AM205 Project - Traffic Flow Modelling

Microscopic Models of Traffic, Comparing the Intelligent Driver Model (with Adaptive Cruise Control) with the Human Driver Model

Anna Hilgard, Sam Daulton and Nicholas Hoernle

1 Introduction

The modeling of traffic at traffic lights, on highways, in dense traffic streams, with varying situations etc can often present a problem that is too complicated to solve analytically. This is mainly due to the inherent random movements of cars, the number of individually moving cars and the many and varying scenarios that can be analyzed. In this project, we present two main models, the 'Intelligent Driver Model' and the 'Human Driver Model' and numerically simulate a number of test cases allowing us to study the underlying dynamics of this system. We focus on modeling the cars using a differential setting where one car's motion is dependent on the car that is in front of it (and sometimes dependent on multiple cars in front of it). We will then extend the scope of this analysis and introduce a number of varying factors into the model to analyze the motion of cars along a straight road, analyze the cars along a circular track and finally include a blend of the intelligent driving cars (comparable to automated cruise control autonomous cars) and cars with the human controlled vehicles. These autonomous cars have more precise, and faster reacting velocity controllers and the inclusion of vehicles such as these in the model is expected to smooth the flow of traffic and ultimately increase the density of traffic flow.

There are generally two ways to model traffic: from a microscopic and from a macroscopic perspective [1, Chapter 34]. In the macroscopic perspective, traffic is viewed as a fluid or gas with a given maximum density moving according to the laws of conservation of mass. Because we'd like to interweave different models for specific cars, these models would be difficult to blend in a way that is easily interpretable.

2 Microscopic Perspective of Single Lane Traffic Interactions

In the microscopic perspective, individual cars are modeled as particles that move according to a relationship with the particle(s) in front of them. Time-continuous microscopic models, commonly referred to as car-following models, model the characteristics of individual vehicles such as acceleration, velocity, position using a systems of coupled ordinary differential equations (ODEs). Car-following models are constained by assumptions that mimic realistic driving behavior such as a vehicle wants to maintain a safe driving gap between itself and vehicle in front of it (commonly called the *leading vehicle*), a vehicle has a certain desired velocity, or there is a maximum comfortable acceleration [2, chapter 10]. In the section, we consider a few variations of the Intelligent Driver Model (IDM), as well as Human Driver Model (HDM). We chose the IDM over other car-following models such as the Gipps Model or Newell's due to the oversimplification of some acceleration assumptions that the alternative models make[2, chapter 11]. The result is that the IDM provides the simplest implementation of plausible driving characteristics.

2.1 Intelligent Driver Models

In this section, we model streams of semi-autonomous vehicles that display the following driving characteristics:

- No estimation errors: each vehicle is aware of its own speed, the gap between itself the car in front of it, and the speed of the car in front of it.

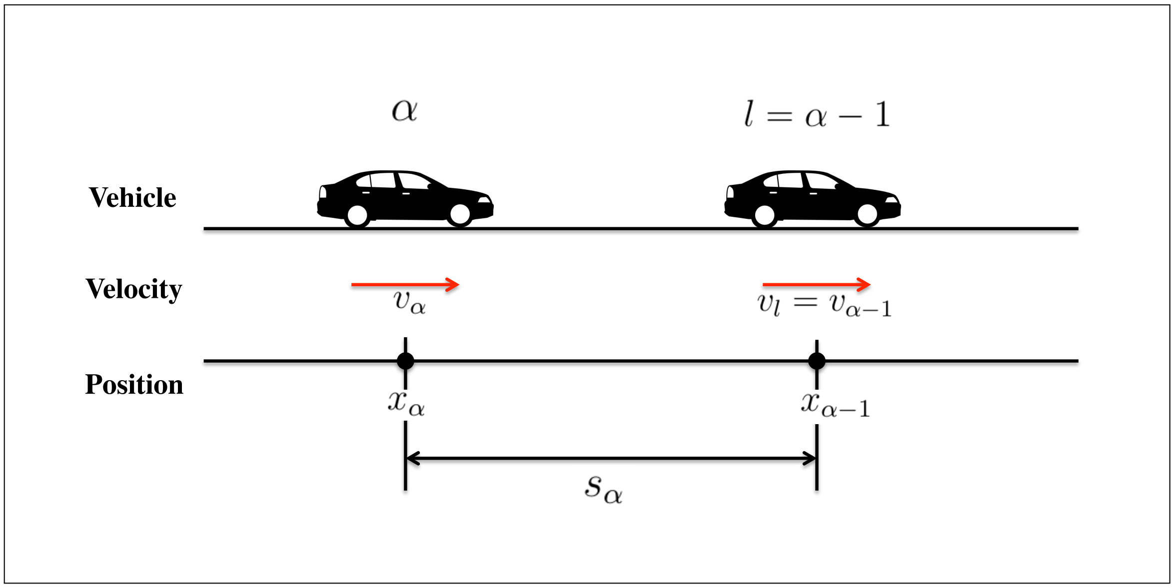

Figure 1: Snapshot of two cars in a system modeled using a car-following model. The leading car with respect to vehicle α is referred to as vehicle $l = \alpha - 1$. Note that vehicles are represented as particles, and therefore, the vehicles themselves have negligible length.

