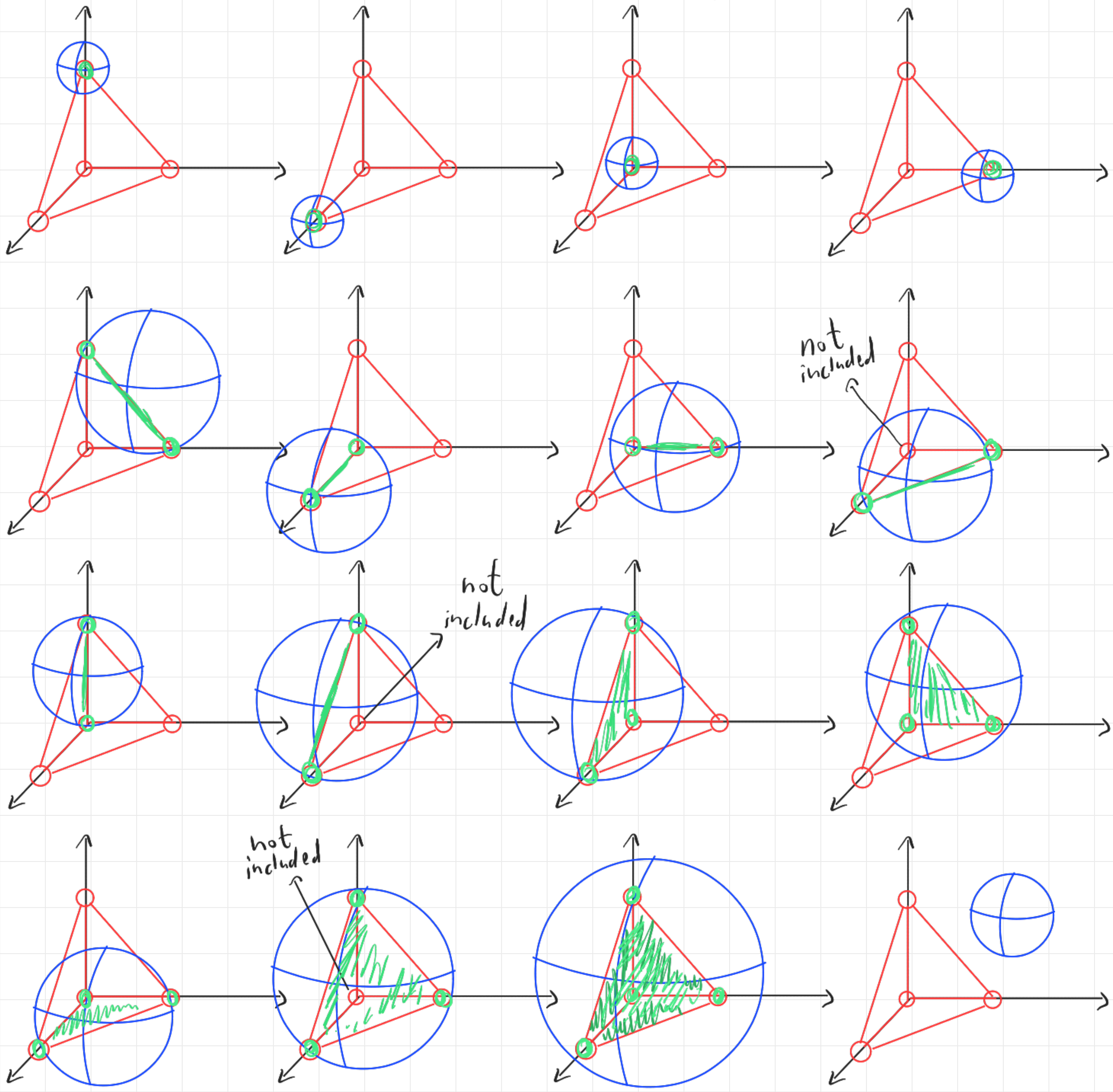
**Problem 1.**

a) What is the VC-dimension of the infinite set of uni-directional balls on three dimensional points? Prove your result.

1a Given the set $B$ of uni-directional 3D balls (assuming inside-green, and outside-red) over the 3D space, we shall prove that this set's VC-dimension is 4.

