AM205 Project - Traffic Flow Modeling

Microscopic Models of Traffic, Comparing the Intelligent Driver Model (with Adaptive Cruise Control) with the Human Driver Model

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1 Introduction

The modeling of traffic in varying situations (at traffic lights, on highways, in dense traffic streams, etc.) can often present a problem that is too complicated to solve analytically. This is mainly due to the inherent stochasticity in the movements of cars and the large number of individually moving cars. In this project we present two main models, the 'Intelligent Driver Model' (IDM) and the 'Human Driver Model' (HDM), for modeling traffic flow within a single lane. We numerically simulate a number of test cases, allowing us to study the underlying dynamics of this system. We focus on modeling the cars using a differential setting where a vehicle's motion is dependent on one or more vehicles front of it. Using each of these models, we analyze the motion of a system of vehicles cars along a straight road and along a circular track. Finally we create a system composed of a mixture intelligent driving vehicles (comparable to automated cruise control semi-autonomous cars) and the human-controlled vehicles. The semi-autonomous vehicles have precise control of their acceleration, instantaneous reactions, and sensors providing accurate estimates of positions and velocities of the vehicles around them. We expect that including of vehicles such as these in the model will smooth the flow of traffic and ultimately increase the density of traffic flow.

There are two common ways to model traffic: from a microscopic and from a macroscopic perspective. In the macroscopic models, traffic is viewed as a fluid or gas (with a given maximum density) and it moves according to the laws of conservation of mass. The microscopic perspective considers systems of individual vehicles each acting according to set of governing differential equations[1, Chapter 34]. In this paper, we consider microscopic models because we are interested in creating systems composed of a mixture of semi-autonomous and human-controlled vehicles. It would be difficult to adapt macroscopic models to model such systems in an interpretable way.

2 Microscopic Perspective of Single Lane Traffic Interactions

In microscopic models, individual cars are modeled as particles that move according to a relationship with the particle(s) in front of them. Time-continuous microscopic models, commonly referred to as car-following models, model the characteristics of individual vehicles using systems of coupled ordinary differential equations (ODEs). These characteristics include the acceleration, velocity, and position of the cars. Car-following models are constrained by assumptions [2, Chapter 10] that attempt to mimic realistic driving behavior such as:

- A vehicle wants to maintain a safe driving gap between itself and vehicle in front of it (commonly called the *leading vehicle*).
- A vehicle has a certain desired velocity[1] (e.g. the speed limit).
- There is a maximum acceleration that can be constrained by either the physical limitations of the car or comfort.

In this project, we chose the Intelligent Driver Model over other car-following models such as the Gipps Model or Newell's due to the oversimplification of some acceleration assumptions that the alternative models make [2, Chapter 11].

[1]Note: we use the terms *speed* and *velocity* interchangeably in this paper since in all models we discuss, vehicles can only have nonnegative velocities.

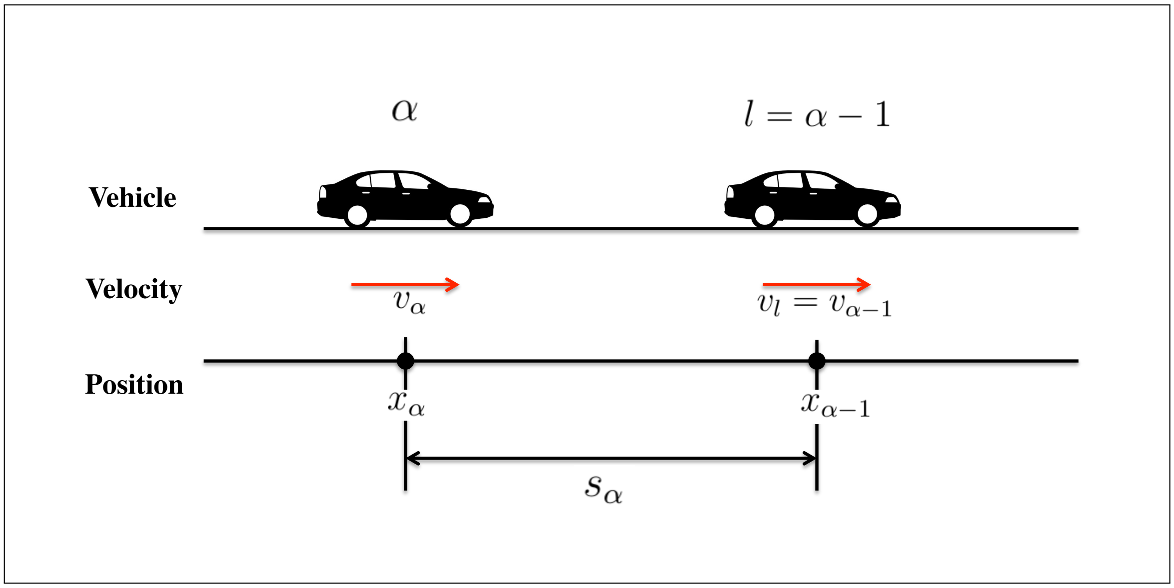

Figure 1: Snapshot of two cars in a system modeled using a car-following model. The leading car with respect to vehicle α is referred to as vehicle $l = \alpha - 1$. Note that vehicles are represented as particles, and therefore, the vehicles themselves have negligible length.

2.1 Intelligent Driver Models

2.1.1 Intelligent Driver Model (IDM)

The IDM is a simple, accident-free model that can model traffic systems under all realistic traffic conditions. The IDM provides a simple implementation for semi-autonomous vehicles that display the following driving characteristics:

- No estimation errors: each vehicle is able to measure exactly its own speed, the gap between itself the car and in front of it, and the speed of the car in front of it.
- Zero response time: there is no delay between observing a stimulus (i.e. a change in gap between itself and vehicle ahead of it or a change the leading vehicle's speed) and responding to it.
- Each vehicle is only aware of itself and the vehicle directly in front of it; it does not account for changes in the speed or gap of a vehicle two cars ahead of it.

- Vehicles are not aware of other indicators such as brake lights, horns, headlight flashes, etc.

The acceleration function, of a particular vehicle α is a function of the distance between itself and its leading vehicle (s_α), the vehicle's desired speed (v_0) and the speed of the leading vehicle (v_l). This is shown in Eq (1) and we further note that s_α is referred to as 'the gap'. [2, Chapter 11]

IDM Parameters

- a: maximum acceleration
- b: maximum comfortable deceleration
- s_0: minimum bumper-to-bumper gap (between vehicle and leading vehicle)
- v_0: desired speed
- T: minimum time gap (between vehicle and leading vehicle)
- δ: acceleration exponent

IDM Governing Equations

$$\dot{v}_\alpha = a\left[1 - \left(\frac{v_\alpha}{v_0}\right)^\delta - \left(\frac{s^*(v_\alpha, \Delta v_\alpha)}{s_\alpha}\right)^2\right] \tag{1}$$

$$s_\alpha = x_{\alpha-1} - x_\alpha = x_l - x_\alpha \tag{2}$$

$$\Delta v_\alpha = v_\alpha - v_{\alpha-1} = v_\alpha - v_l \tag{3}$$

$$s^*(v_\alpha, \Delta v_\alpha) = s_0 + \max\left(0, v_\alpha T + \frac{v_\alpha \Delta v_\alpha}{2\sqrt{ab}}\right) \tag{4}$$

The first term in Eq (1) is the *free acceleration* term. This is the acceleration that a vehicle would take if it were on a wide open road and had no considerations of desired speed or gap. The second term in Eq (1) is the damping effect on the vehicle's acceleration as it approaches its desired speed. The third term in Eq (1) is the effect of the relative difference between the desired gap distance $s^*(v_\alpha, \Delta v_\alpha)$ and the current gap distance on the car's acceleration. The difference in position and speed of a vehicle α and the leading vehicle are given by Eq (2) and (3). The desired gap distance, given by Eq (4), is the sum of a desired safe distance $s_0 + v_\alpha T$ and a term representing the intelligent vehicle's braking strategy [3] ($\frac{v_\alpha \Delta v_\alpha}{2\sqrt{ab}}$).

2.1.2 Improved Intelligent Driver Model (IIDM)

The Improved Intelligent Driver Model is an IDM model with an enhanced acceleration function that aims to correct two unrealistic qualities of the IDM that occur when a vehicle's speed is close to the desired speed. This model makes the following improvements [2, Chapter 11]:

1. If a vehicle exceeds the desired speed, the IDM vehicle's acceleration controller will effectively slam on the brakes. This is unrealistic since drivers on a highway tend to oscillate around their desired speed.

