Guia 3 - WP e Invariante

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1. Parte 1 WP sin ciclos

1.1. Ejercicio 1

- a) True
- b) $b \neq 0$
- c) $b \neq 0 \ \wedge \ a/b \geq 0$
- d) $0 \leq i < |A|$
- e) $0 \leq i < |A| - 3$
- f) $0 \leq i < |A|$

1.2. Ejercicio 2

- a) $\equiv$ wp(a:= a + 1;wp(b:=a/2, $b \geq 0$) $\equiv$ wp(a:= a + 1, $a \geq 0$) $\equiv a \geq -1$
- b) $\equiv$ wp(a:= A[i] + 1,$a \neq \sqrt{2} \ \wedge \ a \neq -\sqrt{2}$) $\equiv 0 \leq i < |A| \ \wedge \ A[i] \neq \sqrt{2} - 1 \ \wedge \ A[i] \neq -\sqrt{2} - 1$
- c) $\equiv$ wp(a:= A[i] + 1,True) $\equiv 0 \leq i < |A|$

1.3. Ejercicio 3

Q $\equiv (\forall j : \mathbb{Z})\ (0 \leq j < |A| \longrightarrow_L A[j] \geq 0)$

- a) $\equiv$ wp(A := setAt(A,i,0),Q)
  $\equiv 0 \leq i < |A| \ \wedge_L \ (\forall j : \mathbb{Z})\ (0 \leq j < setAt(A, i, 0) \longrightarrow_L setAt(A, i, 0)[j] \geq 0)$
  $\equiv 0 \leq i < |A| \ \wedge_L \ ((\forall j : \mathbb{Z})\ (0 \leq j < |A| \ \wedge \ j \neq i \longrightarrow_L A[j] \geq 0) \ \wedge (setAt(A, i, 0) \geq 0))$
  $\equiv 0 \leq i < |A| \ \wedge_L \ ((\forall j : \mathbb{Z})\ (0 \leq j < |A| \ \wedge \ j \neq i \longrightarrow_L A[j] \geq 0))$
- b) $\equiv False$
- c) $\equiv$ wp(A:= setAt(A,i,A[i-1]),Q)
  $\equiv 1 \leq i < |A| \ \wedge_L \ (\forall j : \mathbb{Z})\ (0 \leq j < setAt(A, i, A[i-1]) \longrightarrow_L setAt(A, i, A[i-1])[j] \geq 0))$
  $\equiv 1 \leq i < |A| \ \wedge_L \ (\forall j : \mathbb{Z})\ (0 \leq j < |A| \ \wedge \ j \neq i \longrightarrow_L A[j] \geq 0 \ \wedge \ (j = i \longrightarrow A[i-1] \geq 0))$
  $\equiv 0 \leq i < |A| \ \wedge_L \ (\forall j : \mathbb{Z})\ (0 \leq j < |A| \ \wedge \ j \neq i \longrightarrow_L A[j] \geq 0)$

1.4. Ejercicio 4

- a) $Q \equiv b = -|a| \ \wedge S \equiv$ if $a < 0$ then $b := a$ else $b = -a$ fi

  wp(S,Q) $\equiv def(B) \wedge_L ((B \ \wedge \ wp(S1,Q) \ \vee \ (\neg B \ \wedge \ wp(S2,Q))$

  $B \ \wedge \ wp(S1,Q) \longrightarrow a < 0 \ \wedge \ wp(b := a, Q) \equiv a < 0 \ \wedge \ a \leq 0 \equiv a < 0$

  $\neg B \ \wedge \ wp(S2,Q) \longrightarrow a \geq 0 \ \wedge \ wp(b := -a, Q) \equiv a \geq 0 \ \wedge \ a \geq 0 \equiv a \geq 0$

  $wp(S,Q) \equiv a < 0 \ \vee \ a \geq 0 \equiv True$

- b) $Q \equiv (\forall j : \mathbb{Z}) \ (0 \leq j < |s| \longrightarrow_L s[j] \geq 0) \ \wedge \ S \equiv$ if $i > 0$ then $s[i] := 0$ else $s[0] := 0$ fi

  $wp(S,Q) \equiv def(B) \wedge_L ((B \ \wedge \ wp(S1,Q) \ \vee \ (\neg B \ \wedge \ wp(S2,Q))$

