Lab 7

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Nonlinear Dynamic Systems , Fall 2020

Goals of the Lab

In this lab, we wanted to learn more about the different numerical ways to solve a differential equation as well as compare them and the errors they would produce on a real life example.

Why study Differential Equations?

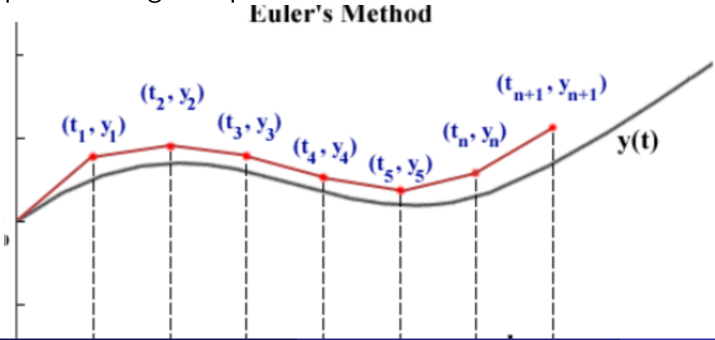
Differential equations help model most everything in day to day life. From rocket flight to water flow to stock price changes to population change, one needs a good grasp on dynamical systems and the differential equations that make them up. Understanding the numerical ways to solve these equations help us better understand situations where there isn't an analytic solution to the problem.

Euler's Method

The algorithm is defined by the function

$$x_{k+1} = x_k + hf(t_k, x_k)$$

Since we are working with an first order iterative function, the error is proportional to the square of the iteration step. Therefore for small values of n, it can be pretty accurate for a lot of iterations. Having such a simple yet, relatively accurate method of solving is important when speed is prioritized against precision.



Modified Euler's Method

Now, one might want to improve Euler's Method. One simple, yet computationally expensive way to do this is by reducing the size of h. A better approach would be to combine the midpoint method in combination with the Euler method, achieved by the formula

$$
\begin{aligned}
x_0 &= \alpha \\
x_{i+1} &= x_i + \frac{h}{2}\left[f(t_i, x_i) + f\Big(t_{i+1}, x_i + hf(t_i, x_i)\Big)\right]
\end{aligned}
$$

Huen's Method

This method also modifies the midpoint method in conjunction with Euler's Method but places a greater weight on the midpoint evaluation, defined by the formula

$$
\begin{aligned}
x_0 &= \alpha \\
x_{i+1} &= x_i + \frac{h}{4}\left[f(t_i, x_i) + 3f\left(t_i + \frac{2}{3}h, x_i + \frac{2}{3}hf(t_i, x_i)\right)\right]
\end{aligned}
$$

Runge Kutta

The RK method uses ideas derived from Taylor's method to create methods for higher order terms without needing to compute those high order derivatives. The method is defined as

$$
\begin{aligned}
x_0 &= \alpha \\
k_1 &= hf(t_i, x_i) \\
k_2 &= hf\left(t_i + \frac{h}{2}, x_i + \frac{1}{2}k_1\right) \\
k_3 &= hf\left(t_i + \frac{h}{2}, x_i + \frac{1}{2}k_2\right) \\
k_4 &= hf(t_{i+1}, x_i + k_3) \\
x_{i+1} &= x_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)
\end{aligned}
$$

Error Analysis and Method Comparison

METHOD	RELATIVE ERROR
Euler	0.722245
Midpoint	0.00268413
Modified	0.00921544
Heun	0.00486124
4th-order R-K	1.40903e-07

By computing the difference between the analytically solved map and each method we arrive at the above result. We can see that in this case the 4th order R-K method is more accurate, but due to its many iterations of the function requirement it is computationally more expensive.

Conclusion

In conclusion different methods were analysed and compared against each other in order to further understand the process that goes into analysing complicated dynamical systems. The trade off between accuracy and computational power was further discussed.