2. If a vehicle is close to the desired speed, the desired gap $s^*(v_\alpha, \Delta v_\alpha)$ becomes much wider than $s_0 + v_\alpha T$. This may preclude vehicles from reaching the desired velocity v_0.

IIDM Governing Functions

$$\left.\frac{dv_\alpha}{dt}\right|_{v_\alpha \le v_0} = \begin{cases} a(1 - z_\alpha^2) & z_\alpha \ge 1 \\ a_{\text{free},\alpha}(1 - z_\alpha^{(2a)/a_{\text{free},\alpha}}) & \text{otherwise} \end{cases} \tag{5}$$

$$\left.\frac{dv_\alpha}{dt}\right|_{v_\alpha > v_0} = \begin{cases} a_{\text{free},\alpha} + a(1 - z_\alpha^2) & z_\alpha \ge 1 \\ a_{\text{free},\alpha} & \text{otherwise} \end{cases} \tag{6}$$

$$a_{\text{free},\alpha}(v_\alpha) = \begin{cases} a\left[1 - (\frac{v_\alpha}{v_0})^\delta\right] & v_\alpha \le v_0 \\ -b\left[1 - (\frac{v_0}{v_\alpha})^{a\delta/b}\right] & v_\alpha > v_0 \end{cases} \tag{7}$$

$$z_\alpha = \frac{s^*(v_\alpha, \Delta v_\alpha)}{s_\alpha} \tag{8}$$

The IIDM governing equations seek to improve the unrealistic qualities of the IDM model [3]. Eq (8) is the free acceleration equation, which limits the deceleration's magnitude when $v_\alpha > v_0$ to be at most b (the maximum deceleration that a vehicle may assume). This prevents vehicles from slamming on the brakes as soon as they go faster than the desired velocity. To improve the acceleration function's behavior near the desired speed, the IIDM sets the acceleration differently depending on whether the speed and gap are above or below their respective desired values. When $v_\alpha < v_0$, the IIDM's acceleration function (Eq (5) and Eq (6)) ensures that the actual gap is no larger than the minimum safe gap $s_\alpha = s_0 + vT$. To maintain the intelligent and comfortable brake method of the IDM, this improved acceleration function mimics the behavior of the IDM acceleration function except for when the actual gap is near the desired gap or when the actual speed is greater than the desired speed [2, Chapter 11].

2.1.3 Adaptive Cruise Control Model (ACC)

The ACC is an improvement on the IIDM model that is designed so that a vehicle can handle discontinuous changes in the gap between itself and the leading vehicle or discontinuous changes in the measured speed of the leading vehicle [2, Chapter 11]. One scenario where such discontinuous changes would occur is when the leading vehicle either enters or exits the lane. Since the IDM based models are only meant for controlling the acceleration of a vehicle within a single lane, this scenario is equivalent to the leading vehicle simply vanishing into or appearing out of thin air. In the IDM and IIDM models, vehicles overreact if the gap is suddenly decreased because the models are designed to avoid accidents under all circumstances. The ACC dampens this response because a vehicle slamming on the brakes when the gap is decreased is unrealistic and can even be dangerous if the system is composed of a mixture of semi-autonomous vehicles and human-controlled vehicles, since most human drivers do not anticipate sudden changes in the acceleration of the vehicle in front of it. It is important to note that the ACC model is not guaranteed to be accident-free; the driving style is less defensive by design [2, Chapter 11][4].

To attenuate the semi-autonomous vehicle's reaction to sudden decreases in the gap distance, vehicles in the ACC model act according to a constant-acceleration heuristic. This is done rather than making decisions based off the worst-case scenario. The constant-acceleration heuristic

(CAH) assumes that vehicles will have constant acceleration in the near future, no safe distance or time gap is necessary at any time, and that vehicles will react instantaneously [2, Chapter 11].

ACC Governing Functions

$$a_{\text{ACC},\alpha} = \begin{cases} a_{\text{IIDM},\alpha} & a_{\text{IIDM},\alpha} \geq a_{\text{CAH},\alpha} \\ (1-c)a_{\text{IIDM},\alpha} + c\left[a_{\text{CAH},\alpha} + b\tanh\left(\frac{a_{\text{IIDM},\alpha} - a_{\text{CAH},\alpha}}{b}\right)\right] & \text{otherwise} \end{cases} \tag{9}$$

$$a_{\text{CAH},\alpha} = \begin{cases} \frac{v_\alpha^2 \tilde{a}_l}{v_l^2 - 2s_\alpha \tilde{a}_l} & v_l(v_\alpha - v_l) \leq -2s_\alpha \tilde{a}_l \\ \tilde{a}_l - \frac{(v_\alpha - v_l)^2 \Theta(v_\alpha - v_l)}{2s_\alpha} & \text{otherwise} \end{cases} \tag{10}$$

$$\tilde{a}_l = \min(\dot{v}_l, a) \tag{11}$$

$$\Theta(x) = \begin{cases} 1 & x \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{12}$$

Under the constant-acceleration heuristic, the vehicle's acceleration is given by Eq (9), where $\tilde{a}_l$ is the effective acceleration of the leading vehicle (limited by the maximum acceleration parameter a). One important distinction between the ACC and the previous intelligent driving models is that the ACC uses the leading vehicle's acceleration as an input, in addition to the leading vehicle's speed and gap. It is important to note that the acceleration under the constant-acceleration heuristic ($a_{\text{CAH},\alpha}$) is only used as an indicator of whether the IIDM acceleration is overreacting to the situation. Hence, the ACC model also calculates the acceleration function from the IIDM model. The acceleration function for the ACC model is given by Eq (9), where c is the 'coolness parameter', which is used to tune the ACC model's behavior ($c = 0$ is equivalent to a pure IIDM model and $c = 1$ is a pure ACC model) [2, Chapter 11]. If the IIDM acceleration is greater than or equal to the CAH acceleration, then the ACC model uses the IIDM acceleration, since the IIDM model is accident-free and thus guaranteed to be safe. Otherwise, the ACC model uses a less cautious acceleration function.

3 Microscopic Perspective of Traffic Interaction: Human Driver Models

The Human Driver Model (HDM) is an extension of the IDM model that models human driving behavior by making five important changes to the IDM [2, Chapter 12]:

1. The *reaction times* are no longer zero because humans have delayed reactions. The HDM requires a reaction time parameter (T_r), which is the delay in the driver's response to changes in the system. The the HDM is therefore governed by a system of time-delayed differential equations.

2. For a given driver, the exact distance to and speed of the leading vehicle are not known. The driver estimates these quantities with inherent *estimation error*. The HDM assumes that the drivers know their own velocities (this assumption is reasonable since vehicles have speedometers).

3. A driver does not update his acceleration exactly to the value given by the acceleration function. This is because a driver controls the car by operating the gas and brake pedals, and there is an inherent error in this input. This leads to imprecise accelerations and induces *driver control errors*.

4. Experienced drivers *anticipate* what will happen in the near future and use this anticipation to inform their own behaviors.

5. Human drivers are able to see multiple vehicles ahead in a single lane of traffic. For example, a human driver often can see brake lights of a vehicle three cars ahead on a highway and will react to the brake lights in the distance. The HDM models this *multi-vehicle anticipation* by allowing the human drivers to update their acceleration based upon perceived changes that are occurring to multiple cars ahead of them.

HDM Parameters

T_r: human reaction time

$\tilde{\tau}$: persistence time for Wiener process autocorrelation function

V_s: variation coefficient–standard deviation of the relative difference between s_α^{est} and s_α

σ_r: standard deviation of the relative approach rate

σ_a: standard deviation of the acceleration error

1. Delayed Reaction Times: The HDM incorporates human reaction time by introducing a new reaction time parameter T_r. Eq (13) shows that vehicles in the model update their acceleration based on delayed information (i.e. the state of the system T_r seconds before). It has been shown that Eq (13) cannot be solved analytically, but $u(t - T_r)$ can be approximated numerically by linear interpolation using the scheme in Eq (14), where u is a placeholder for any of $s_\alpha, v_\alpha, \Delta v_\alpha, v_l$. In this numerical scheme, Δt is the update time and $i = \frac{t}{\Delta t}$ is the update step number.