- Zero response time: there is no delay between observing a stimulus (i.e. change in gap between itself and vehicle ahead of it or change the vehicle ahead's speed) and responding to it.
- Constant attention: the vehicle is constantly looking for changes in speed or gap
- Each vehicle is only aware of itself and the vehicle directly in front of it. I.e. it does not account for changes in speed or gap of a vehicle two cars ahead of it.
- Each vehicle is not aware of vehicles behind it or in other lanes
- Vehicles are not aware other indicators such as brake lights, horns, headlight flashes, etc.

2.1.1 Intelligent Driver Model (IDM)

The IDM is a simple, accident-free model that can model traffic systems under all realistic traffic conditions. The acceleration function, shown in Eq (1), of a particular vehicle α is a function the current gap between itself and the leading vehicle (s_α), the desired speed v_0, and the speed of the vehicle leading vehicle v_l.

IDM Parameters

- a: maximum acceleration
- b: maximum comfortable deceleration
- s_0: minimum bumper-to-bumper gap (between vehicle and leading vehicle)
- v_0: desired speed
- T: minimum time gap (between vehicle and leading vehicle)
- δ: acceleration exponent

IDM Governing Equations

$$\dot{v}_\alpha = a\left[1 - \left(\frac{v_\alpha}{v_0}\right)^\delta - \left(\frac{s^*(v_\alpha, \Delta v_\alpha)}{s_\alpha}\right)^2\right] \tag{1}$$

$$s_\alpha = x_{\alpha-1} - x_\alpha = x_l - x_\alpha \tag{2}$$

$$\Delta v_\alpha = v_\alpha - v_{\alpha-1} = v_\alpha - v_l \tag{3}$$

$$s^*(v_\alpha, \Delta v_\alpha) = s_0 + \max\left(0, v_\alpha T + \frac{v_\alpha \Delta v_\alpha}{2\sqrt{ab}}\right) \tag{4}$$

The first term in Eq (1) is the *free acceleration*–the acceleration that a vehicle would take if on an open road, without considerations of desired speed or gap. The second term in Eq (1) is the effect on the acceleration of the vehicle approaching its desired speed. The third term in Eq(1) is the effect on the acceleration of relative difference between the desired gap distance $s^*(v_\alpha, \Delta v_\alpha)$ and the current gap distance. The desired gap distance given by Eq (4) the sum of two terms the steady state desired safe distance $s_0 + v_\alpha T$ and a dynamical term $\frac{v_\alpha \Delta v_\alpha}{2\sqrt{ab}}$ representing the intelligent vehicle's braking strategy [3].

2.1.2 Improved Intelligent Driver Model (IIDM)

The Improved Intelligent Driver Model is an IDM model with an improved acceleration function that aims to correct three unrealistic qualities of the IDM:

1. If a vehicle exceeds the desired speed, the IDM acceleration function will return a very large negative acceleration. This is unrealistic since drivers on a highway tend to oscillate around their desired speed.
2. If a vehicle is close to the desired speed, the desired gap $s^*(v_\alpha, \Delta v_\alpha)$ becomes much greater than $s_0 + v_\alpha T$. This causes the gaps between vehicles to be unrealistically large and can preclude vehicles from reaching the desired velocities.
3. If the actual gap is less than the desired gap distance (e.g. when another vehicle merges into the particular vehicle's lane), then the acceleration function will return an excessively large negative acceleration. This unrealistic as drivers typically brake lightly to increase the gap.

IIDM Governing Functions

$$\left.\frac{dv_\alpha}{dt}\right|_{v_\alpha \le v_0} = \begin{cases} a(1-z^2) & z_\alpha \ge 1 \\ a_{\text{free},\alpha}(1 - z_\alpha^{(2a)/a_{\text{free},\alpha}}) & \text{otherwise} \end{cases} \tag{5}$$

$$\left.\frac{dv_\alpha}{dt}\right|_{v_\alpha > v_0} = \begin{cases} a_{\text{free},\alpha} + a(1-z_\alpha^2) & z_\alpha \ge 1 \\ a_{\text{free},\alpha} & \text{otherwise} \end{cases} \tag{6}$$

$$a_{\text{free},\alpha}(v_\alpha) = \begin{cases} a\left[1 - (\frac{v_\alpha}{v_0})^\delta\right] & v_\alpha \le v_0 \\ -b\left[1 - (\frac{v_0}{v_\alpha})^{a\delta/b}\right] & v_\alpha > v_0 \end{cases} \tag{7}$$

$$z_\alpha = \frac{s^*(v_\alpha, \Delta v_\alpha)}{s_\alpha} \tag{8}$$

The IIDM governing equations seek to improve the unrealistic qualities of the IDM model. Eq (8) is the free acceleration equation, which limits the deceleration's magnitude when $v_\alpha > v_0$

to be at most b. To improve the acceleration function near the desired speed, we use z_α to indicate whether the desired gap distance is greater than the current gap distance or not. When $v_\alpha < v_0$, the IIDM's acceleration function (Eq (5) and Eq (6)) ensures that the actual gap is no larger than the minimum safe gap $s_\alpha = s_0 + vT$. To maintain the intelligent and comfortable brake method of the IDM, the improved acceleration function only changes when the actual gap is near the desired gap or when the actual velocity is greater than the desired velocity.

