(2^1) * T^(2^1)
    done in 32.73s                                               k))====
]  15% ETA 22 s (5.35 GB)
    size of C = 5.28 MB

━━ H-matrix multiplication C=A*B
    done in 34.24s
    size of C = 7.69 MB

━━ H-matrix inverse ( rtol =  1e-07 , atol =  1e-12 , overwrite= False  )
    done in  24.184748649597168
    size of inverse =  17725073  bytes

computing A_plus = Pi * A

━━ H-matrix multiplication C=A*B
    done in 39.20s
    size of C = 20.57 MB
building PCH25 preconditioner
Computing impulse response moments
getting spatially varying volume
getting spatially varying mean
getting spatially varying covariance
Preparing sample point batch stuff
Preparing c++ object
Building initial sample point batches

Making row and column cluster treesUsing ProductConvolutionKernelRBFColsOnly!

Making block cluster trees
Building A kernel hmatrix
Making input and output mass matrix hmatrices
━━ building H-matrix ( tol = 1e-06 )
[==-                               Computing A_hmatrix = M_out_hmatrix * A_kerne
l_hmatrix * M_in_hmatrix
                              making hmatrix spd
[========-

    done in 14.98s
    size of H-matrix = 4.42 MB
lambda_min= -32200.061569526715
scaling_at_lambda_min= 0.9878048780487805
scaling_at_zero= 0.5
Setting up operator T = (2*A - (b+a) I) / (b-a)
computing T^(2^k)
computing T^(2^1) = T^(2^0) * T^(2^0)
computing T^(2^2) = T^(2^1) * T^(2^1)

━━ H-matrix multiplication C=A*B
    done in 32.94s
    size of C = 5.09 MB

━━ H-matrix multiplication C=A*B
[================                    computing negative spectral projector Pi_mi
nus = I / (I + T^(2^k))
    done in 34.64s
```

```
    size of C = 7.47 MB
━━ H-matrix inverse ( rtol =  1e-07 , atol =  1e-12 , overwrite= False  )
[                                        ]   0%  (5.38 GB)    done in  26.092469
215393066
    size of inverse =  17345569  bytes
computing absolute value projector Pi = I - 2*Pi_minus
computing A_plus = Pi * A

━━ H-matrix multiplication C=A*B
    done in 39.24s
    size of C = 20.28 MB
```

In [46]:

```python
REG.update_gamma(gamma_morozov)

# Rebuild reg hmatrix with same block cluster tree as PCH data misfit hmatrix
print('Building Regularization H-Matrix')
R_hmatrix = REG.make_R_hmatrix(all_Hd_pch[0].bct, rtol=1e-6)

all_H_pch = [Hd + R_hmatrix for Hd in all_Hd_pch]
all_preconditioners = [H.factorized_inverse() for H in all_H_pch]
```

```
/home/nick/anaconda3/envs/fenics4/lib/python3.9/site-packages/scipy/sparse/linal
g/dsolve/linsolve.py:318: SparseEfficiencyWarning: splu requires CSC matrix form
at
  warn('splu requires CSC matrix format', SparseEfficiencyWarning)
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 1.1247e+06
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 1.1247e+06
 |S-A|_2 = 3.13924e-09
  matrix has dimension ━━ H-matrix inverse ( rtol =  1e-06 , atol =  1e-14 , ove
rwrite= False  )
1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
    done in  15.561947107315063
    size of inverse =  3361874  bytes

━━ H-matrix multiplication C=A*B
    done in 25.59s
    size of C = 3.88 MB

━━ H-matrix multiplication C=A*B
    done in 35.28s
    size of C = 8.54 MB
```

```
━━ LU factorisation ( rtol = 1e-07 )
   done in 34.12s                                           B)
   size of LU factor = 23.06 MB

━━ LU factorisation ( rtol = 1e-07 )
   done in 33.98s
   size of LU factor = 22.16 MB

━━ LU factorisation ( rtol = 1e-07 )
   done in 34.10s
   size of LU factor = 21.89 MB
```

In [50]:

```
def cg_wrapper(A_linop, b, M_linop, tol=1e-10, x_true=None):
    if x_true is None:
        x_true = np.nan * np.zeros(b.shape)
    all_relres = list()
    all_err = list()

    ii_list = [0]
    def callback(xk):
        relres = np.linalg.norm(b - A_linop.matvec(xk)) / np.linalg.norm(b)
        all_relres.append(relres)
        err = np.linalg.norm(xk - x_true) / np.linalg.norm(x_true)
        all_err.append(err)
        print('ii=', ii_list[0], ', relres=', relres, ', err=', err)
        ii_list[0] += 1

    x, info = spla.cg(A_linop, b, M=M_linop, tol=tol, callback=callback, maxiter
    all_relres = np.array(all_relres)
    all_err = np.array(all_err)
    return x, info, all_relres, all_err
```

In [48]:

```
H_linop = spla.LinearOperator((Vh2.dim(), Vh2.dim()), matvec=apply_hessian)
Hgn_linop = spla.LinearOperator((Vh2.dim(), Vh2.dim()), matvec=apply_gauss_newto
ireg_linop = spla.LinearOperator((Vh2.dim(), Vh2.dim()), matvec=REG.solve_hessia
all_p_linop = [spla.LinearOperator((Vh2.dim(), Vh2.dim()), matvec=p.matvec) for

b = np.random.randn(Vh2.dim())

print('GN: get truth:')
x_true_gn, _, _, _ = cg_wrapper(Hgn_linop, b, all_p_linop[-1])

all_relres_gn = list()
all_errs_gn = list()
for nb, p_linop in zip(all_num_batches, all_p_linop):
    print('GN: psf'+str(nb))
    _, _, relres_gn, errs_gn = cg_wrapper(Hgn_linop, b, p_linop, x_true=x_true_g
    all_relres_gn.append(relres_gn)
    all_errs_gn.append(errs_gn)

print('GN: reg')
_, _, relres_gn_reg, errs_gn_reg = cg_wrapper(Hgn_linop, b, ireg_linop, x_true=x

print('GN: none')
_, _, relres_gn_none, errs_gn_none = cg_wrapper(Hgn_linop, b, None, x_true=x_tru

plt.semilogy(errs_gn_none[:-1])
plt.semilogy(errs_gn_reg[:-1])
```

```python
for errs_gn in all_errs_gn:
    plt.semilogy(errs_gn[:-1])
plt.title('Error vs. Krylov iteration: Gauss-Newton')
plt.xlabel('Iteration k')
plt.ylabel('relative error')
plt.legend(['None', 'REG'] + ['PSF'+str(nb) for nb in all_num_batches])

plt.figure()
plt.semilogy(relres_gn_none)
plt.semilogy(relres_gn_reg)
for relres_gn in all_relres_gn:
    plt.semilogy(relres_gn)
plt.title('Residual vs. Krylov iteration: Gauss-Newton')
plt.xlabel('Iteration k')
plt.ylabel('relative residual')
plt.legend(['None', 'REG'] + ['PSF'+str(nb) for nb in all_num_batches])
```

```
GN: get truth:
ii= 0 , relres= 0.3176683818809919 , err= nan
ii= 1 , relres= 0.12219419583314826 , err= nan
ii= 2 , relres= 0.05929028993565218 , err= nan
ii= 3 , relres= 0.029223360826238765 , err= nan
ii= 4 , relres= 0.014927410112437286 , err= nan
ii= 5 , relres= 0.008249646977940985 , err= nan
ii= 6 , relres= 0.004492700717803115 , err= nan
ii= 7 , relres= 0.002434905698076133 , err= nan
ii= 8 , relres= 0.0013472210970393488 , err= nan
ii= 9 , relres= 0.0006446787123482165 , err= nan
ii= 10 , relres= 0.000331846225939158 , err= nan
ii= 11 , relres= 0.00015445681454121193 , err= nan
ii= 12 , relres= 7.087694748843961e-05 , err= nan
ii= 13 , relres= 3.473889744332784e-05 , err= nan
ii= 14 , relres= 1.4946417517103958e-05 , err= nan
ii= 15 , relres= 7.4650943849711214e-06 , err= nan
ii= 16 , relres= 3.6027466418852616e-06 , err= nan
ii= 17 , relres= 1.6374987115869782e-06 , err= nan
ii= 18 , relres= 7.260859931916766e-07 , err= nan
ii= 19 , relres= 3.1071291058666855e-07 , err= nan
ii= 20 , relres= 1.362927040644032e-07 , err= nan
ii= 21 , relres= 5.8710800599917436e-08 , err= nan
ii= 22 , relres= 2.4995763683535774e-08 , err= nan
ii= 23 , relres= 1.171649291665167e-08 , err= nan
ii= 24 , relres= 4.8960399468540766e-09 , err= nan
ii= 25 , relres= 1.8426316010750442e-09 , err= nan
ii= 26 , relres= 6.992902245448452e-10 , err= nan
ii= 27 , relres= 2.8468496756831994e-10 , err= nan
ii= 28 , relres= 1.2942732288445515e-10 , err= nan
ii= 29 , relres= 6.771541589629453e-11 , err= nan
GN: psf1
ii= 0 , relres= 0.4303236626802175 , err= 0.2717020210717325
ii= 1 , relres= 0.2555265124959215 , err= 0.12252313046023418
ii= 2 , relres= 0.14672933088858342 , err= 0.06775500624745748
ii= 3 , relres= 0.09387532650646126 , err= 0.03694507657155087
ii= 4 , relres= 0.05749066166412285 , err= 0.024681849207210088
ii= 5 , relres= 0.03536503325229412 , err= 0.01414281762348893
ii= 6 , relres= 0.023291299659775777 , err= 0.008928124218214066
ii= 7 , relres= 0.014891503622148458 , err= 0.005704552293839755
ii= 8 , relres= 0.009680569051942728 , err= 0.003412379371716328
ii= 9 , relres= 0.00620422379816278 , err= 0.00216622079075269
ii= 10 , relres= 0.004020462044569462 , err= 0.0012934516287395888
ii= 11 , relres= 0.002424874462499395 , err= 0.0007960616330352545
```

