

1. Question 1

We know from definition 1.5 in the lecture notes that if A_1, A_2, A_3 are pairwise disjoint, then $P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3)$. Now, let $A, B \in \mathbb{U}$. We can split up $A \cup B$ as $A \cup B = (A - B) \cup (A \cap B) \cup (B - A)$. These are clearly pairwise disjoint. Hence

$$P(A \cup B) = P((A - B) \cup (A \cap B) \cup (B - A)) = P(A - B) + P(A \cap B) + P(B - A)$$

But we know that $P(A - B) = P(A) - P(A \cap B)$ and $P(B - A) = P(B) - P(A \cap B)$. Thus, we end up with

$$P(A \cup B) = P(A) - P(A \cap B) + P(B)$$

2. Question 2

We know from definition 1.9 in the lecture notes that two events are independent if $P(A \cap B) = P(A)P(B)$. Hence, two events are dependent on each other if the equality doesn't hold. Thus, if A and B are exclusive but $P(A) > 0$ and $P(B) > 0$, then $P(A \cap B) = P(\emptyset) = 0$ but $P(A)P(B) > 0$. Thus, they are dependent.