2.1.3 Adaptive Cruise Control Model (ACC)

$$a_{CAH}(s, v, v_l, \dot{v}_l) = \begin{cases} \frac{v^2 \tilde{a}_l}{v_l^2 - 2\tilde{(a)}_l} & v_l(v - v_l) \leq -2s\tilde{a}_l \\ \tilde{a}_l - \frac{(v-v_l)^2 \Theta(v-v_l)}{2s} & otherwise \end{cases}$$

$$\tilde{a}_l(\dot{(v)}_l) = min(\dot{v}_l, a)$$

$$a_{ACC} = \begin{cases} a_{IIDM} & a_{IIDM} \geq a_{CAH} \\ (1-c)a_{IIDM} + c\left[a_{CAH} + btanh(\frac{a_{IIDM} - a_{CAH}}{b})\right] & otherwise \end{cases}$$

$$\dot{v}(t) = a_{mic}\left[s(t - T_r), v(t - T_r), v_l(t - T_r)\right]$$

$$u(t - Tr) = ru_{i-j-1} + (1-r)u_{i-j},\ j = \text{int}\left(\frac{T_r}{\Delta t}\right),\ r = \frac{T_r}{\Delta t} - j$$

$$\text{ln}s^{est} - \text{ln}s = V_s w_s(t)$$

$$v^{est} - v_l = -s\sigma_r w_l(t)$$

3 Microscopic Perspective of Traffic Interaction : Human Driver Models

4 Numerical Simulations of Traffic Systems

To generate numerical simulations of Traffic Systems, we first consider the case of 50 cars on a circular track. We explored scenarios of all IDM cars and all HDM cars. Under these conditions, we see the IDM models generate more ‘phantom‘ traffic waves than the HDM models, especially for larger numbers of HDM car lookahead. This is because the human drivers tend to appear more conservative in a situation like this, with no external perturbations, as the multi-car lookahead prevents them from getting overly close to the car directly in front of them.

We then moved on to simulations of a platoon of cars with some cars being controlled by IDM-type models (in particular, Adaptive Cruise Control) and some cars being controlled by multi-car lookahead HDM models. After allowing the models to run for some time to develop their natural spacing, we then perturbed the acceleration of the lead car, causing it to brake and then resume again. We then observe both the magnitude and length of the volatility of the velocities in the following cars after the event. We find that in general, as one would expect, a larger proportion of IDM-controlled vehicles results in a quicker dissipation of the perturbation as well as a lowered probability of a vehicle collision.

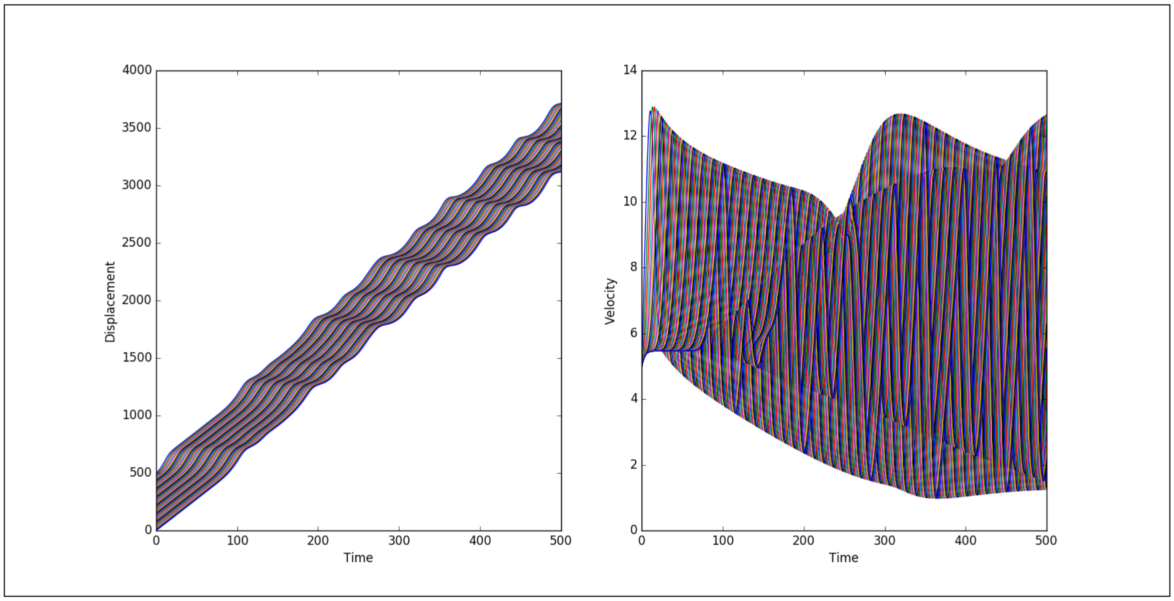

Figure 2: System of IDM cars on a circular track optimizing for maximum distance traveled. The more aggressive driving of the cars results in wave formation.