```
ii= 12 , relres= 0.0014979762685737301 , err= 0.0004660584158228963
ii= 13 , relres= 0.0008939464212390416 , err= 0.00029481167226135386
ii= 14 , relres= 0.0005391273464371393 , err= 0.0001858644088597927
ii= 15 , relres= 0.0003422553740079775 , err= 0.00011192514957659832
ii= 16 , relres= 0.00021207179816944643 , err= 7.699028415455345e-05
ii= 17 , relres= 0.00013239873720282807 , err= 4.773709776935418e-05
ii= 18 , relres= 8.228939438767952e-05 , err= 2.8363142281393274e-05
ii= 19 , relres= 4.769484675429377e-05 , err= 1.8083419494714644e-05
ii= 20 , relres= 2.9204278233260142e-05 , err= 9.04493560396812e-06
ii= 21 , relres= 1.7110497475221808e-05 , err= 6.115674690108275e-06
ii= 22 , relres= 9.650734568833551e-06 , err= 3.4067583315337225e-06
ii= 23 , relres= 5.8191751717731e-06 , err= 2.1940663286548443e-06
ii= 24 , relres= 3.5687023738934375e-06 , err= 1.3673827622071813e-06
ii= 25 , relres= 2.2087218797760863e-06 , err= 8.100138806098674e-07
ii= 26 , relres= 1.422394049685037e-06 , err= 4.739309811416228e-07
ii= 27 , relres= 8.743639694047915e-07 , err= 3.026299399956638e-07
ii= 28 , relres= 5.214805395438914e-07 , err= 1.6665609245351856e-07
ii= 29 , relres= 3.012419089327925e-07 , err= 1.0535178492941126e-07
ii= 30 , relres= 1.6027321874843957e-07 , err= 5.932240133183649e-08
ii= 31 , relres= 9.474400335824318e-08 , err= 2.979386630636426e-08
ii= 32 , relres= 5.593200567609568e-08 , err= 1.9232650037924314e-08
ii= 33 , relres= 3.091973074006108e-08 , err= 1.0578363647943844e-08
ii= 34 , relres= 1.7951443503208012e-08 , err= 5.890671617420606e-09
ii= 35 , relres= 1.0096109499473212e-08 , err= 3.640301706989164e-09
ii= 36 , relres= 6.5552737332496135e-09 , err= 2.1331008329346033e-09
ii= 37 , relres= 4.322184530407875e-09 , err= 1.4591763262835361e-09
ii= 38 , relres= 2.6846461348090248e-09 , err= 8.968458265824674e-10
ii= 39 , relres= 1.728313525691744e-09 , err= 5.355869909418848e-10
ii= 40 , relres= 9.221454397331823e-10 , err= 3.141224497095239e-10
ii= 41 , relres= 4.756205669793359e-10 , err= 1.615457063718629e-10
ii= 42 , relres= 2.70531236399651e-10 , err= 9.686271284055208e-11
ii= 43 , relres= 1.5650947281481934e-10 , err= 5.4713383236149084e-11
ii= 44 , relres= 1.041729472145198e-10 , err= 3.2004823493610865e-11
ii= 45 , relres= 6.70098045127641e-11 , err= 2.5165472434190912e-11
GN: psf5
ii= 0 , relres= 0.4011092431371095 , err= 0.2893812407776159
ii= 1 , relres= 0.1890284813593398 , err= 0.08535446932302092
ii= 2 , relres= 0.12482077844273369 , err= 0.04542358676473394
ii= 3 , relres= 0.06941615825608709 , err= 0.02990196295505023
ii= 4 , relres= 0.03932048727482909 , err= 0.015404818615934344
ii= 5 , relres= 0.027527567593108457 , err= 0.008502118209189155
ii= 6 , relres= 0.0160073765645773 , err= 0.005653201176890141
ii= 7 , relres= 0.009806627424953349 , err= 0.003599797836124551
ii= 8 , relres= 0.006037427746509738 , err= 0.0021424523963990126
ii= 9 , relres= 0.0036193142095174236 , err= 0.0013113638392699824
ii= 10 , relres= 0.002023761045460334 , err= 0.0006979901951075163
ii= 11 , relres= 0.001198432211879224 , err= 0.00036648046258855153
ii= 12 , relres= 0.0006798127403473902 , err= 0.0002361519328738246
ii= 13 , relres= 0.0003640274038694264 , err= 0.00013082968349142782
ii= 14 , relres= 0.00020271523448636575 , err= 7.364012682890258e-05
ii= 15 , relres= 0.00011239780290271259 , err= 4.1419863855806154e-05
ii= 16 , relres= 6.280887143496206e-05 , err= 2.2459431228097592e-05
ii= 17 , relres= 3.68870363205681e-05 , err= 1.2263878753017295e-05
ii= 18 , relres= 2.0995643739567486e-05 , err= 7.337221822804505e-06
ii= 19 , relres= 1.2642029051025374e-05 , err= 4.217530843548029e-06
ii= 20 , relres= 6.854541814701528e-06 , err= 2.410726536395615e-06
ii= 21 , relres= 3.698805873227959e-06 , err= 1.224803109954489e-06
ii= 22 , relres= 1.818896904636552e-06 , err= 6.408609687011261e-07
ii= 23 , relres= 9.665566419339014e-07 , err= 3.186821215252854e-07
ii= 24 , relres= 4.930141582176686e-07 , err= 1.6375237832038025e-07
ii= 25 , relres= 3.061142801516709e-07 , err= 9.010701855263016e-08
```

```
ii= 26 , relres= 1.9875157574325685e-07 , err= 5.5313044229807776e-08
ii= 27 , relres= 1.0521116151025253e-07 , err= 3.6003033108737843e-08
ii= 28 , relres= 6.249452662583066e-08 , err= 1.921702096831036e-08
ii= 29 , relres= 3.690709220219193e-08 , err= 1.1231712990827585e-08
ii= 30 , relres= 2.154831454077728e-08 , err= 6.555872804673522e-09
ii= 31 , relres= 1.0689509895331447e-08 , err= 3.6802332309686628e-09
ii= 32 , relres= 5.829928125485472e-09 , err= 1.994203821391325e-09
ii= 33 , relres= 2.967362986062361e-09 , err= 1.0111126848797098e-09
ii= 34 , relres= 1.5708865094066604e-09 , err= 5.374300370604884e-10
ii= 35 , relres= 9.079545077723813e-10 , err= 2.895145668323176e-10
ii= 36 , relres= 5.559844088915752e-10 , err= 1.6433358850787509e-10
ii= 37 , relres= 3.0652229575559607e-10 , err= 8.895060199317302e-11
ii= 38 , relres= 1.727363804489731e-10 , err= 5.350021885712507e-11
ii= 39 , relres= 1.3446967290053136e-10 , err= 3.283248335942117e-11
ii= 40 , relres= 5.558307064708 99e-11 , err= 2.4867307832569578e-11
GN: psf25
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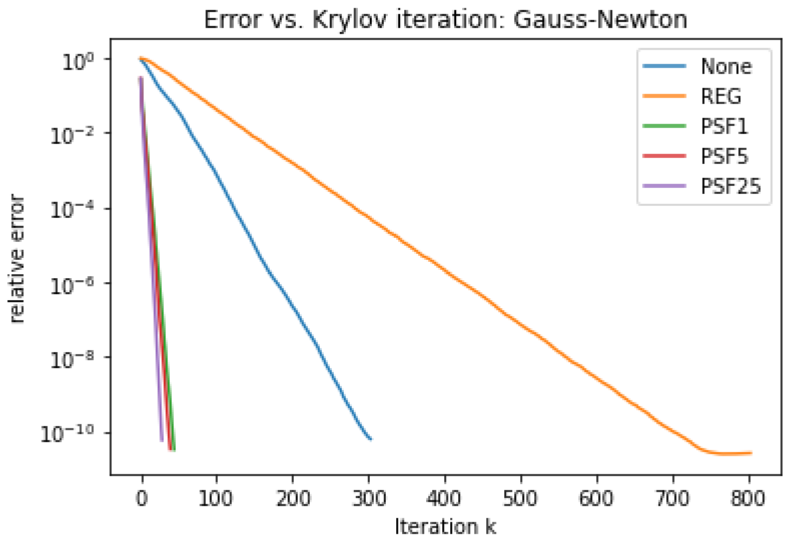
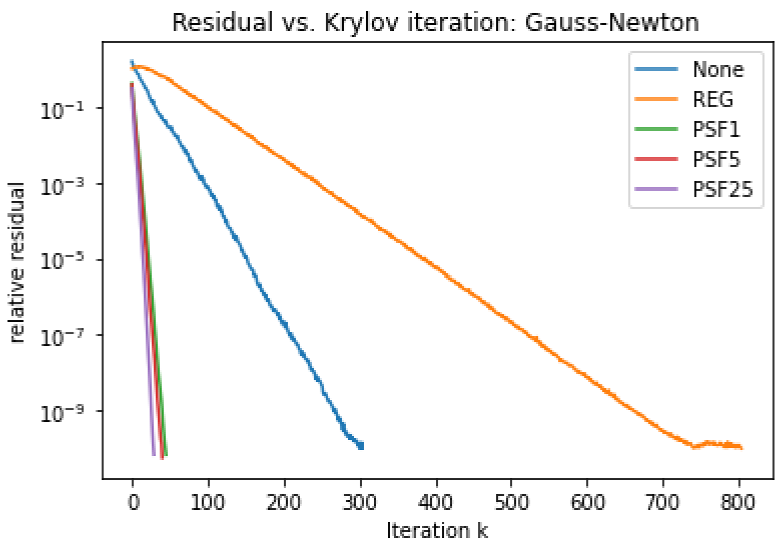
Out[48]:





In [51]:

```
print('FULL: get truth:')
x_true, _, _, _ = cg_wrapper(H_linop, b, all_p_linop[-1])

all_relres = list()
all_errs = list()
for nb, p_linop in zip(all_num_batches, all_p_linop):
    print('FULL: psf'+str(nb))
    _, _, relres, errs = cg_wrapper(H_linop, b, p_linop, x_true=x_true)
    all_relres.append(relres)
    all_errs.append(errs)

print('FULL: reg')
_, _, all_relres_reg, all_err_reg = cg_wrapper(H_linop, b, ireg_linop, x_true=x_

print('FULL: none')
_, _, all_relres_none, all_err_none = cg_wrapper(H_linop, b, None, x_true=x_true

plt.semilogy(all_err_none[:-1])
plt.semilogy(all_err_reg[:-1])
for errs in all_errs:
    plt.semilogy(errs[:-1])
plt.title('Error vs. Krylov iteration: Full Hessian')
plt.xlabel('Iteration k')
```

```
plt.ylabel('relative error')
plt.legend(['None', 'REG'] + ['PSF'+str(nb) for nb in all_num_batches])

plt.figure()
plt.semilogy(all_relres_none[:-1])
plt.semilogy(all_relres_reg[:-1])
for relres in all_relres:
    plt.semilogy(relres)
plt.title('Residual vs. Krylov iteration: Full Hessian')
plt.xlabel('Iteration k')
plt.ylabel('relative residual')
plt.legend(['None', 'REG'] + ['PSF'+str(nb) for nb in all_num_batches])
```

```
FULL: get truth:
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Out[51]:

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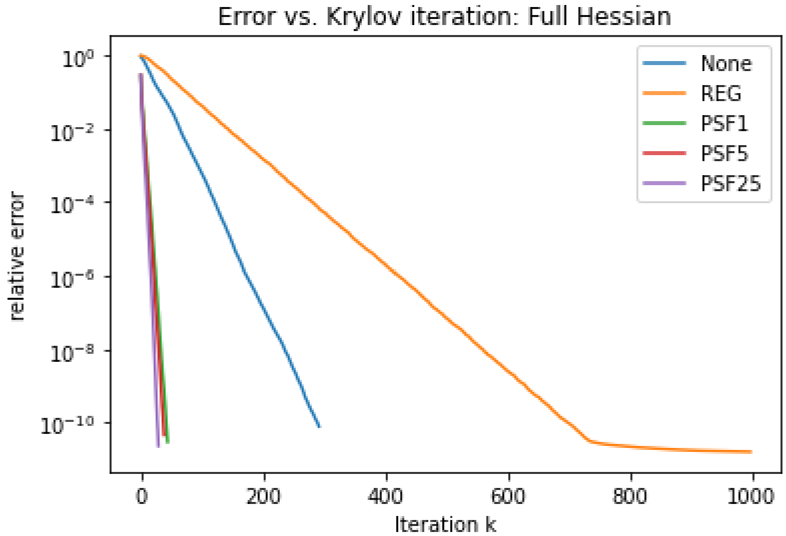

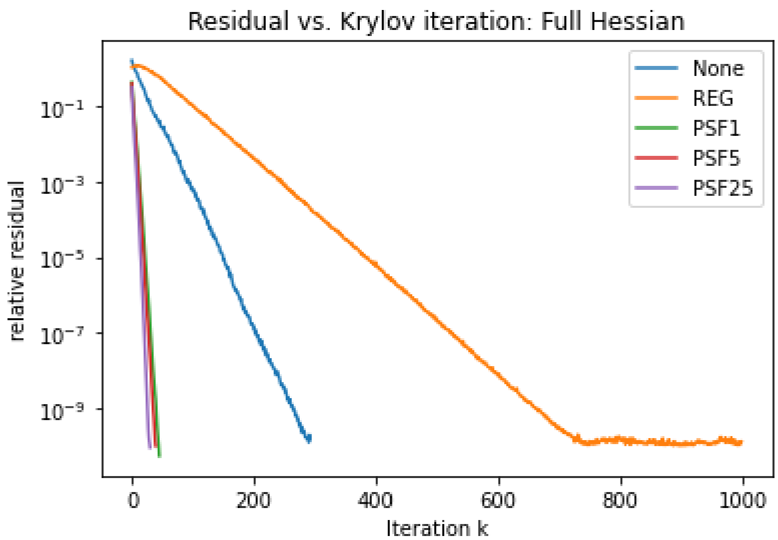

Plots to make (5):

- CG convergence (Full Newton, full page width, residual)

In [74]:
```
np.savetxt(save_dir_str + '/cg_relres_none.txt', all_relres_none)
np.savetxt(save_dir_str + '/cg_relres_reg.txt', all_relres_reg)
for nb, relres in zip(all_num_batches, all_relres):
    np.savetxt(save_dir_str + '/cg_relres_p' + str(nb) + '.txt', relres)

