

# Oplossingen Oefeningen Grondslagen 1: Propositielogica: Semantiek II

## Oefening 64

$$\forall x(\varphi(x) \wedge \psi(x)) \vdash \forall x\varphi(x) \wedge \forall x\psi(x)$$

$$\frac{\frac{\frac{\forall x(\varphi(x) \wedge \psi(x))}{\varphi(d) \wedge \psi(d)} \forall E}{\varphi(d)} \wedge E}{\forall x\varphi(x)} \forall I \quad \frac{\frac{\frac{\forall x(\varphi(x) \wedge \psi(x))}{\varphi(d) \wedge \psi(d)} \forall E}{\psi(d)} \wedge E}{\forall x\psi(x)} \forall I$$

$$\frac{\forall x\varphi(x) \quad \forall x\psi(x)}{\forall x\varphi(x) \wedge \forall x\psi(x)} \wedge I$$

$$\forall x\varphi(x) \wedge \forall x\psi(x) \vdash \forall x(\varphi(x) \wedge \psi(x))$$

$$\frac{\frac{\frac{\forall x\varphi(x) \wedge \forall x\psi(x)}{\forall x\varphi(d)} \wedge E}{\varphi(d)} \forall E}{\varphi(d) \wedge \psi(d)} \wedge I \quad \frac{\frac{\frac{\forall x\varphi(x) \wedge \forall x\psi(x)}{\forall x\psi(d)} \wedge E}{\psi(d)} \forall E}{\psi(d)} \forall E$$

$$\frac{\varphi(d) \wedge \psi(d)}{\forall x(\varphi(x) \wedge \psi(x))} \forall I$$

$$\exists x(\varphi(x) \vee \psi(x)) \vdash \exists x\varphi(x) \vee \exists x\psi(x)$$

$$\frac{\frac{\frac{\frac{\frac{\frac{\exists x(\varphi(x) \vee \psi(x))}{\varphi(d) \vee \psi(d)} \exists I}{\varphi(d)} \vee I}{\exists x\varphi(x)} \exists I}{\varphi(d) \vee \psi(d)} \vee I}{\exists x\varphi(x) \vee \exists x\psi(x)} \vee I \quad \frac{\frac{\frac{\frac{\frac{\frac{\exists x(\varphi(x) \vee \psi(x))}{\psi(d) \vee \psi(d)} \exists I}{\psi(d)} \vee I}{\exists x\psi(x)} \exists I}{\psi(d) \vee \psi(d)} \vee I}{\exists x\varphi(x) \vee \exists x\psi(x)} \vee I$$

$$\frac{\exists x\varphi(x) \vee \exists x\psi(x)}{\exists x\varphi(x) \vee \exists x\psi(x)} \vee E[-2, -3]$$

$$\frac{\exists x\varphi(x) \vee \exists x\psi(x)}{\exists x\varphi(x) \vee \exists x\psi(x)} \exists E[-1]$$

$$\exists x\varphi(x) \vee \exists x\psi(x) \vdash \exists x(\varphi(x) \vee \psi(x))$$

$$\frac{\frac{\frac{\frac{\frac{\frac{\exists x\varphi(x) \vee \exists x\psi(x)}{\exists x\varphi(x)} \vee I}{\varphi(d)} \vee I}{\varphi(d) \vee \psi(d)} \vee I}{\exists x\varphi(x) \vee \exists x\psi(x)} \vee I \quad \frac{\frac{\frac{\frac{\frac{\frac{\exists x\varphi(x) \vee \exists x\psi(x)}{\exists x\psi(x)} \vee I}{\psi(d)} \vee I}{\psi(d) \vee \psi(d)} \vee I}{\exists x\varphi(x) \vee \exists x\psi(x)} \vee I$$

$$\frac{\exists x\varphi(x) \vee \exists x\psi(x)}{\exists x(\varphi(x) \vee \psi(x))} \vee E[-3] \quad \frac{\exists x\varphi(x) \vee \exists x\psi(x)}{\exists x(\varphi(x) \vee \psi(x))} \vee E[-4]$$

$$\frac{\exists x(\varphi(x) \vee \psi(x))}{\exists x(\varphi(x) \vee \psi(x))} \vee E[-1, -2]$$