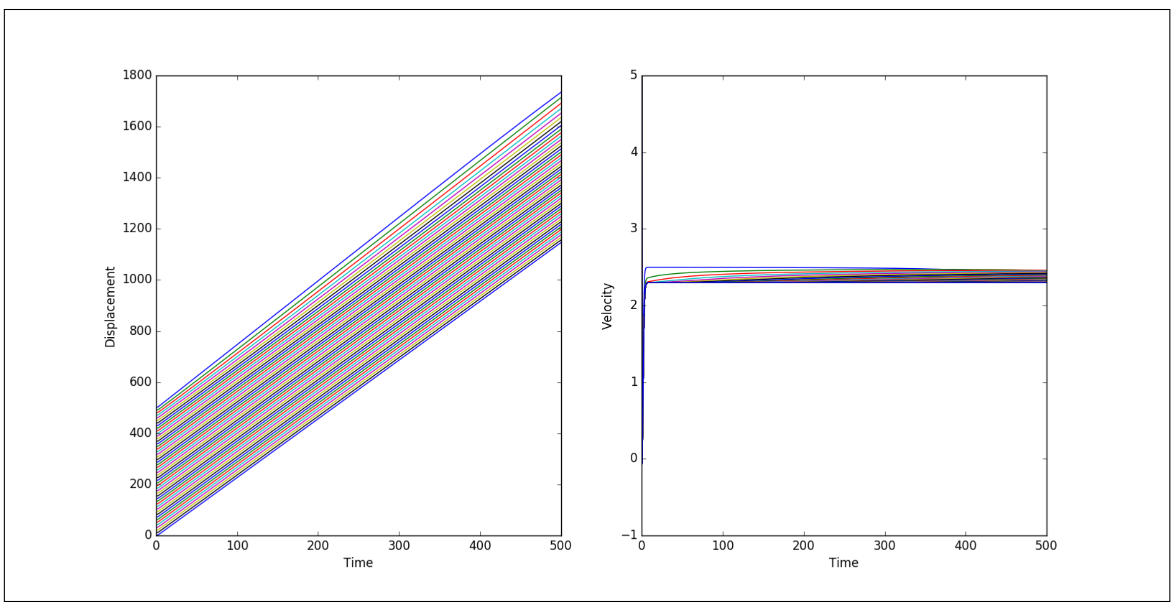

Figure 3: System of IDM cars on a circular track optimizing for maximum 'comfort' (minimum jerk). As one might expect, they choose to go rather slowly and thereby achieve a very stable velocity.

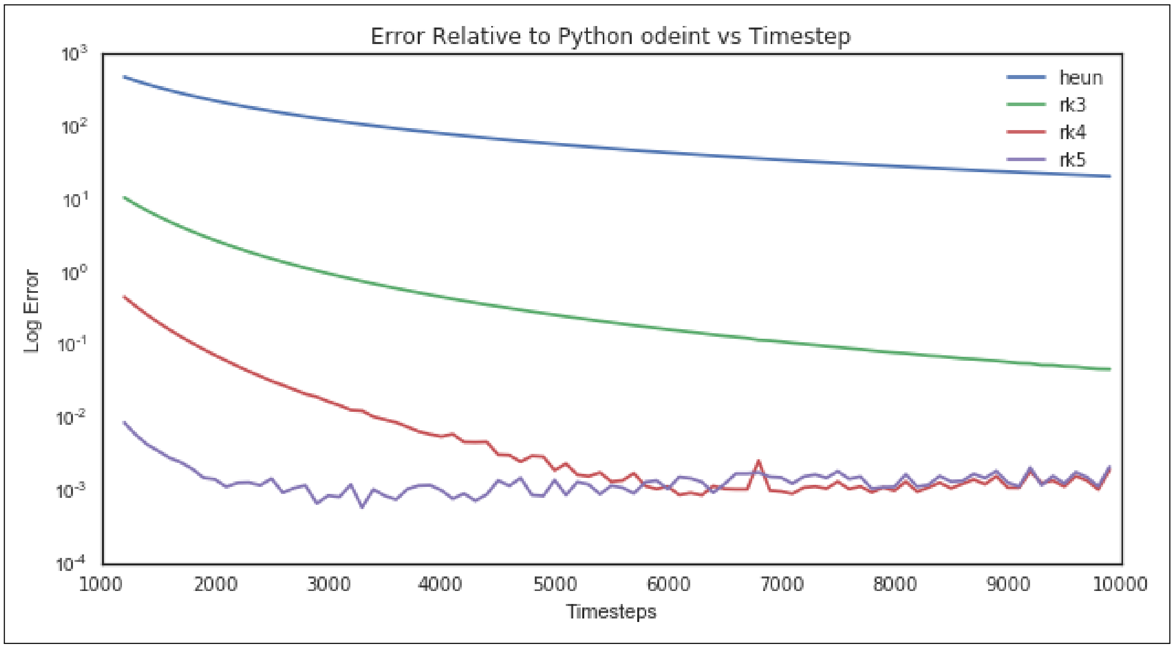

Figure 4: Error to odeint decreases with accuracy of finite differencing methods.

5 Stability and Error Comparison of a Variety of Numerical Integration Schemes

Because there is no analytical solution given for our highly-nonlinear problem, to analyze the error of our numerical integration schemes, we compared to the python `odeint` solver function. For large enough time steps, this shows us error decreasing on a log scale for each of our higher order methods. However, for small time steps, the ‘error‘ for 4th and 5th order methods converges, as the `odeint` solver presumably decides that 4th order is sufficiently accurate at these time steps and stops using higher order methods.

For our purposes, we desired to use higher order methods such that we could use larger timesteps and hopefully decrease running time of our programs while still maintaining stability in the differential equation systems. To display the effectiveness of these higher order methods, we ran one of our simple models for a variety of time steps over the same total time with the different solvers and noted the number of time steps at which each method stabilized. We notice here that we don't see an additional benefit from using RK5, which in this case we suspect is because the extremely complicated coefficients are contributing to existing numerical overflow issues.