np.savetxt(save_dir_str + '/cg_relres_none_gn.txt', relres_gn_none)
np.savetxt(save_dir_str + '/cg_relres_reg_gn.txt', relres_gn_reg)
for nb, relres_gn in zip(all_num_batches, all_relres_gn):
    np.savetxt(save_dir_str + '/cg_relres_p' + str(nb) + '_gn.txt', relres_gn)

print('all_relres_none=', all_relres_none)
print('all_relres_reg=', all_relres_reg)
print('all_relres=', all_relres)

print('relres_gn_none=', relres_gn_none)
print('relres_gn_reg=', relres_gn_reg)
print('all_relres_gn=', all_relres_gn)
```

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 7.56576038e-01 7.37747503e-01 5.96015906e-01 5.65079336e-01
 5.34023057e-01 4.81020978e-01 4.55647309e-01 4.07719440e-01
 3.88506260e-01 3.35173522e-01 3.36514524e-01 2.96578704e-01
 2.86489100e-01 2.56040122e-01 2.16579802e-01 1.96831555e-01
 1.71185648e-01 1.56045391e-01 1.51715165e-01 1.30913947e-01
 1.48693674e-01 1.18372875e-01 1.08275366e-01 1.00746306e-01
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 6.35378592e-02 6.10063288e-02 5.51065820e-02 5.69459369e-02
 5.13610185e-02 4.45903834e-02 4.31607225e-02 4.18811104e-02
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```

Preconditioned spectrum plots

In [53]:
```
all_H_pch_dense = list()
for nb, H in zip(all_num_batches, all_H_pch):
    print('building p' + str(nb) + '_dense')
    all_H_pch_dense.append(build_dense_matrix_from_matvecs(H.matvec, Vh2.dim()))
```

```
building p1_dense

building p5_dense

building p25_dense
```

In [54]:
```
print('building reg_dense')
reg_dense = build_dense_matrix_from_matvecs(REG.apply_R_numpy, Vh2.dim())
```

```
building reg_dense
```

In [55]:
```
print('building H_dense')
H_dense = build_dense_matrix_from_matvecs(H_linop.matvec, Vh2.dim())

print('building Hgn_dense')
Hgn_dense = build_dense_matrix_from_matvecs(Hgn_linop.matvec, Vh2.dim())
```

```
building H_dense

building Hgn_dense
```

In [56]:
```
all_ee_gn = list()
all_P_gn = list()
for nb, Hpch_dense in zip(all_num_batches, all_H_pch_dense):
    print('computing ee' + str(nb) + '_gn')
    ee_gn, P_gn = sla.eigh(Hgn_dense, Hpch_dense)
    all_ee_gn.append(ee_gn)
    all_P_gn.append(P_gn)

print('computing eeR_gn')
eeR_gn, PR_gn = sla.eigh(Hgn_dense, reg_dense)
print('computing ee_none_gn')
ee_none_gn, P_none_gn = sla.eigh(Hgn_dense)

all_cond_gn = list()
for ee_gn in all_ee_gn:
    all_cond_gn.append(np.max(ee_gn) / np.min(ee_gn))
condR_gn = np.max(eeR_gn) / np.min(eeR_gn)
cond_none_gn = np.max(ee_none_gn) / np.min(ee_none_gn)

for nb, cond_gn in zip(all_num_batches, all_cond_gn):
    print('cond' + str(nb) + '_gn=', cond_gn)

print('condR_gn=', condR_gn)
print('cond_none_gn=', cond_none_gn)
```

```
computing ee1_gn
computing ee5_gn
computing ee25_gn
computing eeR_gn
computing ee_none_gn
cond1_gn= 19.09179376713973
```

```
cond5_gn= 17.80228689923624
cond25_gn= 8.932119380135207
condR_gn= 7640.964640412505
cond_none_gn= 972.1505567784867
```

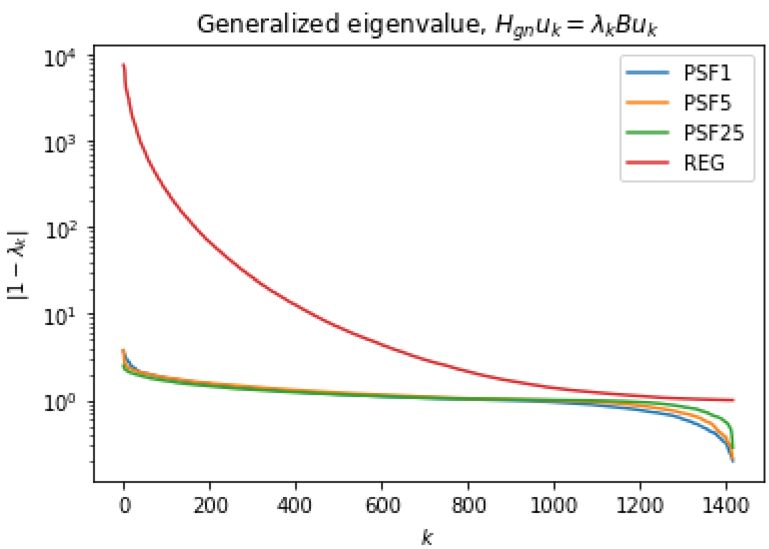
In [57]:

```
plt.figure()
for ee_gn in all_ee_gn:
#     plt.semilogy(np.sort(np.abs(1.0 - ee_gn))[::-1])
    plt.semilogy(np.sort(ee_gn)[::-1])

# plt.semilogy(np.sort(np.abs(1.0 - eeR_gn))[::-1])
plt.semilogy(np.sort(eeR_gn)[::-1])
plt.legend(['PSF' + str(nb) for nb in all_num_batches] + ['REG', 'None'])
plt.xlabel(r'$k$')
plt.ylabel(r'$|1 - \lambda_k|$')
plt.title(r'Generalized eigenvalue, $H_{gn} u_k = \lambda_k B u_k$')
```

Out[57]:

```
Text(0.5, 1.0, 'Generalized eigenvalue, $H_{gn} u_k = \\lambda_k B u_k$')
```



In [58]:

```
all_ee = list()
all_P = list()
for nb, Hpch_dense in zip(all_num_batches, all_H_pch_dense):
    print('computing ee' + str(nb))
    ee, P = sla.eigh(H_dense, Hpch_dense)
    all_ee.append(ee)
    all_P.append(P)

print('computing eeR')
eeR, PR = sla.eigh(H_dense, reg_dense)
print('computing ee_none')
ee_none, P_none = sla.eigh(H_dense)

all_cond = list()
for ee in all_ee:
    all_cond.append(np.max(ee) / np.min(ee))
condR = np.max(eeR) / np.min(eeR)
cond_none = np.max(ee_none) / np.min(ee_none)

for nb, cond in zip(all_num_batches, all_cond):
    print('cond' + str(nb) + '=', cond)
```

```
print('condR=', condR)
print('cond_none=', cond_none)
```

```
computing ee1
computing ee5
computing ee25
computing eeR
computing ee_none
cond1= 19.00142342245143
cond5= 17.795941453773036
cond25= 8.889097756657046
condR= 7640.986530049078
cond_none= 917.6858215976631
```

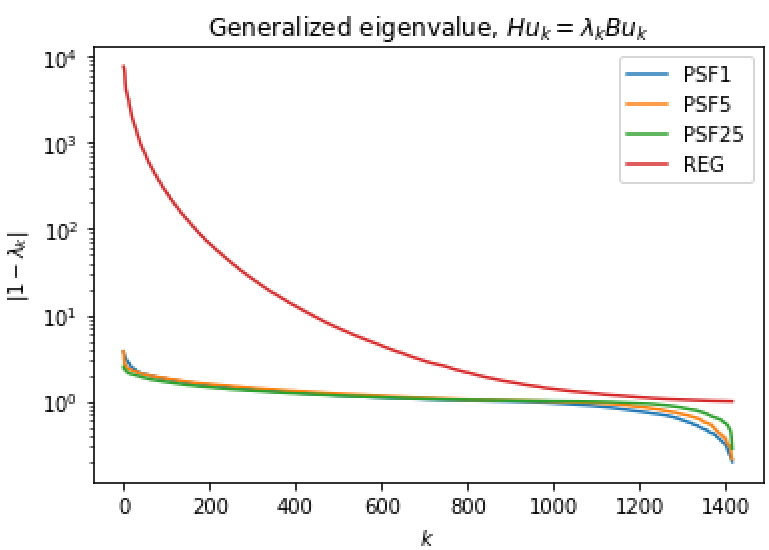
In [59]:

```
plt.figure()
for ee in all_ee:
#     plt.semilogy(np.sort(np.abs(1.0 - ee))[::-1])
    plt.semilogy(np.sort(ee)[::-1])

# plt.semilogy(np.sort(np.abs(1.0 - eeR))[::-1])
plt.semilogy(np.sort(eeR)[::-1])
plt.legend(['PSF' + str(nb) for nb in all_num_batches] + ['REG', 'None'])
plt.xlabel(r'$k$')
plt.ylabel(r'$|1 - \lambda_k|$')
plt.title(r'Generalized eigenvalue, $H u_k = \lambda_k B u_k$')
```

Out[59]:

```
Text(0.5, 1.0, 'Generalized eigenvalue, $H u_k = \\lambda_k B u_k$')
```



Plots to make (6)

- Preconditioned spectrum (Full newton)
- Condition number table (Gauss-Newton and full Newton

In [61]:

```
for nb, ee in zip(all_num_batches, all_ee):
    np.savetxt(save_dir_str + '/generalized_eigenvalues_psf' + str(nb) + '.txt',
np.savetxt(save_dir_str + '/generalized_eigenvalues_reg.txt', eeR)

for nb, ee_gn in zip(all_num_batches, all_ee_gn):
```

```
    np.savetxt(save_dir_str + '/generalized_eigenvalues_psf' + str(nb) + '_gn.tx
np.savetxt(save_dir_str + '/generalized_eigenvalues_reg_gn.txt', eeR_gn)

np.savetxt(save_dir_str + '/all_num_batches.txt', all_num_batches)
np.savetxt(save_dir_str + '/cond_psf.txt', all_cond)
np.savetxt(save_dir_str + '/condR.txt', np.array([condR]))
np.savetxt(save_dir_str + '/cond_none.txt', np.array([cond_none]))

np.savetxt(save_dir_str + '/cond_psf_gn.txt', all_cond_gn)
np.savetxt(save_dir_str + '/condR_gn.txt', np.array([condR_gn]))
np.savetxt(save_dir_str + '/cond_none_gn.txt', np.array([cond_none_gn]))

ps = " "*14 + " | " + "{:>8}".format("None") + " " + "{:>8}".format("REG") + " '
for nb in all_num_batches:
    ps += "{:>8}".format("PSF" + str(nb)) + " "
ps += "\n"
ps += "-"*(34 + 9*len(all_num_batches)) + "\n"
ps += "cond(P^-1 H)   | "
ps += "{:>8}".format(np.format_float_scientific(cond_none, precision=2, trim='k'
ps += "{:>8}".format(np.format_float_scientific(condR, precision=2, trim='k', ur
for c in all_cond:
    ps += "{:>8}".format(np.format_float_scientific(c, precision=2, trim='k', ur
ps += "\n"
ps += "cond(P^-1 Hgn) | "
ps += "{:>8}".format(np.format_float_scientific(cond_none_gn, precision=2, trim=
ps += "{:>8}".format(np.format_float_scientific(condR_gn, precision=2, trim='k',
for c_gn in all_cond_gn:
    ps += "{:>8}".format(np.format_float_scientific(c_gn, precision=2, trim='k',
ps += "\n"

print(ps)

