**INTRODUCTION**

Energy is a common necessity for both commercial and residential purposes and is a major component for both industrial and household budgets. Electrical energy is mainly produced in power plants that use turbines, which are engines that convert fluid flow into work, in order to produce energy. The two main sources of energy in the United States are Coal and Crude Oil, aka liquid petroleum.

Coal is used to produce over 50% of the electricity in the United States.  If consumption continues at the same rate, the current reserves will last for more than 200 years. The burning of coal results in significant atmospheric pollution. The sulfur contained in coal forms sulfur dioxide when burned.  Harmful nitrogen oxides, heavy metals, and carbon dioxide are also released into the air during coal burning. The harmful emissions can be reduced by installing scrubbers and electrostatic precipitators in the smokestacks of power plants. The toxic ash remaining after coal burning is also an environmental concern and is usually disposed into landfills.[1]

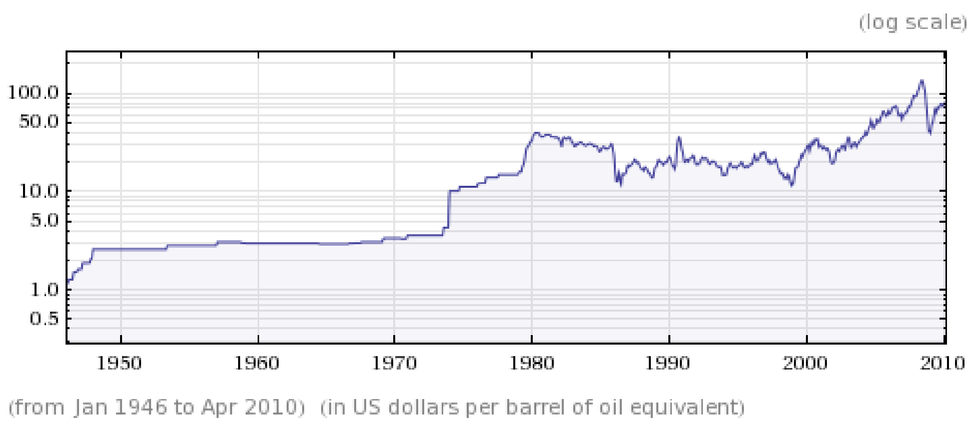
Despite its limited supply, oil is a relatively inexpensive fuel source. It is a preferred fuel source over coal. An equivalent amount of one gallon of oil produces more kilowatts of energy than coal. It also burns cleaner, producing about 50 percent less sulfur dioxide. The burning of oil releases atmospheric pollutants such as sulfur dioxide, nitrogen oxides, carbon dioxide and carbon monoxide. These gases are smog-precursors that pollute the air and greenhouse gases that contribute to global warming. Another environmental issue associated with the use of oil is the impact of oil drilling. Substantial oil reserves lie under the ocean. Oil spill accidents involving drilling platforms kill marine organisms and birds. Some reserves such as those in northern Alaska occur in wilderness areas. The building of roads, structures and pipelines to support oil recovery operations can severely impact the wildlife in those natural areas.[1]

It takes millions of years for coal and oil to be produced in nature, and we are burning it faster than it is being made.  Because of this, we are projected to used up these sources. Scientists have developed alternative resources in order to use renewable energy to produce electricity.  This also helps to reduce the amount of emissions given off from the burning of fossil fuels.

Global warming is a serious concern. The concern ranges from climatologists to average citizens who partake in environmental activism. The obtaining and burning of fossil fuels is harmful to our environment; for this reason, many individuals and companies are “going green” to reduce the amount of energy they consume from oil and coal plants.

Another main factor why people seek out alternative energy sources is the fact that the prices of oil and coal have been rising over the course of the years. It’s a simple supply and demand situation, the demands will always increase as manufacturers produce more elecronic devices with high energy consumputions. Since the supply is getting lower the price is going to consistently increase which makes energy and electricity a bigger factor in people’s budgets. The financial burden that comes from the use of oil and coal plants is a big reason why people are looking for alternative energy sources which are less expensive than the current sources.  Below is a graph of the price of per barrel of oil which shows its growth over time.

