Bone Cancer Biopsy Navigation for CISC 330

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1) Tool Tip Calibration

Note: both bonus questions were completed for this portion of the assignment, resulting in 3 total solutions.

All solutions produced the expected output:

```
-------------Running Tool_Tip_Calibration_Testing Script-------------
Expected ground truth tool tip coordinates: [0, -20, 0]
Calculated tool tip coordinates with Moore-Penrose: [0.0000, -20.0000, -0.0000]
Calculated tool tip coordinates with sphere fitting: [-0.0000, -20.0000, -0.0000]
Calculated tool tip coordinates with plane intersection and Moore-Penrose: [-0.0000, -20.0000, 0.0000]
```

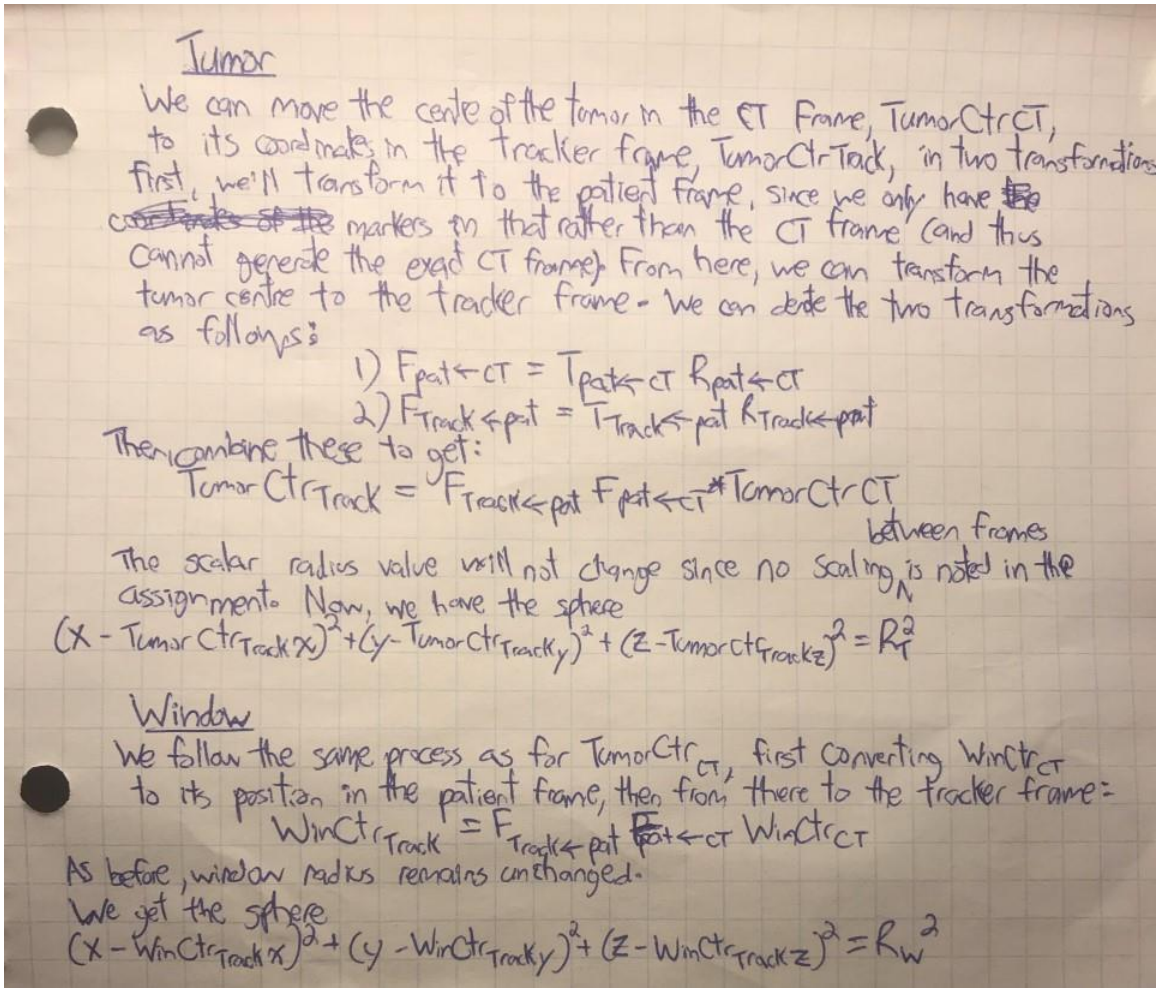
2) Tool Axis Calibration

Program produced the expected output:

```
-------------Running Tool_Axis_Calibration_Testing Script-------------
Expected ground truth tool axis direction vector: [0, 1, 0]
Calculated tool axis direction vector: [-0.0000, 1.0000, -0.0000]
```

