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GPS/INS SENSOR FUSION FOR ACCURATE POSITIONING AND NAVIGATION BASED ON KALMAN FILTERING

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Abstract: This paper presents the position and velocity determination by using INS and GPS. The measurement results from INS and GPS sensors are fused by using Kalman filter. Dilution of Precision (DOP) technique is used to select a combination of satellites to be used as measurement data. Two implementations of Kalman filter, feedforward and feedback are used. The experiment shows that the selection of the satellites affects the measurements. For biased and correlated data an adaptive technique, developed earlier was used. This paper is a demonstration of known techniques rather than an introduction of new ones.

Keywords—Sensor Fusion, GPS, INS, Kalman Filtering, Accurate Positioning

1 INTRODUCTION

The Inertial Navigation System (INS) is widely used as one component in guidance, navigation, and control systems. It includes accelerometers and gyroscopes to provide velocity and angular rate information. Integrating this information, the position and orientation of the vehicle can be calculated.
Global positioning system (GPS) is commonly combined with the INS to bind its propagation error. These two types of signals are fused together to produce one accurate navigational information. Kalman filter is the common algorithm used to fuse the measurements (Rumeliotis *et al* 1999, and Santini *et al*, 1997).
Some authors (Jetto, *et al.*, 1999) and (Sasiadek and Wang, 1999 and Sasiadek and Wang, 2003) used advanced techniques of Kalman filter gain tuning based on fuzzy logic.
There are many research works already conducted to improve position estimation when implementing INS and GPS, whether as a standalone estimation method, or as an integrated system. Sasiadek and Hartana (2004) made a more extensive introduction to these problems. Pozo-Ruz *et al* (1998), a satellite selection criterion was introduced in order to improve accuracy of the estimation when using GPS.
The distributed Kalman filter simulator (DKFSIM) was introduced Lawrence (1996). This filter is used to evaluate the performance of several different filter architectures and sensor model conditions for advanced, multi-sensor navigation systems. In (Sukkarieh *et al,* 1998), authors examined the integration of INS-GPS for autonomous land vehicle.

Currently, there are 28 global positioning satellites constellation available to be used for navigation. From any particular location, a receiver can view around 10 satellites at any given time. Although, the estimation is usually improved when using more satellites, the channels available to receive the measurement signals are usually restricted. Moreover, not all the visible satellites are good to be used as the sources of measurement. Therefore, the receiver has to choose the best combination of satellites in view to get the optimal estimation.
In this paper, INS and GPS measurements data are combined to find the optimal measurement results by using Kalman filter. The best combination of satellites is chosen from a number of visible satellites by using dilution of precision (DOP) technique. The effects of the change of satellites in combination to the estimation are examined. If the data are found to be biased and/or correlated, a special adaptive techniques described by Sasiadek and Wang (1999) was used.

2 POSITION DETERMINATION BY USING INERTIAL NAVIGATION SYSTEM

Position determination by using the INS can be briefly explained by the following steps:

- Measurement of the accelerations in the directions of the navigation axes. For vertical acceleration, the effect of the earth gravitation has to be included in calculation.
- Determination of distance and velocity by integrating the acceleration. This integration involves time interval, so this interval must be known accurately.
- Measurement of the rotation rates (from gimbals motion in a stabilized platform or from gyroscopes in a strapdown system) as a process to find the direction of the measured distance and velocity. This should include the compensation of the earth's rotation.
- Combining the distance and heading data gives an updated dead reckoned position to display.

3 POSITION DETERMINATION BY USING GPS PSEUDORANGE

To determine user position, GPS uses time-of-arrival of signals broadcast by satellites. The signals contain two types of ranging codes, which are pseudorandom noise (PRN), and navigation data.

In three dimensional spaces, user position can be determined from GPS satellites constellation by measuring the ranges from minimal four satellites positions. Since, the satellites and user receiver clock will generally have a bias error from GPS system time, the ranges determined from this process are called pseudorange (ρ), and contains the geometric satellite-to-user range, an offset caused by the difference between user clock and GPS system time, and an offset caused by the difference between satellites clock and GPS system time.
In three dimensions space, the range can be determined by using the following equations:

$$\rho_1 = \sqrt{(x_1 - x_u)^2 + (y_1 - y_u)^2 + (z_1 - z_u)^2} + ct_u \quad (1)$$

$$\rho_2 = \sqrt{(x_2 - x_u)^2 + (y_2 - y_u)^2 + (z_2 - z_u)^2} + ct_u \quad (2)$$

$$\rho_3 = \sqrt{(x_3 - x_u)^2 + (y_3 - y_u)^2 + (z_3 - z_u)^2} + ct_u \quad (3)$$

$$\rho_4 = \sqrt{(x_4 - x_u)^2 + (y_4 - y_u)^2 + (z_4 - z_u)^2} + ct_u, \quad (4)$$

where x_i, y_i, and z_i denote the satellites position in three dimensions, x_u , y_u , z_u are user position, c is the speed of light.