#     ps += "{:>9}".format(np.format_float_scientific(g_psf, precision=2, trim='
#     ps += " | "
#     ps += "{:<3d}".format(ncg_reg)

#            |  None    Reg   PSF3   PSF6   PSF9
#---------------------------------------------------
# P^-1 H     |  923.5   7762  15.59  14.16  13.96
# P^-1 Hgn   |  978.0   7760  15.57  14.12  13.98
```

```
               |     None      REG     PSF1     PSF5    PSF25
-------------------------------------------------------------
cond(P^-1 H)   | 9.18e+02 7.64e+03 1.90e+01 1.78e+01 8.89e+00
cond(P^-1 Hgn) | 9.72e+02 7.64e+03 1.91e+01 1.78e+01 8.93e+00
```

In [75]:

```
print('all_ee=', all_ee)
print('eeR', eeR)

print('all_ee_gn=', all_ee_gn)
print('eeR_gn', eeR_gn)

print('all_num_batches=', all_num_batches)
```

```
all_ee= [array([0.19842947, 0.20892042, 0.21550906, ..., 3.52461612, 3.54900351,
       3.77044232]), array([0.21340901, 0.21487324, 0.23179099, ..., 2.76840479,
3.14229576,
       3.79781422]), array([0.28277853, 0.32940431, 0.38604362, ..., 2.36111911,
2.36561666,
```

```
       2.51364603])]
eeR [1.00058127e+00 1.00070518e+00 1.00103769e+00 ... 7.03456055e+03
 7.25709269e+03 7.64542797e+03]
all_ee_gn= [array([0.19752471, 0.20523518, 0.21515482, ..., 3.47864946, 3.545046
03,
       3.77110096]), array([0.21266263, 0.21388609, 0.23161424, ..., 2.76801111,
3.13049279,
       3.78588115]), array([0.28225006, 0.32866337, 0.38545191, ..., 2.32594184,
2.36370811,
       2.52109123])]
eeR_gn [1.00035138e+00 1.00059224e+00 1.00064244e+00 ... 7.03300005e+03
 7.25439669e+03 7.64364950e+03]
all_num_batches= [1, 5, 25]
```

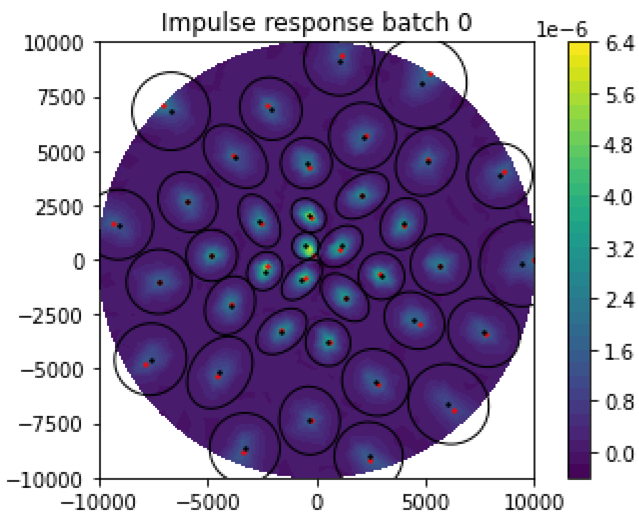
Plots to make (7):

- Batches of impulse responses (Nick will do this, save as hi-res PNG)

In [62]:

```python
PCH1.PCK.col_batches.visualize_impulse_response_batch(0)
```

```
/home/nick/repos/nalger_helper_functions/nalger_helper_functions/plot_ellipse.p
y:16: RuntimeWarning: divide by zero encountered in double_scalars
  theta = np.arctan(v_big[1] / v_big[0]) * 180. / np.pi
```



Regularization robustness plot

In [108…

```python
all_gammas = np.logspace(np.log10(5e3), np.log10(5e5), 7)[::-1]

b = np.random.randn(Vh2.dim())

sweep_tol = 1e-6
def first_good_iter(rr): # 1-based indexing
    return np.argwhere(rr < sweep_tol).reshape(-1)[0]+1
```

In [109…

```python
all_num_iter_sweep_gn = list()
num_iter_sweep_reg_gn = list()
num_iter_sweep_none_gn = list()
```

```
for gamma in all_gammas:
    print('---  GAMMA  ---:', gamma)
    REG.update_gamma(gamma)

    # Rebuild reg hmatrix with same block cluster tree as PCH data misfit hmatri
    print('Building Regularization H-Matrix')
    R_hmatrix = REG.make_R_hmatrix(all_Hd_pch[0].bct, rtol=1e-6)

    all_H_pch = [Hd + R_hmatrix for Hd in all_Hd_pch]
    all_preconditioners = [H.factorized_inverse() for H in all_H_pch]

    ireg_linop = spla.LinearOperator((Vh2.dim(), Vh2.dim()), matvec=REG.solve_he
    all_p_linop = [spla.LinearOperator((Vh2.dim(), Vh2.dim()), matvec=p.matvec)

    print('GN: get truth:')
    x_true_gn_sweep, _, _, _ = cg_wrapper(Hgn_linop, b, all_p_linop[-1], tol=1e-

    all_relres_gn_sweep = list()
    all_errs_gn_sweep = list()
    for nb, p_linop in zip(all_num_batches, all_p_linop):
        print('GN: psf'+str(nb))
        _, _, relres_gn_sweep, errs_gn_sweep = cg_wrapper(Hgn_linop, b, p_linop,
                                                         x_true=x_true_gn_sweep
                                                         tol=1e-7)
        all_relres_gn_sweep.append(relres_gn_sweep)
        all_errs_gn_sweep.append(errs_gn_sweep)

    print('GN: reg')
    _, _, relres_gn_reg_sweep, errs_gn_reg_sweep = cg_wrapper(Hgn_linop, b, ireg
                                                             x_true=x_true_gn_s
                                                             tol=1e-7)

    print('GN: none')
    _, _, relres_gn_none_sweep, errs_gn_none_sweep = cg_wrapper(Hgn_linop, b, No
                                                               x_true=x_true_gn
                                                               tol=1e-7)

    print('gamma=', gamma)
    all_num_iter_sweep_gn.append([])
    for k, relres_gn_sweep in enumerate(all_relres_gn_sweep):
        nb = all_num_batches[k]
        good_iter_psf = first_good_iter(relres_gn_sweep)
        all_num_iter_sweep_gn[-1].append(good_iter_psf)
        print('nb=', nb, ', good_iter_psf=', good_iter_psf)

    good_iter_reg = first_good_iter(relres_gn_reg_sweep)
    num_iter_sweep_reg_gn.append(good_iter_reg)
    print('good_iter_reg=', good_iter_reg)

    good_iter_none = first_good_iter(relres_gn_none_sweep)
    num_iter_sweep_none_gn.append(good_iter_none)
    print('good_iter_none=', good_iter_none)
```

```
---  GAMMA  ---: 499999.99999999994
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 8.06144e+07
```

```
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 8.06144e+07
 |S-A|_2 = 2.38579e-09
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
━━ H-matrix inverse ( rtol =  1e-06 , atol =  1e-14 , overwrite= False  )
    done in  18.86290669441223
    size of inverse =  3361874  bytes
                                                                       ==
]   8% ETA 20 s (5.84 GB)
━━ H-matrix multiplication C=A*B
    done in 27.73s
    size of C = 4.02 MB

━━ H-matrix multiplication C=A*B
    done in 38.39s
    size of C = 8.67 MB

━━ LU factorisation ( rtol = 1e-07 )
GN: get truth:==========-               ]  59% ETA 14 s (5.84 GB)
    done in 35.01s
    size of LU factor = 19.16 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 34.60s                                                      ]  69% ETA
10 s (5.84 GB)
    size of LU factor = 17.60 MB

━━ LU factorisation ( rtol = 1e-07 )
                                            0 , relres= 0.43711318982839875 , er
r= nan
    done in 34.66s                                                    ]  34% ETA 2
2 s (5.84 GB)
    size of LU factor = 17.49 MB
ii= 1 , relres= 0.34785913089575005 , err= nan
ii= 2 , relres= 0.16203156252868048 , err= nan
ii= 3 , relres= 0.063971797919686 , err= nan
ii= 4 , relres= 0.03007166315316233 , err= nan
ii= 5 , relres= 0.005686745892353261 , err= nan
ii= 6 , relres= 0.0034247916909714952 , err= nan
ii= 7 , relres= 0.000713214522113516 , err= nan
ii= 8 , relres= 0.00025240562711629084 , err= nan
ii= 9 , relres= 6.737090368278595e-05 , err= nan
ii= 10 , relres= 1.7584602694645174e-05 , err= nan
ii= 11 , relres= 7.952751180460031e-06 , err= nan
ii= 12 , relres= 4.4476248851469585e-06 , err= nan
ii= 13 , relres= 1.490343640926807e-06 , err= nan
ii= 14 , relres= 3.140934189365392e-07 , err= nan
ii= 15 , relres= 8.33450021176544e-08 , err= nan
ii= 16 , relres= 2.561249120889199e-08 , err= nan
ii= 17 , relres= 6.891165487446775e-09 , err= nan
ii= 18 , relres= 3.2820908923789766e-09 , err= nan
```

```
ii= 19 , relres= 1.3060126429797654e-09 , err= nan
ii= 20 , relres= 3.1961696968097513e-10 , err= nan
ii= 21 , relres= 9.927817079391021e-11 , err= nan
GN: psf1
ii= 0 , relres= 0.44261953359038 , err= 0.250200427740746
ii= 1 , relres= 0.35828310833620874 , err= 0.17245189173893938
ii= 2 , relres= 0.16551814396929118 , err= 0.13328779518827247
ii= 3 , relres= 0.06462232841614773 , err= 0.05527924441070741
ii= 4 , relres= 0.030328335909351172 , err= 0.014450768707643386
ii= 5 , relres= 0.005680236319203395 , err= 0.006213146430058274
ii= 6 , relres= 0.0034458333657437553 , err= 0.001328229170755505
ii= 7 , relres= 0.0007164397190623917 , err= 0.0006876234794481907
ii= 8 , relres= 0.0002628271450715272 , err= 0.0001406406957717083
ii= 9 , relres= 6.708884204030992e-05 , err= 3.9867304610395755e-05
ii= 10 , relres= 2.1736131639426278e-05 , err= 9.730537093054517e-06
ii= 11 , relres= 1.1174737063585904e-05 , err= 1.0288988683049074e-05
ii= 12 , relres= 5.189134472773039e-06 , err= 3.313977374958066e-06
ii= 13 , relres= 1.6154765221862242e-06 , err= 9.482559471479703e-07
ii= 14 , relres= 3.29938370511706e-07 , err= 2.1489404829350166e-07
ii= 15 , relres= 8.215634002466847e-08 , err= 4.544451655050858e-08
GN: psf5
ii= 0 , relres= 0.43739892433918137 , err= 0.24123325221844752
ii= 1 , relres= 0.34825424263012034 , err= 0.16616114576992627
ii= 2 , relres= 0.1625978766175718 , err= 0.13351467082827856
ii= 3 , relres= 0.06403695219011948 , err= 0.05467202195335507
ii= 4 , relres= 0.03024597323614624 , err= 0.013986897730158581
ii= 5 , relres= 0.005680860258746497 , err= 0.0062634488923232565
ii= 6 , relres= 0.0034470312326415325 , err= 0.0013202423501577749
ii= 7 , relres= 0.0007167835471102118 , err= 0.0006859402226891935
ii= 8 , relres= 0.00025178126493853045 , err= 0.00014136910329473374
ii= 9 , relres= 6.902574023295636e-05 , err= 3.5089809179472446e-05
ii= 10 , relres= 1.5580691620298714e-05 , err= 1.3413405221897782e-05
ii= 11 , relres= 1.6072025958185072e-05 , err= 8.311664653393313e-06
ii= 12 , relres= 4.544845146181365e-06 , err= 2.714534345547628e-06
ii= 13 , relres= 1.3670590964126241e-06 , err= 8.759254108383331e-07
ii= 14 , relres= 3.0857994118000654e-07 , err= 2.09008313006985e-07
ii= 15 , relres= 8.147706576692592e-08 , err= 4.1604721927487e-08
GN: psf25
ii= 0 , relres= 0.43711318982839875 , err= 0.240639206630425
ii= 1 , relres= 0.34785913089575005 , err= 0.16615482204876747
ii= 2 , relres= 0.16203156252868048 , err= 0.1330929146298968
ii= 3 , relres= 0.063971797919686 , err= 0.05429944358559951
ii= 4 , relres= 0.03007166315316233 , err= 0.013918955740206912
ii= 5 , relres= 0.005686745892353261 , err= 0.006256324364009815
ii= 6 , relres= 0.0034247916909714952 , err= 0.0013287544747111024
ii= 7 , relres= 0.000713214522113516 , err= 0.0006799772679293569
ii= 8 , relres= 0.00025240562711629084 , err= 0.0001390147605444905
ii= 9 , relres= 6.737090368278595e-05 , err= 3.6033603368409826e-05
ii= 10 , relres= 1.7584602694645174e-05 , err= 1.0025570708800359e-05
ii= 11 , relres= 7.952751180460031e-06 , err= 3.3209111042090802e-06
ii= 12 , relres= 4.4476248851469585e-06 , err= 3.0191744276304533e-06
ii= 13 , relres= 1.490343640926807e-06 , err= 8.894208060107737e-07
ii= 14 , relres= 3.140934189365392e-07 , err= 2.1210700977233572e-07
ii= 15 , relres= 8.33450021176544e-08 , err= 4.2971417523493895e-08
GN: reg
ii= 0 , relres= 0.18250051029763176 , err= 0.2691434741398484
ii= 1 , relres= 0.040852464897886506 , err= 0.061673113640854996
ii= 2 , relres= 0.006385591383766411 , err= 0.011927735502802921
ii= 3 , relres= 0.0012615064745439618 , err= 0.002642954015487068
ii= 4 , relres= 0.00030287154004338227 , err= 0.0005123690072643312
ii= 5 , relres= 5.2336117210063997e-05 , err= 9.044119527417037e-05
```

