Quadratic spline.

$$y(x) = \begin{cases} \sum_{j=1}^{3} \beta_j x^{j-1} + \sum_{K=1}^{K} \theta_K (x-\xi)_+^2 & x \in [\xi_1, \xi_K] \\ \sum_{j=1}^{2} \beta_j x^{j-1} + \sum_{K=1}^{K} \theta_K (x-\xi)_+^2 & x \text{ outside } (\xi_1, \xi_K) \end{cases}$$

Outside interval so second term

$$y'(x) = \sum_{j=1}^{2} (j-1)\beta_j x^{j-2} + 2\sum_{K=1}^{K} \theta_K (x-\xi)$$

$$y''(x) = \sum_{j=1}^{2} (j-2)(j-1)\beta_j x^{j-3} + 2\sum_{K=1}^{K} \theta_K$$

→ always 0

$$0 = 2\sum_{K=1}^{K} \theta_K$$

$$0 = \theta_K + \sum_{K=1}^{K-1} \theta_K$$

$$\theta_K = -\sum_{K=1}^{K-1} \theta_K$$ sub back into $y(x)$

$$y(x) = \beta_1 + \beta_2 x + \sum_{K=1}^{K-1} \theta_K (x-\xi_K)^2 - (x-\xi_K)^2 \sum_{K=1}^{K-1} \theta_K$$

$f(x) = \sum_{i=0}^{2} B_{i,2}(x) P_i$ 5 Knots

$B_{i,0}(x) = \begin{cases} 1, & \tau_i \le x < \tau_{i+1} \\ 0, & \text{else} \end{cases}$

$B_{i,j}(x) = \frac{x - \tau_i}{\tau_{i+j} - \tau_i} B_{i,j-1}(x) + \frac{\tau_{i+j+1} - x}{\tau_{i+j+1} - \tau_{i+1}} B_{i+1,j-1}(x)$

$X = \{\tau_0, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5\} = \{0, 1, 2, 3, 4, 5\}$

for $B_{0,2}$ need $B_{0,1}$ + $B_{1,1}$

$B_{0,1}$ need $B_{0,0}$ + $B_{1,0}$

$B_{1,1}$ need $B_{1,0}$ + $B_{2,0}$ ✓

$B_{0,0}(x) = \begin{cases} 1, & 0 \le x < 1 \\ 0, & \text{else} \end{cases}$

$B_{1,0}(x) = \begin{cases} 1, & 1 \le x < 2 \\ 0, & \text{else} \end{cases}$

$B_{2,0}(x) = \begin{cases} 1, & 2 \le x < 3 \\ 0, & \text{else} \end{cases}$

$\vdots$

$B_{0,1}(x) = \frac{x-0}{1-0} B_{0,0} + \frac{2-x}{2-1} B_{1,0} = x B_{0,0} + (2-x) B_{1,0}$

$B_{1,1}(x) = \frac{x-1}{2-1} B_{1,0} + \frac{3-x}{3-2} B_{2,0} = (x-1) B_{1,0} + (3-x) B_{2,0}$

$B_{0,2}(x) = \frac{x-0}{2-0} B_{0,1} + \frac{3-x}{3-1} B_{1,1} = \frac{x}{2} B_{0,1} + \left(\frac{3-x}{2}\right) B_{1,1}$

$B_{2,1}(x) = \frac{x-2}{3-1} B_{2,0} + \frac{4-x}{4-3} B_{3,0} = \left(\frac{x-2}{2}\right) B_{2,0} + (4-x) B_{3,0}$

$B_{1,2}(x) = \frac{x-1}{3-1} B_{1,1} + \frac{3-x}{4-2} B_{2,1} = \left(\frac{x-1}{2}\right) B_{1,1} + \left(\frac{3-x}{2}\right) B_{2,1}$

$B_{3,1}(x) = \frac{x-3}{4-1} B_{3,0} + \frac{5-x}{5-4} B_{4,0} = (x-3) B_{3,0} + (5-x) B_{4,0}$

$B_{2,2}(x) = \frac{x-2}{3-2} B_{2,1} + \frac{5-x}{5-3} B_{3,1} = (x-2) B_{2,1} + \left(\frac{5-x}{2}\right) B_{3,1}$

$B_{1,2}$ need $B_{1,1}$ + $B_{2,1}$

$B_{2,1}$ need $B_{2,0}$ + $B_{3,0}$ ✓

$B_{2,2}$ need $B_{2,1}$ + $B_{3,1}$

$B_{3,1}$ need $B_{3,0}$ + $B_{4,0}$ ✓