4. GPS ERRORS

In determination of position by using GPS, the accuracy of the sensors fusion performance is also influenced by the geometry of satellites distribution, which is known as dilution of precision (DOP). It is defined as geometry factors that relate parameters of the user position and time bias errors to the pseudorange errors.
The more usable satellites that are available result in higher accuracy. In this case, the DOP parameter is calculated from all available satellites. However, the DOP criteria can also be used to find the four best satellites out of a geometric constellation consisting of more than four satellites. This gives advantages particularly if only a four-channel receiver is available.
The error of the pseudorange measurement from the approximate user position in this case can be formulated as:

$$\begin{aligned}\rho_i - \hat{\rho}_i = &-\frac{x_i - \hat{x}_u}{\hat{r}_i}\Delta x_u - \frac{y_i - \hat{y}_u}{\hat{r}_i}\Delta y_u \\ &-\frac{z_i - \hat{z}_u}{\hat{r}_i}\Delta z_u + c\Delta t_u,\end{aligned} \quad (5)$$

or in matrix form as:

$$\begin{bmatrix}\Delta\rho_1\\ \Delta\rho_2\\ \Delta\rho_3\\ \Delta\rho_4\end{bmatrix} = \begin{bmatrix}a_{x1} & a_{y1} & a_{z1} & 1\\ a_{x2} & a_{y2} & a_{z2} & 1\\ a_{x3} & a_{y3} & a_{z3} & 1\\ a_{x4} & a_{y4} & a_{z4} & 1\end{bmatrix}\begin{bmatrix}\Delta x_u\\ \Delta y_u\\ \Delta z_u\\ c\Delta t_u\end{bmatrix} \quad (6)$$

Equation (6) can be written in compact form as:

$$\Delta\boldsymbol{\rho} = \mathbf{H}\Delta\mathbf{x}. \quad (7)$$

From this equation, geometric dilution of precision (GDOP) is defined as:

$$\mathrm{GDOP} = \sqrt{\mathrm{trace}(\mathbf{H}^T\mathbf{H})^{-1}}, \quad (8)$$

where $\mathrm{trace}[\cdot]$ indicates the sum of diagonal elements of the matrix. Not all of the available satellites used in this work are visible all the time during the flight. Sasiadek and Hartana, (2004) presented the satellites visibility chart as a function of time when the INS and GPS measurement data are recorded.
The other possibility for localization using the GPS sensor is the carrier-phase method. Farell *et al* (2003) used this method to calculate lateral displacement of the vehicle.

5. SENSOR FUSION PROCEDURE

When the GPS measurement result is used with INS, the characteristics of GPS measurement binds the error introduced by INS.
In this work, the GPS and INS sensors are used to find the user position. The problem can be summarized how to the optimal signal on the basis of two different signals available.
In this case, the Kalman filter is implemented to fuse the measurement output and feed the result to the control system.

When implementing Kalman filter in inertial navigation problems (or any other applications), two states can be used: total state space and error state space formulation. In this work, error state space formulation will be used, where the difference between INS and GPS measurement data is used as the input to the Kalman filter. The advantage of this configuration is the filter does not need to work at the same frequency as the INS, which is usually very high. The frequency at which the filter works is the same as the frequency of the availability of the GPS data, which is usually lower than the frequency of INS.

In general, there are two different implementations of the error state space configuration: feedback and feedforward. These implementations are based on how the estimated errors resulted from the filter are combined with Fig.1 and Fig. 2 display the appropriate configurations as block diagrams.