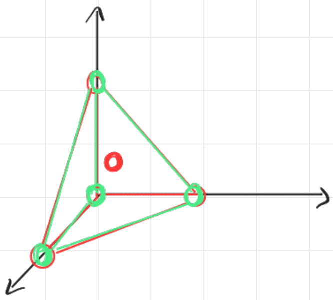
* Shattering an example of 4 points (and therefore proving that the set also shatters 1, 2, 3 points):



* Proof there's no 5 points B shatters:

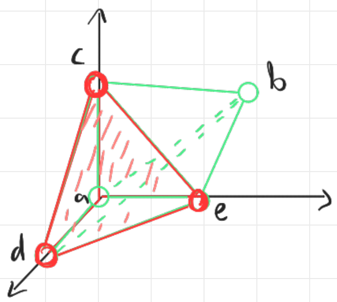
In this proof we will refer to the 2 general cases for 4 points and an additional point:

1) The additional point is inside an arbitrary convex hull made by the 4 points:



There is no way to exclude the red point since any of the infinite no. of points contained in the convex hull made by the 4 green points will always be contained in a Ball from B that contains them.

2) The opposite of one:




Due to the property of a ball's radius, which makes a ball grow at the same length at all directions from it's center, there is no way to include both a and b without also including c, d, e.
An ovoid, for example, which has the main properties of a ball but lacks the property above, can shatter these points.

b) Hypothesis class **C** contains both the infinite set of uni-directional balls and the infinite set of hyperplanes on three dimensional points. What is its VC-dimension? Prove your result.

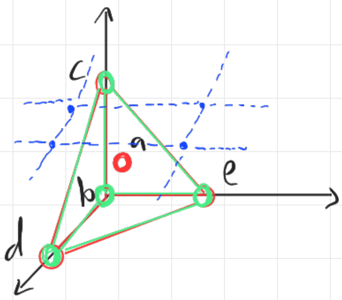
(1b) The VC dimension of C will also be 4.

To prove this, we will go over why uni-directional hyperplanes cannot shatter any 5 points.

Proving that the addition of the uni-directional hyperplanes does not help solve any of the 2 general cases shown in 1a, will suffice in order to conclude that they do not add classifying power to B's.

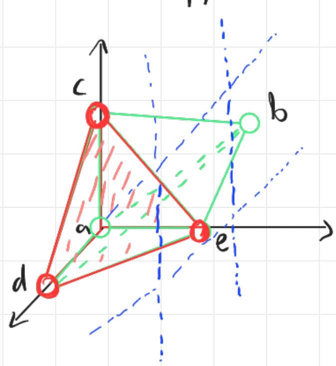
Since we've proved B cannot solve them, proving that they also can't will suffice.

1) The additional point is inside an arbitrary convex hull made by the 4 points:




A hyper plane in the 3D plane can cut at most 3 sides of a pyramid.
Therefore there is no way to place a hyperplane that has the red point at one side of it without also having at least one green point at the same side, therefore this case can't be solved for this classification of the points.

2) The opposite of one:




For the same reason as case 1, this case cannot be solved.

c) Hypothesis class **D** contains the infinite set of half-balls on three dimension points. Give an upper bound on its VC-dimension.

1c We can see that a half-ball is really a union between a ball and hyperplanes over the 3D space.

We've also seen throughout the lectures that the VC-Dimension of hyperplanes in the 3D space is 4 (because $\dim = 2 \rightarrow D = 4$)

Since we've also showed in this exercise that B's VC-dimension is 4 we can utilize the Union of Hypothesis as seen in our course to create a union over these 2 sets of rules and get the following bound:

$$VC\text{-}dim(D) \leq 2 \cdot d \cdot s \cdot \log_2(3 \cdot s) = 2 \cdot 4 \cdot 2 \cdot \log_2(6) \sim 41.35$$

d) How many different labels can **D** give to 100 points? Give an upper bound.

1d Using Sauer's Lemma and the Union of Hypotheses, the bound $\Pi$ of $D$ over set $P$ of 100 points is:

$$\Pi(D, P) \leq \left(\frac{e \cdot n}{d}\right)^{d \cdot s} = \left(\frac{100e}{4}\right)^{4 \cdot 2} = (25 \cdot e)^8$$

DEMIZ DESIGN