Delayed Reaction Time Equations Equations

$$\dot{v}_\alpha(t) = a_\alpha \left[s_\alpha(t - T_r), v_\alpha(t - T_r), v_l(t - T_r) \right] \tag{13}$$

$$u(t - T_r) = r u_{i-j-1} + (1 - r) u_{i-j}, \;\; j = \text{int}\left(\frac{T_r}{\Delta t}\right), \;\; r = \frac{T_r}{\Delta t} - j \tag{14}$$

2. Estimation Errors: The HDM addresses a driver's error in estimating the distance to and the speed of the vehicle in front of it by adding Gaussian noise to the true value of the estimand. However, it is likely that a particular driver's estimation error will be correlated over time (e.g. if you perceive that you are a certain distance from the car in front, this estimate will inform your belief at one time step in the future) [5]. Hence, for each driver α, the HDM creates two stochastic processes ($w_{s,\alpha}(t), w_{l,\alpha}(t)$) to represent the driver's error in estimating the distance to and speed of the leading vehicle at each time step, respectively. Each of the stochastic processes is independent and comes from the standard Gaussian distribution. Moreover, each stochastic process uses the autocorrelation function given by Eq (15). These processes are created by approximating a Wiener Process using the numerical scheme given in Eq (16), where $i = \frac{t}{\Delta t}$ is the update step number and $\eta_i \sim N(0, 1)$. Using these stochastic processes, the estimation errors (or log errors) of distance to and speed of the leading vehicle are given by Eq (17, 18), respectively, where s_α^{est} and v_l^{est} are the estimates [2, Chapter 12].

Estimation Error Equations

$$\langle w(t)w(t')\rangle = \exp\left(-\frac{|t-t'|}{\tilde{\tau}}\right) \tag{15}$$

$$w_i = e^{\frac{-\Delta t}{\tilde{\tau}}} w_{i-1} + \sqrt{\frac{2\Delta t}{\tilde{\tau}}}\eta_i \tag{16}$$

$$\ln s_\alpha^{\text{est}} - \ln s_\alpha = V_s w_{s,\alpha}(t) \tag{17}$$

$$v^{est} - v_l = -s_\alpha \sigma_r w_{l,\alpha}(t) \tag{18}$$

3. Driver Control Errors: The HDM includes driver control errors by adding noise to the acceleration function at each time step [2, Chapter 12]. Specifically, another stochastic process $w_{a,\alpha}(t)$ with the same properties as $w_{s,\alpha}(t)$ and $w_{l,\alpha}(t)$ is created for each driver. This noise is added to the acceleration function as shown in the Eq (19).

Driver Control Error Equation Equations

$$\dot{v}_\alpha(t) = a_\alpha(s_\alpha^{\text{est}}, v_\alpha, v_l^{\text{est}}) + \sigma_a w_{a,\alpha}(t) \tag{19}$$

4. Temporal Anticipation: The HDM models an experienced driver's anticipation by having drivers assume that their own acceleration is constant over the reaction time T_r. This assumption allows a driver to predict what their own speed will be T_r seconds in the future. This prediction is given by Eq (20). Moreover, in the HDM, a driver assumes that the velocity of the leading vehicle is constant over next T_r seconds [2, Chapter 12]. This allows the driver to anticipate the distance to and speed of the leading vehicle, as shown in Eq (21, 22) respectively, where $\Delta v_\alpha^{\text{est}} = v_\alpha - v_l^{\text{est}}$.

Temporal Anticipation Equations

$$v_\alpha^{\text{prog}}(t) = v_\alpha(t - T_r) + T_r \dot{v}_\alpha(t - T_r) \tag{20}$$

$$v_l^{\text{prog}}(t) = v_l^{\text{est}}(t - T_r) \tag{21}$$

$$s_\alpha^{\text{prog}}(t) = s_\alpha^{\text{est}}(t - T_r) - T_r \Delta v_\alpha^{\text{est}}(t - T_r) \tag{22}$$

5. Multi-Vehicle Anticipation: The HDM allows a driver to be aware of multiple vehicles ahead of it. The acceleration function (shown in Eq (23)) is broken up into two terms: a free acceleration term that represents the acceleration on an open road and an interaction term that models the effect, on the acceleration, of a driver's awareness of multiple leading vehicles. The free acceleration and interaction terms directly follow from adapting the IDM acceleration function to include the distances to and speeds of multiple leading vehicles. The coefficient of the interaction term c_{IDM} is used to keep the interaction term from blowing up in the negative direction as n_a, the number of vehicles anticipated, increases [2, Chapter 12].

Multi-Vehicle Anticipation Equations

$$\dot{v}_\alpha = a_{\text{free},\alpha} + c_{\text{IDM}} \sum_{\beta=\alpha-n_a}^{\alpha-1} a_{\text{int}}(s_{\alpha\beta}, v_\alpha, v_\beta) \tag{23}$$

$$a_{\text{free},\alpha}^{\text{IDM}} = a\left[1 - \left(\frac{v}{v_0}\right)^\delta\right] \tag{24}$$

$$a_{\text{int}}^{\text{IDM}}(s_{\alpha\beta}, v_\alpha, v_\beta) = -a\left(\frac{s^*(v_\alpha, v_\alpha - v_\beta)}{s_{\alpha\beta}}\right)^2 \tag{25}$$

$$s_{\alpha\beta} = \sum_{j=0}^{\alpha-\beta-1} s_{\alpha-j} \tag{26}$$

$$c_{\text{IDM}} = \left(\sum_{j=1}^{n_a} \frac{1}{j^2}\right)^{-1} \tag{27}$$

HDM Model: Assembling the various components of the HDM model, the HDM governing equation is obtained (Eq (28)).

HDM Governing Equation

$$\dot{v}_\alpha = a^{\text{IDM}}_{\text{free},\alpha} + c_{\text{IDM}} \sum_{\beta=\alpha-n_a}^{\alpha-1} a^{\text{IDM}}_{\text{int}}(s^{\text{prog}}_{\alpha\beta}, v^{\text{prog}}_{\alpha}, v^{\text{prog}}_{\beta}) \tag{28}$$

4 Numerical Simulations of Traffic Systems

To generate numerical simulations of traffic systems, we first considered the case of 50 vehicles on a circular track. We explored scenarios where the system is a platoon of either entirely IDM vehicles or entirely HDM vehicles. Under these conditions, we see the IDM models generate more ‘phantom’ traffic waves than the HDM models, especially when set larger values for the number of vehicles ahead that a human driver anticipates HDM car lookahead. This is because the human drivers tend to be more more conservative in a situation like this (i.e. with no external perturbations) as the multi-car lookahead prevents them from getting overly close to the car directly in front of them.

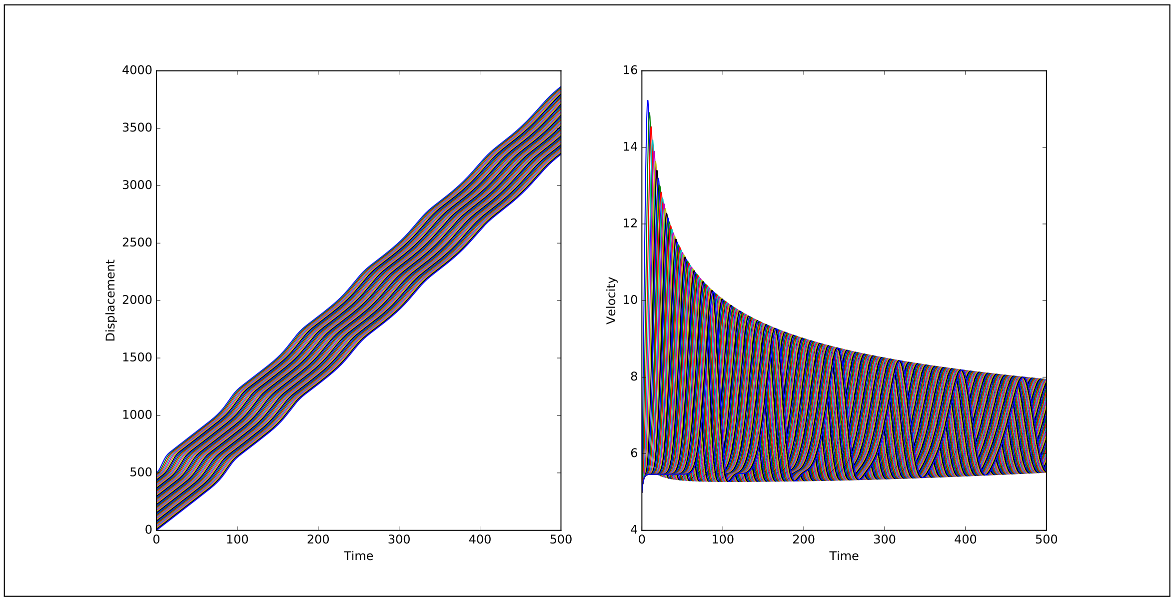

Figure 2: System of 50 IDM-controlled cars on a circular track.

