

1. Question 1

For this ODE, we can solve it using Separation of Variables because it is of the form $\frac{dy}{dt} = f(t)g(y)$ where $f(t) = 1$ and $g(y) = 3y + 1$. Applying the formula, we get

$$\int \frac{dy}{3y+1} = \int 1dt$$

which integrates to

$$\frac{1}{3} \ln(3y+1) = t + C_1$$

Simplifying, we get

$$\frac{e^{3t+C_2} - 1}{3} = \frac{C * e^{3t} - 1}{3}$$

where C_1, C_2, C are all constants.

2. Question 2

This equation can be solved using the characteristic equation. $F(\lambda) = \lambda^2 + 2\lambda + 4 = 0$. Solving this quadratic, we get

$$\lambda = \frac{-2 \pm \sqrt{2^2 - 4 * 1 * 4}}{2 * 1} = \frac{-2 \pm 2i\sqrt{3}}{2} = -1 \pm i\sqrt{3}$$

Since these are complex, the solutions to the ODE are $e^{-1t}(c_1 \cos(\sqrt{3}t) + c_2 \sin(\sqrt{3}t))$ where $c_1, c_2 \in \mathbb{R}$.

3. Question 3

Applying the same process as above,

$$\lambda = \frac{-4 \pm \sqrt{4^2 - 4 * 1 * 4}}{2 * 1} = \frac{-4 \pm 0}{2} = -2$$

with multiplicity two. Thus, the general solution is $y(t) = c_1 e^{-2t} + t c_2 e^{-2t}$. But we can use the initial values to figure out what the constants are. $y(0) = c_1 e^{-2*0} + 0 * c_2 e^{-2*0} = c_1 = 1$. So $c_1 = 1$. Taking the derivative, we get that $y'(t) = -2e^{-2t} + c_2 e^{-2t} - 2c_2 t e^{-2t}$. Thus, $y'(0) = -2e^{-2*0} + c_2 e^{-2*0} - 2c_2 * 0 * e^{-2*0} = -2 + c_2 = 3$ so $c_2 = 5$. Thus, the particular solution is $y(t) = e^{-2t} + t * 5e^{-2t}$.