Table 1: Timesteps for Stability with T=500s

Method	Minimum Timesteps for Stability
Heun's Method	325
Runge-Kutta 3	275
Runge-Kutta 4	250
Runge-Kutta 5	250

6 String Stability of Traffic Systems to Perturbation

The concept of string stability refers to whether a perturbation to the beginning of the system will result in a wave that dissipates or is enhanced as it propagates through the system. As we have seen in previous experiments that the IDM models are excellent for adjusting to and dissipating traffic perturbations, we expect them to be string stable and indeed they are.

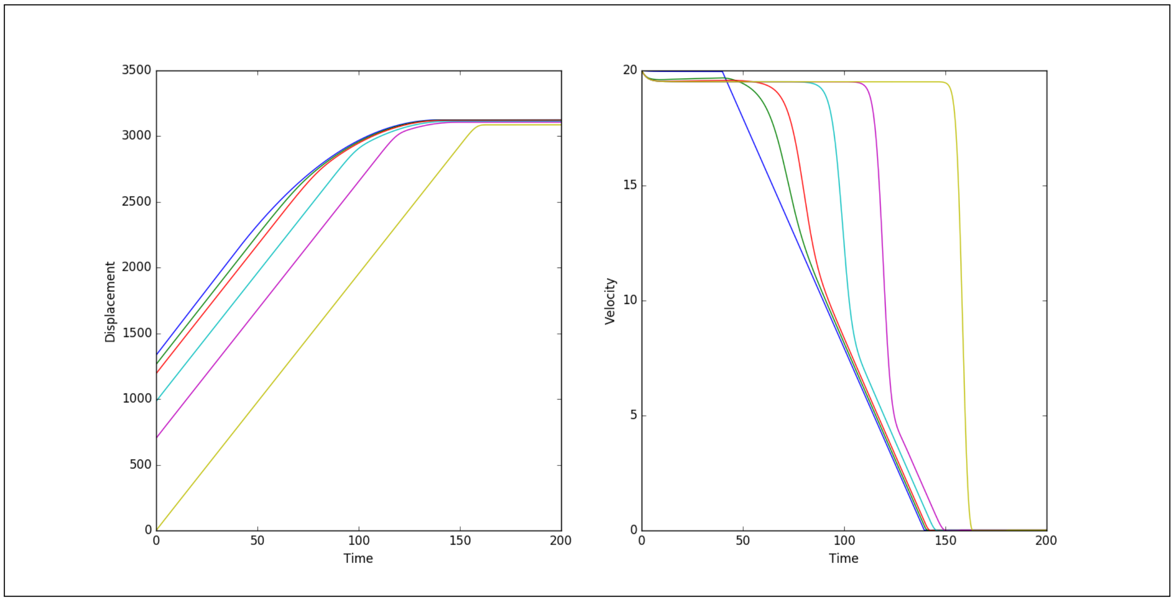

Figure 5: Plot of cars 1,2,3,6,10, and 20 for IDM. String stable.

For HDM models, the presence or lack thereof of string stability depends on a number of parameters: number of leading cars being considered, time step (Δt), and reaction time (T_r). In general, string stability increases with number of leading cars being considered, although the marginal contribution of each car decreases to the point where there is not much additional benefit beyond 5 cars. String stability is inversely related to time step and reaction time, although reaction time has a greater effect. Below, we show two plots for parameters where the HDM system is string stable and string unstable.

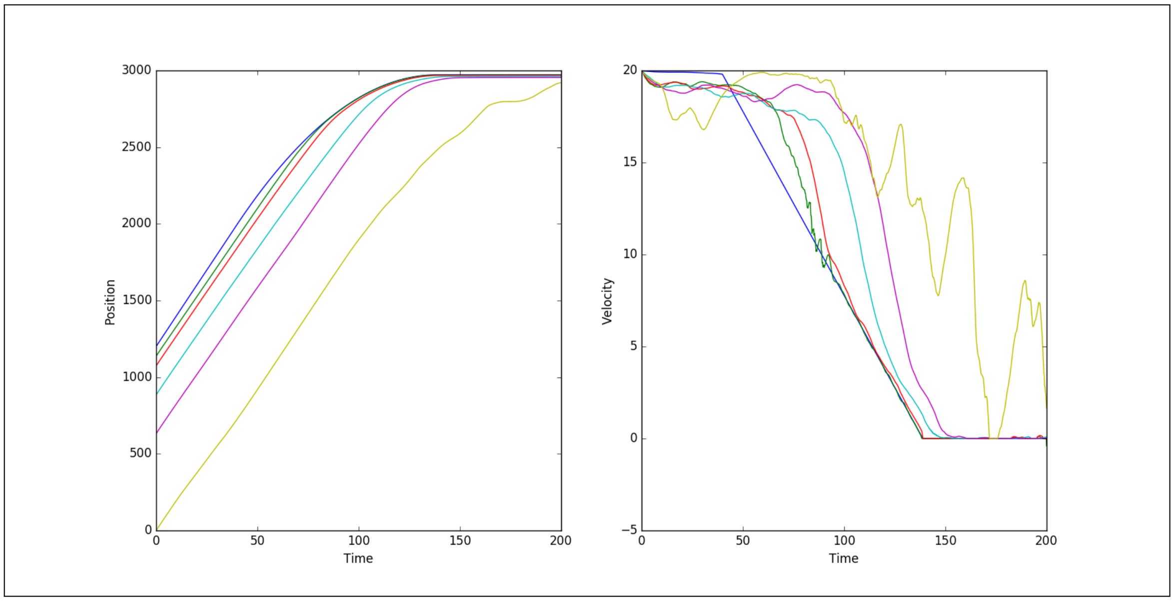

Figure 6: Plot of cars 1,2,3,6,10, and 20 for HDM with two car lookahead. We see that for parameters T_r = .6 and Δt = .2, the system is string unstable, as evidenced by the large wave in car 20's velocity.