  $B \ \wedge \ wp(S1,Q):$

  $\equiv i > 0 \ \wedge \ wp(s := setAt(s,i,0), (\forall j : \mathbb{Z}) \ (0 \leq j < |s| \longrightarrow_L s[j] \geq 0)$

  $\equiv i > 0 \ \wedge \ 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z}) \ (0 \leq j < |s| \longrightarrow_L setAt(s,i,0)[j] \geq 0)$

  $\equiv 0 < i < |s| \ \wedge_L \ (\forall j : \mathbb{Z}) \ (0 \leq j < |s| \ \wedge \ i \neq j \longrightarrow_L s[j] \geq 0) \wedge$

  $(\forall j : \mathbb{Z}) \ (0 \leq j < |s| \ \wedge \ i = j \longrightarrow_L 0 \geq 0)$

  $\equiv 0 < i < |s| \ \wedge_L \ (\forall j : \mathbb{Z}) \ (0 \leq j < |s| \ \wedge \ i \neq j \longrightarrow_L s[j] \geq 0)$

  $\neg B \ \wedge \ wp(S2,Q):$

  $\equiv i \leq 0 \ \wedge \ wp(s := setAt(s,0,0), Q)$

  $\equiv i \leq 0 \ \wedge \ 0 \leq i < |s| \wedge_L (\forall j : \mathbb{Z}) \ (0 \leq j < |s| \longrightarrow_L setAt(s,0,0)[j] \geq 0)$

  $\equiv i = 0 \wedge_L (\forall j : \mathbb{Z}) \ (0 \leq j < |s| \ \wedge \ i \neq j \longrightarrow_L s[j] \geq 0) \wedge$

  $(\forall j : \mathbb{Z}) \ (0 \leq j < |s| \ \wedge \ i = j \longrightarrow_L s[j] \geq 0)$

  $\equiv (\forall j : \mathbb{Z}) \ (0 \leq j < |s| \ \wedge \ i = j \longrightarrow_L s[j] \geq 0)$

  $wp(S,Q) \equiv (0 \leq i < |s| \ \wedge_L \ (\forall j : \mathbb{Z}) \ (0 \leq j < |s| \longrightarrow_L s[j] \geq 0))$

- c) $Q \equiv (\forall j : \mathbb{Z}) \ (1 \leq j < |s| \longrightarrow_L s[j] = s[j-1]) \wedge \ S \equiv$ if $i > 1$ then $s[i] = s[i-1]$ else $s[i] = 0$ fi

  wp(S,Q) $\equiv def(B) \wedge_L ((B \ \wedge \ wp(S1,Q) \ \vee \ (\neg B \ \wedge \ wp(S2,Q))$

  $B \ \wedge \ wp(S1,Q):$

  $\equiv i > 1 \ \wedge \ wp(s := setAt(s,i,s[i-1]), Q)$

  $\equiv i > 1 \ \wedge \ 1 \leq i < |s| \ \wedge \ (\forall j : \mathbb{Z}) \ (1 \leq j < |s| \longrightarrow_L setAt(s,i,s[i-1])[j] = setAt(s,i,s[i-1])[j-1])$

  $\equiv 1 < i < |s| \ \wedge \ (\forall j : \mathbb{Z}) \ (1 \leq j < |s| \wedge i = j \longrightarrow_L s[i-1] = s[i-1])$

  $\wedge \ (\forall j : \mathbb{Z}) \ (1 \leq j < |s| \ \wedge \ j - 1 = i \longrightarrow_L s[i+1] = s[i-1])$

  $\wedge \ (\forall j : \mathbb{Z}) \ (1 \leq j < |s| \wedge j \neq i \ \wedge \ j - 1 \neq i \longrightarrow_L s[j] = s[j-1]))$

  $\equiv 1 < i < |s| \ \wedge \ (\forall j : \mathbb{Z}) \ (1 \leq j < |s| \ \wedge \ j - 1 = i \longrightarrow_L s[i+1] = s[i-1])$

  $\wedge \ (\forall j : \mathbb{Z}) \ (1 \leq j < |s| \wedge j \neq i \ \wedge \ j - 1 \neq i \longrightarrow_L s[j] = s[j-1])$

