

\Rightarrow)

$$\frac{}{\forall X. \forall Y. P(X, Y) \mid - \forall X. \forall Y. P(X, Y)} \text{ax}$$

$$\frac{}{\forall X. \forall Y. P(X, Y) \mid - \forall Y. P(X, Y)} \forall e$$

$$\frac{}{\forall X. \forall Y. P(X, Y) \mid - P(X, Y)} \forall e$$

$$\frac{}{\forall X. \forall Y. P(X, Y) \mid - \forall X. P(X, Y)} \forall i$$

$$\frac{}{\forall X. \forall Y. P(X, Y) \mid - \forall Y. \forall X. P(X, Y)} \forall i$$

$$\frac{}{\forall X. \forall Y. P(X, Y) \Rightarrow \forall Y. \forall X. P(X, Y)} \rightarrow i$$

\Leftarrow)

$$\frac{}{\forall Y. \forall X. P(X, Y) \mid - \forall Y. \forall X. P(X, Y)} \text{ax}$$

$$\frac{}{\forall Y. \forall X. P(X, Y) \mid - \forall X. P(X, Y)} \forall e$$

$$\frac{}{\forall Y. \forall X. P(X, Y) \mid - P(X, Y)} \forall e$$

$$\frac{}{\forall Y. \forall X. P(X, Y) \mid - \forall Y. P(X, Y)} \forall i$$

$$\frac{}{\forall Y. \forall X. P(X, Y) \mid - \forall X. \forall Y. P(X, Y)} \forall i$$

$$\frac{}{\forall Y. \forall X. P(X, Y) \Rightarrow \forall X. \forall Y. P(X, Y)} \rightarrow i$$

$\frac{}{R \mid - \exists X. \forall Y. P(X, Y)} \text{ax}$ $\frac{}{R \mid - \exists X. P(X, Y)} \exists e$ $\frac{}{R = \{\exists X. \forall Y. P(X, Y)\} \mid - \forall Y. \exists X. P(X, Y)} \forall i$ $\frac{}{\exists X. \forall Y. P(X, Y) \Rightarrow \forall Y. \exists X. P(X, Y)} \rightarrow i$	$\frac{}{R, \forall Y. P(X, Y) \mid - \forall Y. P(X, Y)} \text{ax}$ $\frac{}{R, \forall Y. P(X, Y) \mid - P(X, Y)} \forall e$ $\frac{}{R, \forall Y. P(X, Y) \mid - \exists X. P(X, Y)} \exists i$
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