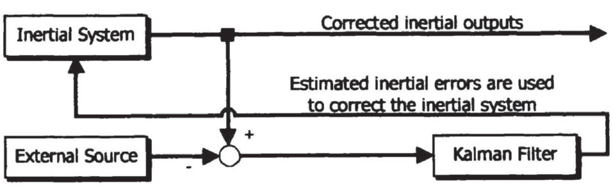

Fig. 1 Error Feedback Kalman Filter

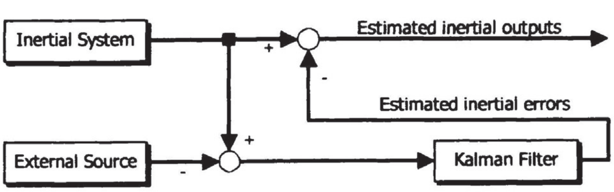

Fig. 2 Error Feedforward Kalman Filter

The measurements obtained from satellites should be unbiased and uncorrelated. Positions and velocities can be represented as a linear model (9) in state space form. In case of 8 states model, the particular states represent East position error, East velocity, North position error, North velocity, altitude error, altitude rate, clock bias, and clock drift respectively. For 8-state Kalman filter, model for the INS measurement can be written as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \\ \dot{x}_7 \\ \dot{x}_8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} + \begin{bmatrix} 0 \\ w_2 \\ 0 \\ w_4 \\ 0 \\ w_6 \\ w_7 \\ w_8 \end{bmatrix} \tag{9}$$

or in matrix form as:

$$\dot{\mathbf{x}}(t) = \mathbf{F}(t)\mathbf{x}(t) + \mathbf{w}(t), \tag{10}$$

where $\mathbf{w}(t)$ represents the accelerations and clock noises of the INS sensor. It is assumed that the noise is purely white noise, zero mean, and Gaussian with covariance $\mathbf{Q}$. Transforming (10) into discrete form results:

$$\mathbf{x}_{k+1} = \mathbf{\Phi}_k \mathbf{x}_k + \mathbf{w}_k, \tag{11}$$

where:

$$\mathbf{\Phi}_k = e^{\mathbf{F}T}$$

$$\mathbf{w}_k = \int_{kT}^{(k+1)T} e^{\mathbf{F}[(k+1)T-\tau]}\mathbf{w}(\tau)d\tau$$

The covariance of $\mathbf{w}_k$ in discrete form ($\mathbf{Q}_k$) can be found as:

$$\mathbf{Q}_k = \int_0^T e^{F\tau}\mathbf{Q}\, e^{F^T\tau}\, d\tau. \tag{12}$$

The observation for the Kalman filter is the difference between pseudorange as measured by GPS and the predicted pseudorange, which can be derived from (6). This equation is defined in Earth-Centered, Earth-Fixed (ECEF) coordinate system. Transforming it into local coordinate frame results:

$$\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} a_{x1} & 0 & a_{y1} & 0 & a_{z1} & 0 & 1 & 0 \\ a_{x2} & 0 & a_{y2} & 0 & a_{z2} & 0 & 1 & 0 \\ a_{x3} & 0 & a_{y3} & 0 & a_{z3} & 0 & 1 & 0 \\ a_{x4} & 0 & a_{y4} & 0 & a_{z4} & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \\ x_8 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ v_4 \end{bmatrix} \tag{13}$$

The last term in (13) is the measurement noise. It is also assumed as zero mean Gaussian white noise with covariance $\mathbf{R}$.

In practice, this assumption is not always valid. In that case the Kalman filter gain would diverge and filter would not perform its role. In that case the adaptive Kalman filter has to be employed. The details of such filter could be found in Sasiadek and Wang (1999 and 2003).

Measurement (13) is already in discrete form, so it can be implemented directly into discrete Kalman filter. Writing this equation in matrix form result:

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{v}_k \quad (14)$$

Therefore, the set of equations for the GPS aided inertial navigation system as shown in (11) and (14).

6. EXPERIMENTS AND RESULTS

The real experiments with aircraft flying and measuring simultaneously position with GPS and INS were conducted. Subsequently, the measurements were fed into the sensor fusion algorithm including the implementation of the Kalman filter in the GPS-INS sensors fusion. Two implementations of the filter were considered: error state feedforward and error state feedback Kalman filter.

The data were recorded on board of an aircraft flying along a designated path. The data in form of the INS measurements, GPS satellites' pseudorange measurements, and the satellites' locations from all visible satellites were recorded.

Number of visible satellite has to be at least four. The selection of the satellites used in the experiment was done by using the DOP method as explained before. The simulation experiment has been conducted for 2.5 hours long. The results of the experiment are displayed as the position and velocity errors. Figures 3 until 10 display these results. The dash line represents the result of the experiment when using error state feedforward Kalman filter and the solid line represents the result when using error state feedback Kalman filter.

From the figures, it can be seen that the error state feedback Kalman filter implementation results in smaller error than the feedforward one. It can also be seen that there are some spikes of the error. These spikes are resulted when the filter switches into different satellites combination. The switches are needed because one or more satellites used in previous combination are not visible anymore, or because the combination will produce greater DOP value than the other possible combinations.