  $\neg B \wedge \ wp(S2,Q):$

  $\equiv i \leq 1 \ \wedge \ wp(s := setAt(s,i,0), Q)$

  $\equiv i \leq 1 \ \wedge \ 0 \leq i < |s| \ \wedge_L \ (\forall j : \mathbb{Z}) \ (1 \leq j < |s| \longrightarrow_L setAt(s,i,0)[j] = setAt(s,i,0)[j-1])$

$$\equiv 0 \leq i \leq 1 \ \wedge_L \ (\forall j:\mathbb{Z})\ (1 \leq j < |s| \ \wedge \ j = i \longrightarrow_L 0 = s[j-1])$$
$$\wedge \ \ (\forall j:\mathbb{Z})\ (1 \leq j < |s| \ \wedge \ j-1 = i \longrightarrow_L s[j] = 0)$$
$$\wedge \ \ (\forall j:\mathbb{Z})\ (1 \leq j < |s| \ \wedge \ j \neq i \longrightarrow_L s[j] = s[j-1])$$
$$\equiv 0 \leq i \leq 1 \ \wedge_L \ (\forall j:\mathbb{Z})\ (1 \leq j < |s| \ \wedge \ j-1 = i \ \vee \ j = i \longrightarrow_L s[j-1] = 0) \ \wedge$$
$$(\forall j:\mathbb{Z})\ (1 \leq j < |s| \ \wedge \ j \neq i \longrightarrow_L s[j] = s[j-1])$$

$$wp(S,Q) \equiv ((1 < i < |s| \ \wedge \ (\forall j:\mathbb{Z})\ (1 \leq j < |s| \ \wedge \ j-1 = i \longrightarrow_L s[j] = s[j-2]) \ \vee$$
$$(0 \leq i \leq 1 \ \wedge \ (\forall j:\mathbb{Z})\ (1 \leq j < |s| \ \wedge \ j-1 = i \ \vee \ j = i \longrightarrow_L s[j-1] = 0) \ \wedge$$
$$(\forall j:\mathbb{Z})\ (1 \leq j < |s| \ \wedge \ i \neq j \longrightarrow_L s[j] = s[j-1])$$

- d) $Q \equiv (\forall j:\mathbb{Z})\ (0 \leq j < |s| \longrightarrow_L s[j] \geq 0) \ \wedge S \equiv \textsf{if } s[i] > 0 \textsf{ then } s[i] := -s[i] \textsf{ else } skip \textsf{ fi}$

  wp(S,Q) $\equiv def(B) \wedge_L ((B \ \wedge \ wp(S1,Q) \ \vee \ (\neg B \ \wedge \ wp(S2,Q))$

  $B \ \wedge \ wp(S1,Q):$

  $\equiv s[i] > 0 \ \wedge \ wp(s := setAt(s,i,-s[i],Q)$

  $\equiv s[i] > 0 \ \wedge \ 0 \leq i < |s| \ \wedge_L \ (\forall j:\mathbb{Z})\ (0 \leq j < |s| \longrightarrow_L setAt(s,i,-s[i])[j] \geq 0)$

  $\equiv s[i] > 0 \ \wedge \ 0 \leq i < |s| \ \wedge_L \ (\forall j:\mathbb{Z})\ (0 \leq j < |s| \ \wedge \ j \neq i \longrightarrow_L s[j] \geq 0) \ \wedge$

  $(\forall j:\mathbb{Z})\ (0 \leq j < |s| \ \wedge \ j = i \longrightarrow_L -s[i] \geq 0)$

  $\equiv False$

  $\neg B \ \wedge \ wp(S2,Q):$

  $\equiv s[i] \leq 0 \ \wedge \ wp(skip,Q)$

  $\equiv s[i] \leq 0 \ \wedge \ 0 \leq i < |s| \ \wedge \ (\forall j:\mathbb{Z})\ (0 \leq j < |s| \longrightarrow_L s[j] \geq 0)$

  $\equiv s[i] = 0 \ \wedge \ 0 \leq i < |s| \ \wedge \ (\forall j:\mathbb{Z})\ (0 \leq j < |s| \longrightarrow_L s[j] \geq 0)$

  $wp(S,Q) \equiv (\forall i:\mathbb{Z})\ (0 \leq i < |s| \longrightarrow_L s[i] = 0)$

