Lab 6 Report

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Abstract

This lab takes a break from the iterative process we have spent the semester examining and taking a look at continuous functions and calculations. We focused on dynamical systems and understanding them as flows and continuous systems.

From the Book

The simplest ODE we can examine is

$$\frac{dx}{dt} = ax$$

for $x = x(t)$ being a real valued function of a real variable t and dx/dt being its derivative. This means

$$x'(t) = ax(t)$$

Using calculus we obtain the unique solution

$$f'(t) = ake^{at} = af(t)$$

Here k is any constant and can only be specified if given an initial condition of the form $x(o) = K$

If $a \neq 0$ then the equation is stable, meaning a small change in a doesn't result in a massive and chaotic change in the output. We sometimes refer to 0 as the bifurcation point.

For the system

$$x_1'(t) = a_1 x_1$$

$$x_2'(t) = a_2 x_2$$

we can immediately write down the solution following the rule above because the equations are not related.

$$x_1(t) = K_1 e^{a_1 t}$$

$$x_2(t) = K_2 e^{a_2 t}$$

Each of these can be observed visually forming a vector field.

We focus on the study of dynamical systems, where the independent variable is taken as time, t, and the solution as some sort of physical phenomena, such as a particle moving in space. Thus we can graph the position of the particle by

$$\phi_t(u) = (u_1 e^{a_1 t}, u_2 e^{a_2 t})$$

We know that this map is a linear transformation, meaning $\phi_t(u+r) = \phi_t(u) + phi_t(r)$ where $\phi_t(\lambda u) = \lambda \phi_t(u)$ for real numbers λ

For coupled systems, one must find the diagonal form of the system (uncoupled), which can be found by applying a change of coordinates and substituting.

Generalizing, instead of working with two coordinates we work with n differential equations each with n many real number constants. Here we are working in the R^n plane, with n-tuples as coordinates. Addition, scalar product, and size are as defined in a general vector sense. A differential equation in this sense can be defined as

$$x' = Ax$$

where x is the vertical n tuple of all xs and A is the matrix of coefficients.

Working in the gravitational field of the sun, the acceleration vector can be modeled as

$$a(t) = x(t)$$

Newton's second law is

$$F(x(t)) = mx(t)x = F(x)/m$$

Where $F(x)$ will be the force function.

For one dimensional harmonic oscillation we get

$$x + p^2 x = 0$$

with solution

$$x(t) = A\cos pt + B\sin pt = a\cos(pt + t_0)$$

With a being the amplitude, which is the size of vector (A, B). The nonhomogenous version of the system with constant K will have solution

$$x(t) = a\cos(pt + t_0) + \frac{K}{p^2}$$

A two dimensional version expands similarly as we have seen in the two dimensional system mentioned previously.

When the force field is given by

$$F(x) = -(\frac{\partial V}{\partial x_1}, \frac{\partial V}{\partial x_2}, \frac{\partial V}{\partial x_3}) = -\text{grad}V(x)$$

for $V : R^3 \to R^3$ it is called conservative.

The force field $F(x) = -mkx$ gives rise to the planar harmonic oscillation, which is conservative. Our V in this case is the potential energy $V(x) = 1/2mk|x|^2$ The kinetic energy is defined by $T = 1/2m|\dot{x}(t)|^2$. The total energy is defined as $E = T + V$

For a particle in a conservative field, the total energy is independent of time.

Lab

In the lab, we examined how changing the initial conditions in a projectile motion model affects the outcome. As we can see, as one changes the value for k, the resulting plot is not chaotic and can be well understood as the solutions to some differential equations.