**Table 1:  US dollars per barrel of oil equivalent[2]**




There are many different sources of alternative energy such as hydroelectric, geothermal, wind and solar power. Hydroelectric power plants produce electrical power through the use of the gravitational force of falling or flowing water. Geothermal energy uses energy from the Earth's core to vaporize water into steam.  This steam is then used to turn turbines and generate electricity.  The reason geothermal and hydroelectric plants are not readily used is because they require a large amount of land and can only be used in specific areas of the planet. Hydroelectric power can only be used at the mouths of rivers and in the ocean; geothermal can only be used in areas of the planet where the molten rock from the inner layers of the Earth is close enough to the surface to be utilized.

The main two sources of alternative energy to be studied in the report are wind and solar energy. Wind turbines that uses wind to produce energy must have a minimum of 7.5 miles per hour. Since wind is renewable resources and most areas of the Earth experience proper wind conditions, wind-powered turbines are becoming more common as an energy source. Solar energy is a renewable energy source that has been developed over the last few decades. Solar energy requires the use of special fuel cells called photovoltaic cells, which when struck by sunlight, cause chemical reactions to occur and generate electricity.  These two technologies are the most convenient means of alternative energy for small companies and even residential applications because of their relatively low cost and space required.

**Problem Statement**

A company is looking to open a new five thousand square foot factory to produce its product.  The company has hired a group of consultants in order to analyze how they may save money by adapting alternative energy means at four selected cities. These four cities are: New York City, Phoenix, Seattle, and Chicago.  The consultants are to compare the energy savings from both solar panels and wind turbines in order to give a final suggestion as to where the company should build their new facility.

## MATHEMATICAL MODEL

We first describe our methodology. The first step was to obtain specifications on solar panels and wind turbines. The turbines selected for the analysis are 1.3 kW Wind Turbine Systems manufactured by Raum Energy.[3] The specification sheets included in the appendix show the correlation of wind speed to energy produced. The solar panel data was obtained from PVWatts, this data was based on a 4kW solar panel system.[4] The data collected from this website was used to find a correlation of how much energy is produced as a function of solar radiation. These two parameters are essential in calculating energy produced.

The variables that are essential for this model are average wind speed, average solar radiation, and average temperature. These values were all obtained as a function of the month of the year. The average wind speed for a given month is used to calculate the energy a wind turbine can produce in a given month, and likewise solar radiation for solar panels. Average temperatures were obtained because solar panel efficiency is a function of temperature, and assume 100% efficiency throughout the course of a year. The data collected for each variable and for the each of the four cities being studied. If the weather is too hot it will lead to inefficient solar panels which would mean less energy is saved.

There are some other variables which are being excluded. Some of those variables include wear and tear of a turbine, cleanliness of the solar panels and proper maintenance of the equipment. This is because at the time of the initial installation of the equipment we assume 100% working equipment, with no need for cleaning or maintenance. Of course over the course of the years the efficiency of the equipment may decrease because of these factors, but for all intents and purposes we can exclude them to show the maximum potential of the equipment.

### *Assumptions*

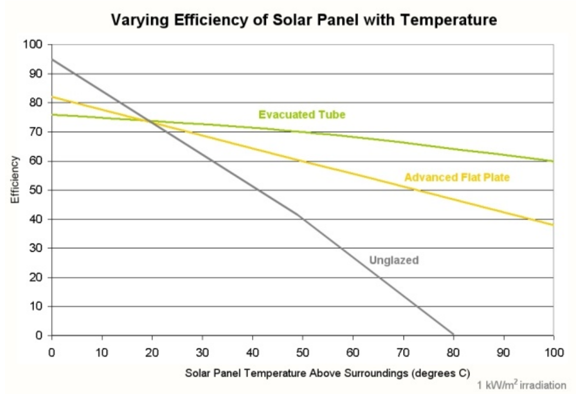
The assumptions made in this model are: the only sources of energy availables are solar or wind, the cost of the solar panels and wind turbines are the same in all areas, the position of the factory in each location is exactly the same in relation to the sun, and that an average speed of 7.5mph is needed for wind turbines to work. The reason for the restiction to solar and wind power is because of the limited areas in which hydroelectric and geothermal energy sources can be found. This assumption generally means that the only alternatives considered are from the wind and the sun. We did not wish to adohers the variations in cost as that is more of a political question.