```
ii= 6 , relres= 1.2245148077347317e-05 , err= 2.4314694823112618e-05
ii= 7 , relres= 2.873011939564902e-06 , err= 5.550558240746515e-06
ii= 8 , relres= 5.509281579876109e-07 , err= 9.604242920593243e-07
ii= 9 , relres= 1.1699118482011062e-07 , err= 2.2725662889988567e-07
ii= 10 , relres= 2.7273077194326127e-08 , err= 5.749303776504525e-08
GN: none
ii= 0 , relres= 1.1526505504884619 , err= 0.9632434895924922
ii= 1 , relres= 1.0386949173865545 , err= 0.9308103328580514
ii= 2 , relres= 0.9415881357232275 , err= 0.892273933920592
ii= 3 , relres= 0.9605642696281357 , err= 0.8462979139252497
ii= 4 , relres= 0.9009583261663666 , err= 0.7996796076030811
ii= 5 , relres= 0.7951822627084615 , err= 0.7547214220906495
ii= 6 , relres= 0.7012044248685837 , err= 0.7124099609964768
ii= 7 , relres= 0.6569818899540232 , err= 0.6689258805203794
ii= 8 , relres= 0.6123798394049023 , err= 0.6267272184568861
ii= 9 , relres= 0.56395240987822 , err= 0.5887696686525032
ii= 10 , relres= 0.5160347287786976 , err= 0.5544835335884396
ii= 11 , relres= 0.47428860297082687 , err= 0.5210961369571704
ii= 12 , relres= 0.44120621464658305 , err= 0.4925895845654563
ii= 13 , relres= 0.3897255482820369 , err= 0.4638498305043264
ii= 14 , relres= 0.36568023755175183 , err= 0.43771751266916326
ii= 15 , relres= 0.3239345457508552 , err= 0.4166117874604724
ii= 16 , relres= 0.2912117930490681 , err= 0.39672468108787656
ii= 17 , relres= 0.2862331136764234 , err= 0.37888912774286254
ii= 18 , relres= 0.24999912326419788 , err= 0.362972172276221
ii= 19 , relres= 0.22133960231147506 , err= 0.34814634733588584
ii= 20 , relres= 0.2065193423515016 , err= 0.3352638341313236
ii= 21 , relres= 0.18397079861580076 , err= 0.3214387770175231
ii= 22 , relres= 0.1815691197067111 , err= 0.30927993541514104
ii= 23 , relres= 0.15890286769518755 , err= 0.2982759642339288
ii= 24 , relres= 0.1489601530047679 , err= 0.28896285360902746
ii= 25 , relres= 0.1403028548532354 , err= 0.2796185701769489
ii= 26 , relres= 0.12280580469361216 , err= 0.26954773415264643
ii= 27 , relres= 0.11897554901677888 , err= 0.26081848633693594
ii= 28 , relres= 0.1070278339345818 , err= 0.2525235536317488
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Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 3.74179e+07
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 3.74179e+07
 |S-A|_2 = 8.63985e-09
  matrix has dimension 1416 x 1416
━━ H-matrix inverse ( rtol =      no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
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    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
1e-06 , atol =  1e-14 , overwrite= False  )
    done in  18.130712032318115
[===

━━ H-matrix multiplication C=A*B
    done in 27.28s                                                ==============
============             ]  66% ETA 10 s (5.84 GB)
    size of C = 3.99 MB

━━ H-matrix multiplication C=A*B
    done in 38.35s
]  10% ETA 27 s (5.84 GB)
    size of C = 8.66 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 35.81s                                                       ]   7%
ETA 32 s (5.84 GB)
    size of LU factor = 20.72 MB

━━ LU factorisation ( rtol = 1e-07 )
    GN: get truth:======================-]  99% ETA 0 s (5.84 GB)
    done in 34.53s
    size of LU factor = 19.22 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 34.79s                                                % ETA 27 s (5.
84 GB)============-        ]  79% ETA 7 s (5.84 GB)
    size of LU factor = 19.01 MB
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ii= 2 , relres= 0.03895417587886728 , err= 0.051683866486858784
ii= 3 , relres= 0.02232122984773521 , err= 0.017074224270177716
ii= 4 , relres= 0.009801030847744533 , err= 0.004636831365910871
ii= 5 , relres= 0.0019005951968310646 , err= 0.0024759591643621725
ii= 6 , relres= 0.0010081827558529783 , err= 0.0005008859028474312
ii= 7 , relres= 0.00019237131770427044 , err= 0.00014096476509347625
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ii= 9 , relres= 1.477178772025519e-05 , err= 6.729583231162293e-06
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ii= 11 , relres= 7.137975506114475e-07 , err= 5.019476329488063e-07
ii= 12 , relres= 1.3941322531116235e-07 , err= 1.0055629380229236e-07
ii= 13 , relres= 3.1371282835395006e-08 , err= 1.8542433175144037e-08
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ii= 1 , relres= 0.23925349255119688 , err= 0.25882215280695114
ii= 2 , relres= 0.0867524726805356 , err= 0.11268097122080355
ii= 3 , relres= 0.036709470145557185 , err= 0.05066100702024013
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ii= 6 , relres= 0.003601446406061829 , err= 0.00457504206770576
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ii= 9 , relres= 0.00034733940910202044 , err= 0.0004472156581258213
ii= 10 , relres= 0.00016313005112633468 , err= 0.00022086232921407907
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ii= 12 , relres= 3.125714042899878e-05 , err= 3.687343940753434e-05
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ii= 14 , relres= 4.7611853602688725e-06 , err= 6.343301707547725e-06
ii= 15 , relres= 1.8538310457287465e-06 , err= 2.2505386433965385e-06
ii= 16 , relres= 5.561832994310441e-07 , err= 7.574974220126709e-07
ii= 17 , relres= 2.1110834040980762e-07 , err= 2.765672320182992e-07
ii= 18 , relres= 7.230144764454313e-08 , err= 1.0729540256908329e-07
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ii= 6 , relres= 0.6042965954163918 , err= 0.6239299095609021
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ii= 169 , relres= 8.549530491967446e-08 , err= 1.1402939645528892e-07
gamma= 232079.44168063859
nb= 1 , good_iter_psf= 12
nb= 5 , good_iter_psf= 12
nb= 25 , good_iter_psf= 12
good_iter_reg= 17
good_iter_none= 145
---  GAMMA  ---: 107721.7345015941
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 1.73678e+07
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 1.73678e+07
 |S-A|_2 = 2.14155e-09
  matrix has dimension ━━ H-matrix inverse ( rtol =  1e-06 , atol =  1e-14 , ove
rwrite= False  )
```

```
    done in  17.85023522377014
    size of inverse =  3361874  bytes
1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09

━━ H-matrix multiplication C=A*B
    done in 27.56s
    size of C = 3.97 MB

━━ H-matrix multiplication C=A*B
    done in 39.14s
    size of C = 8.65 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 35.09s
    size of LU factor = 21.93 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 35.08s
    size of LU factor = 20.53 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 34.89s
    size of LU factor = 20.30 MGN: get truth:
B
ii= 0 , relres= 0.14407437159172248 , err= nan
ii= 1 , relres= 0.04369991780214835 , err= nan
ii= 2 , relres= 0.011167994775569207 , err= nan
ii= 3 , relres= 0.003548767152608874 , err= nan
ii= 4 , relres= 0.0007393960497692093 , err= nan
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ii= 6 , relres= 6.10668057841616e-05 , err= nan
ii= 7 , relres= 1.5474752764541806e-05 , err= nan
ii= 8 , relres= 6.46466960547122e-06 , err= nan
ii= 9 , relres= 1.596491142789587e-06 , err= nan
ii= 10 , relres= 2.474286679726451e-07 , err= nan
ii= 11 , relres= 6.156907800244857e-08 , err= nan
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ii= 13 , relres= 1.4712727397057307e-09 , err= nan
ii= 14 , relres= 2.734696661376043e-10 , err= nan
ii= 15 , relres= 3.530191558541122e-11 , err= nan
GN: psf1
ii= 0 , relres= 0.14995908882804646 , err= 0.10339651580439359
ii= 1 , relres= 0.047043545684990085 , err= 0.03701434869274175
ii= 2 , relres= 0.012074562744568935 , err= 0.012719293942465652
ii= 3 , relres= 0.004067527352334396 , err= 0.002618466663777071
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ii= 9 , relres= 2.062463858505103e-06 , err= 9.938895643734155e-07
ii= 10 , relres= 2.8024707115236286e-07 , err= 3.1402048488450125e-07
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GN: psf5
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ii= 1 , relres= 0.04387253100135546 , err= 0.03576123991207128
ii= 2 , relres= 0.011522896703326839 , err= 0.011573369899898276
ii= 3 , relres= 0.0036162656799792066 , err= 0.0025256043777281234
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nb= 1 , good_iter_psf= 11
nb= 5 , good_iter_psf= 11
nb= 25 , good_iter_psf= 11
good_iter_reg= 34
good_iter_none= 116
---  GAMMA  ---: 49999.99999999999
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 8.06144e+06
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 8.06144e+06
 |S-A|_2 = 2.29781e-09
  matrix has dimension ━━ H-matrix inverse ( rtol =  1e-06 , atol =  1e-14 , ove
rwrite= False  )
    done in  17.927733421325684
    size of inverse =  3361874  bytes
1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09

━━ H-matrix multiplication C=A*B
    done in 27.44s
    size of C = 3.95 MB

━━ H-matrix multiplication C=A*B
    done in 38.54s
    size of C = 8.64 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 35.81s
    size of LU factor = 22.69 MB
```

```
━━ LU factorisation ( rtol = 1e-07 )
    done in 35.36s
    size of LU factor = 21.51 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 35.22s
    size of LU factor = 21.31 MB
GN: get truth:
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ii= 4 , relres= 0.00012358910979791096 , err= nan
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ii= 3 , relres= 0.0015011458318775314 , err= 0.001285865954101106
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nb= 1 , good_iter_psf= 9
nb= 5 , good_iter_psf= 9
nb= 25 , good_iter_psf= 8
good_iter_reg= 67
good_iter_none= 98
---  GAMMA  ---: 23207.944168063907
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 3.74179e+06
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 3.74179e+06
 |S-A|_2 = 5.56695e-09
  matrix has dimension ━━ H-matrix inverse ( rtol =  1e-06 , atol =  1e-14 , ove
rwrite= False  )
    done in 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
 17.96379852294922
    size of inverse =  3361874  bytes