For filter covariance, shown only for velocity error in East direction, in general the feedback Kalman filter gives smaller uncertainty. It can also be seen that the uncertainty increases before the change of satellite combination occurs. This is reasonable, because when the satellites become not visible, the measurements produced from GPS receiver also become unreliable. From the plot of covariance can be seen that the third combination gives smaller uncertainty than the first two. The third combination also gives good results until the end of simulation. This pattern is valid for position error covariance as well as velocity error covariance.

For Kalman gain, in the first combination, the feedback configuration gives more weight to the INS measurement than the feedforward configuration. The opposite is resulted in the third combination, where the feedback gives more weight to the GPS measurements.

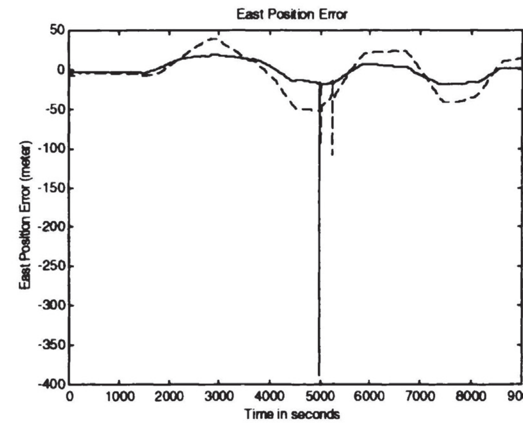

Fig. 3 East position error.

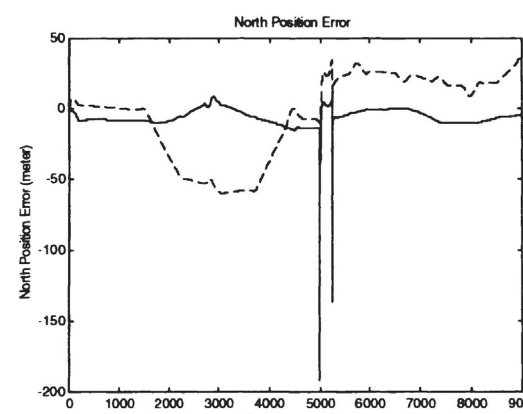

Fig. 4 North position error.

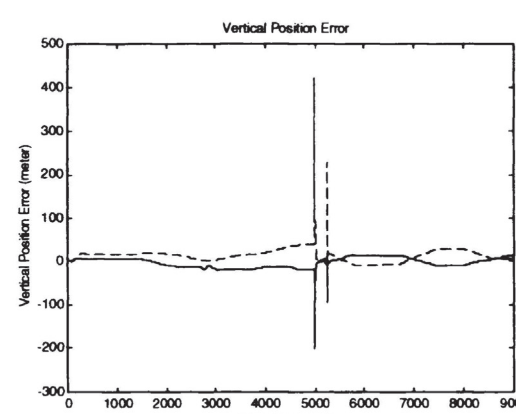

Fig. 5 Vertical position error.

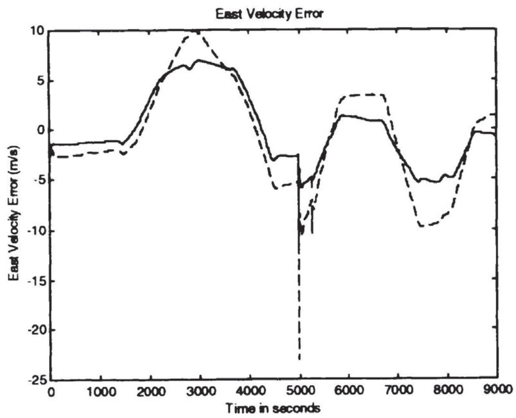

Fig. 6 East velocity error.

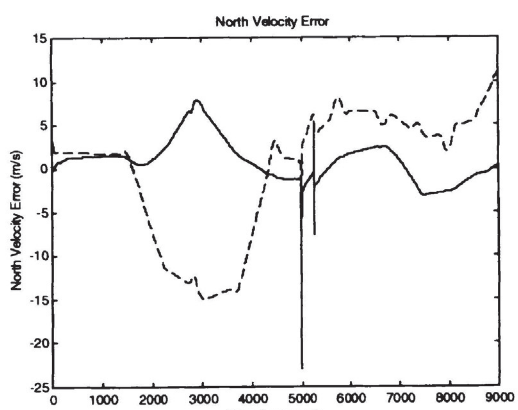

Fig. 7 North velocity error.