***Methods***

Our mathematical model is for the most part fairly simple.  After compiling a large array of data points for different variables, the FindFit function of Mathematica to fit these data points as functions.  The collected data were monthly averages; the fitting allowed us to have a smooth function over the years which allowed us to integrated the function. The area under the curve is a measure of the yearly amount of energy produced in kilowatt hours.

We used Mathematica to first plot the data points and then obtain a suitable function for the set of data points.  After plotting the data points, one must inspect the points and determine what type of correlation these data points seem to follow.  The first set of data points we analyzed was how solar radiation is converted into actual energy produced. Using the data found in the Appendix A, a function for the curves of radiation was found by plotting the data and using Mathematica to do a linear regression. The data  was checked to make sure that it fit the initial data points and the function. The next step was to use the data from the radiation function and convert it into a function that would show the amount of energy that could be produced by a solar panel. The data from the radiation function was plotted again and another linear regression was done to the function to determine the amount of energy from the radiation called the solar energy function. Once the new data for the solar energy function was determined it was plotted to show how the graph matched the data points.

The next step was to determine a formula for the temperature in each city. The data was plotted and a temperature function was obtained again via Mathematica's FindFit command. Once the fit determined the constants a temperature function was made for each city and the function was graphed along with its data points. This temperature function is used in order to determine the efficiency of a solar panel.  The efficiency of a solar panel is a linear function which was obtained from the following graph, varying efficiency of solar Panel with Temperature.[5]




This function was visually obtained to be:

$$\text{Efficiency} = -4/9\ T + 82 \qquad (1)$$

The function would require the conversion of Fahrenheit to Celsius and also into decimal form, this is because the temperature profiles for the cities are obtained in Fahrenheit.

As one may notice, there are functions which are dependent on other functions. These functions were then combined into one function and plotted. A function of solar energy obtained from solar panels was determined for each city and a plot of this data was made.

The next step was to determine the amount of energy produced from wind. The wind speeds obtained was put into a graph with data from a BWC Class turbine and then was fitted in order to produce a function. The function was then plotted against the initial data points to produce a graph, to ensure its validity. In order to calculate the amount of energy a city can produce from wind speed, one must obtain a function of the monthly behavior of wind speed for a specific city.  The same routine above was done here for each city. The two formulas were combined into one in order to express the energy produced from wind from each city at a given time in the year.

Once the energy from solar panels and the energy from wind were determined, the two formulas summed in order to observe what would happen if the factory had both solar and wind energy. The formulas were plotted together and then Mathematica was used to determine the area under the curve, which represents the yearly annual energy production of both a solar panel and wind turbine. A comparison of increasing the number of solar panels and wind turbines was done to show how one may obtain a certain desired energy output.

**RESULTS**

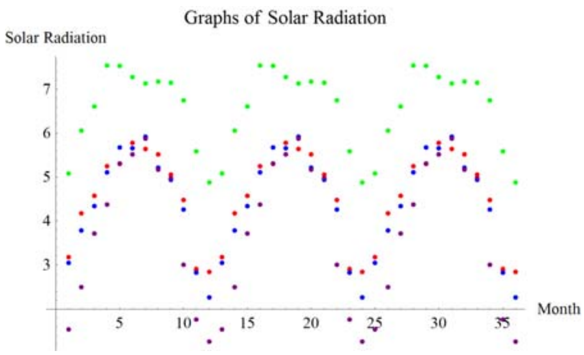
Legend:

Red: New York City, NY Green: Phoenix, AZ

Blue: Chicago, IL Purple: Seattle, WA

***Solar Panels***

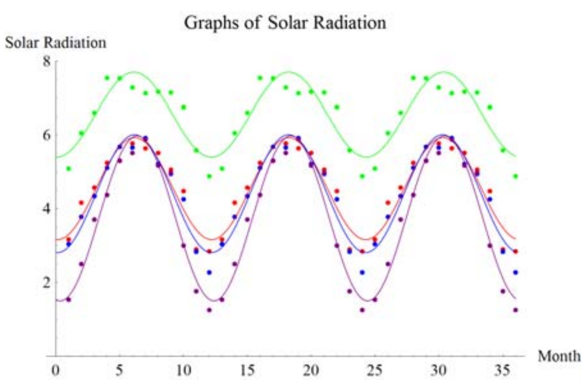
The plot of incoming solar radiation from the data obtained from NREL gave the following Graph 1:




After Plotting the initial data points for the incoming solar radiation, from NREL, a fit was done to the line. The constants for the function to these graphs were found using mathematica (see appendix) and the function that represents the best fit for these graphs was:

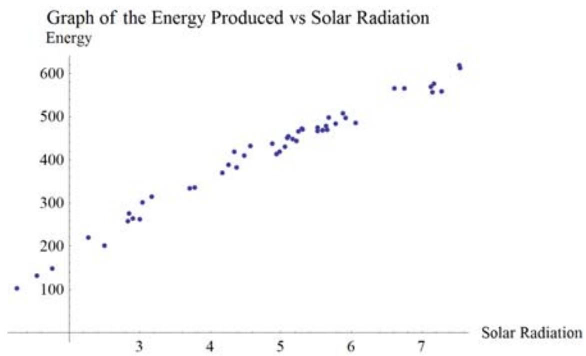
$$y = A + S \, \text{Sin}[C(x-D)] \qquad (2)$$

The Mathematica plot for the data about the incoming solar radiation for each city gave the following Graph 2:

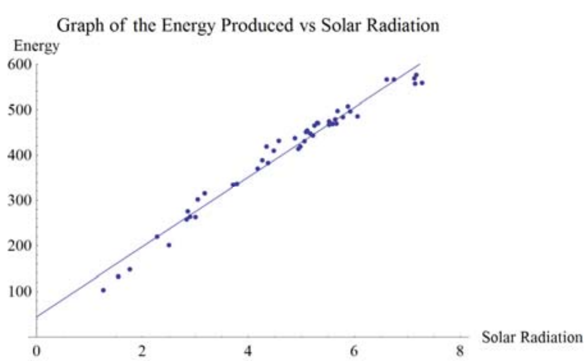


Graph 2 shows a parabolic curve that very nicely fits the data for each city. It does however make derivations for the cities of Phoenix and Seattle. The trend of the graph shows that in the winter months there is less radiation than in the summer months, this result was expected. There is actually one problem with the Seattle's (purple) function obtained, this is that there is 0 radiation in the month of January. We acknowledge this, but will not correct this because of the little impact it will have on the results.

The information obtained above, can be used to obtain a particular solar radiation for a given month. After having this value, one may use the graph below in order to obtain a value of how much that particular month can produce in actual energy. The data points for the conversion of solar radiation into energy produced was obtained from NREL: Renewable Resource Data Center[4], (see ACEnergy in appendix). The plot of the data points gave the following Graph 3.




Since the points in Graph 3 seem to follow a linear correlation, a linear fit was done to evaluate the constants and give a function for the amount of energy you can get from solar panels. The graph of the fitted function is shown below in Graph 4:

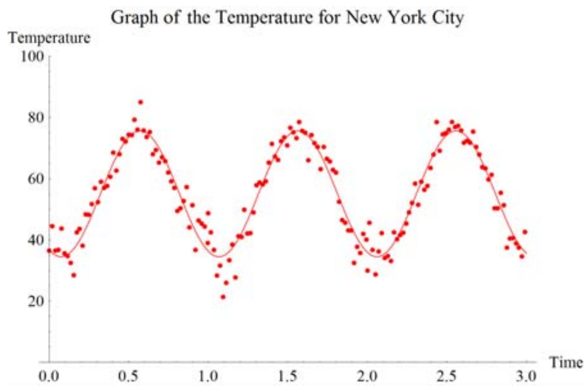


The line is a fairly good representation of the data points, there are many data points which are outside the region of the line but for all intent and purposed this line fits the data well enough to utilize in the investigation.

The temperatures of all four cities were the analyzed, this would be used to calculate the efficiency. The temperature curves of all 4 regions behaved with sinusoidal characteristics and therefore the general form of the sin function was used to fit these data points. The general form of a sin function is as follows.