3) Surgical Navigation

Derivation of Math for program



Tumor

We can move the centre of the tumor in the CT Frame, TumorCtrCT, to its coordinate in the tracker frame, TumorCtrTrack, in two transformations. First, we'll transform it to the patient frame, since we only have the coordinates of the markers on that rather than the CT frame (and thus cannot generate the exact CT frame). From here, we can transform the tumor centre to the tracker frame. We can denote the two transformations as follows:

$$1)\ F_{pat \leftarrow CT} = T_{pat \leftarrow CT} R_{pat \leftarrow CT}$$
$$2)\ F_{Track \leftarrow pat} = T_{Track \leftarrow pat} R_{Track \leftarrow pat}$$

Then, combine these to get:

$$TumorCtr_{Track} = F_{Track \leftarrow pat} F_{pat \leftarrow CT} * TumorCtrCT$$

The scalar radius value will not change since no scaling between frames is noted in the assignment. Now, we have the sphere

$$(X - TumorCtr_{Track\,x})^2 + (y - TumorCtr_{Track\,y})^2 + (z - TumorCtr_{Track\,z})^2 = R_T^2$$

Window

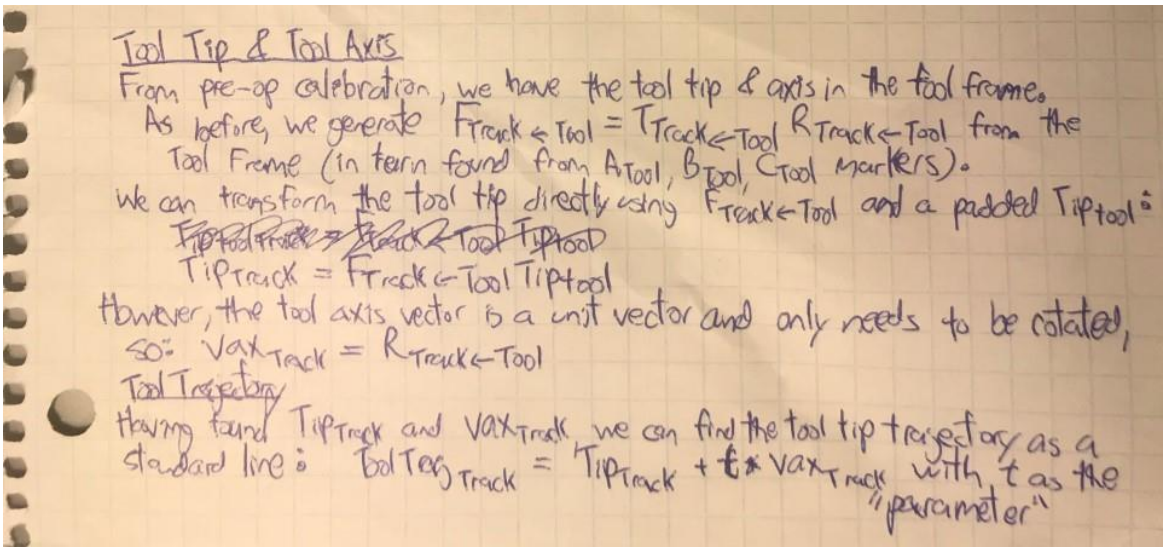
We follow the same process as for $TumorCtr_{CT}$, first converting $WinCtr_{CT}$ to its position in the patient frame, then from there to the tracker frame:

$$WinCtr_{Track} = F_{Track \leftarrow pat} F_{pat \leftarrow CT} WinCtrCT$$

As before, window radius remains unchanged.

