Lab 8 Report

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Abstract

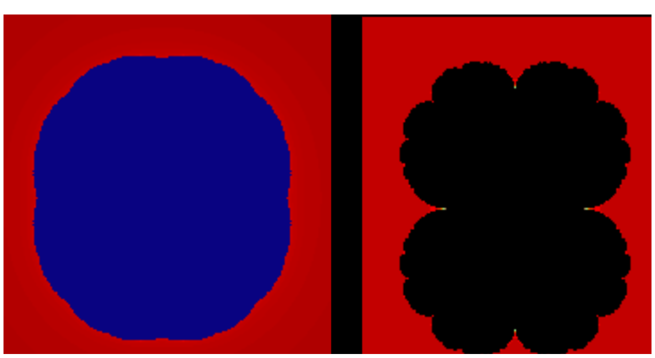
This lab focus's on the Mandelbrot set and plotting it. The Mandelbrot set is a type of plot of the recursive function $z_{n+1} = z_n^2 + c$. If, after many iterations of the function at an initial value for a point $c \in C$ doesn't diverge to infinity, the point is filled in black. If the point goes off to infinity instead, the point is colored in based on how quickly it diverges.

Introduction

The Mandelbrot set is a type of fractal discovered by Benoit B. Mandelbrot in 1980. Like most of the labs in the course, complexity comes from simplicity as the recursive function has just 5 symbols and follows one of the simplest operations possible. Just by adding and multiplying complex numbers would one be able to understand the Mandelbrot set on their own. Even though it is extremely simple, the only way to truely get a grasp on it is to iterate the process thousands or even millions of times. Only through such intense computation would one see the detail that makes the set remarkable.

Julia Sets

Julia sets are a predecessor to the Mandelbrot set and what Dr. Mandelbrot studied before he discovered the set. The Julia set for a constant k is the set where you allow z_0 to vary and for all such initial values, set $c = k$. This set generally takes more circular shapes like the ones shown below.



Mandelbrot Set

The Mandelbrot set is the set of points $c \in C$ such that when setting $z_0 = 0$, the function doesn't diverge. For example, consider the two values, $c = 1$ and $c = -1$. Applying the rule to $c = 1$, we get 0,1,2,5,26.... It is clear that this sequence is not bounded and diverges to infinity. Thus we know 1 is not in the set. Applying the rule to -1 we get: 0,-1,0,1,0,.... It is clear that this sequence doesn't diverge to infinity so the -1 is in the set. Another interesting fact about the set is that all points within the set are at most a distance of 2 from the origin. The amazing aspect of the set is the self similarity that occurs as one zooms in. Imagine for instance, zooming in on the boarder of the points in the set and the points outside. No matter how far in you go, you would see smaller versions of the whole set, often called islands, along with other wonderful shapes. One could zoom in infinitely and the whole time see new and clear details.

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Connection to the Logistic Map We can recall that the logistic map is as follows: $x_{n+1} = rx_n(1 - x_n)$. The Mandelbrot set, M, intersects with the real line along the interval $[-2, \frac{1}{4}]$. The values along this interval have a one to one correspondence with the parameters in the logistic map. They are as follows,

$$z = r(\frac{1}{2} - x)$$

$$c = \frac{r}{2}(1 - \frac{r}{2})$$

Another connection is that when plotting the function on the vertical axis, one sees what is essentially the logistic map! It bifurcates at the points where the function is finite.

Plotting

First, we created two vectors, one of x coordinates and another of y coordinates. Then, using the meshgrid function, we created a matrix of the intersection points. Then we applied the iteration function to the set of intersection points. For points that were outside the set, meaning the ones that diverged to ∞, we stored the iteration at which they grew past a boundary such that we could plot the color correctly. The coloring of a point is correlated to the speed of divergence. Then we plotted the function. The figure below is the result.