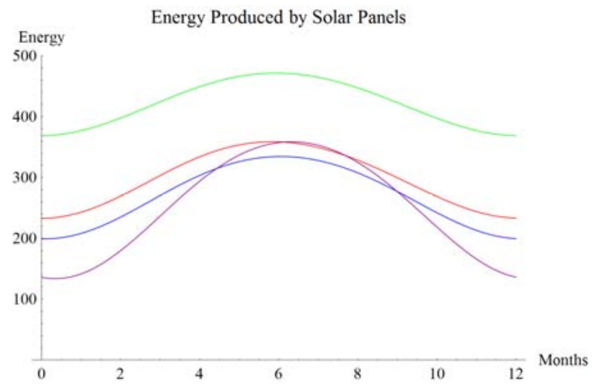
$$\text{Temperature} = A + B\,\mathrm{Sin}[C\,(x - D\,)] \tag{3}$$

A plot of the temperature profile of New York City was then plotted to ensure Mathematica had obtained the correct coefficients. The graph of the temperature profile of New York City is shown below in Graph 5:




Plotting the functions along with the data points is extremely important. Mathematica initially obtained coefficients which did not fit the data points and if this step was not done, the results would have not been accurate. This problem was solved by inputting initial guess values of the coefficients in order for Mathematica to converge its solution to the appropriate values.

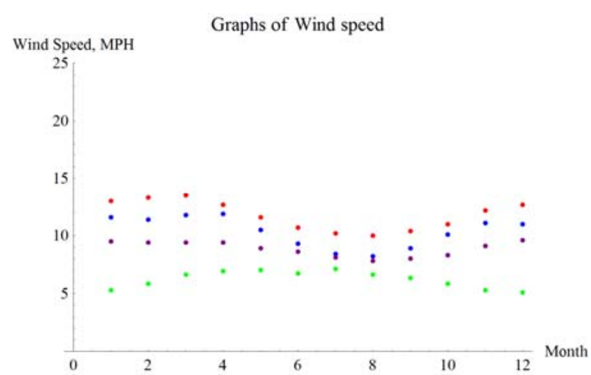
All data has now been properly fit and the energy produced by a solar panel can be compiled. Plotting the compiled function shows the energy produced from a solar panel at a given time in the year, this can be seen in Graph 6.



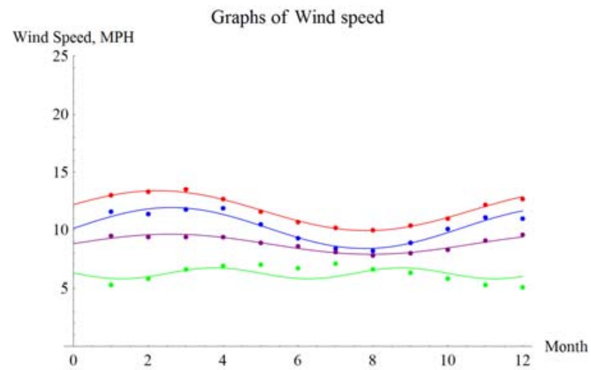

Once again, one may notice that the optimal time for solar panels is during the summer months where the sun is more directly overhead and more solar radiation hits the Northern Hemisphere.

***Wind Energy***

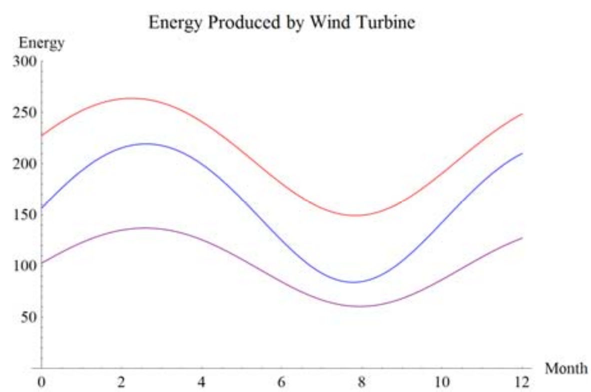
Using the data provided by NREL, the average wind speeds in the cities of New York, Phoenix, Seattle and Chicago.[6] The data was put into mathematica and then put into a graph in order to set-up a function that would best fit the graphs of the average wind speed of each city. The plot of the data gave the following graph, Graph 7:




This graph also seems to have a sinusoidal trend and therefore the general form of the Sin function is used to fit this data.  Below is a graph of the fitted functions with the data points, Graph 8.