We get the sphere

$$(X - WinCtr_{Track\,x})^2 + (y - WinCtr_{Track\,y})^2 + (z - WinCtr_{Track\,z})^2 = R_W^2$$



Tool Tip & Tool Axis

From pre-op calibration, we have the tool tip & axis in the tool frame.

As before, we generate $F_{Track \leftarrow Tool} = T_{Track \leftarrow Tool} R_{Track \leftarrow Tool}$ from the Tool Frame (in turn found from A_{Tool}, B_{Tool}, C_{Tool} markers).

We can transform the tool tip directly using $F_{Track \leftarrow Tool}$ and a padded Tip_{Tool}:

$$Tip_{Track} = F_{Track \leftarrow Tool} Tip_{Tool}$$

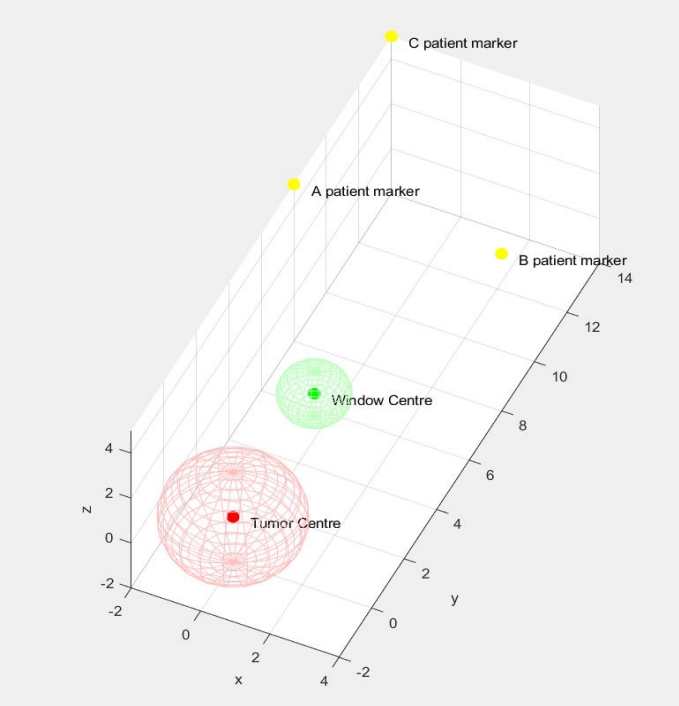
However, the tool axis vector is a unit vector and only needs to be rotated, so: $Vax_{Track} = R_{Track \leftarrow Tool}$

Tool Trajectory

Having found Tip_{Track} and Vax_{Track}, we can find the tool tip trajectory as a standard line: $ToolTraj_{Track} = Tip_{Track} + t * Vax_{Track}$ with t as the "parameter"