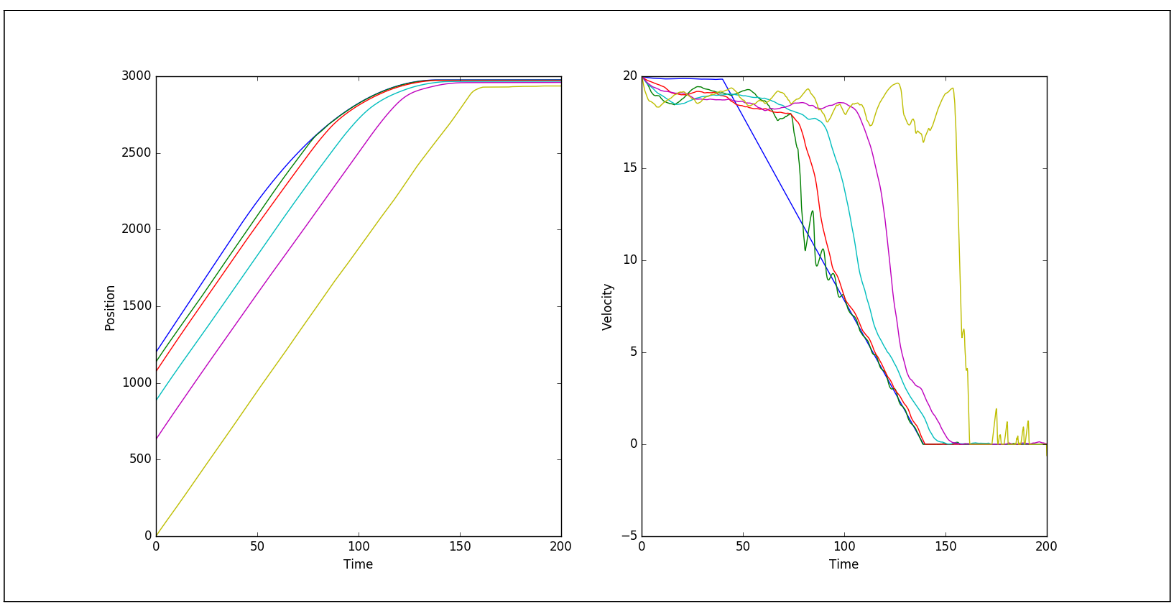

Figure 7: Plot of cars 1,2,3,6,10, and 20 with two car lookahead. We see that for parameters T_r = .2 and Δt = .15, the system is string stable.

7 Optimization of Traffic Systems

We set up a number of optimization programs to attempt to search for the optimal values of some of the many parameters for the system as a whole and for individual drivers for a number of cost functions, including maximizing displacement and maximizing comfort (minimizing jerk). The initial trials for these systems were run on IDM models, and the results are largely what one would suspect. When attempting to maximize total displacement, the cars drive quickly and close together, resulting in the creation of traffic waves in the system. When attempting to minimize jerk, the cars drive much more conservatively, as one would expect, never coming within the range of another car that would cause them to change acceleration.
RESULTS HERE

We then attempted to optimize behavior for a single driver and were interested to see what implications this would have for the system as a whole. However, the following behaviors in the IDM are such that a single driver can tailgate and maximize target speed basically as much as we will allow without ever disrupting the rest of the system. Even when adding some stochasticity to the lead car, we were unable to see the rest of the system suffer from traffic waves due to the behavior of the second car, because that behavior would simply be quickly dissipated by the third car. In fact, it seems to be true generally of these systems that unless the aggressive driving behavior of a car causes other cars to collide behind it, the greater distance achieved by one car inevitably results in other cars also achieving a greater distance/average velocity.

Next, we attempted to replicate these experiments with the more complicated Human Driver Model. However, due to the highly piecewise nature of the functions we are attempting to optimize over, we were unable to use traditional minimization solvers. Instead, we discretized the parameter space we were interested in exploring and searched for the set of parameters which would minimize our target cost function.
Again, we find that in general maximizing displacement for a single car leads to greater displacement for the rest of the system unless the driving behavior is so aggressive that it results in

a crash. Given that we have seen that the Human Driver Model is more likely to propagate perturbations, we had expected that there might be in this model a penalty to other cars in greater volatility of velocity, i.e. a less pleasant ride, but this effect was also not observed in the data. We conclude that the model is largely insensitive to individual parameters in this context and that the structure of the model tends not to penalize aggressive driving unless a collision results.

