CS 7446 Project 1: martingale

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1 EXPERIMENT 1

1.1 Question 1

Based on my experimental results from experiment 1 which employs the simple gambling simulator, the number of episodes that terminated with $80 in winnings was 1000. I can do this quite simply by looking at the last value of the NumPy array or the last column in the 2D NumPy array aggregate. This suggest that there is/is near a 100% probability that one would win $80 within 1000 sequential bets. For this analysis I use the 1000 episode case as it is a more accurate representation compared to the 10 episode case (related to the weak law of large numbers).

1.2 Question 2

In a similar light, I can now estimate the expected value of winnings after 1000 sequential bets. I can achieve this by looking at the distribution of winnings for all 1000 episodes in experiment 1. As stated in the previous section, every episode ended with a winnings of $80 so the expected value estimate is quite simply, $80. This can be seen visually in Figures 1-3 below where there is a clear saturation of winnings after just a few hundred bets.

1.3 Question 3

It is quite clear from the experimental results in Figures 2 3 that the upper and lower standard deviation lines have an extremely wide range and high variance. They do converge once the many of the episodes reach the $80 winnings mark which appears to be around bet number 200. It is clear that both of these lines would converge to the mean (and have value 0) right around when most of the episodes reach the $80 winnings mark because there is no deviation of winnings from the mean value of $80.

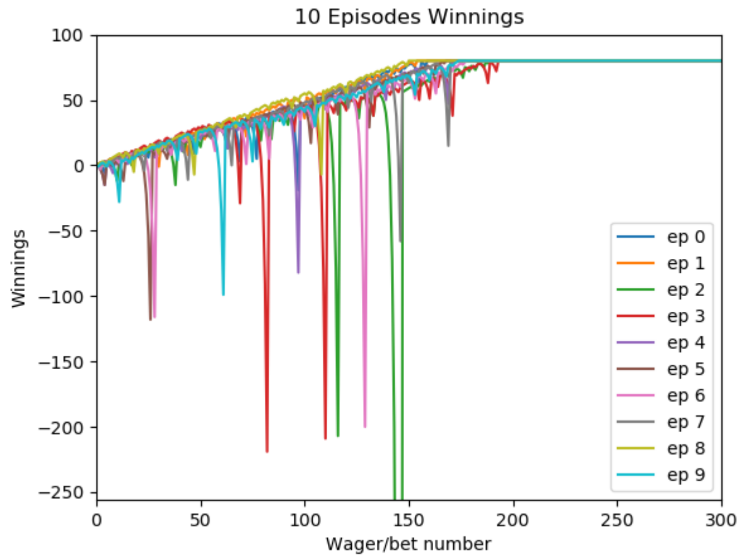

Figure 1—The winnings trajectory of 10 episodes using the simple gambling simulator.

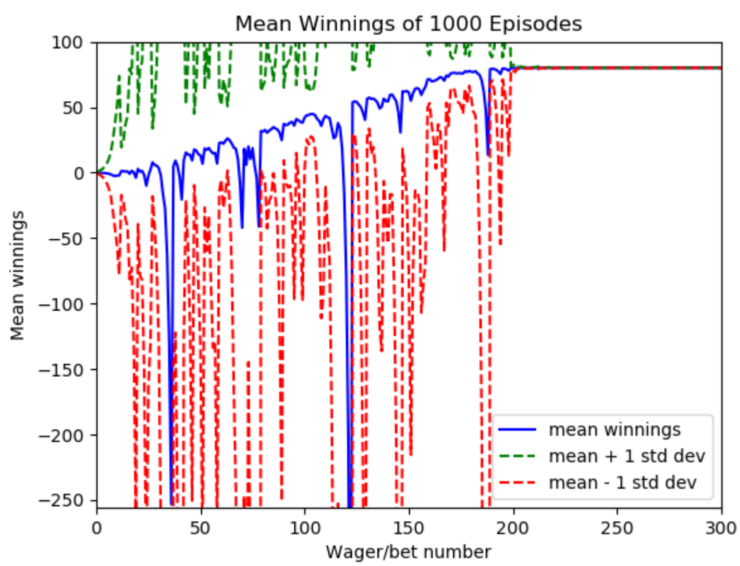

Figure 2—The mean winnings of each bet number of 1000 episodes using the simple gambling simulator. The mean ± 1 standard deviation is also shown.

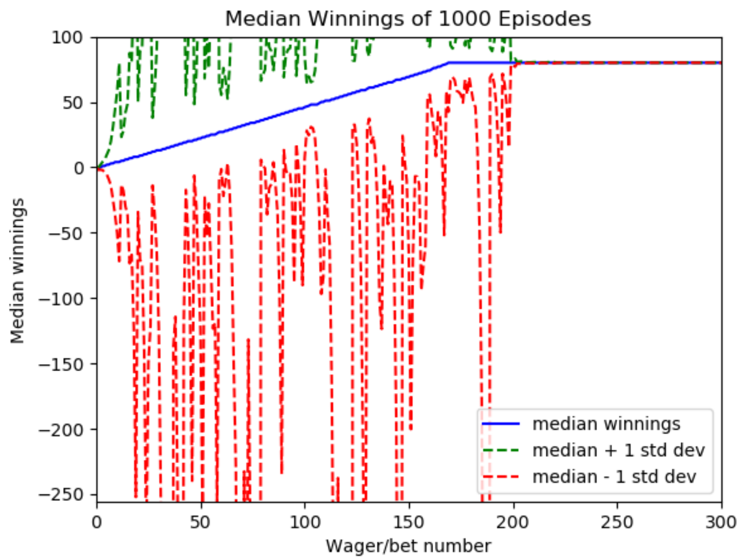

Figure 3—The median winnings of each bet number of 1000 episodes using the simple gambling simulator. The mean ± 1 standard deviation is also shown.