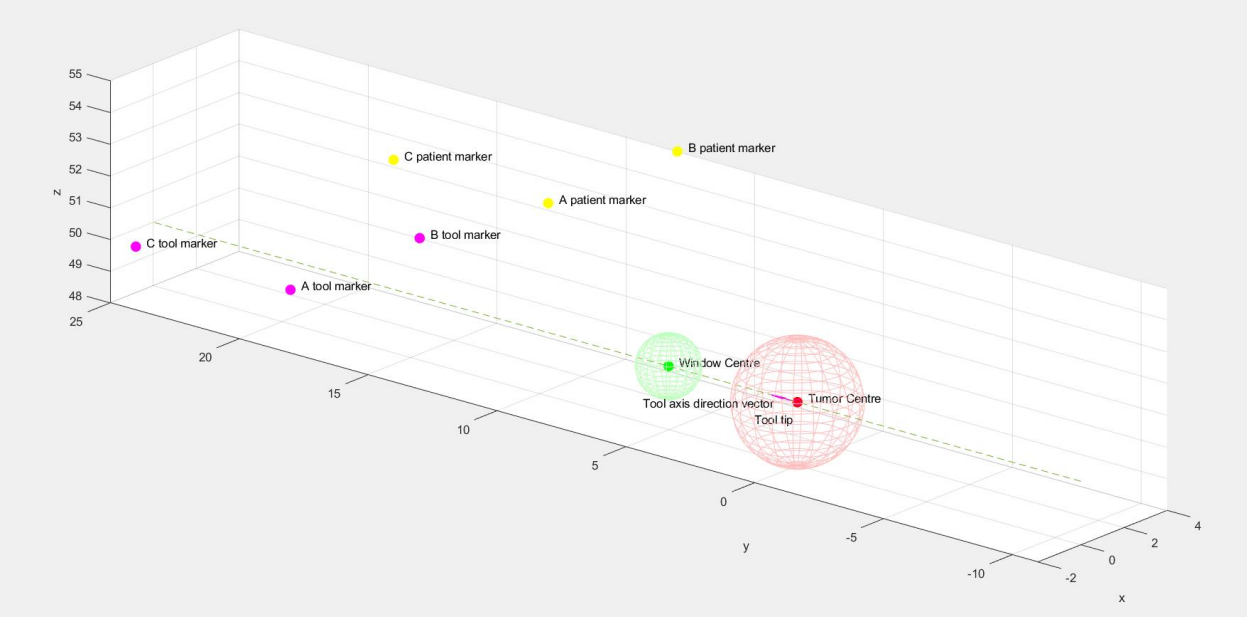
Testing

The testing script produced the following pre-operative plan:



Then, as a requirement for both the surgical navigation module and its testing thereafter, the surgical navigation function produced the following 3 navigation scenes for each navigation case:

Navigation Case 1



The program produced the expected output:

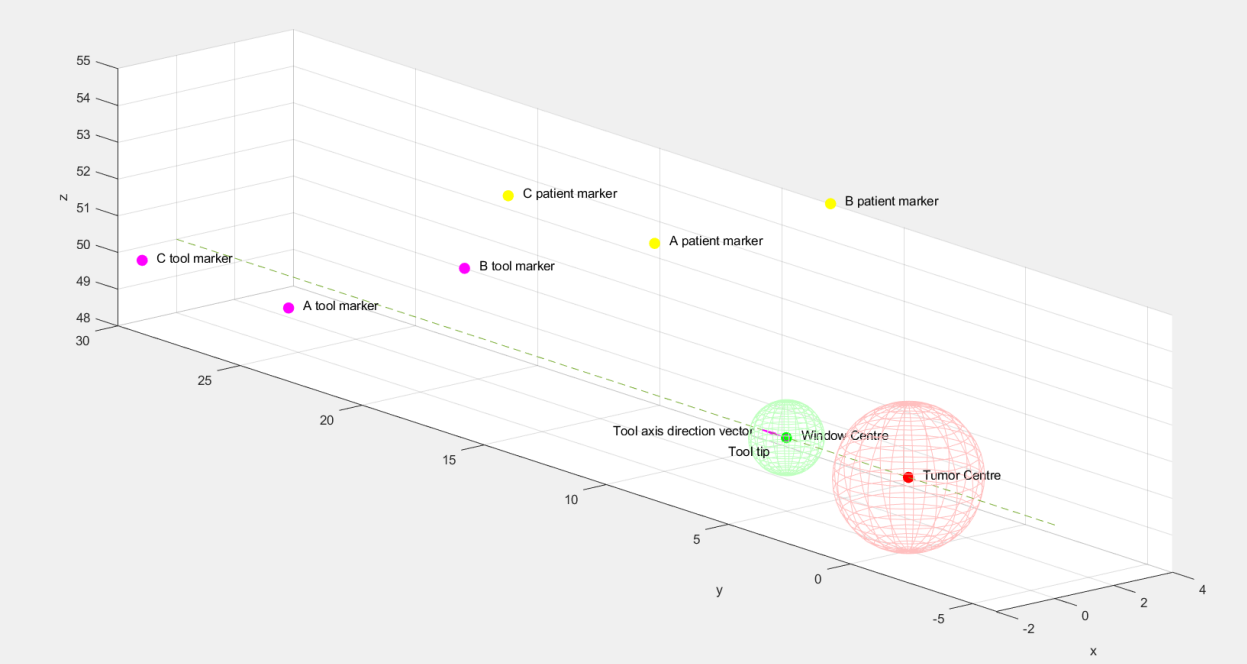
```
-------------Running Surgical_Navigation_Testing Script-------------
NAVIGATION CASE 1
Tool tip in the tracker frame: [0.0000,0.0000,50.0000]
Tool axis direction vector in the tracker frame: [0.0000,1.0000,0.0000]
Tool trajectory line:
Tool_Traj_track = [0.0000,0.0000,50.0000] + t[0.0000,1.0000,0.0000]

Equation of tumor sphere in tracker frame:
(x - 0) + (y - 0) + (z - 50) = 2^2
Equation of window sphere in tracker frame:
(x - 0) + (y - 5) + (z - 50) = 1^2

Current tool trajectory passes through window and tumor

Required further drilling distance to reach depth of tumor centre: 0
```