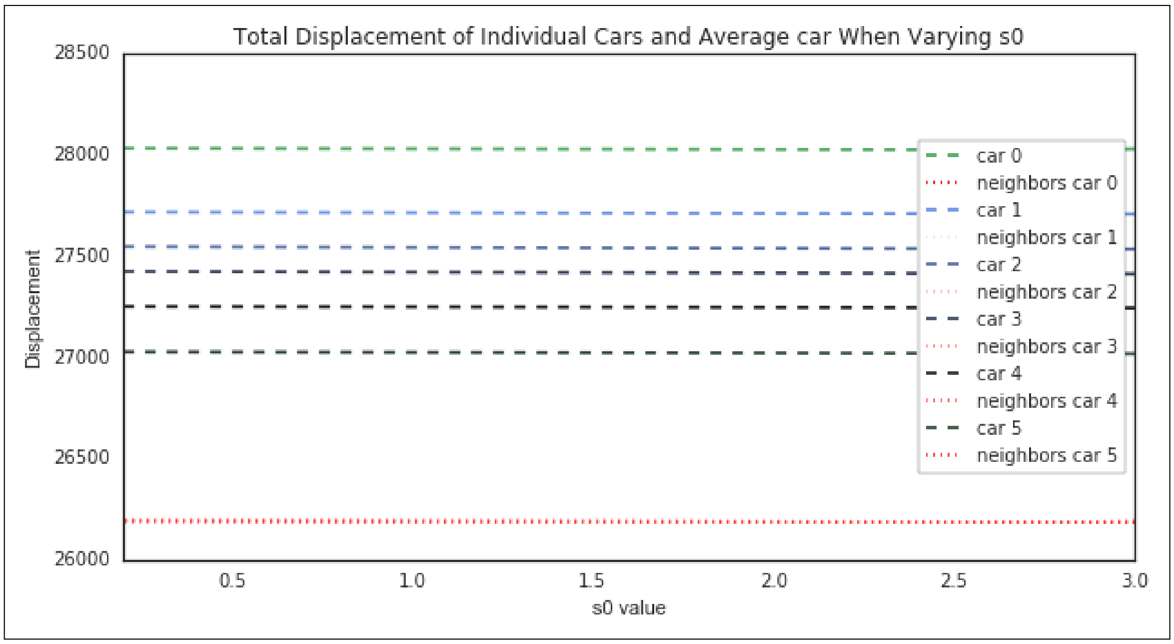

Figure 8: Changing Safe Following Distance: Changes are minimal but overall distance increases for both the car in question and the system of cars with smaller distance.

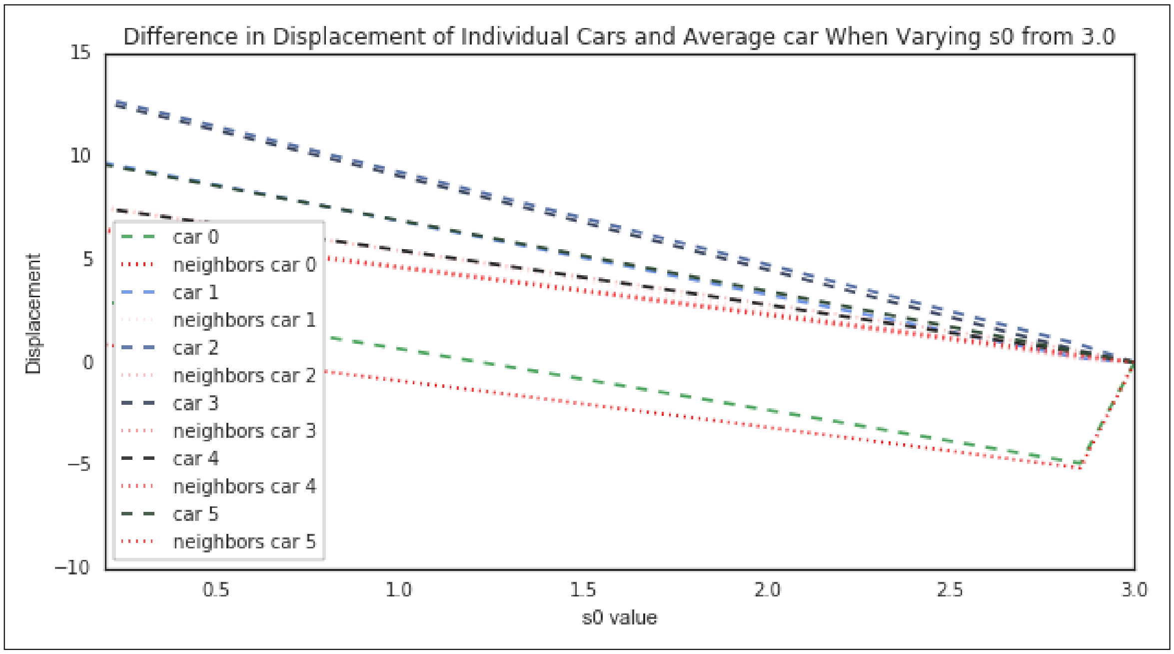

Figure 9: Variation in Distance Traveled when Changing Safe Following Distance From Initial Value of 3.0

