

-- a) Reglas de tipado

$$\frac{}{\Gamma \vdash []t : [t]} \text{t-vacia} \qquad \frac{\Gamma \vdash M1 : t \quad \Gamma \vdash M2 : [t]}{\Gamma \vdash M1 :: M2 : [t]} \text{t-lista}$$
$$\frac{\Gamma \vdash M : [t]}{\Gamma \vdash \text{head}(M) : t} \text{t-head} \qquad \frac{\Gamma \vdash M : [t]}{\Gamma \vdash \text{tail}(M) : [t]} \text{t-tail}$$

-- b) Valores:

$V ::= \dots \mid []t \mid V :: V$

-- Reglas de Reducción:

$$\frac{M \rightarrow M'}{M :: M2 \rightarrow M' :: M2} \text{e-lista1} \qquad \frac{M2 \rightarrow M2'}{V :: M2 \rightarrow V :: M2'} \text{e-lista2}$$
$$\frac{M \rightarrow M'}{\text{head}(M) \rightarrow \text{head}(M')} \text{e-head1} \qquad \frac{}{\text{head}(V :: []t) \rightarrow V} \text{e-head2}$$
$$\frac{}{\text{head}(V1 :: V2 :: V3) \rightarrow \text{if } V1 < V2 \text{ then } \text{head}(V1 :: V3) \text{ else } \text{head}(V2 :: V3)} \text{e-head3}$$
$$\frac{M \rightarrow M'}{\text{tail}(M) \rightarrow \text{tail}(M')} \text{e-tail1} \qquad \frac{}{\text{tail}(V :: []t) \rightarrow []t} \text{e-tail2}$$
$$\frac{}{\text{tail}(V1 :: V2 :: V3) \rightarrow \text{if } V1 < V2 \text{ then } V2 :: \text{tail}(V1 :: V3) \text{ else } V1 :: \text{tail}(V2 :: V3)} \text{e-tail3}$$

c) $\text{head } ((\lambda x:\text{Nat}. 2) \text{ Zero} :: 1 :: []\text{nat}) \rightarrow \text{e-head1}, \beta, \text{e-lista1}$
 $\text{head}(2 :: 1 :: []\text{nat}) \rightarrow \text{e-head3}$
 $\text{if } 2 < 1 \text{ then } \text{head}(2 :: []\text{nat}) \text{ else } \text{head}(1 :: []\text{nat}) \rightarrow \text{if-false}$
 $\text{head}(1 :: []\text{nat}) \rightarrow \text{e-head2}$
1.