━━ H-matrix multiplication C=A*B
    done in 27.24s
    size of C = 3.92 MB

━━ H-matrix multiplication C=A*B
    done in 37.88s
    size of C = 8.61 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 35.62s
    size of LU factor = 23.04 MB

━━ LU factorisation ( rtol = 1e-07 )
[=========-                                ]  24% ETA 26 s (5.84 GB)28 s (5.84 GB)
    done in 35.44s
    size of LU factor = 21.97 MB

━━ LU factorisation ( rtol = 1e-07 )
                      , relres= 0.11282798104107807 , err= nan GB)
    done in 35.07s
    size of LU factor = 21.79 MB
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gamma= 23207.944168063907
nb= 1 , good_iter_psf= 12
nb= 5 , good_iter_psf= 12
nb= 25 , good_iter_psf= 9
good_iter_reg= 140
good_iter_none= 87
---  GAMMA  ---: 10772.17345015941
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 1.73678e+06
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 1.73678e+06
 |S-A|_2 = 3.96925e-09
  matrix has dimension ━━ H-matrix inverse ( rtol =  1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
1e-06 , atol =  1e-14 , overwrite= False  )
    done in  17.80538821220398
    size of inverse =  3361874  bytes
                                                                   =
]  15% ETA 18 s (5.84 GB)
━━ H-matrix multiplication C=A*B
    done in 27.10s                                                 6% ETA 29 s
 (5.84 GB)
    size of C = 3.90 MB

━━ H-matrix multiplication C=A*B
    done in 37.27s
    size of C = 8.57 MB

━━ LU factorisation ( rtol = 1e-07 )
                                             44% ETA 19 s (5.84 GB)21 s (5.84 GB)
    done in 34.67s
    size of LU factor = 23.13 MB
```

```
━━ LU factorisation ( rtol = 1e-07 )
    done in 34.20s
]  47% ETA 18 s (5.84 GB)
    size of LU factor = 22.18 MB

━━ LU factorisation ( rtol = 1e-07 )
        0 , relres= 0.22417919838994443 , err= nanA 22 s (5.84 GB)GB)
    done in 34.30s                                                  ==============
=-                       ]  42% ETA 19 s (5.84 GB)
    size of LU factor = 21.95 MB
ii= 1 , relres= 0.06638823103635193 , err= nan
ii= 2 , relres= 0.02404845561475376 , err= nan
ii= 3 , relres= 0.010273184624516945 , err= nan
ii= 4 , relres= 0.005166836378327075 , err= nan
ii= 5 , relres= 0.002450328865145924 , err= nan
ii= 6 , relres= 0.0008758852286455534 , err= nan
ii= 7 , relres= 0.0002979731490602179 , err= nan
ii= 8 , relres= 9.763186837020906e-05 , err= nan
ii= 9 , relres= 3.1555018839746554e-05 , err= nan
ii= 10 , relres= 1.2616197030725942e-05 , err= nan
ii= 11 , relres= 4.876790032486029e-06 , err= nan
ii= 12 , relres= 1.6435327730736072e-06 , err= nan
ii= 13 , relres= 4.488291924486252e-07 , err= nan
ii= 14 , relres= 1.6954118537759827e-07 , err= nan
ii= 15 , relres= 7.558735018657903e-08 , err= nan
ii= 16 , relres= 2.9852918720645106e-08 , err= nan
ii= 17 , relres= 1.0294830279438417e-08 , err= nan
ii= 18 , relres= 3.087886667006815e-09 , err= nan
ii= 19 , relres= 1.0128913672784837e-09 , err= nan
ii= 20 , relres= 3.406275085900517e-10 , err= nan
ii= 21 , relres= 1.1427343197430834e-10 , err= nan
ii= 22 , relres= 4.379031937803236e-11 , err= nan
GN: psf1
ii= 0 , relres= 0.3265714513551525 , err= 0.21985721459384883
ii= 1 , relres= 0.15636423350658876 , err= 0.06387699467718416
ii= 2 , relres= 0.08972373379153512 , err= 0.03765780912999794
ii= 3 , relres= 0.04486110828749767 , err= 0.0207266511723002
ii= 4 , relres= 0.02506714723396819 , err= 0.010319885885649931
ii= 5 , relres= 0.013681131243262206 , err= 0.006043049362439034
ii= 6 , relres= 0.007258532751406085 , err= 0.0028083430940146737
ii= 7 , relres= 0.003914065364149573 , err= 0.001635453150951271
ii= 8 , relres= 0.001931605876214334 , err= 0.0007744903838776152
ii= 9 , relres= 0.0010472180622887886 , err= 0.00041301079629199117
ii= 10 , relres= 0.0005386177470474429 , err= 0.00021538850417761213
ii= 11 , relres= 0.0002762716264029277 , err= 0.00010421737028318418
ii= 12 , relres= 0.00013281977980680062 , err= 5.205233457292766e-05
ii= 13 , relres= 6.342617779886638e-05 , err= 2.621523443150617e-05
ii= 14 , relres= 3.174052225992403e-05 , err= 1.3314960242837017e-05
ii= 15 , relres= 1.700793573371981e-05 , err= 7.809925198288215e-06
ii= 16 , relres= 9.683520705956494e-06 , err= 3.850773060993146e-06
ii= 17 , relres= 4.617522999848045e-06 , err= 1.9141153572326936e-06
ii= 18 , relres= 2.094045424595943e-06 , err= 8.584985619734241e-07
ii= 19 , relres= 9.557012934887276e-07 , err= 3.4561828911396826e-07
ii= 20 , relres= 4.2372477569667485e-07 , err= 1.7043122479287164e-07
ii= 21 , relres= 2.0181687972284357e-07 , err= 7.354912306461461e-08
ii= 22 , relres= 1.0627231601746559e-07 , err= 3.922948669707669e-08
ii= 23 , relres= 5.431394061980124e-08 , err= 1.93540805095436e-08
GN: psf5
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ii= 1 , relres= 0.1167501599876759 , err= 0.05540241470819096
ii= 2 , relres= 0.06552267501534612 , err= 0.023175549778893762
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ii= 4 , relres= 0.018904981714452755 , err= 0.006895523503297053
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gamma= 10772.17345015941
nb= 1 , good_iter_psf= 20
nb= 5 , good_iter_psf= 18
nb= 25 , good_iter_psf= 14
good_iter_reg= 296
good_iter_none= 144
```

```
---  GAMMA  ---: 4999.999999999999
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 806144
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 806144
 |S-A|_2 = 4.35686e-09
  matrix has dimension ━━ H-matrix inverse ( rtol =  1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
1e-06 , atol =  1e-14 , overwrite= False  )
    done in  17.329407691955566
    size of inverse =  3361874  bytes

━━ H-matrix multiplication C=A*B
    done in 26.96s
    size of C = 3.86 MB

━━ H-matrix multiplication C=A*B
    done in 36.72s
    size of C = 8.53 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 34.31s
    size of LU factor = 23.02 MB

━━ LU factorisation ( rtol = 1e-07 )
[============================-          GN: get truth:
    done in 34.24s
    size of LU factor = 22.09 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 34.09s
    size of LU factor = 21.86 MB
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ii= 222 , relres= 6.22144138871579e-07 , err= 5.431567641470845e-07
ii= 223 , relres= 5.438290787150173e-07 , err= 5.193274209134652e-07
ii= 224 , relres= 4.774517507636067e-07 , err= 4.880977827646244e-07
ii= 225 , relres= 5.190357344688632e-07 , err= 4.683095415612918e-07
ii= 226 , relres= 4.5733710425882936e-07 , err= 4.4494223371661085e-07
ii= 227 , relres= 4.258157497779002e-07 , err= 4.251868350795875e-07
ii= 228 , relres= 4.524872543157936e-07 , err= 4.052691496825803e-07
ii= 229 , relres= 3.4563605168040816e-07 , err= 3.8826199589357317e-07
ii= 230 , relres= 4.128594830415908e-07 , err= 3.672101175866215e-07
ii= 231 , relres= 3.598577440205839e-07 , err= 3.560040165342033e-07
ii= 232 , relres= 3.6298755448236617e-07 , err= 3.4017337569885197e-07
ii= 233 , relres= 2.8331501574492307e-07 , err= 3.238521725086966e-07
ii= 234 , relres= 3.003897800147897e-07 , err= 3.0484025708700077e-07
ii= 235 , relres= 2.8352259705379653e-07 , err= 2.8983239656088013e-07
ii= 236 , relres= 2.4231150891419744e-07 , err= 2.74321123075376e-07
ii= 237 , relres= 2.2336489640190759e-07 , err= 2.5924881038811364e-07
ii= 238 , relres= 1.8685849409897247e-07 , err= 2.4252473812924697e-07
ii= 239 , relres= 2.0250490956096114e-07 , err= 2.2564388492533517e-07
ii= 240 , relres= 2.1213949718030417e-07 , err= 2.13440691985964e-07
```

```
ii= 241 , relres= 1.6858640856829658e-07 , err= 2.011150123158448e-07
ii= 242 , relres= 1.5959530493474274e-07 , err= 1.8454171641701302e-07
ii= 243 , relres= 1.7552834179610597e-07 , err= 1.7456667177093074e-07
ii= 244 , relres= 1.8334403668270056e-07 , err= 1.6342457315051652e-07
ii= 245 , relres= 1.5831007891396815e-07 , err= 1.5669712144487995e-07
ii= 246 , relres= 1.547507010598693e-07 , err= 1.4627731529419425e-07
ii= 247 , relres= 1.3965615074039616e-07 , err= 1.3849218054347948e-07
ii= 248 , relres= 1.375093283368299e-07 , err= 1.3024641615102128e-07
ii= 249 , relres= 1.231795324799832e-07 , err= 1.2232499281722096e-07
ii= 250 , relres= 1.6705792168091726e-07 , err= 1.156278620297991e-07
ii= 251 , relres= 1.0006913105187982e-07 , err= 1.0925512722206442e-07
ii= 252 , relres= 1.1466762248568117e-07 , err= 1.0132023353747015e-07
ii= 253 , relres= 1.0149054130723109e-07 , err= 9.694568634904712e-08
ii= 254 , relres= 1.1178727128621149e-07 , err= 9.12461921067729e-08
ii= 255 , relres= 8.075931246835305e-08 , err= 8.565530939128331e-08
gamma= 4999.999999999999
nb= 1 , good_iter_psf= 36
nb= 5 , good_iter_psf= 32
nb= 25 , good_iter_psf= 23
good_iter_reg= 630
good_iter_none= 214
```

In [110…

```
print('all_num_iter_sweep_gn=', all_num_iter_sweep_gn)
print('num_iter_sweep_reg_gn=', num_iter_sweep_reg_gn)
print('num_iter_sweep_none_gn=', num_iter_sweep_none_gn)
```

```
all_num_iter_sweep_gn= [[15, 15, 15], [12, 12, 12], [11, 11, 11], [9, 9, 8], [1
2, 12, 9], [20, 18, 14], [36, 32, 23]]
num_iter_sweep_reg_gn= [9, 17, 34, 67, 140, 296, 630]
num_iter_sweep_none_gn= [184, 145, 116, 98, 87, 144, 214]
```

In [111…

