1. Question 1
The second order Legendre Polynomials are given by $P_2(x) = \frac{1}{2}(3x^2 - 1)$. The nodes are the x such that $P_2(x) = \frac{1}{2}(3x^2 - 1) = 0$. Solving for x, we get $x_{1,2} = \pm\frac{1}{\sqrt{3}}$. Since these nodes are found for the interval $[-1, 1]$ but we need them for $[-2, 0]$, we need to transform them to find the new nodes. They would be $x' = \frac{0-(-2)}{2}x + \frac{0+(-2)}{2}$. This would result in $x'_1 = -1 - \frac{1}{\sqrt{3}}$ and $x'_2 = -1 + \frac{1}{\sqrt{3}}$. The weights would be $w_1 = w_2 = 1$ on $[-1, 1]$. On $[-2, 0]$, these would still be $w'_1 = w'_2 = 1$. The integral can now be approximated as $I = w'_1 f(x'_1) + w'_2 f(x'_2)$. Thus we get

$$\begin{aligned} I &= w'_1 f(x'_1) + w'_2 f(x'_2) \\ &= 1 * e^{-(-1-\frac{1}{\sqrt{3}})} + 1 * e^{-(-1+\frac{1}{\sqrt{3}})} \\ &= e^{1+\frac{1}{\sqrt{3}}} + e^{1-\frac{1}{\sqrt{3}}} \end{aligned}$$

2. Question 2
The corresponding RK Scheme would be

$$\xi_1 = y_n + \frac{1}{2}hf(t_n + \frac{1}{2}h, \xi_1)$$

$$y_{n+1} = y_n + hf(t_n + \frac{1}{2}h, \xi_1)$$

To find the relationship between the two, we can multiply ξ_1 by 2 and then subtract from y_{n+1}. This gives us

$$\begin{aligned} 2\xi_1 - y_{n+1} &= 2y_n + hf(t_n + \frac{1}{2}h, \xi_1) - y_n + hf(t_n + \frac{1}{2}h, \xi_1) \\ &= y_n \end{aligned}$$

This implies $2\xi_1 = y_n + y_{n+1}$ or that $\xi_1 = \frac{1}{2}(y_n + y_{n+1})$. This means that

$$y_{n+1} = y_n + hf(t_n + \frac{1}{2}h, \xi_1) = y_{n+1} = y_n + hf(t_n + \frac{1}{2}h, \frac{1}{2}(y_n + y_{n+1}))$$

as intended. Thus, this gives us the same information as we had originally.