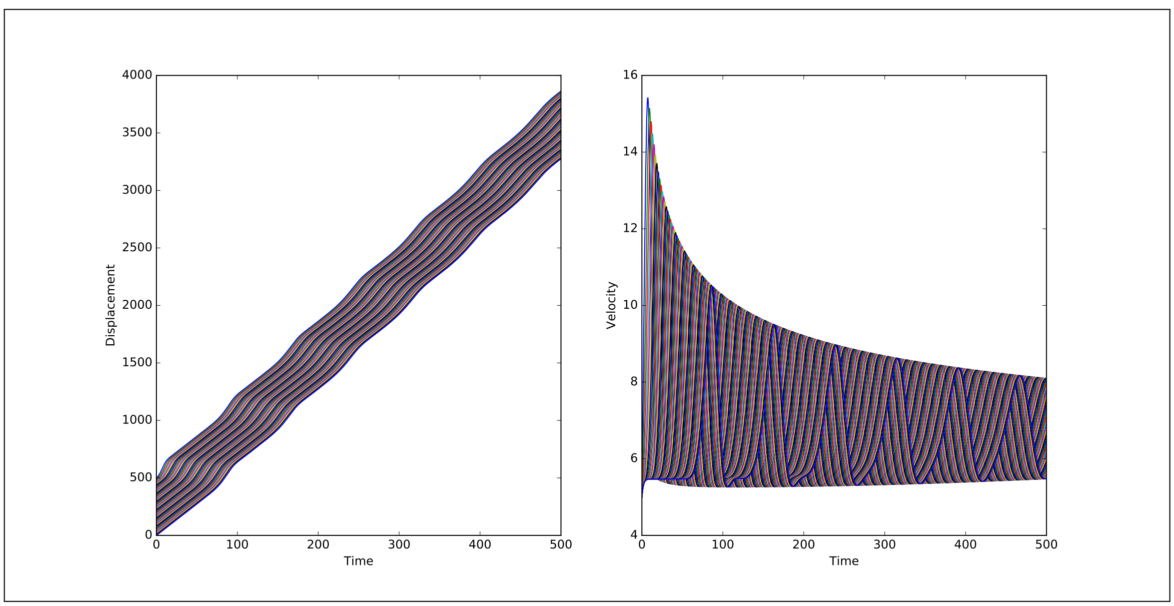

Figure 3: System of 50 ACC-controlled cars on a circular track.

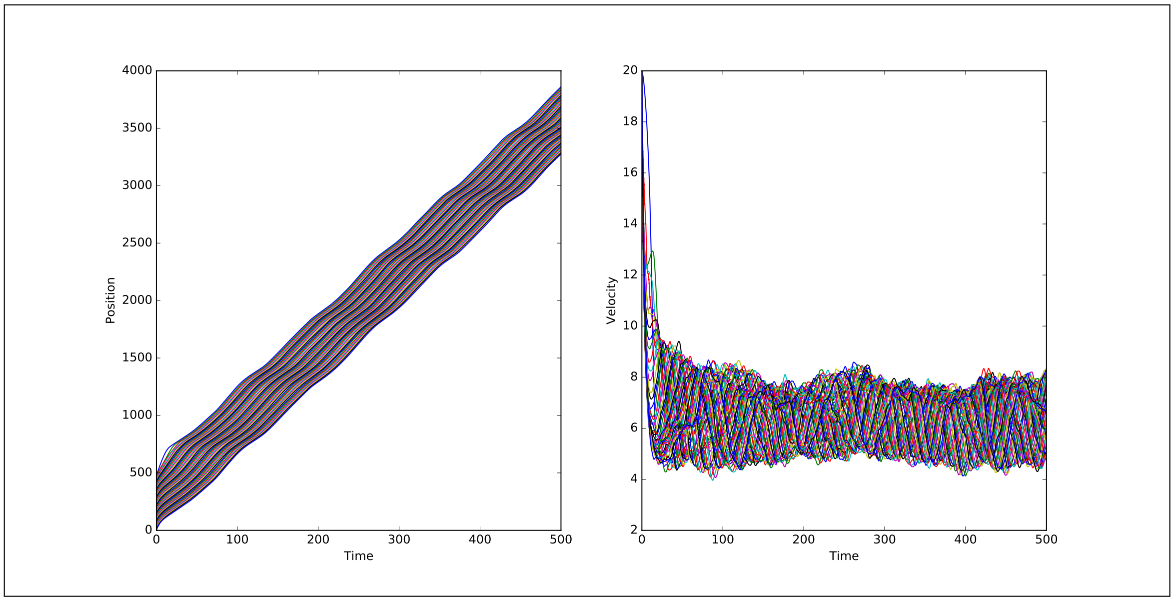

Figure 4: System of 50 HDM-controlled cars with one car-lookahead on a circular track.

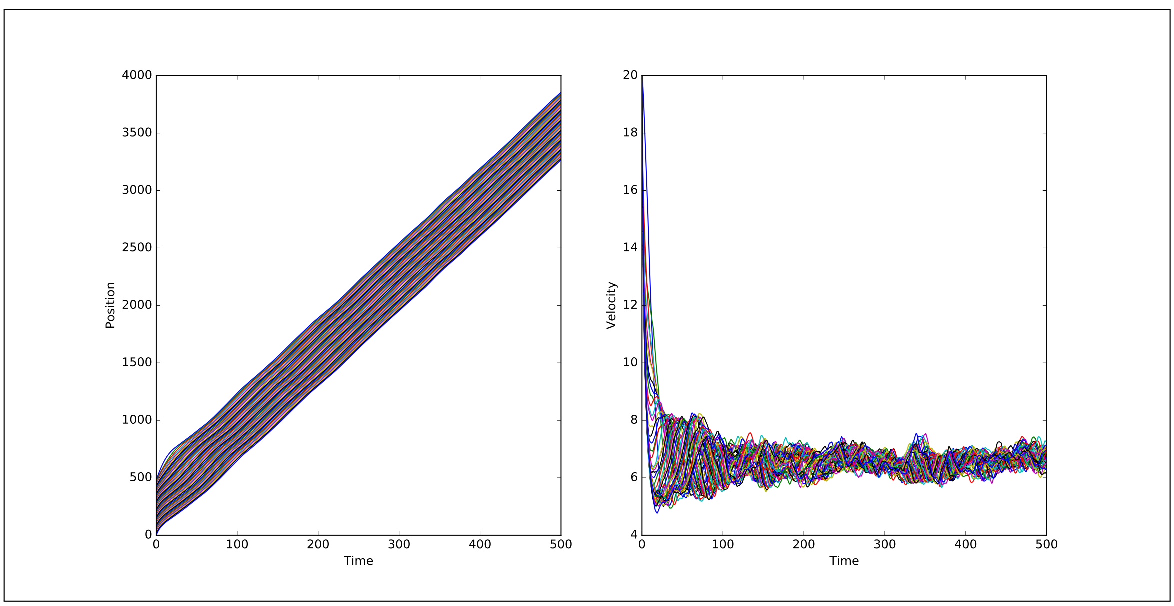

Figure 5: System of 50 HDM-controlled cars with five car-lookahead on a circular track. Note the significant decrease in volatility of velocity.

We then moved on to simulations of a platoon of cars with some cars being controlled by IDM-type models (in particular, Adaptive Cruise Control) and some cars being controlled by multi-car lookahead HDM models. After allowing the models to run for some time to develop their natural spacing, we then perturbed the acceleration of the lead car, causing it to brake and then resume again. We then observe both the magnitude and length of the volatility of the velocities in the following cars after the event. We find that in general, as one would expect, a larger proportion of IDM-controlled vehicles results in a quicker dissipation of the perturbation as well as a lowered probability of a vehicle collision.

