ISYE 8803 Homework 3 Problem 3

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1 Problem 2

In this problem, I analyze a heat transfer process. At each time, t, and image is recorded representing intensity values which results in a 21x21x10 tensor. As the problem states, the thermal diffusivity coefficient depends on the material being heated. Here I have three datasets, a tensor that represents a heat transfer process in material 1 (T1), and tensor that represents a heat transfer process in material 2 (T2), and a tensor from a material that is either 1 or 2 but is unknown (T3). I use CP and Tucker decomposition to make this prediction. The tensors can be seen in Figure 1. Each row represents one of the three unique tensors and each image from left to right is the next time step.

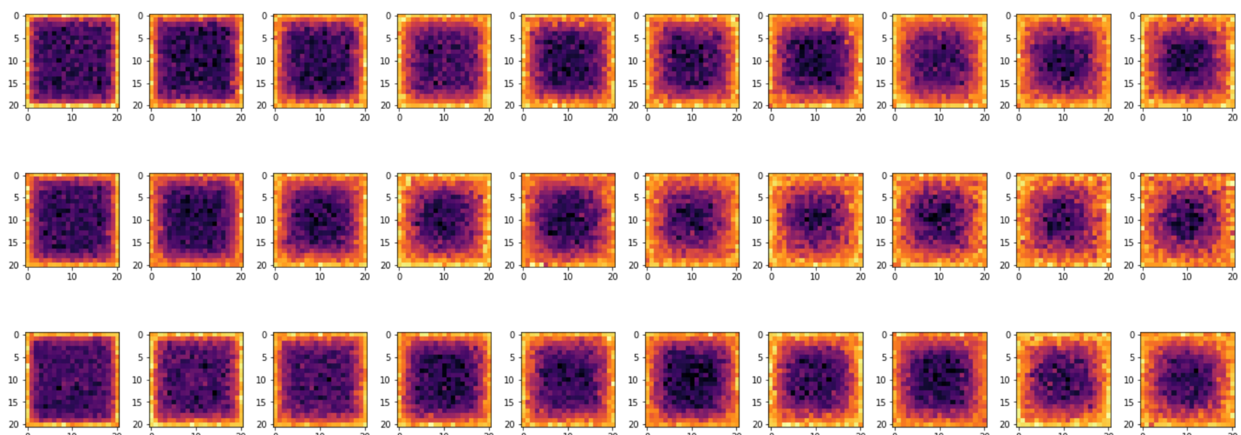

Figure 1: The three unique tensors shown as series of intensity images.

1.1 CP and Tucker decomposition

I use the Akaike information criterion (AIC) to select the optimal rank for the decomposition methods (rank which minimizes the AIC). Figures 2 and 3 show examples of this for CP decomposition. I then decompose the two tensors of known processes (T1 and T2) and use their optimal ranks to decompose T3 respectively. I can analyze the temporal and spatial patterns of the decomposition. Figures 4 and 5 show this for CP decomposition as an example and Figures 6 and 7 show visuals of the spatial components in 2D. I can then reconstruct the tensors and find the distance between T1 and T3 and T2 and T3 to determine if the unknown T3 is classified as material 1 or 2. Table 1 summarizes all of the results that lead me to the conclusion that CP decomposition seems to yield the better result and T3 can be classified as material 1.

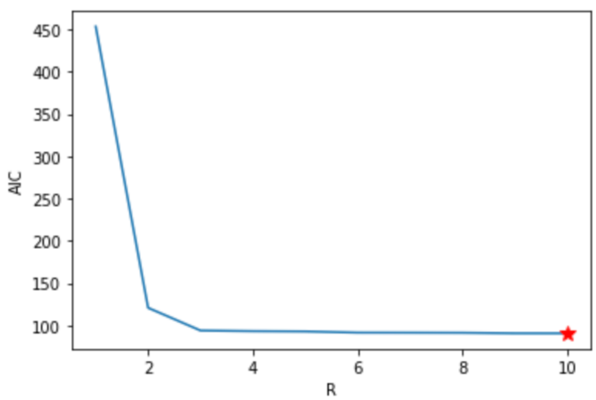

Figure 2: AIC as a function of rank for T1 using CP decomposition.

