

$\neg_L: \Phi, \neg \alpha \circ \Psi$ $\quad \quad \quad \downarrow$ $\quad \quad \quad \Phi \circ \alpha, \Psi$	$\neg_R: \Phi \circ \neg \alpha, \Psi$ $\quad \quad \quad \downarrow$ $\quad \quad \quad \Phi, \alpha \circ \Psi$	$\forall_R: \quad \quad \quad \Phi \circ \forall x \varphi, \psi$ $\quad \quad \quad \downarrow$ $\quad \quad \quad \Phi \circ [d_{k+1}/x] \varphi, \psi$ <div> <div>(1) $D = \{d_1, \dots, d_k\}$</div> <div>(2) ...</div> </div> <div> <div>(1) $D = \{d_1, \dots, d_k, d_{k+1}\}$</div> <div>(2) ...</div> </div>
$\wedge_L: \Phi, \alpha \wedge \beta \circ \Psi$ $\quad \quad \quad \downarrow$ $\quad \quad \quad \Phi, \alpha, \beta \circ \Psi$	$\wedge_R: \Phi \circ \alpha \wedge \beta, \Psi$ $\quad \quad \quad \downarrow$ $\quad \quad \quad \Phi \circ \alpha, \Psi \quad \Phi \circ \beta, \Psi$	<div> <div>(1) $D = \{d_1, \dots, d_k\}$</div> <div>(2) ...</div> </div> $\forall_L: \quad \quad \quad \Phi, \forall x \varphi \circ \psi$ $\quad \quad \quad \downarrow$ $\quad \quad \quad \Phi, \forall x \varphi, [d_1/x] \varphi, \dots, [d_k/x] \varphi \circ \psi$ <div> <div>(1) $D = \{d_1, \dots, d_k\}$</div> <div>(2) $\forall x \varphi: \{d_1, \dots, d_k\}$</div> </div>
$\vee_L: \Phi, \alpha \vee \beta \circ \Psi$ $\quad \quad \quad \downarrow$ $\quad \quad \quad \Phi, \alpha \circ \Psi \quad \Phi, \beta \circ \Psi$	$\vee_R: \Phi \circ \alpha \vee \beta, \Psi$ $\quad \quad \quad \downarrow$ $\quad \quad \quad \Phi \circ \alpha, \beta, \Psi$	<div> <div>(1) $D = \{d_1, \dots, d_k\}$</div> <div>(2) ...</div> </div> $\exists_L: \quad \quad \quad \Phi, \exists x \varphi \circ \psi$ $\quad \quad \quad \downarrow$ $\quad \quad \quad \Phi, [d_{k+1}/x] \varphi \circ \psi$ <div> <div>(1) $D = \{d_1, \dots, d_k, d_{k+1}\}$</div> <div>(2) ...</div> </div>
$\rightarrow_L: \Phi, \alpha \rightarrow \beta \circ \Psi$ $\quad \quad \quad \downarrow$ $\quad \quad \quad \Phi, \beta \circ \Psi \quad \Phi \circ \alpha, \Psi$	$\rightarrow_R: \Phi \circ \alpha \rightarrow \beta, \Psi$ $\quad \quad \quad \downarrow$ $\quad \quad \quad \Phi, \alpha \circ \beta, \Psi$	<div> <div>(1) $D = \{d_1, \dots, d_k\}$</div> <div>(2) ...</div> </div> $\exists_R: \quad \quad \quad \Phi \circ \exists x \varphi, \psi$ $\quad \quad \quad \downarrow$ $\quad \quad \quad \Phi \circ [d_1/x] \varphi, \dots, [d_k/x] \varphi, \exists x \varphi, \psi$ <div> <div>(1) $D = \{d_1, \dots, d_k\}$</div> <div>(2) $\exists x \varphi: \{d_1, \dots, d_k\}$</div> </div>
$\Leftrightarrow_L: \Phi, \alpha \Leftrightarrow \beta \circ \Psi$ $\quad \quad \quad \downarrow$ $\quad \quad \quad \Phi, \alpha, \beta \circ \Psi \quad \Phi \circ \alpha, \beta, \Psi$	$\Leftrightarrow_R: \Phi \circ \alpha \Leftrightarrow \beta, \Psi$ $\quad \quad \quad \downarrow$ $\quad \quad \quad \Phi, \alpha \circ \beta, \Psi \quad \Phi, \beta \circ \alpha, \Psi$	<div> <div>(1) $D = \{d_1, \dots, d_k\}$</div> <div>(2) ...</div> </div>