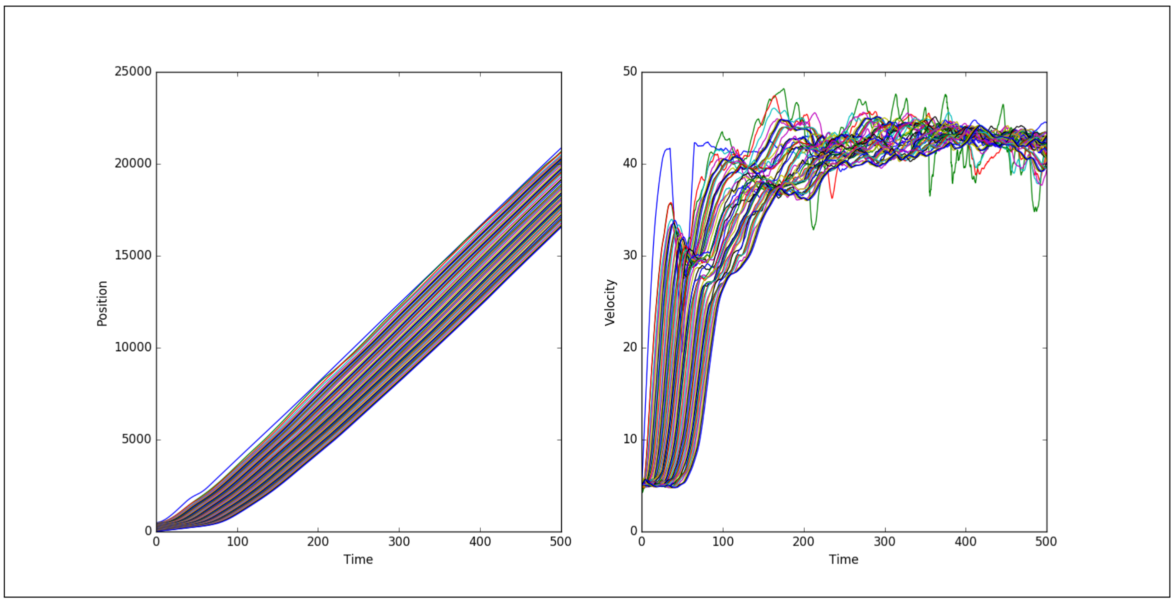

Figure 6: System with 35 % IDM cars on a straight road with acceleration of first car perturbed. System maintains a high degree of volatility in velocity long after the initial perturbation.

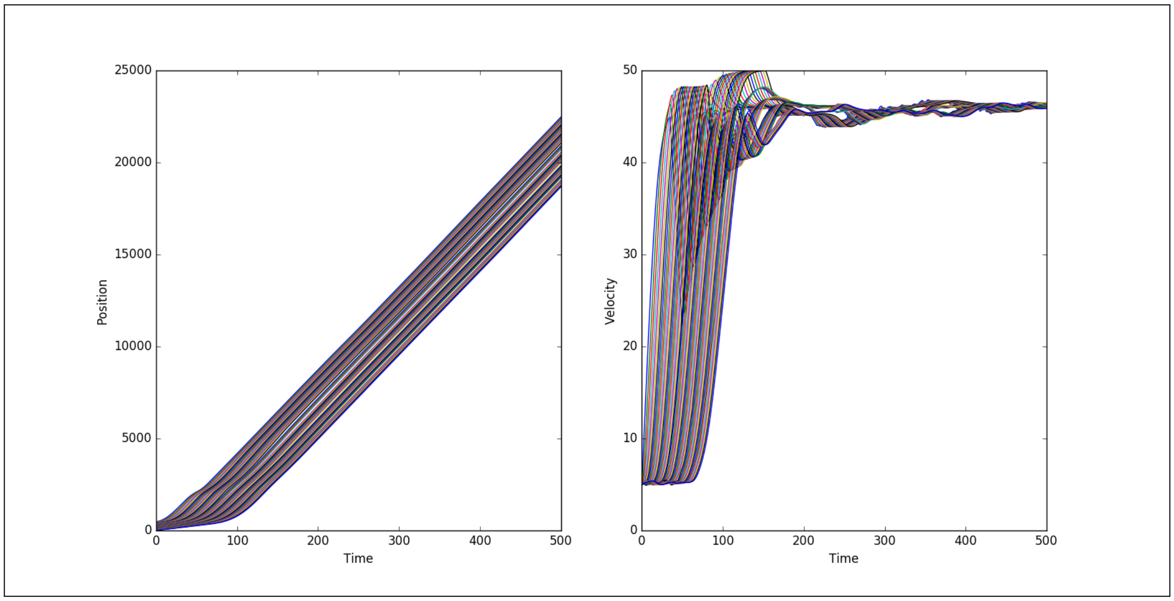

Figure 7: System with 90 % IDM cars on a straight road with acceleration of first car perturbed. System quickly dissipates the majority of the volatility in velocity.

5 Stability and Error Comparison of a Variety of Numerical Integration Schemes

Because there is no analytical solution given for our highly-nonlinear problem, we compared several numerical integration schemes to the python `odeint` solver function to analyze the error of the different numerical schemes. For large enough time steps, the error decreases on a log scale for each of our higher order methods. However, for small time steps, the 'error' for 4th and 5th order methods converges, as the `odeint` solver presumably decides that 4th order is sufficiently accurate at these time steps and stops using higher order methods.

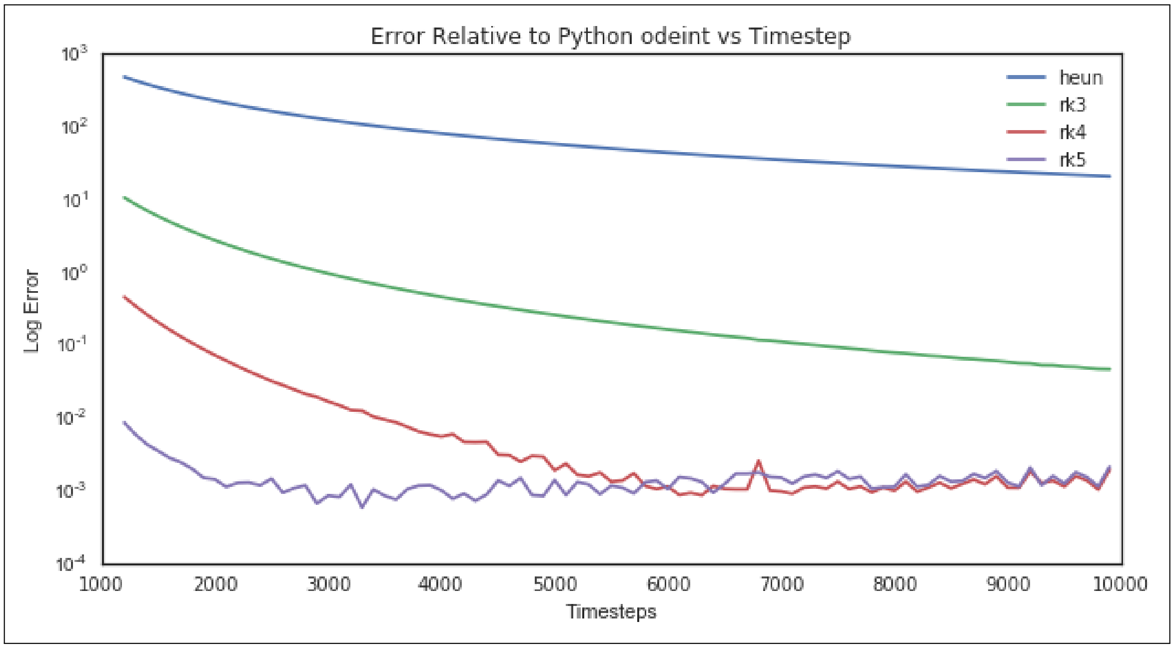

Figure 8: Error to `odeint` decreases with accuracy of finite differencing methods.

For our purposes, we used higher order methods as we were then able use larger timesteps and hopefully decrease the running time of our programs while still maintaining stability in the differential equation systems. To display the effectiveness of these higher order methods, we ran one of our simple models for a variety of time steps over the same total time with the different solvers and noted the number of time steps at which each method stabilized. We notice here that we don't see an additional benefit from using RK5, which in this case we suspect is because the extremely complicated coefficients are contributing to existing numerical overflow issues.

Table 1: Timesteps for Stability with T=500s

Method	Minimum Timesteps for Stability
Heun's Method	325
Runge-Kutta 3	275
Runge-Kutta 4	250
Runge-Kutta 5	250

6 String Stability of Traffic Systems to Perturbation

The concept of string stability refers to whether a perturbation to the beginning of the system will result in a wave that dissipates or is enhanced as it propagates through the system. In previous experiments, we have observed that the IDM models are excellent for adjusting to and dissipating traffic perturbations, so we expect them to be string stable and indeed they are.