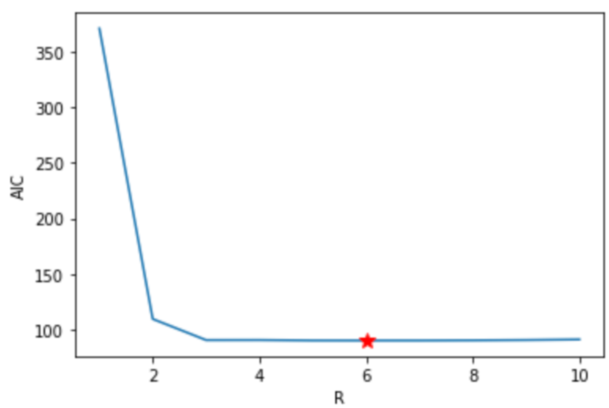

Figure 3: AIC as a function of rank for T2 using CP decomposition.

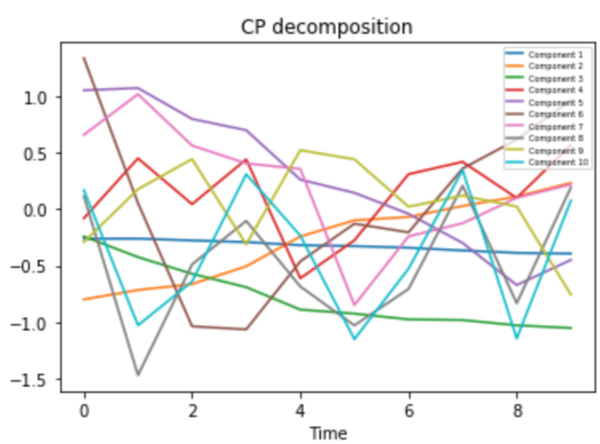

Figure 4: Visual of decomposed components for T1 using CP decomposition.

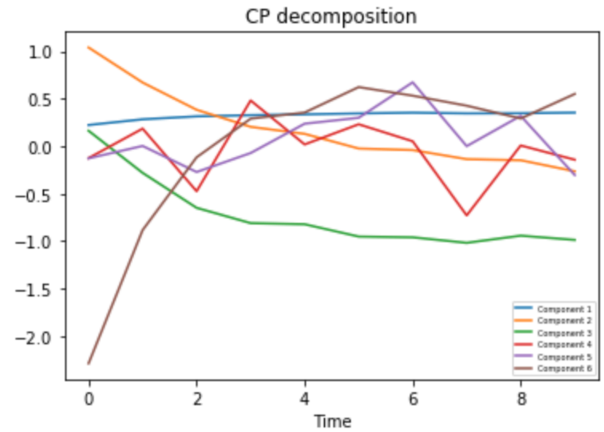

Figure 5: Visual of decomposed components for T2 using CP decomposition.

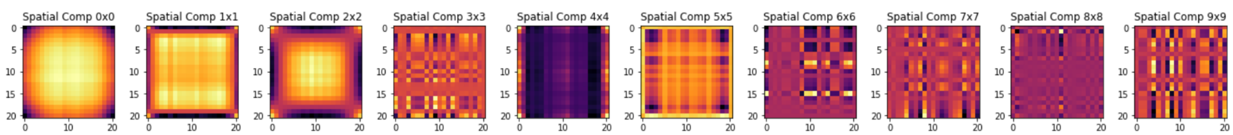

Figure 6: Spatial and temporal patterns for T1 using CP decomposition.

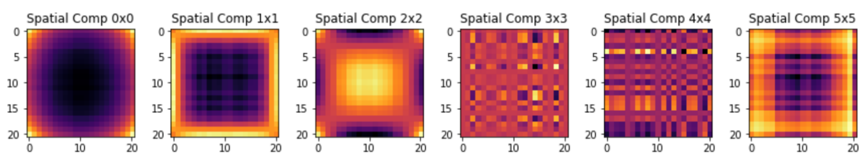

Figure 7: Spatial and temporal patterns for T2 using CP decomposition.

Tensor	Decomposition method	Optimal Rank	Min AIC	Difference to T3
T1	CP	10	90.62	4.22
T1	Tucker	2	339.74	1.43
T2	CP	6	90.46	10.08
T2	Tucker	2	329.07	9.42

Table 1: Summary of decomposition methods to classify T3.