Navigation Case 2:



The program produced the expected output:

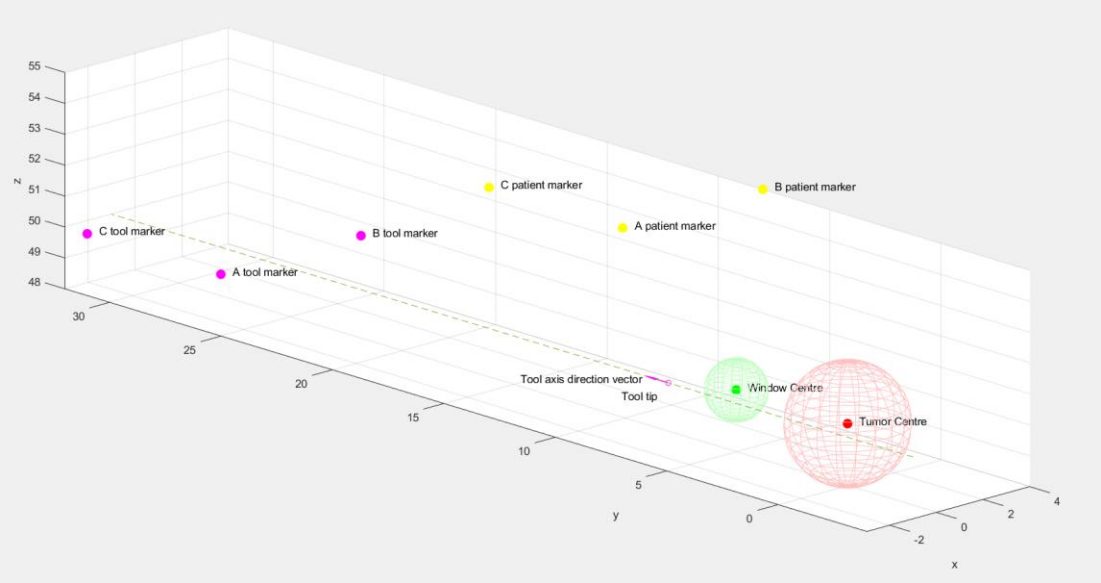
```
NAVIGATION CASE 2
Tool tip in the tracker frame: [0.0000,5.0000,50.0000]
Tool axis direction vector in the tracker frame: [0.0000,1.0000,0.0000]
Tool trajectory line:
Tool_Traj_track = [0.0000,5.0000,50.0000] + t[0.0000,1.0000,0.0000]

Equation of tumor sphere in tracker frame:
(x - 0) + (y - 0) + (z - 50) = 2^2
Equation of window sphere in tracker frame:
(x - 0) + (y - 5) + (z - 50) = 1^2

Current tool trajectory passes through window and tumor

Required further drilling distance to reach depth of tumor centre: 5
```

Navigation Case 3:



The program produced the expected output:

```
NAVIGATION CASE 3
Tool tip in the tracker frame: [-1.0000,7.0000,50.0000]
Tool axis direction vector in the tracker frame: [0.0000,1.0000,0.0000]
Tool trajectory line:
Tool_Traj_track = [-1.0000,7.0000,50.0000] + t[0.0000,1.0000,0.0000]

Equation of tumor sphere in tracker frame:
(x - 0) + (y - 0) + (z - 50) = 2^2
Equation of window sphere in tracker frame:
(x - 0) + (y - 5) + (z - 50) = 1^2

Current tool trajectory passes through tumor but not window

Required further drilling distance to reach depth of tumor centre: 7
```