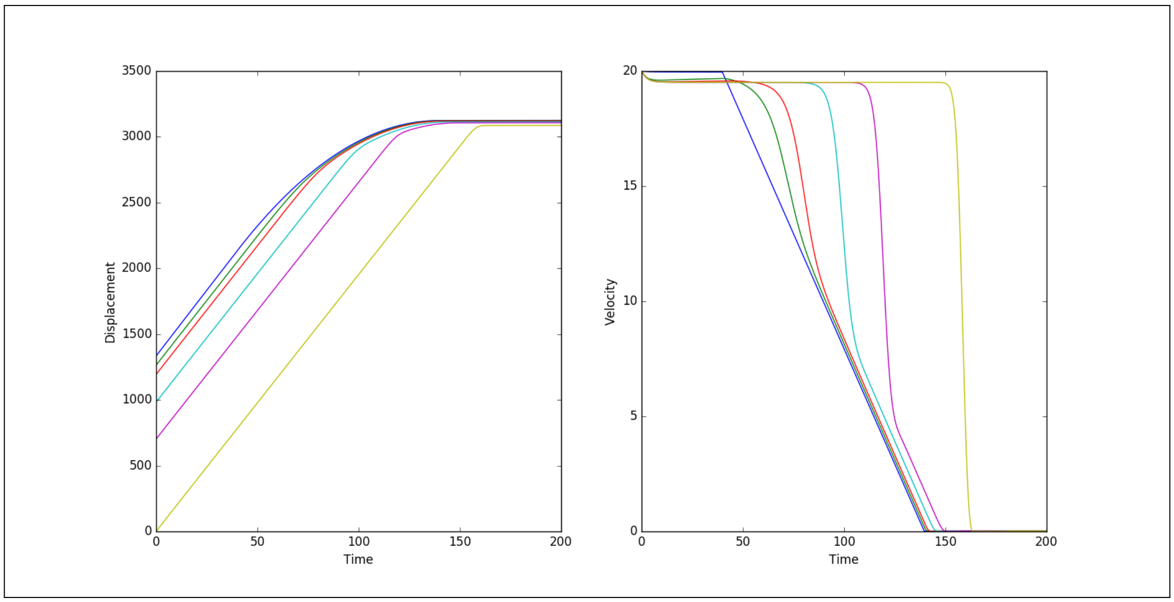

Figure 9: Plot of cars 1, 2, 3, 6, 10, and 20 for IDM. String stable.

For HDM models, the presence or lack thereof of string stability depends on a number of parameters: number of leading cars being considered, time step (Δt), and reaction time (T_r) [2, Chapter 15]. In general, string stability increases with number of leading cars being considered, although the marginal contribution of each car decreases to the point where there is not much additional benefit beyond 5 cars. String stability is inversely related to time step and reaction time, although reaction time has a greater effect. Below, we show two plots for parameters where the HDM system is string stable and string unstable.

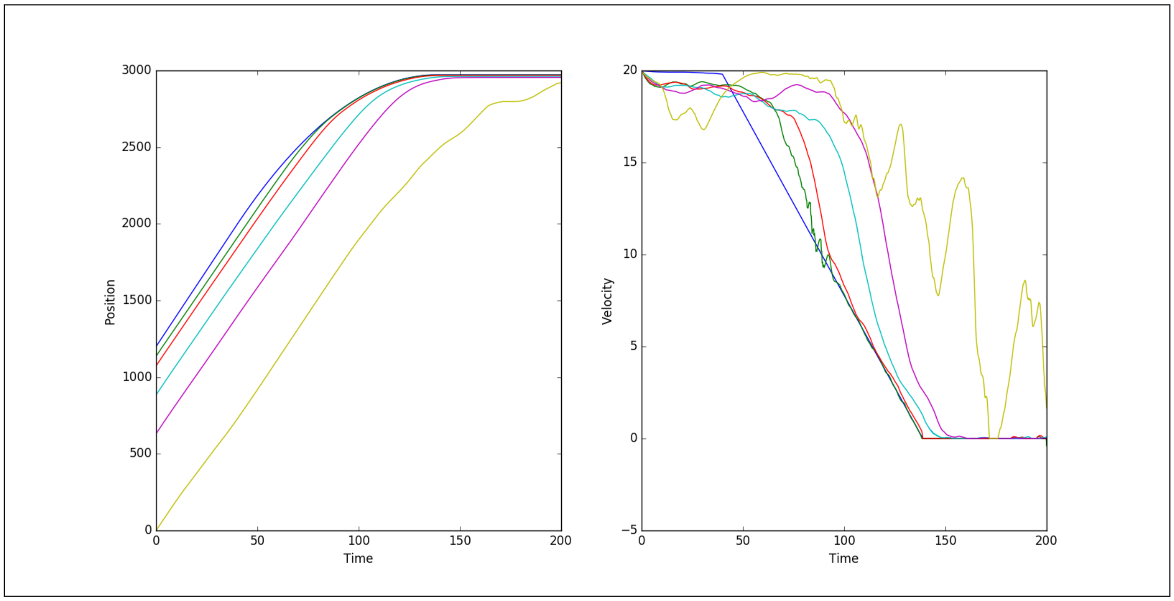

Figure 10: Plot of cars 1, 2, 3, 6, 10, and 20 for HDM with two car lookahead. We see that for parameters T_r = .6 and Δt = .2, the system is string unstable, as evidenced by the large wave in car 20's velocity.

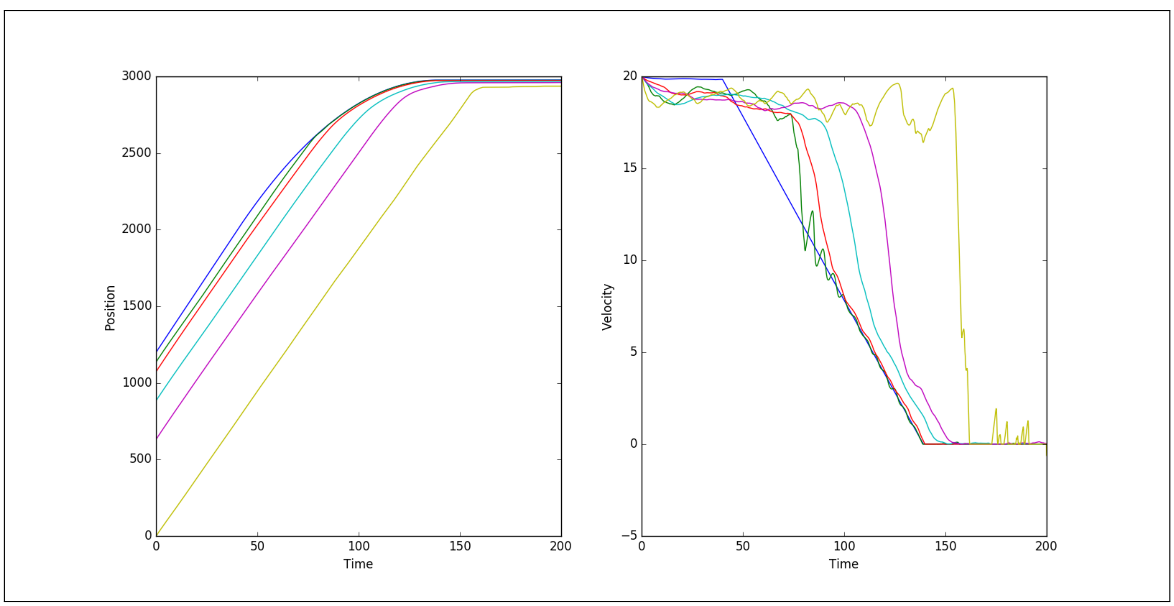

Figure 11: Plot of cars 1, 2, 3, 6, 10, and 20 with two car lookahead. We see that for parameters T_r = .2 and Δt = .15, the system is string stable.

7 Optimization of Traffic Systems

We set up a number of optimization programs to attempt to search for the optimal values of some of the many parameters for the system as a whole and for individual drivers for a number of cost functions, including maximizing displacement and maximizing comfort (minimizing jerk factor[2]) [2, Chapter 21]. The initial trials for these systems were run on IDM models, and the results are largely what one would suspect. When attempting to maximize total displacement, the cars drive quickly and close together, resulting in the creation of traffic waves in the system. When attempting to minimize jerk, the cars drive much more conservatively, as one would expect, never coming within the range of another car that would cause them to change acceleration.