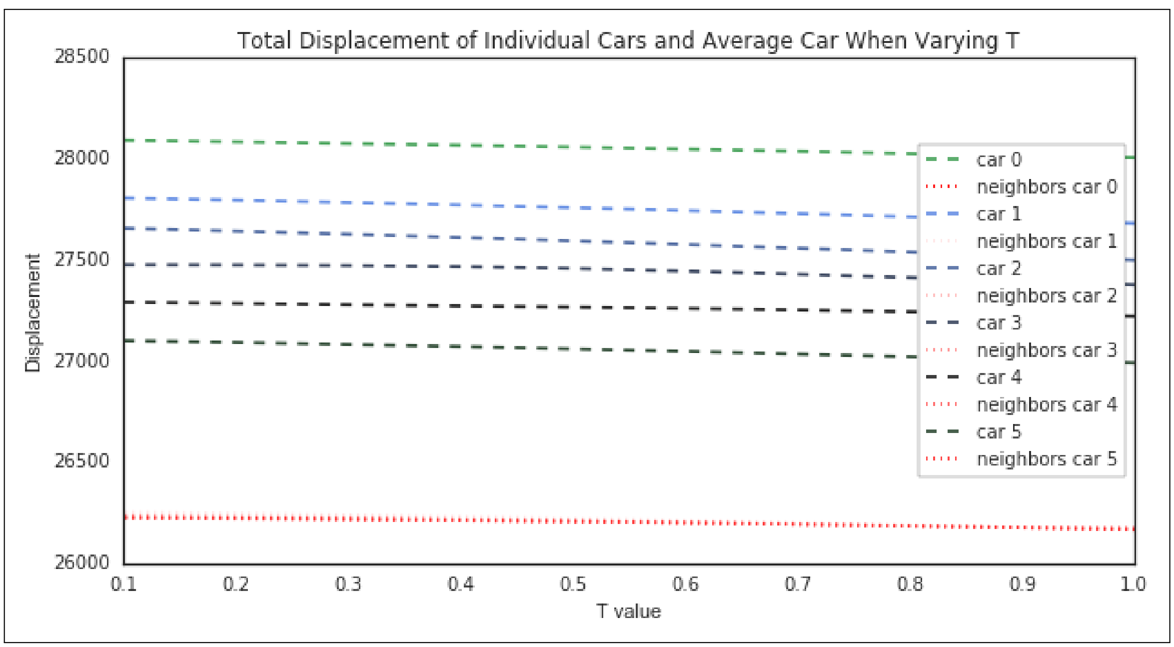

Figure 10: Changing Safe Following Time: Changes are minimal but overall distance increases for both the car in question and the system of cars with smaller.

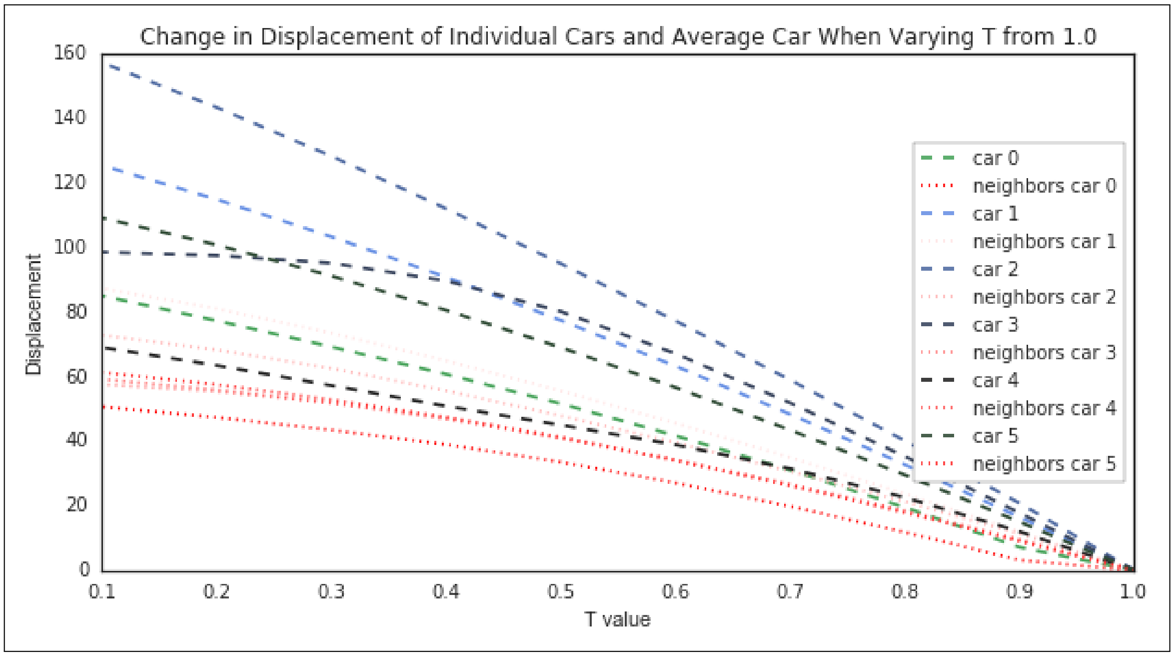

Figure 11: Variation in Distance Traveled when Changing Safe Following Time From Initial Value of 1.0

8 TODO

fuck this to do list, we've done enough.
develop a cost function for emissions/energy usage and optimize over that
can we find the parameters that will cause a phantom traffic jam?

9 Discussion

10 Conclusion

References and Notes

[1] R. Mathew, *Introduction to Transportation Engineering*. NPTEL, 2007.

[2] M. Treiber and A. Kesting, *Traffic Flow Dynamics: Data, Models and Simulation*. Springer, 2013.

[3] R. Malinauskas, "The intelligent driver model: Analysis and application to adaptive cruise control," *All Theses*, vol. Paper 1934, 2014.