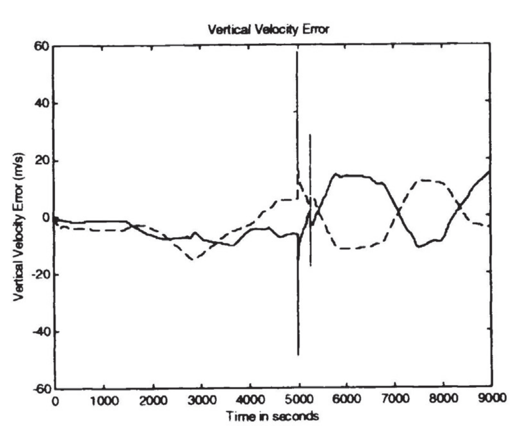

Fig. 8 Vertical velocity error.

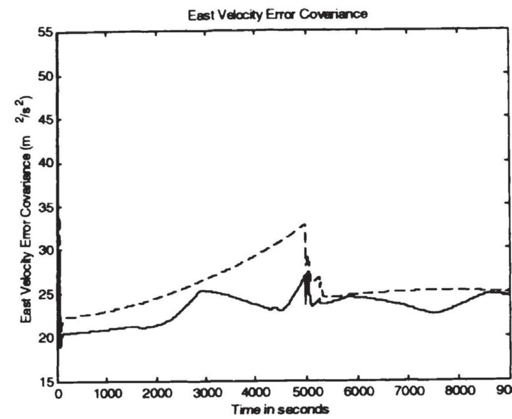

Fig. 9 Velocity error covariance in East direction

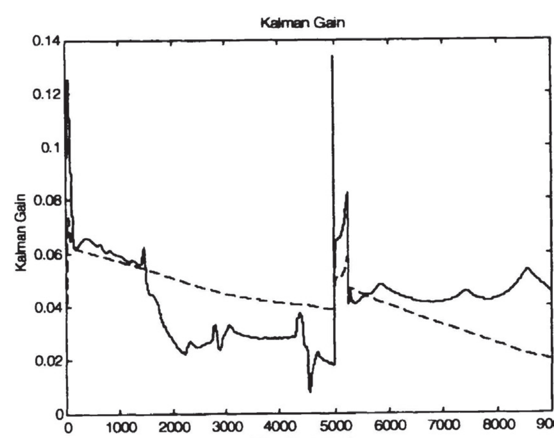

Fig. 10 Kalman filter gain.

The results shown in Figs. 3 to 10 are obtained from the experimental data involving the aircraft flying over a test range during several hours.

Sometimes, the data from GPS as well as INS cannot be considered to be a unbiased and uncorrelated, and therefore the basic assumption of the Kalman filter – white noise would not hold. The colored noise will cause the filter gain to diverge. The adaptive gain has to be used. Earlier, the authors suggested an efficient method of adaptive Kalman filtering based on fuzzy logic method. Sasiadek and Wang (1999) used fuzzy logic adapted controller (FLAC) to prevent the filter from divergence when fusing signals coming from INS and GPS on flying vehicle. Nine rules were used. There were two inputs, which are the mean value and covariance of innovation, and the output is a constant that is used to weight exponentially the model and measurement noise covariance.

Figure 11 explains the idea of fuzzy adaptive Kalman filtering.

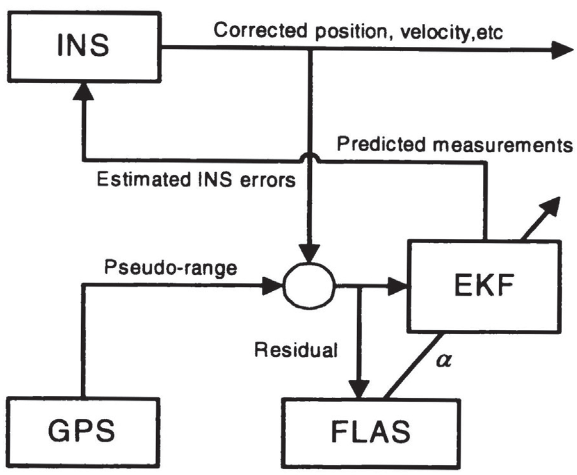

Fig. 11 Adaptive sensor fusion based on fuzzy logic

The adaptation of Kalman gain is done on the base of covariance and mean value. This adaptation prevents filter from divergence and improves the performance of the algorithm. The details of this method were reported by Sasiadek and Wang (1999, 2003).

7. CONCLUSIONS

This paper presents methods of sensor fusion based on Kalman Filter and analyzes various design approaches. The method was applied to the GPS/INS integration. The experimental results were presented and discussed. The objective of this paper was to demonstrate a known engineering technique and verify procedure developed by authors earlier. The performance of this method was checked and it was possible to verify the applicability in various conditions.

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