```
# all_num_iter_sweep= [[15, 15, 15], [12, 11, 11], [9, 9, 9], [16, 14, 11], [37,
# num_iter_sweep_reg= [9, 23, 68, 205, 627]
# num_iter_sweep_none= [187, 132, 97, 110, 220]
all_num_iter_sweep_gn = np.array(all_num_iter_sweep_gn)
num_iter_sweep_reg_gn = np.array(num_iter_sweep_reg_gn)
num_iter_sweep_none_gn = np.array(num_iter_sweep_none_gn)

print('all_gammas', all_gammas)
print('all_num_iter_sweep_gn', all_num_iter_sweep_gn)
print('num_iter_sweep_reg_gn=', num_iter_sweep_reg_gn)
print('num_iter_sweep_none_gn=', num_iter_sweep_none_gn)

np.savetxt(save_dir_str + '/all_gammas.txt', all_gammas)
np.savetxt(save_dir_str + '/all_num_iter_sweep_gn.txt', all_num_iter_sweep_gn)
np.savetxt(save_dir_str + '/num_iter_sweep_reg_gn.txt', num_iter_sweep_reg_gn)
np.savetxt(save_dir_str + '/num_iter_sweep_none_gn.txt', num_iter_sweep_none_gn)
```

```
all_gammas [500000.         232079.44168064 107721.73450159  50000.
  23207.94416806  10772.17345016   5000.        ]
all_num_iter_sweep_gn [[15 15 15]
 [12 12 12]
 [11 11 11]
 [ 9  9  8]
 [12 12  9]
 [20 18 14]
 [36 32 23]]
```

```
num_iter_sweep_reg_gn= [  9  17  34  67 140 296 630]
num_iter_sweep_none_gn= [184 145 116  98  87 144 214]
```

In [ ]:

```
plt.loglog(all_gammas, num_iter_sweep_none_gn)
plt.loglog(all_gammas, num_iter_sweep_reg_gn)
for k in range(len(all_num_batches)):
    plt.loglog(all_gammas, all_num_iter_sweep_gn[:,k])

plt.title(r'CG iterations to achieve $||H_{gn}x-b||/||b|| < ' + str(sweep_tol) +
plt.xlabel(r'$\gamma$')
plt.ylabel('CG iterations')
plt.legend(['None', 'REG'] + ['PSF'+str(nb) for nb in all_num_batches])
```

In [ ]:

```
all_num_iter_sweep = list()
num_iter_sweep_reg = list()
num_iter_sweep_none = list()
for gamma in all_gammas:
    print('---  GAMMA  ---:', gamma)
    REG.update_gamma(gamma)

    # Rebuild reg hmatrix with same block cluster tree as PCH data misfit hmatri
    print('Building Regularization H-Matrix')
    R_hmatrix = REG.make_R_hmatrix(all_Hd_pch[0].bct, rtol=1e-6)

    all_H_pch = [Hd + R_hmatrix for Hd in all_Hd_pch]
    all_preconditioners = [H.factorized_inverse() for H in all_H_pch]

    ireg_linop = spla.LinearOperator((Vh2.dim(), Vh2.dim()), matvec=REG.solve_he
    all_p_linop = [spla.LinearOperator((Vh2.dim(), Vh2.dim()), matvec=p.matvec)

    print('FULL: get truth:')
    x_true_sweep, _, _, _ = cg_wrapper(H_linop, b, all_p_linop[-1], tol=1e-10)

    all_relres_sweep = list()
    all_errs_sweep = list()
    for nb, p_linop in zip(all_num_batches, all_p_linop):
        print('FULL: psf'+str(nb))
        _, _, relres_sweep, errs_sweep = cg_wrapper(H_linop, b, p_linop,
                                                    x_true=x_true_sweep,
                                                    tol=1e-7)
        all_relres_sweep.append(relres_sweep)
        all_errs_sweep.append(errs_sweep)

    print('FULL: reg')
    _, _, relres_reg_sweep, errs_reg_sweep = cg_wrapper(H_linop, b, ireg_linop,
                                                        x_true=x_true_sweep,
                                                        tol=1e-7)

    print('FULL: none')
    _, _, relres_none_sweep, errs_none_sweep = cg_wrapper(H_linop, b, None,
                                                          x_true=x_true_sweep,
                                                          tol=1e-7)

    print('gamma=', gamma)
    all_num_iter_sweep.append([])
    for k, relres_sweep in enumerate(all_relres_sweep):
        nb = all_num_batches[k]
        good_iter_psf = first_good_iter(relres_sweep)
        all_num_iter_sweep[-1].append(good_iter_psf)
```

```
        print('nb=', nb, ', good_iter_psf=', good_iter_psf)

    good_iter_reg = first_good_iter(relres_reg_sweep)
    num_iter_sweep_reg.append(good_iter_reg)
    print('good_iter_reg=', good_iter_reg)

    good_iter_none = first_good_iter(relres_none_sweep)
    num_iter_sweep_none.append(good_iter_none)
    print('good_iter_none=', good_iter_none)
```

```
---  GAMMA  ---: 499999.99999999994
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 8.06144e+07
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 8.06144e+07
 |S-A|_2 = 2.38579e-09
  matrix has dimension ━━ H-matrix inverse ( rtol = 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
 1e-06 , atol =  1e-14 , overwrite= False  )

/home/nick/anaconda3/envs/fenics4/lib/python3.9/site-packages/scipy/sparse/lina
g/dsolve/linsolve.py:318: SparseEfficiencyWarning: splu requires CSC matrix form
at
  warn('splu requires CSC matrix format', SparseEfficiencyWarning)
    done in  17.111899852752686
    size of inverse =  3361874  bytes

━━ H-matrix multiplication C=A*B
    done in 27.32s
    size of C = 4.02 MB

━━ H-matrix multiplication C=A*B
    done in 37.81s
    size of C = 8.67 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 34.57s
    size of LU factor = 19.16 MB

━━ LU factorisation ( rtol = 1e-07 )
            FULL: get truth:             ]   9% ETA 29 s (5.85 GB)
    done in 34.25s                                                   -
]  17% ETA 27 s (5.85 GB)
    size of LU factor = 17.60 MB

━━ LU factorisation ( rtol = 1e-07 )
```

```
    done in 34.23s
    size of LU factor = 17.49 MB
ii= 0 , relres= 0.4371024642107277 , err= nan
ii= 1 , relres= 0.347856815097376 , err= nan
ii= 2 , relres= 0.16202377657044942 , err= nan
ii= 3 , relres= 0.06397194599433267 , err= nan
ii= 4 , relres= 0.03006710845309418 , err= nan
ii= 5 , relres= 0.0056854950943026955 , err= nan
ii= 6 , relres= 0.00342390838282088 , err= nan
ii= 7 , relres= 0.0007132422565788598 , err= nan
ii= 8 , relres= 0.0002522720715259203 , err= nan
ii= 9 , relres= 6.735373452848496e-05 , err= nan
ii= 10 , relres= 1.786700839124333e-05 , err= nan
ii= 11 , relres= 4.72535117028113e-06 , err= nan
ii= 12 , relres= 4.840263184108101e-06 , err= nan
ii= 13 , relres= 1.4816726307468622e-06 , err= nan
ii= 14 , relres= 3.1394558707881166e-07 , err= nan
ii= 15 , relres= 8.326825138836459e-08 , err= nan
ii= 16 , relres= 2.5593641391416978e-08 , err= nan
ii= 17 , relres= 7.483769308013486e-09 , err= nan
ii= 18 , relres= 3.6624237689836588e-09 , err= nan
ii= 19 , relres= 1.3086510777248923e-09 , err= nan
ii= 20 , relres= 3.183110369523604e-10 , err= nan
ii= 21 , relres= 1.2708055607827445e-10 , err= nan
ii= 22 , relres= 8.123353552760929e-11 , err= nan
FULL: psf1
ii= 0 , relres= 0.44260811901338404 , err= 0.2502036630162524
ii= 1 , relres= 0.35828044020742156 , err= 0.17246694916139033
ii= 2 , relres= 0.1655096345503034 , err= 0.1332857629496461
ii= 3 , relres= 0.06462102666098501 , err= 0.05526476515177078
ii= 4 , relres= 0.030324495889876513 , err= 0.014447178287425037
ii= 5 , relres= 0.005679052717201478 , err= 0.006212793523961303
ii= 6 , relres= 0.003444974415420327 , err= 0.0013275194989280188
ii= 7 , relres= 0.0007165016882066503 , err= 0.0006874590382707105
ii= 8 , relres= 0.0002626622450099581 , err= 0.000140619953623694
ii= 9 , relres= 6.719652617849372e-05 , err= 3.973023071451421e-05
ii= 10 , relres= 2.0285940242384327e-05 , err= 1.0829001779830334e-05
ii= 11 , relres= 1.687148189939294e-05 , err= 6.665122293763567e-06
ii= 12 , relres= 5.078680654417829e-06 , err= 3.400044759537999e-06
ii= 13 , relres= 1.6161394751498156e-06 , err= 9.474247985552454e-07
ii= 14 , relres= 3.298897652998864e-07 , err= 2.1497640404737782e-07
ii= 15 , relres= 8.217865266375351e-08 , err= 4.5126319563132076e-08
FULL: psf5
ii= 0 , relres= 0.4373881276390904 , err= 0.24123777822802864
ii= 1 , relres= 0.34825192373402 , err= 0.16617624515857585
ii= 2 , relres= 0.1625899561853565 , err= 0.1335119309306187
ii= 3 , relres= 0.06403711204180365 , err= 0.054657307361604936
ii= 4 , relres= 0.030241444201619852 , err= 0.013983534724855518
ii= 5 , relres= 0.0056796782027813564 , err= 0.006263075415688111
ii= 6 , relres= 0.00344609490873036 , err= 0.0013194758232033391
ii= 7 , relres= 0.0007167667471938175 , err= 0.0006858117286683961
ii= 8 , relres= 0.0002516950674617034 , err= 0.0001413219338070971
ii= 9 , relres= 6.862610071745716e-05 , err= 3.545541349151669e-05
ii= 10 , relres= 2.023852364759406e-05 , err= 9.365707538042006e-06
ii= 11 , relres= 1.3491484474578116e-05 , err= 1.0077195398234716e-05
ii= 12 , relres= 4.568518045513093e-06 , err= 2.6909862215004843e-06
ii= 13 , relres= 1.3638303331243622e-06 , err= 8.755003921989787e-07
ii= 14 , relres= 3.0834759899775166e-07 , err= 2.0904647127760268e-07
ii= 15 , relres= 8.160708188140944e-08 , err= 4.131914908563542e-08
FULL: psf25
ii= 0 , relres= 0.4371024642107277 , err= 0.24064342506903696
```

