1. 1.3.1
   Since E is a finite set, we can list its contents and say $E = \{x_1, ..., x_n\}$. Then
   $$E = \bigcup_{i=1}^{n} x_i$$
   But each x_i is a singleton and hence closed. Thus E is the finite union of closed sets and is thus closed.

2. 1.3.2
   Let E be a subset of X. If E is the empty set, it is trivially both open and closed. Assume it is non-empty. Then it must contain some element $a \in X$. If we construct a ball with radius $\frac{1}{2}$, we can see that $B_{\frac{1}{2}}(a) \subseteq E$. Thus, since we can construct an open ball around a such that the ball is a subset of E and a was arbitrary, then E is open. Now choose a set $F \subseteq X$ such that $F^c = E$. We know that F must be open. But since it is open, its complement must be closed. Hence, E is also closed.

3. 1.3.4
   By way of contradiction, assume that B is a closed set such that $E \subseteq B \subseteq \bar{E}$ and $B \neq \bar{E}$. This means that B contains all the elements of E but not all of its limit points. But this means that there is at least one convergent sequence entirely in B whose limit point is not in B. But this is a contradiction to the assumption that B is closed. Hence, $B = \bar{E}$.

4. 1.3.5
   First, assume $E \subseteq X$ is closed. Then we know that E^c is open. Now assume that $\bar{x}$ is a limit point of x_n but $\bar{x} \notin E$. This means that $\bar{x} \in E^c$. Since E^c is open, we can find a neighborhood around $\bar{x}$ that is entirely within E^c. But this is a contradiction since it would have no other points from E in that neighborhood, contradicting the fact that $\bar{x}$ is a limit point of x_n. Thus $\bar{x}$ must be in E and thus, E contains its limit points. Now, to prove the other direction, assume that E contains all of its limit points. We will show that E^c is open. Let $a \in E^c$. Since $a \notin E$, it can't be a limit point of E by our assumption. So this means we can find a neighborhood around a that contains no points from E. This means that the ball is entirely contained within E^c and hence E^c is open. Thus E is closed.

5. 1.3.6
   To show that if each $E_k \subseteq X_k$ is open, E would be open, take an element $x = (x_1, x_2, ..., x_m) \in E$. Then we know that we can find balls $B_k = B(x_k, \varepsilon_k) \subseteq E_k$ for each k. Take the minimum radius, $\varepsilon = \min(\{\varepsilon_k\})$ and make a ball $B^* = B((x_1, ..., x_n), \varepsilon)$. Clearly, $B^* \subseteq E$ since it contains elements from each B_k and hence B^* is in E. Thus E is open.
   To show if each $E_k \subseteq X_k$ is closed, then so must E be, we will look at limit points. Since each E_k is closed, they contain all of their limit points. Let $x_n = (x_1^n, x_2^n, ..., x_m^n) \in X$ be a sequence in $E_1 \times E_2 \times ... \times E_m$ that converges to $x = (x_1, x_2, ..., x_m) \in X$. We want to show that $x \in E$. Since the sequence converges, it converges in the product metric.

Thus $d(x_n, x) = \sum_{k=1}^{m} d_k(x_k^n, x_k) \to 0$ as $n \to \infty$. Since the sum goes to 0, so must each individual term, since all the terms are positive. Thus, each $x_k^n \to x_k$. But since $x_k^n \in E_k$ and E_k is closed, we know that $x_k \in E_k$. Thus $x \in E$ and E is closed.

6. 1.3.7
   Since G is open in $\mathbb{R}$, we know that for every $x \in G$, we can find an open interval $I \subset G$ such that $x \in I \subset G$. We set $I = (a_x, b_x)$ such that $a_x = \inf\{y \in \mathbb{R} : (y, x] \subset G\}$ and $b_x = \sup\{y \in \mathbb{R} : [x, y) \subset G\}$. The interval $I = (a_x, b_x)$ is by construction the largest open interval that contains x. These intervals are disjoint for x_1, x_2 parts of different connected components. Clearly, G is covered by the open intervals because each element of G is contained in an intervals. **NEED TO EXTEND TO** R^N

7. 1.4.6

8. 1.4.7

9. 1.4.8

10. 1.4.10

11. 1.4.11