The function above fit very nicely for 3 of the 4 cities. To our advantage, Phoenix (green), does not maintain the required 7.5mph needed to initially start the turbine. Therefore, we can exclude Phoenix from the wind turbine results. We can now show a graph of how much energy is produced over the course of a year from a wind turbine, as shown below in Graph 9.

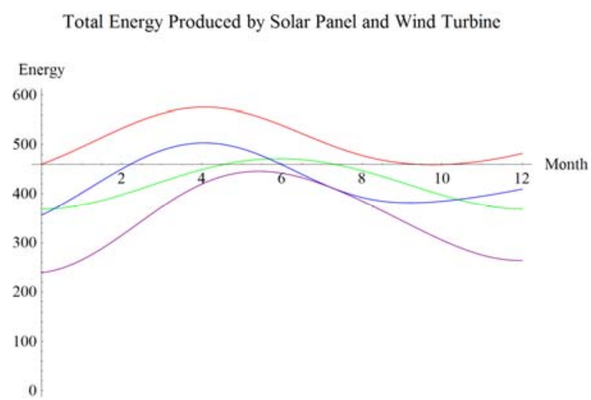



New York City, Seattle, and Chicago were the only three cities that could generate wind energy. This data is very nice, from what was obtained from the solar panels, one may notice in the summer months and for this set of data one may notice higher outputs in the winter months. This can be used to balance the yearly annual energy production close to a desired value for each month.

The two energy production plots can now be combined in order to show the total energy production. Below is an excerpt from Mathematica to show this.

$$Energy_{total} = Energy_{wind} + Energy_{solar} \tag{4}$$

The graph of the total energy produced is shown below in Graph 10.




The area under the curve is the total yearly energy production of both one solar panel and one wind turbine. Mathematica gave errors with the direct approach to integrate the function, and therefore the Mote Carlo simulation within Mathematica was used to find the area under the curves. The following table shows how much money the company can save by using either solar or wind energy, the units are Unites States Dollars (USD):

| City | Solar Savings, USD | Wind Savings, USD | Combined Savings, USD |
|---|---|---|---|
| NEW YORK | 543.975 | 391.215 | 935.19 |
| PHOENIX | 428.961 | 0. | 428.961 |
| CHICAGO | 257.081 | 150.923 | 408.004 |
| SEATTLE | 217.417 | 86.5728 | 303.99 |

This table suggests that the best place to put the new factory is in New York City because it has the highest amount of money that can be saved by both solar and wind energies.

The maximum number of solar panels for the given roof area is 12 and the maximum number of wind turbines is 10. Below is a table showing the year savings with varying solar panels and wind turbines. The Top axis is Solar Panels and the Side Axis is the Wind Turbines.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 935.19 | 1479.16 | 2023.14 | 2567.11 | 3111.09 | 3655.06 | 4199.04 | 4743.01 | 5286.99 | 5830.96 | 6374.94 | 6918.91 |
| 2 | 1326.4 | 1870.38 | 2414.35 | 2958.33 | 3502.3 | 4046.28 | 4590.25 | 5134.23 | 5678.2 | 6222.18 | 6766.15 | 7310.13 |
| 3 | 1717.62 | 2261.59 | 2805.57 | 3349.54 | 3893.52 | 4437.49 | 4981.47 | 5525.44 | 6069.42 | 6613.39 | 7157.37 | 7701.34 |
| 4 | 2108.83 | 2652.81 | 3196.78 | 3740.76 | 4284.73 | 4828.71 | 5372.68 | 5916.66 | 6460.63 | 7004.61 | 7548.58 | 8092.56 |
| 5 | 2500.05 | 3044.02 | 3588. | 4131.97 | 4675.95 | 5219.92 | 5763.9 | 6307.87 | 6851.85 | 7395.82 | 7939.8 | 8483.77 |
| 6 | 2891.26 | 3435.24 | 3979.21 | 4523.19 | 5067.16 | 5611.14 | 6155.11 | 6699.09 | 7243.06 | 7787.04 | 8331.01 | 8874.99 |
| 7 | 3282.48 | 3826.45 | 4370.43 | 4914.4 | 5458.38 | 6002.35 | 6546.33 | 7090.3 | 7634.28 | 8178.25 | 8722.23 | 9266.2 |
| 8 | 3673.69 | 4217.67 | 4761.64 | 5305.62 | 5849.59 | 6393.57 | 6937.54 | 7481.52 | 8025.49 | 8569.47 | 9113.44 | 9657.42 |
| 9 | 4064.91 | 4608.88 | 5152.86 | 5696.83 | 6240.81 | 6784.78 | 7328.76 | 7872.73 | 8416.71 | 8960.68 | 9504.66 | 10048.6 |
| 10 | 4456.12 | 5000.1 | 5544.07 | 6088.05 | 6632.02 | 7176. | 7719.97 | 8263.95 | 8807.92 | 9351.9 | 9895.87 | 10439.8 |

