Interest Rate Swap Compounding Formulae

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In this short paper, we outline geometric and arithmetic compound formulae for interest rate swaps. We also present ISDA protocol when compounding with a floating spread.

Geometric compounded rates are calculated as,

$$Geometric\ Compounded\ Rate,\ r_G = \left(\prod_{i=1}^{n} (1 + r_i\, \tau_i) - 1 \right) \Big/ \tau_C \tag{1a}$$

, where r_i denotes a rate fixing or reset, τ_i the fixing year fraction and τ_c the coupon year fraction

If we assume discount factors (DF) have simple compounding[1] this reduces to,

$$Geometric\ Compounded\ Rate,\ r_G = \left(\left(\frac{1}{DF_n} \right) - 1 \right) / \tau_C \tag{1b}$$

Arithmetic compounded rates are calculated as,

$$Arithmetic\ Compounded\ Rate,\ r_A = \left(\sum_{i=1}^{n} r_i\, \tau_i \right) \Big/ \tau_C \tag{2}$$

Next we briefly outline how to compound floating rates that have a spread, which is often the case for swap transactions. ISDA[2] define three compounding approaches for floating interest rates with a spread as follows,

[1] Computationally Efficient Zero Coupon Swap Formulae. Available at: https://ssrn.com/abstract=3881983

[2] Alternative compounding methods for over-the-counter derivative transactions. Available at: https://www.isda.org/a/2KiDE/isda-compounding-memo.pdf

Method 1: **Compounding** (Include Spread)

Floating rates compound <u>with</u> the spread.

$$C_1 = N\,(r_1 + s)\,\tau_1$$
$$C_2 = (N + C_1)\,(r_2 + s)\,\tau_2$$
$$C_3 = (N + C_2)\,(r_3 + s)\,\tau_3$$
$$FA = C_1 + C_2 + C_3$$

(3)

, where C denotes the compound amount and FA the total float amount.

Method 2: **Flat Compounding** (Exclude Spread)

Floating rates compound <u>without</u> the spread.

$$C_1 = N\,(r_1 + s)\,\tau_1$$
$$C_2 = N\,(r_2 + s)\,\tau_2 \;+\; C_1\,r_2\,\tau_2$$
$$C_3 = N\,(r_3 + s)\,\tau_3 + \;C_2\,r_3\,\tau_3$$
$$FA = C_1 + C_2 + C_3$$

(4)

, where C denotes the compound amount and FA the total float amount.

Method 3: **None** (Simple Interest)

The spread is excluded from compounding and accrues simple interest.

$$C_1 = N\,r_1\,\tau_1$$
$$C_2 = (N + C_1)\,r_2\,\tau_2$$
$$C_3 = (N + C_1 + C_2)\;\,r_3\,\tau_3$$
$$SA = N\,s\,(\,\tau_1 + \tau_2 + \tau_3)$$
$$FA = C_1 + C_2 + C_3 + SA$$

(5)

, where C denotes the compound amount, SA the spread amount and FA the total float amount.