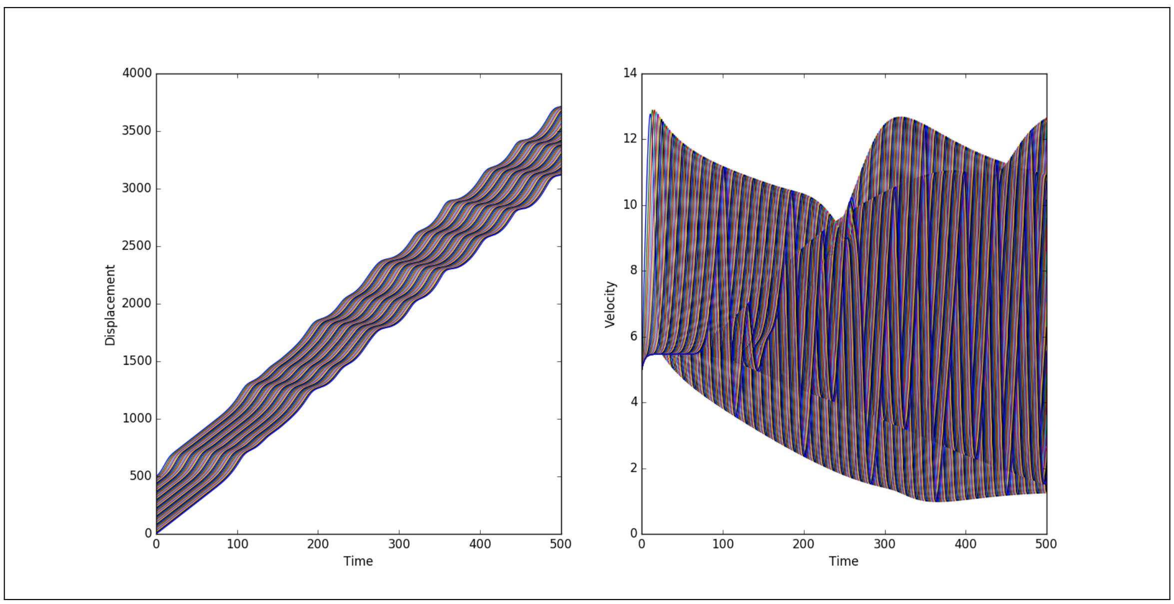

Figure 12: System of IDM cars on a circular track optimizing for maximum distance traveled. The more aggressive driving of the cars results in wave formation.

[2]Jerk factor is a combination of a car's acceleration and the derivative of acceleration. The aim is to minimize any fast changes to the car's acceleration.

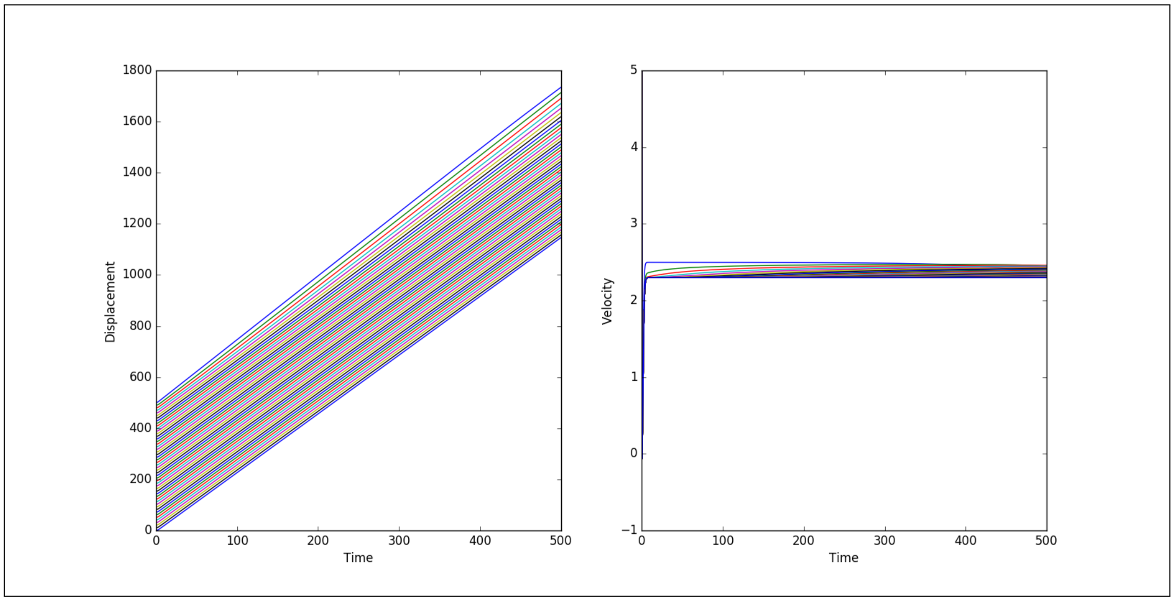

Figure 13: System of IDM cars on a circular track optimizing for maximum ‘comfort’ (minimum jerk). As one might expect, they choose to go rather slowly and thereby achieve a very stable velocity.

We then attempted to optimize behavior for a single driver and were interested to see what implications this would have for the system as a whole. However, the car-following behaviors in the IDM are such that a single driver can tailgate and maximize target speed basically as much as we will allow without ever disrupting the rest of the system. Even when adding some stochasticity to the lead car, we were unable to see the rest of the system suffer from traffic waves due to the behavior of the second car, because that behavior would simply be quickly dissipated by the third car. In fact, it seems to be true generally of these systems that unless the aggressive driving behavior of a car causes other cars to collide behind it, the greater distance achieved by one car inevitably results in other cars also achieving a greater distance/average velocity.

Next, we attempted to replicate these experiments with the more complicated Human Driver Model. However, due to the highly piecewise nature of the functions we are attempting to optimize over, we were unable to use traditional minimization solvers. Instead, we discretized the parameter space that we were interested in exploring and searched for the set of parameters which would minimize our target cost function.

Again, we find that in general, maximizing displacement for a single car leads to a greater displacement for the rest of the system unless the driving behavior is so aggressive that it results in a crash. Given that we have seen that the Human Driver Model is more likely to propagate perturbations to the system, we had expected that in this model there might be a penalty to other cars in greater volatility of velocity, i.e. a less pleasant ride, but this effect was also not observed in the data. We conclude that the model is largely insensitive to individual parameters in this context and that the structure of the model tends not to penalize aggressive driving unless a collision results.

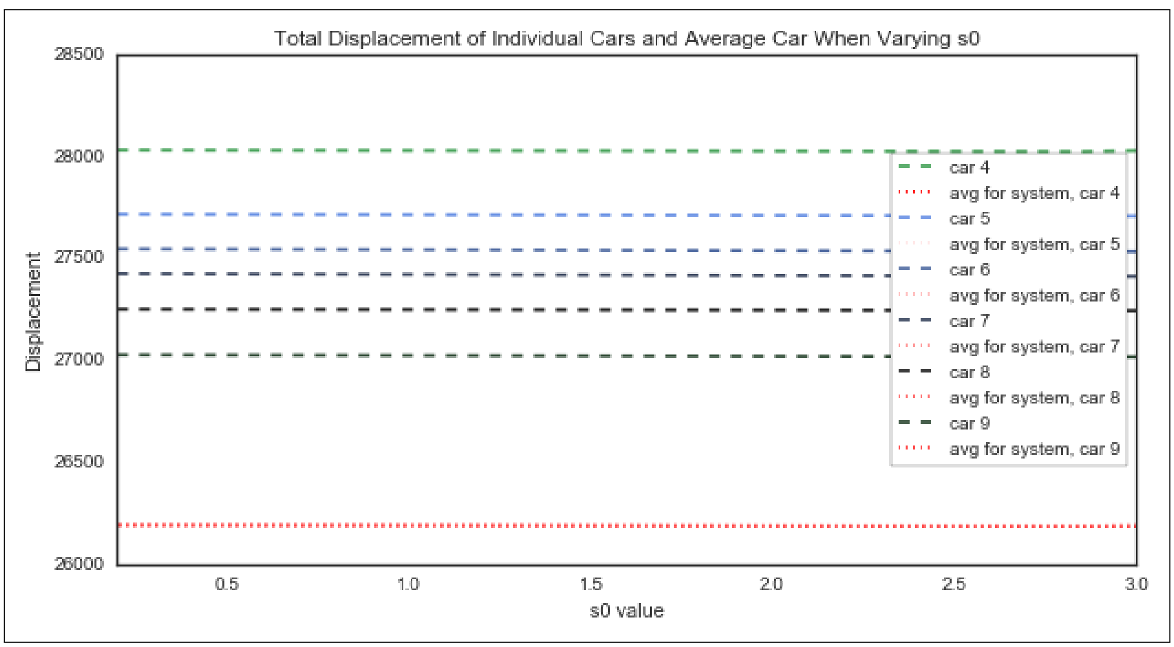

Figure 14: Changing Safe Following Distance: Changes are minimal but overall distance increases for both the car in question and the system of cars with smaller distance.