1.5. Ejercicio 5

- a) Nombre : sumaHasta
- $S \equiv$

  If $(i > 0)$
  a := a + s[i]
  else
  a:= s[i]
  endif

- $wp(S,Q) \equiv (i > 0 \ \wedge \ wp(a := a + s[i], a = \sum_{j=0}^{i} s[i])) \ \vee \ (i \leq 0 \ \wedge \ wp(a := s[0], a = \sum_{j=0}^{i} s[i]))$

  $wp(S,Q) \equiv (i > 0 \ \wedge \ 0 \leq i < |s| \ \wedge_L \ a + s[i] = \sum_{j=0}^{i} s[i]) \ \vee \ (i \leq 0 \ \wedge_L \ s[i] = \sum_{j=0}^{i} s[i])$

  $wp(S,Q) \equiv 0 < i < |s| \wedge_L a = \sum_{j=0}^{i-1} s[i]$

- b)Nombre : todosPositivosHasta
- $S \equiv$

  If $(s[i] \geq 0)$

  res:= true

  else

  res:= false

  endif
- $wp(S,Q) \equiv 0 \leq i < |s| \wedge_L ((s[i] = true \ \wedge \ wp(S1,Q) \ \vee \ (s[i] = false \ \wedge \ wp(S2,Q))$

  $s[i] = true \ \wedge \ wp(S1,Q) \equiv s[i] = true \ \wedge \ true = true \iff (\forall j : \mathbb{Z}) \ (0 \leq j \leq i \longrightarrow_L s[j] = true)$

  $\equiv (\forall j : \mathbb{Z}) \ (0 \leq j < i \longrightarrow_L s[j] = true)$

  $s[i] = false \ \wedge \ wp(S2,Q) \equiv s[i] = false \ \wedge \ false = true \iff (\forall j : \mathbb{Z}) \ (0 \leq j \leq i \longrightarrow_L s[j] = true)$

  $\equiv True$

  $wp(S,Q) \equiv 0 \leq i < |s| \ \wedge_L \ res = true \iff (\forall j : \mathbb{Z}) \ (0 \leq j < i \longrightarrow_L s[j] = true)$

1.6. Ejercicio 6

- a) $|s| = 3 \ \wedge \ i = 3 \longrightarrow$ Cumple P pero se indefine cuando entra al ciclo.Esta mal el rango de i.
- b) $|s| = 3 \ \wedge \ i = 1 \longrightarrow$ Cumple P pero al salir del programa no cumple Q.
- c)La unica razon seria que existe un P mas debil aun.
- d) $|s| = 3 \ \wedge \ i = 9 \longrightarrow$ No acoto i , puede salir de rango de s.
- e) Esta siendo muy restrictivo lo que no la volveria la condicion mas debil sino que hay otra mas debil.

2. Parte 2 - Invariante

2.1. Ejercicio 1

- a) $Pc \equiv res = 0 \ \wedge \ i = 0$

  $Qc \equiv res = \sum_{j=0}^{|s|-1} s[j]$
- b) Falla en $I \ \wedge \ \neg B \longrightarrow Qc$ ya que $\neg B$ indica que i debe ser $\geq |s|$ y la condicion de invariante dice que i debe ser menor que $|s|$.
- c) Falla en $Pc \longrightarrow I$ ya que $res \neq 0$ cuando el limite superior de la sumatoria es 0.
- d) Al cambiar el orden de las instrucciones el limite inferior deberia arrancar en $j = 1$ por que con $j = 0$ estarias sumando siempre 1 extra.
- e) $Pc \longrightarrow I \equiv res = 0 \ \wedge \ i = 0 \longrightarrow 0 \leq i \leq |s| \wedge_L res = \sum_{j=0}^{i-1} s[j]$