```
ii= 1 , relres= 0.347856815097376 , err= 0.16616949775714512
ii= 2 , relres= 0.16202377657044942 , err= 0.13308997548144771
ii= 3 , relres= 0.06397194599433267 , err= 0.05428476548829491
ii= 4 , relres= 0.03006710845309418 , err= 0.013915639843661442
ii= 5 , relres= 0.0056854950943026955 , err= 0.006255943805335876
ii= 6 , relres= 0.00342390838282088 , err= 0.0013280052037084695
ii= 7 , relres= 0.0007132422565788598 , err= 0.0006798313185221535
ii= 8 , relres= 0.0002522720715259203 , err= 0.0001389966240081108
ii= 9 , relres= 6.735373452848496e-05 , err= 3.6024151852372324e-05
ii= 10 , relres= 1.786700839124333e-05 , err= 9.78731184381991e-06
ii= 11 , relres= 4.72535117028113e-06 , err= 4.698229519410513e-06
ii= 12 , relres= 4.840263184108101e-06 , err= 2.630515769173895e-06
ii= 13 , relres= 1.4816726307468622e-06 , err= 8.924600288911608e-07
ii= 14 , relres= 3.1394558707881166e-07 , err= 2.1196915572021866e-07
ii= 15 , relres= 8.326825138836459e-08 , err= 4.2939338215421026e-08
FULL: reg
ii= 0 , relres= 0.1825734844789773 , err= 0.2691818869202921
ii= 1 , relres= 0.040873992234545256 , err= 0.06170614114727593
ii= 2 , relres= 0.006390133828755188 , err= 0.011935453902086487
ii= 3 , relres= 0.0012625313262540272 , err= 0.0026442401148237373
ii= 4 , relres= 0.0003031967810558532 , err= 0.0005129059386550644
ii= 5 , relres= 5.239162631339509e-05 , err= 9.055610506689429e-05
ii= 6 , relres= 1.226283731354563e-05 , err= 2.4329271421236693e-05
ii= 7 , relres= 2.8786229190681236e-06 , err= 5.558958092753653e-06
ii= 8 , relres= 5.523777370012941e-07 , err= 9.622820566598361e-07
ii= 9 , relres= 1.1700615966308288e-07 , err= 2.2728924349199652e-07
ii= 10 , relres= 2.72940815557024e-08 , err= 5.756751241381467e-08
FULL: none
ii= 0 , relres= 1.152648113705038 , err= 0.9632362066695127
ii= 1 , relres= 1.038689936629488 , err= 0.9307970394960375
ii= 2 , relres= 0.9415802113470243 , err= 0.8922539364654242
ii= 3 , relres= 0.9605519105900987 , err= 0.846270488613722
ii= 4 , relres= 0.9009416079809828 , err= 0.7996453554230912
ii= 5 , relres= 0.7951618667283572 , err= 0.7546813800681983
ii= 6 , relres= 0.7011800173906306 , err= 0.7123653629784
ii= 7 , relres= 0.6569517067995406 , err= 0.6688776763870584
ii= 8 , relres= 0.6123442736890726 , err= 0.6266766648734974
ii= 9 , relres= 0.5639121881000605 , err= 0.5887181023308409
ii= 10 , relres= 0.5159910549196975 , err= 0.5544320311303068
ii= 11 , relres= 0.47424192086609973 , err= 0.5210456331564591
ii= 12 , relres= 0.4411564372574846 , err= 0.4925407520336361
ii= 13 , relres= 0.38967528568981785 , err= 0.46380351039869955
ii= 14 , relres= 0.3656273722943484 , err= 0.437674258032415
ii= 15 , relres= 0.32388275979521963 , err= 0.4165715366978816
ii= 16 , relres= 0.29116072651726826 , err= 0.39668768169571356
ii= 17 , relres= 0.2861786736384113 , err= 0.37885537943800157
ii= 18 , relres= 0.24994861316852376 , err= 0.3629414944381152
ii= 19 , relres= 0.22129228188135375 , err= 0.3481186269213754
ii= 20 , relres= 0.20647318131242315 , err= 0.33523868292520237
ii= 21 , relres= 0.18392810171198246 , err= 0.32141630181732633
ii= 22 , relres= 0.18152568027628485 , err= 0.30925974601441725
ii= 23 , relres= 0.15886414903528231 , err= 0.29825767363177075
ii= 24 , relres= 0.14892335925840727 , err= 0.28894604204571284
ii= 25 , relres= 0.1402679742423323 , err= 0.2796030663980791
ii= 26 , relres= 0.12277557355365055 , err= 0.2695333959121875
ii= 27 , relres= 0.11894688369559248 , err= 0.26080490629606384
ii= 28 , relres= 0.10700262550011638 , err= 0.2525104492011999
ii= 29 , relres= 0.09895155651385301 , err= 0.24411784752950702
ii= 30 , relres= 0.09828632789259809 , err= 0.23560928009302337
ii= 31 , relres= 0.09033186008066832 , err= 0.22657842646708387
ii= 32 , relres= 0.08820001567665763 , err= 0.2178649878286926
```

```
ii= 33 , relres= 0.09034443525866322 , err= 0.20886580742647815
ii= 34 , relres= 0.08225841565564138 , err= 0.19963256149698744
ii= 35 , relres= 0.07814468445884264 , err= 0.19023791730041859
ii= 36 , relres= 0.07391593565896327 , err= 0.18091984918071047
ii= 37 , relres= 0.07326409779512047 , err= 0.17233557453885037
ii= 38 , relres= 0.07076611892522469 , err= 0.16293154240462496
ii= 39 , relres= 0.0643437182486391 , err= 0.15432301146200259
ii= 40 , relres= 0.058561528854456375 , err= 0.1460302656790306
ii= 41 , relres= 0.0587158626928254 , err= 0.1380289202771556
ii= 42 , relres= 0.058152828714025814 , err= 0.12925360461135607
ii= 43 , relres= 0.05579107954975136 , err= 0.12176427771351388
ii= 44 , relres= 0.05526312860398601 , err= 0.11404331991855476
ii= 45 , relres= 0.059336152140793255 , err= 0.10744772426720914
ii= 46 , relres= 0.0558216799675333 , err= 0.10152151274325992
ii= 47 , relres= 0.04760198108314797 , err= 0.094877539173168
ii= 48 , relres= 0.04462343841459878 , err= 0.08809723451264638
ii= 49 , relres= 0.041312333726888535 , err= 0.08235347774770108
ii= 50 , relres= 0.03884148306602096 , err= 0.07689787447495194
ii= 51 , relres= 0.03729465094565502 , err= 0.0717355218038806
ii= 52 , relres= 0.03382879730898894 , err= 0.0666478782474048
ii= 53 , relres= 0.033637755203328776 , err= 0.06138654213525204
ii= 54 , relres= 0.03313567719556325 , err= 0.056317199993568275
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gamma= 499999.99999999994
nb= 1 , good_iter_psf= 15
nb= 5 , good_iter_psf= 15
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nb= 25 , good_iter_psf= 15
good_iter_reg= 9
good_iter_none= 184
---  GAMMA  ---: 232079.44168063859
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 3.74179e+07
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 3.74179e+07
 |S-A|_2 = 8.63985e-09
  matrix has dimension ━━ H-matrix inverse ( rtol =  1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
1e-06 , atol =  1e-14 , overwrite= False  )
    done in  17.18127942085266
    size of inverse =  3361874  bytes

━━ H-matrix multiplication C=A*B
    done in 27.03s
    size of C = 3.99 MB

━━ H-matrix multiplication C=A*B
    done in 38.03s
    size of C = 8.66 MB

━━ LU factorisation ( rtol = 1e-07 )
                 FULL: get truth:===-    ]  89% ETA 3 s (5.85 GB)
    done in 34.50s
    size of LU factor = 20.72 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 34.28s                                                -
]  17% ETA 27 s (5.85 GB)TA 15 s (5.85 GB)
    size of LU factor = 19.21 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 34.34s                                                 28 s (5.85 G
B)
    size of LU factor = 19.01 MB
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ii= 2 , relres= 0.038965773326310495 , err= nan
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ii= 13 , relres= 3.135782846187948e-08 , err= nan
ii= 14 , relres= 7.2686225859237915e-09 , err= nan
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ii= 16 , relres= 6.565386161851986e-10 , err= nan
ii= 17 , relres= 2.782133033099569e-10 , err= nan
ii= 18 , relres= 4.648324437424162e-11 , err= nan
FULL: psf1
ii= 0 , relres= 0.2666489071498253 , err= 0.1506933930240778
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ii= 2 , relres= 0.04266587286528455 , err= 0.05529091284532726
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FULL: psf5
ii= 0 , relres= 0.26340166185246744 , err= 0.14460120785745634
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ii= 3 , relres= 0.022595950626136306 , err= 0.017545194398988172
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ii= 2 , relres= 0.038965773326310495 , err= 0.05166402092752238
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ii= 4 , relres= 0.009797402876634628 , err= 0.004635135110475905
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ii= 2 , relres= 0.08679121932632435 , err= 0.11273984621337885
ii= 3 , relres= 0.03672892461045517 , err= 0.05067960026416916
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nb= 5 , good_iter_psf= 12
nb= 25 , good_iter_psf= 12
good_iter_reg= 17
good_iter_none= 145
---  GAMMA  ---: 107721.7345015941
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 1.73678e+07
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 1.73678e+07
 |S-A|_2 = 2.14155e-09
  matrix has dimension ━━ H-matrix inverse ( rtol =  1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
1e-06 , atol =  1e-14 , overwrite= False  )
[========-                              ]  22% ETA 17 s (5.85 GB)    done in  1
7.40661859512329
    size of inverse =  3361874  bytes

━━ H-matrix multiplication C=A*B
    done in 27.21s
    size of C = 3.97 MB

━━ H-matrix multiplication C=A*B
    done in 37.85s
    size of C = 8.65 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 34.60s
    size of LU factor = 21.93 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 34.61s
    size of LU factor = 20.53 MB

━━ LU factorisation ( rtol = 1e-07 )
[====================
    done in 34.32s
    size of LU factor = 20.30 MB
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nb= 5 , good_iter_psf= 11
nb= 25 , good_iter_psf= 11
good_iter_reg= 34
good_iter_none= 116
---  GAMMA  ---: 49999.99999999999
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 8.06144e+06
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 8.06144e+06
 |S-A|_2 = 2.29781e-09
  matrix has dimension ━━ H-matrix inverse ( rtol = 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
```

```
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
 1e-06 , atol =  1e-14 , overwrite= False  )
    done in  17.347942113876343
    size of inverse =  3361874  bytes
                                                                   ==
]   8% ETA 15 s (5.85 GB)
━━ H-matrix multiplication C=A*B
    done in 26.76s
    size of C = 3.95 MB

━━ H-matrix multiplication C=A*B
    done in 37.86s
    size of C = 8.64 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 34.93s
    size of LU factor = 22.69 MB

━━ LU factorisation ( rtol = 1e-07 )
[===========================FULL: get truth:
    done in 34.80s
    size of LU factor = 21.51 MB

━━ LU factorisation ( rtol = 1e-07 )
    done in 34.58s
]   2% ETA 24 s (5.85 GB) (5.85 GB)
    size of LU factor = 21.32 MB
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nb= 5 , good_iter_psf= 9
nb= 25 , good_iter_psf= 8
good_iter_reg= 67
good_iter_none= 97
---  GAMMA  ---: 23207.944168063907
Building Regularization H-Matrix
  matrix has dimension 1416 x 1416
    no of non-zeroes    = 9746
    matrix is             real valued
    format              = non symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 3.74179e+06
    sparsity constant = 20
    size of H-matrix  = 4.06 MB
    |A|_F             = 3.74179e+06
 |S-A|_2 = 5.56695e-09
  matrix has dimension 1416 x 1416
    no of non-zeroes    = ━━ H-matrix inverse ( rtol =  9746
    matrix is             real valued
    format              = symmetric
  size of sparse matrix = 163.52 kB
  |S|_F                 = 5.18561e+06
    sparsity constant = 20
    size of H-matrix  = 2.29 MB
    |A|_F             = 5.18561e+06
 |S-A|_2 = 4.66653e-09
1e-06 , atol =  1e-14 , overwrite= False  )
          17.24683380126953              ]   7% ETA 19 s (5.85 GB)
    size of inverse =  3361874  bytes
                                                       s (5.85 GB)
```

In [ ]:
```
print('all_num_iter_sweep=', all_num_iter_sweep)
print('num_iter_sweep_reg=', num_iter_sweep_reg)
print('num_iter_sweep_none=', num_iter_sweep_none)
```

In [ ]:
```
all_num_iter_sweep = np.array(all_num_iter_sweep)
num_iter_sweep_reg = np.array(num_iter_sweep_reg)
num_iter_sweep_none = np.array(num_iter_sweep_none)

print('all_num_iter_sweep', all_num_iter_sweep)
print('num_iter_sweep_reg=', num_iter_sweep_reg)
print('num_iter_sweep_none=', num_iter_sweep_none)

np.savetxt(save_dir_str + '/all_num_iter_sweep.txt', all_num_iter_sweep)
np.savetxt(save_dir_str + '/num_iter_sweep_reg.txt', num_iter_sweep_reg)
np.savetxt(save_dir_str + '/num_iter_sweep_none.txt', num_iter_sweep_none)
```

In [ ]:
```
plt.loglog(all_gammas, num_iter_sweep_none)
plt.loglog(all_gammas, num_iter_sweep_reg)
for k in range(len(all_num_batches)):
    plt.loglog(all_gammas, all_num_iter_sweep[:,k])

plt.title(r'CG iterations to achieve $||Hx-b||/||b|| < ' + str(sweep_tol) + '$')
plt.xlabel(r'$\gamma$')
plt.xlabel('CG iterations')
plt.legend(['None', 'REG'] + ['PSF'+str(nb) for nb in all_num_batches])
```