As one may notice the maximum total yearly savings for this facility in NYC is $10,439.8.40.

**DISCUSSION**

This model is useful for people and businesses trying to examine which alternate energy sources they should use and how much money they can save from alternate energy. Qualities which make a model if it has its strengths, is descriptively realistic, and if it is fruitful. The strengths of the model rely on the amount of data obtained and how well the functions fit the data.

This model contains a wealth of information that was obtained form many sources. A main component of the data was from the National Renewable Energy Lab but other data were collected from other sources and also from phone interviews. This large amount of data was useful because it was used to ensure there was a correlation, and this data helped to eliminate false data which could have affected our results. The fact that there was no false information and that the data correlated with each other meant that the results we obtained were accurate.

A common theme of this model was how well a function fit the data provided. Mathematica was the main tool in providing these fits, but the function fit the data accurately meant that there was an obtainable answer .Since there was an obtainable answer to the problem statement it means the final answer obtained was suitable and correct.

The model gave a definite answer to the problem statement is a strength because it tells the company exactly how much money it will save by using these forms of alternative energy. Some models are designed to give a function as an answer to a problem and some may giving an array of answers. The problem that would occur if this model had those same results would be that it might be unclear as to how much a company could save using either wind or solar energy. Another issue would be how much money a company could save using different scenarios which would mean multiple linear approximations. The problem with multiple linear approximations would be that the errors that can occur from doing a linear approximation such as miscounting, over counting or round-up errors would occur multiple times. This means the model solving which city is the best city for the company to build a new factory has a simple uncomplicated answer.

Even though this model has strengths it also has some weaknesses to it. The nature of the modeling process tends to lead to some weaknesses because of the multiple steps and techniques that are used during the process. Some of the weaknesses of this model are the visual approximation of efficiency, only looking at specific cities rather than regions, and also possible errors made in the modeling process. This model has many regressions which are fit to data, each regression introduces some error. As the amount of regressions increase, the amount of errors also increases.

The main problem with the assumptions made in the modeling process that some had to be made in order for the model to make sense. The assumption that was too simple was the assumption that solar panels and wind turbines cost the same in each city. The fact that different cities tax solar panels and wind turbines differently needs to be considered in a more thorough analysis been taken into account. The taxes applied to solar panels and wind turbines are variable on city and state levels of government in addition, there are permits and other regulations imposed by different city governments that add to the cost of solar and wind energy. These costs would have had to be deducted from the amount of money saved from solar and wind energy but were left out because of the assumption made in the beginning of the modeling process.

Another weakness of the model is only very small and specific areas were looked at in the model. Instead of looking at the entire southwest or northeast, the cities of Phoenix and New York were chosen to represent them. Data could have been obtained for each region of the United States and that could have been modeled to make a recommendation for where the company should build its factory so it can get the most profit from alternative energy. This would have meant a lot more work and data collection would have to be done and it is a recommendation of what can be done next with this model.

Errors can occur in the modeling process and tend to weaken the model because some of the errors lead to inaccurate results. The error that could have occurred in this model was an error in data collection. The amount of data collected was substantial but there is the possibility that all the data does not reflect what is going to occur in the future. Another possible error in the data could be manipulation of data. Sometimes data is manipulated in order to progress someone's own political or social views and that leads to inaccurate data being distributed.