  $\equiv 0 = \sum_{j=0}^{-1} s[j] \longrightarrow True$

$\{I \ \wedge \ B\}S\{I\} \equiv I \ \wedge \ B \longrightarrow wp(S, I)$

wp(S,I) $\equiv wp(res := res + s[i]; i := i+1, I) \equiv wp(res := res + s[i], 0 \leq i+1 \leq |s| \wedge_L res = \sum_{j=0}^{i} s[j]$

$$\equiv 0 \leq i < |s| \wedge_L 0 \leq i+1 \leq |s| \wedge_L res + s[i] = \sum_{j=0}^{i} s[j]$$

$$\equiv 0 \leq i < |s| \wedge_L res = \sum_{j=0}^{i-1} s[j] \longrightarrow True$$

$$I \wedge \neg B \longrightarrow Qc \equiv 0 \leq i \leq |s| \wedge_L res = \sum_{j=0}^{i-1} s[j] \ \wedge \ i \geq |s| \longrightarrow res = \sum_{j=0}^{i-1} s[j]$$

$$\equiv i = |s| \ \wedge \ res = \sum_{j=0}^{|s|-1} s[j] \longrightarrow res = \sum_{j=0}^{|s|-1} s[j] \longrightarrow True$$

- f) $fv = |s| - i$

  $\{I \ \wedge fv = v0\} \ S \ \{fv < v0\} \equiv I \ \wedge \ fv = v0 \longrightarrow wp(S, fv < v0)$

  $wp(S, fv < v0) \equiv wp(res := res + s[i]; i := i+1, |s| - i < v0)$

  $\equiv wp(res := res + s[i], |s| - i - 1 < v0)$

  $\equiv 0 \leq i < |s| \wedge_L |s| - i - 1 < v0$

  $I \ \wedge \ |s| - i = v0 \longrightarrow 0 \leq i < |s| \wedge_L |s| - i < |s| - i + 1 \longrightarrow True$

  $I \ \wedge \ fv \leq 0 \longrightarrow \neg B \equiv I \ \wedge \ |s| - i \leq 0 \longrightarrow i \geq |s| \equiv I \ \wedge \ |s| \leq i \longrightarrow |s| \leq i \longrightarrow True$

2.2. Ejercicio 2

- $Pc \equiv res = 0 \ \wedge \ i = 0 \ \wedge \ n \geq 0$
- $Qc \equiv res = \sum_{j=0}^{n-1}(\text{if } j \ mod \ 2 \ = 0 \text{ then } j \text{ else } 0 \text{ fi}$
- $I \equiv 0 \leq i \leq n+1 \ \ \wedge \ i \ mod \ 2 \ = 0 \ \wedge_L \ res = \sum_{j=0}^{i-1}(\text{if } j \ mod \ 2 \ = 0 \text{ then } j \text{ else } 0 \text{ fi}$
- $Pc \longrightarrow I \equiv res = 0 \ \wedge \ i = 0 \ \wedge \ n \geq 0 \longrightarrow 0 = \sum_{j=0}^{-1}(\text{if } j \ mod \ 2 \ = 0 \text{ then } j \text{ else } 0 \text{ fi} \longrightarrow True$
- $\{I \ \wedge \ B\} \ S \ \{I\} \equiv (I \ \wedge \ B) \longrightarrow wp(S, I)$

  $\equiv (I \ \wedge \ B) \longrightarrow wp(res := res + i; i := i+2, I)$

  $\equiv (I \ \wedge \ B) \longrightarrow wp(res := res + i, I|_{i+2}^{i})$

  $\equiv (I \ \wedge \ B) \longrightarrow 0 \leq i+2 \leq n+1 \ \wedge \ i \ mod \ 2 \ = 0 \ \wedge_L \ res + i = \sum_{j=0}^{i+1}(\text{if } j \ mod \ 2 \ = 0 \text{ then } j \text{ else } 0 \text{ fi})$

  $\equiv (I \ \wedge \ B) \longrightarrow 0 \leq i \leq n-1 \ \ \wedge_L \ res = \sum_{j=0}^{i-1}(\text{if } j \ mod \ 2 \ = 0 \text{ then } j \text{ else } 0 \text{ fi}) \equiv True$
- $I \ \wedge \ \neg B \longrightarrow Qc \equiv n \leq i \leq n+1 \ \wedge \ i \ mod \ 2 \ = 0 \longrightarrow res = \sum_{j=0}^{n-1}(\text{if } j \ mod \ 2 \ = 0 \text{ then } j \text{ else } 0 \text{ fi}) \equiv True$

2.3. Ejercicio 3

- a) res:= 0

  i:= 1

  While $(i \leq n)$ do:

  If $(n \; mod \; i = 0)$

  res:= res + i

  Else

  skip

  Endif

  i:= i + 1

  Endwhile

- b) $Pc \equiv res = 0 \;\wedge\; i = 0$

  $Qc \equiv res = \sum_{j=1}^{n} \text{if } n \; mod \; j = 0 \text{ then } j \text{ else } 0 \text{ fi}$

- c) $I \equiv 1 \leq i \leq n+1 \;\wedge\; res = \sum_{j=1}^{i-1} \text{if } n \; mod \; j = 0 \text{ then } j \text{ else } 0 \text{ fi}$

2.4. Ejercicio 4

- a) Debo aclarar el rango en que puede moverse i , tambien deberian aclarar los valores de s y r en Qc.
- b) $I \equiv 0 \leq i \leq |s| \;\wedge\; (\forall j : \mathbb{Z})\ (0 \leq j < i \longrightarrow_L s[j] = r[j])$

  como i inicia en 0 ese es el valor minimo que puede tomar y como B dice que i debe ser menor que $|s|$ necesito ver que pasa con el invariante cuando sale del ciclo por eso lo llevo hasta $|s|$.Ademas como el ciclo recorre todos los valores de $|s|$ usando el indice i , eso significa que j llega hasta i - 1.
- c) fv $= |s| - i \longrightarrow$ cuando i llegue al final del ciclo la funcion variante valdra 0.

2.5. Ejercicio 5

- a) $I \equiv |s|/2 - 1 \leq i \leq |s| - 1 \;\wedge_L\; suma = \sum_{j=0}^{|s|-2-i} s[j]$

  i inicia en $|s| - 1$ y va decreciendo en el ciclo hasta llegar a $|s|/2 - 1$ donde sale del ciclo.
  Suma en Qc como maximo llega hasta $|s|/2 - 1$ que es el valor que tomar i al salir del ciclo , pero como i inicia en $|s| - 1$ debo tenerlo en cuenta para que I sume lo mismo hasta llegar a Qc.
- b) fv $= i - |s|/2 + 1 \longrightarrow$ cuando i sale del ciclo fv vale 0.

2.6. Ejercicio 6

- a) $I \equiv 0 \leq i \leq |s| \ \wedge \ res = \sum_{j=0}^{i-1} s[j]$
- b) $I \equiv 0 \leq i \leq |s| \ \wedge \ res = \sum_{j=0}^{i-1} s[|s|-1-j]$
- c) $I \equiv -1 \leq i \leq |s|-1 \ \wedge \ res = \sum_{j=0}^{|s|-2-i} s[|s|-1-j]$
- d) $I \equiv 0 \leq i \leq |s|/2-1 \ \wedge \ |s| \ mod\ 2 \ = 0 \ \wedge \ res = \sum_{j=0}^{|s|/2-1} s[j] + s[|s|-1-j]$

2.7. Ejercicio 7

- i:= 0

  While $(i < |s|)$ do:

  If $(i \ mod\ 2 = 0)$

  $s[i]$:= 2*i

  Else

  $s[i]$ = 2*i + 1

  Endif

  i:= i + 1

  Endwhile
- $Pc \equiv i = 0$
- $Qc \equiv (\forall j : \mathbb{Z}) \ (0 \leq j < |s| \longrightarrow_L ((j \ mod\ 2 \ = 0 \ \wedge \ s[j] = 2*j) \ \vee \ (j \ mod\ 2 \neq 0 \ \wedge \ s[j] = 2*j+1)))$
- $fv = |s| - i$

2.8. Ejercicio 8

- i:= 0

  While $(i < |s|/2)$ do:

  s[i]:= 0

  $s[|s|-1-i] := 0$

  i:= i + 1

  Endwhile
- $Pc \equiv i := 0 \ \wedge \ |s| \ mod\ 2 \ = 0$
- $Qc \equiv 0 \leq i \leq |s|/2 \ \wedge \ (\forall j : \mathbb{Z}) \ (0 \leq j < |s|/2 \longrightarrow_L (s[j] = 0 \ \wedge \ s[|s|-1-j] = 0))$
- $fv = |s|/2 - i$

2.9. Ejercicio 9

- Es verdadero ya que si k ≤ 0 significa que el programa debe salir del ciclo , y si eso implica $\neg B$ significa que el ciclo termina.