

Logistic Map

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Basis of Logistic Map

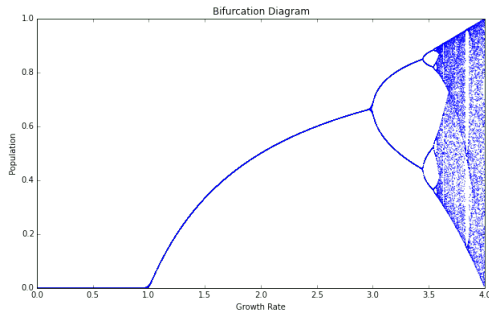
The logistic map function is modeled by the equation pictured below. By its recursive nature, the function takes two inputs and modifies one of them using the previous iteration of one input and the scaling of the other input. While the initial population (x_0) input does not affect the function much, the growth rate (r) input largely determines the shape of the outputs.

$$x_{n+1} = r \cdot x_n \cdot (1 - x_n)$$

The Parameters

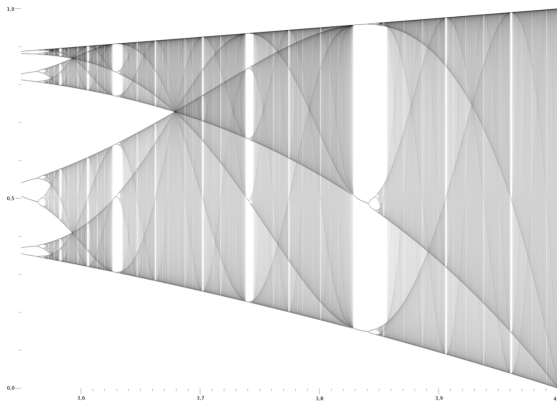
Changing the parameters does change the the outputs but one matters more than the other. The initial condition is the less important one as while in the beginning the outputs are altered, the end behavior converges to the same value. The r value however does wildly change the value of the convergence. For $0 \leq r < 1$, the function will converge to 0. For values $1 \leq r < 3.56995$, the function converges to a single value. However, for values higher than about 3.56, the function cycles through different values. This can be seen in the graphs shown in the following slides.

Bifurcation Diagram



The above image visualizes the values of the function that are visited by the iterations of the function dependent on the initial parameters.

Chaos of the Logistic Map



The above image shows a magnification of the latter end of the previous image, and examines the critical points of the model, especially after the significant value of 3.56995, where chaos begins to become prevalent. Afterwards, there are specific parameter sets produce cyclical iterations that act like simple systems, but these are simply islands of stability.