Total variation in image

$$\sum_{i=1}^{t} (x_i - \mu_T)^2$$

μ_T = avg intensity

Within cluster variation

$$\sum_{j=1}^{K} \sum_{i=1}^{t} w_{ij} (x_i - \mu_j)^2$$

μ_j = mean intensity C_j

Between cluster variation = total variation − within cluster variation

$$= \sum_{i=1}^{t} (x_i - \mu_T)^2 - \sum_{j=1}^{K} \sum_{i=1}^{t} w_{ij} (x_i - \mu_j)^2$$

$$= \sum_{i=1}^{t} x_i^2 - 2 x_i \mu_T + \mu_T^2 - \sum_{j=1}^{K} \sum_{i=1}^{t} w_{ij} x_i^2 - 2 w_{ij} x_i \mu_j + \mu_j^2 w_{ij}$$

$$= \sum_{i=1}^{t} x_i^2 - 2\mu_T \sum_{i=1}^{t} x_i + \sum_{i=1}^{t} \mu_T^2 - \sum_{j=1}^{K} \sum_{i=1}^{t} w_{ij} x_i^2 + 2 \sum_{j=1}^{K} \sum_{i=1}^{t} w_{ij} x_i \mu_j - \sum_{j=1}^{K} \sum_{i=1}^{t} \mu_j^2 w_{ij}$$

$$= \cancel{\sum_{i=1}^{t} x_i^2} - 2 \mu_T t \mu_T + t \mu_T^2 - \cancel{\sum_{i=1}^{t} x_i^2} + 2 \sum_{j=1}^{K} t_j \mu_j^2 \mu_T - \sum_{j=1}^{K} t_j \mu_j^2$$

$$= -t \mu_T^2 + \sum_{j=1}^{K} 2 t_j \mu_j^2 \mu_T - \sum_{j=1}^{K} t_j \mu_j^2$$

can make this sum

$$= -\sum_{j=1}^{K} t_j \mu_T^2 + \sum_{j=1}^{K} 2 t_j \mu_j \mu_T - \sum_{j=1}^{K} t_j \mu_j^2$$

$$= \sum_{j=1}^{K} \left(-t_j (\mu_j^2) + 2 t_j \mu_j \mu_T - t_j \mu_j^2 \right)$$

$$= -\sum_{j=1}^{K} t_j (\mu_j - \mu_T)^2 \quad \checkmark$$