2 EXPERIMENT 2

2.1 Question 4

Now I turn to analyze experiment 2 which employs the realistic gambling simulator which caps the bettor at a $256 bankroll. Similar to the previous section, all 1000 episodes ended with a winnings of $80 which suggests that there is a 100% chance or probability of winning $80 withing 1000 sequential bets. Thinking about this a bit deeper, it can be seen that an episode would terminate in a $256 or greater loss if there were several lost bets in a row i.e. right off the bat losing eight straight bets ($256 = 2^8$). Considering the win probability is $\frac{18}{38}$, the probability of this happening is $\approx (\frac{18}{38})^8 \approx 0.0025$ and hence not very likely.

2.2 Question 5

Similar to experiment 1, and as it can be seen by experimental data in Figures 4 5, each episode ended with a winnings of $80 and so the expected value of winnings after 1000 sequential bets. Based on the formula for expected value, because the value for each episode is $80 and each have equal weight, the expectation is also $80.

2.3 Question 6

Although there are notable differences in the shape of the mean winnings curve when comparing the two gambling methods, the experimental results for standard deviation in Figures 4 5 are similar to that of the simple gambling simulator in the sense that they vary greatly. Additionally, the standard deviation converges to 0 around 200 bets as most and then eventually all of the episodes end in a $80 winnings. After this point there is no variance because all values are $80 so the curves hug the mean and median lines.

2.4 Question 7

As I eluded to earlier, there are extreme benefits to using a large number of experiments compared to just one, namely the fact that it provides a better representation of what will happen in expectation because of the weak law of large numbers. As it can be seen from all of the above figures, there can be extremely large variance in some single measurements compared to the mean and it may not properly depict what is happening at that bet number. For example, compare the shape of the curve of episode 2 in Figure 1 to that of the mean in Figure 2. Expectation is valuable in the betting realm because it accurately depicts the value of playing from a probabilistic standpoint.

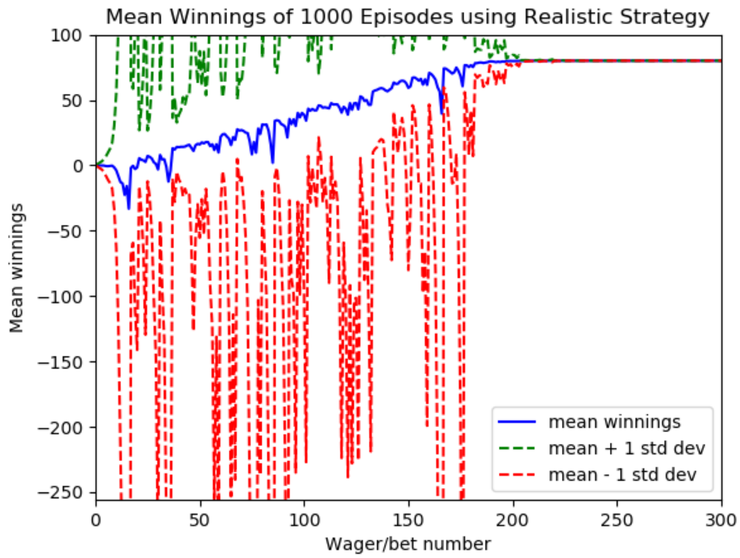

Figure 4—The mean winnings of each bet number of 1000 episodes using the realistic gambling simulator. The mean ± 1 standard deviation is also shown.

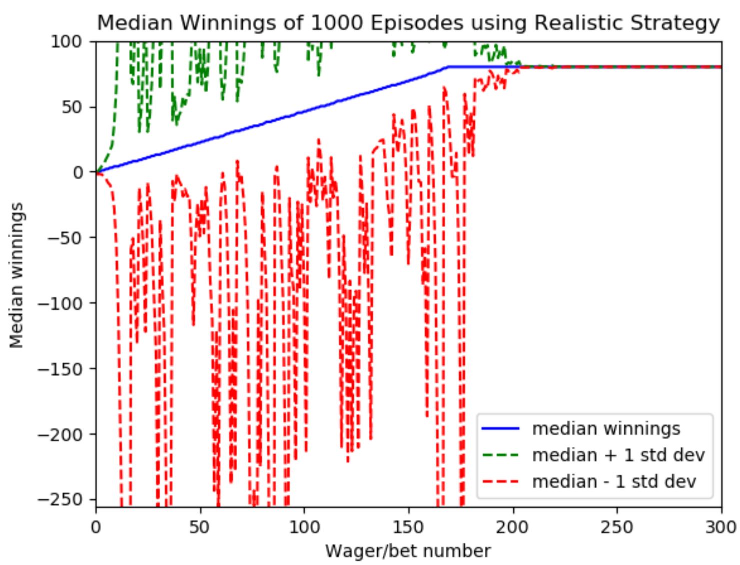

Figure 5—The median winnings of each bet number of 1000 episodes using the realistic gambling simulator. The mean ± 1 standard deviation is also shown.