Quadratic spline

$$y(x) = \begin{cases} \sum_{j=1}^{3} \beta_j x^{j-1} + \sum_{K=1}^{K} \theta_K (x-\xi)_+^2 & x \in [\xi_1, \xi_K] \\ \sum_{j=1}^{2} \beta_j x^{j-1} + \sum_{K=1}^{K} \theta_K (x-\xi)_+^2 & x \text{ outside } (\xi_1, \xi_K) \end{cases}$$

Outside interval so second term

$$y'(x) = \sum_{j=1}^{2} (j-1)\beta_j x^{j-2} + 2\sum_{K=1}^{K} \theta_K (x-\xi)$$

$$y''(x) = \sum_{j=1}^{2} (j-2)(j-1)\beta_j x^{j-3} + 2\sum_{K=1}^{K} \theta_K$$

→ always 0

$$0 = 2\sum_{K=1}^{K} \theta_K$$

$$0 = \theta_K + \sum_{K=1}^{K-1} \theta_K$$

$$\theta_K = -\sum_{K=1}^{K-1} \theta_K$$ sub back into $y(x)$

$$y(x) = \beta_1 + \beta_2 x + \sum_{K=1}^{K-1} \theta_K (x-\xi_K)^2 - (x-\xi_K)^2 \sum_{K=1}^{K-1} \theta_K$$