The model is realistic because the amount of incoming solar radiation and temperatures of the cities are items that occur in the real world and are measured accurately. The model is also realistic because the installation of wind turbines and solar panels are becoming more common in the United States and the assumptions the model makes are realistic to the process of building solar panels and wind turbines. The model is not descriptively realistic because there was no formulation of equations. The equations obtained were based on data points collected. A model does not have to be descriptively realistic in order to be a good model; in fact there may be some descriptively realistic models which are not as good.

This model can be considered fruitful because the results are useful and this model can be used to make other good models. The results are useful to anyone who is looking to use alternative energy in each of the four cities that were evaluated in this model. It can also be useful to the governments of those cities to see how much money their citizens can save with wind and solar energies and could lead to them advocating their citizens' switch to these sources. The way the governments can endorse these forms of energy would be to grant tax credits or grants to those citizens that want to buy wind turbines or solar panels. This model could also be useful to current businesses within each of the four cities evaluated and other companies looking to open up a new factory. Businesses that are currently in one of the four cities in the model can find out how much money that company can save by switching to alternative energy. The businesses could find a way to start budgeting properly in order to buy solar panels and wind turbines which would eventually save money. Companies that are looking to open a new factory can look at this model in order to get an idea of how much they can save if they open up a factory in one of the four cities and plan to use alternative energy.

This model can be used to make future models such as the regional model that was discussed earlier. This model could also be used in a model that could determine which city is the best city to choose for the maximum amount of alternative energy and the maximum amount of profit from alternative energy. The model can also be adapted to account for other sources of alternative energy such as geothermal and hydroelectric power. These ideas for future models based off the general idea of this model are a reason this model is very fruitful.

## CONCLUSION

Since energy costs from sources such as oil and coal are bound to increase it is essential to look into alternative sources of energy. This model looked at only two sources of alternative energy and found that both of them can save money and cut down on the amount of energy consumed from oil and coal. Functions for solar energy and wind energy produced were found using mathematica and these functions were used to evaluate the total amount of energy produced from both sources. A Monte Carlo simulation was done on the graph of the total amount of energy produced by wind and solar energy to find which city had the most energy produced by solar and wind. The model showed that New York City was the best spot for a company to build a factory solely for the purpose of maximize savings from alternative energy.

**ABSTRACT**

The prices of oil and coal energies have been rising and are bound to keep rising due to the fact that they are not renewable energy sources. The best way to alleviate the cost of these energy sources is to look into alternative energy. A company is looking to open a new factory in either New York City, Phoenix, Seattle, or Chicago and wants to know which city can save the most using solar panels and wind turbines. Data was collected about incoming solar radiation and average wind speed in each of the cities and each data plot was fitted with a function to determine the amount of solar and wind energy produced. The functions for wind and solar energy were plotted for each city and a Monte Carlo simulation was run the find which city produced the most alternative energy and how much the company can save. The result was New York City produced the most energy form solar panels and wind turbines and could save a maximum of $10,343.40 per year.

## REFERENCES


1. *Connexions - Sharing Knowledge and Building Communities*. University of California College Prep, University of California. Web. 24 Apr. 2010. <http://cnx.org/content/m16730/latest/>

2. *WolframAlpha: Computational Knowledge Engine*. Wolfram Alpha LLC—A Wolfram Research Company. Web. 24 May 2010. <http://www.wolframalpha.com/input/?i=oil+price>

3. *Raum Energy | Saskatoon, SK | Wind Turbines, Inverters and Towers*. Web. 24 Apr. 2010. <http://www.raumenergy.com>.

4. "NREL: Renewable Resource Data Center - PVWattts." *National Renewable Energy Laboratory (NREL) Home Page*. Web. 24 Apr. 2010. <http://www.nrel.gov/rredc/pvwatts/>.

5. *Viridian Solar Roof Integrated solar water heating panels*. Web. 24 Apr. 2010. <http://viridiansolar.co.uk>.

6. Dellinger, Dan. "Wind- Average Wind Speed- (MPH)." *NCDC: * National Climatic Data Center (NCDC) **. Web. 24 Apr. 2010. <http://lwf.ncdc.noaa.gov/oa/climate/online/ccd/avgwind.html>.