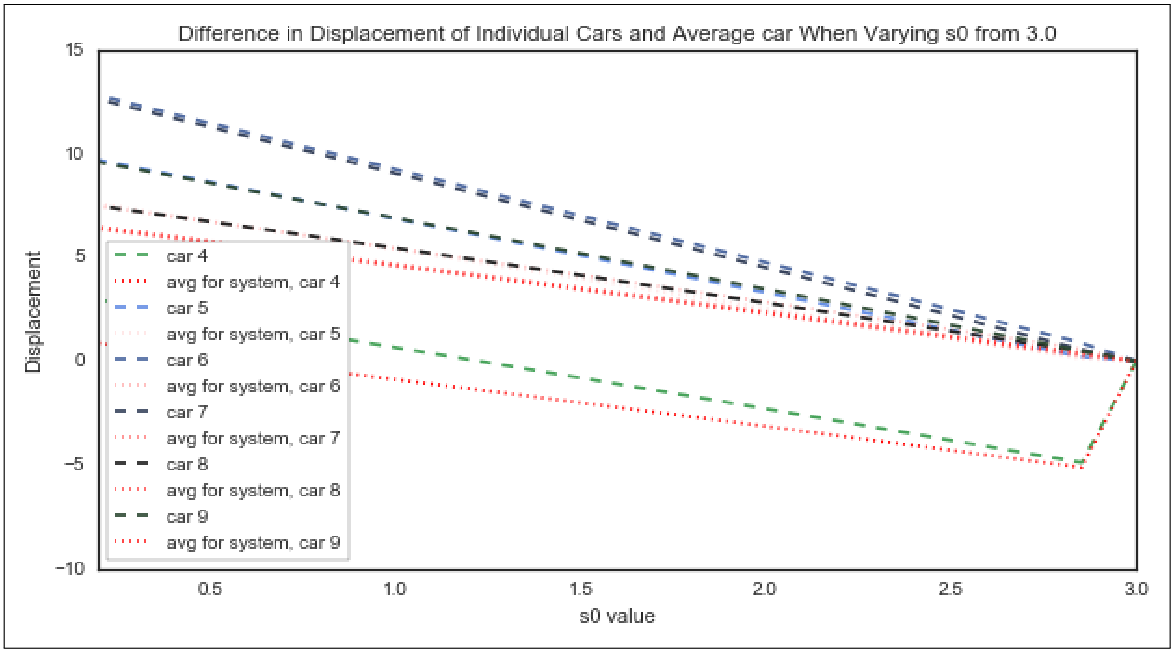

Figure 15: Variation in Distance Traveled when Changing Safe Following Distance Relative to Initial Value of 3.0

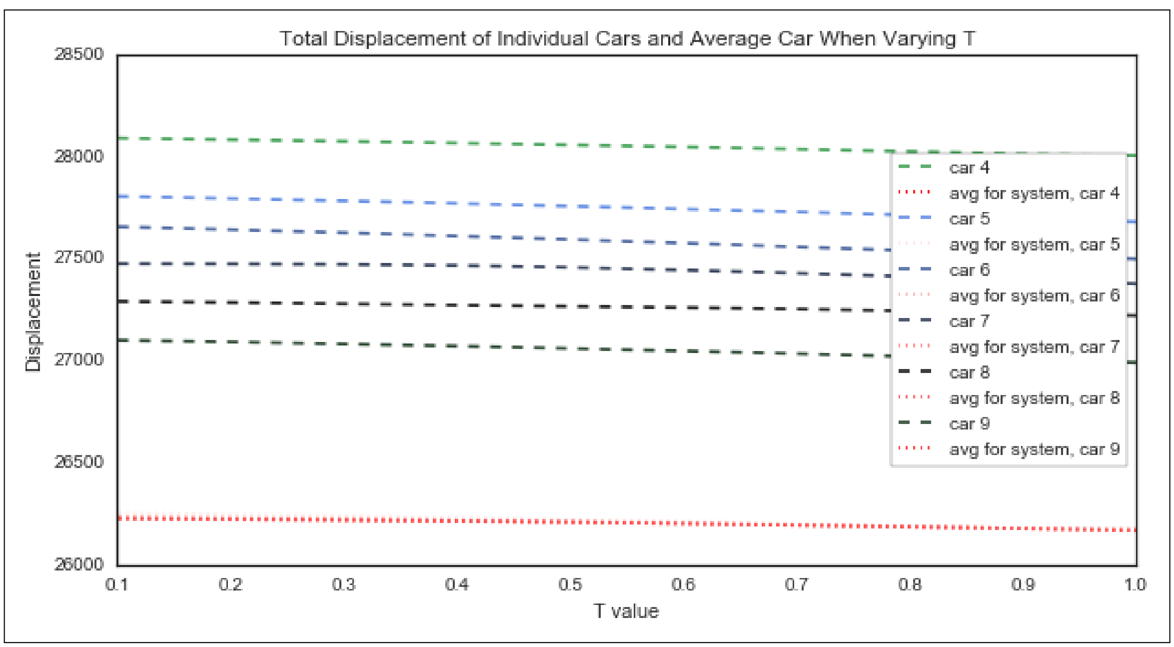

Figure 16: Changing Safe Following Time: Changes are minimal but overall distance increases for both the car in question and the system of cars with smaller safe following times.

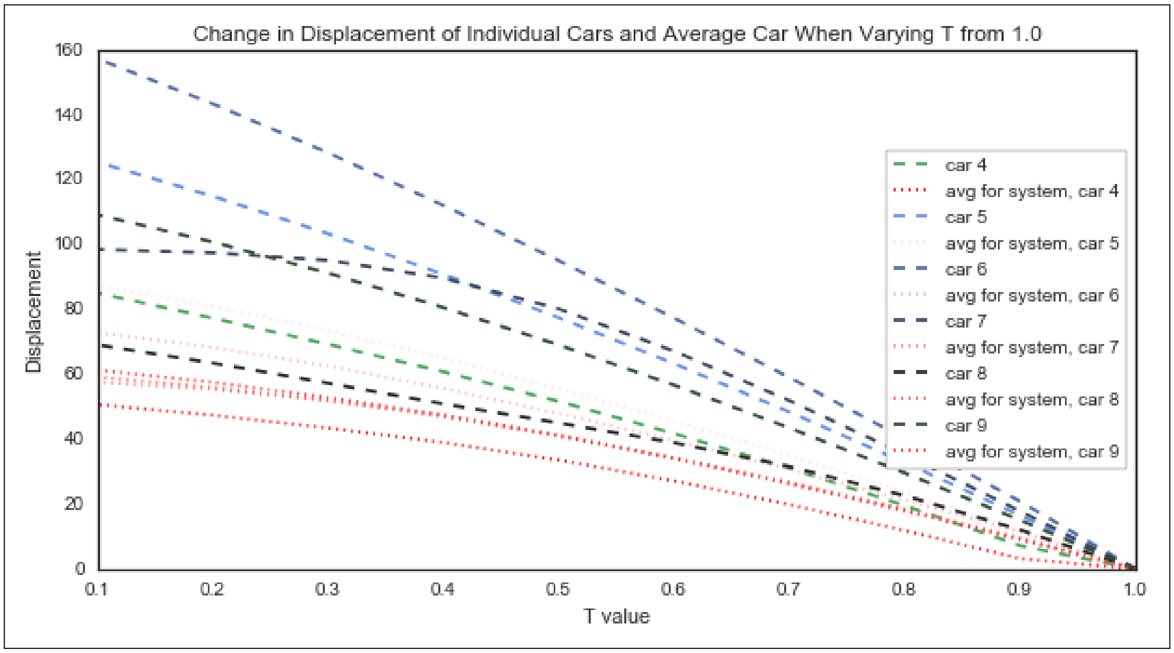

Figure 17: Variation in Distance Traveled when Changing Safe Following Time Relative to Initial Value of 1.0

8 Conclusion

The models used within this paper necessarily make many simplifying assumptions, many of which vary somewhat between models. Because of this, it was often difficult to maintain stability in the systems composed of a mixture of the two models, as one model may not fully satisfy the assumptions of the other under all parameters. However, within these restrictions, we were still able to construct a series of simulations with stable parameter values and model the reactions of the systems in reasonable ways. We find that in general ACC-controllers are better than human drivers at adapting to and dissipating traffic disturbances and that introducing more self-driving cars in a simplified scenario such as this is likely to improve traffic patterns by reducing the propagation and amplification of traffic waves.

References and Notes

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