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An agenda-based framework for multi-issue negotiation

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Abstract

This paper presents a new model for multi-issue negotiation under time constraints in an incomplete information setting. The issues to be bargained over can be associated with a single good/service or multiple goods/services. In our *agenda-based model*, the order in which issues are bargained over and agreements are reached is determined endogenously, as part of the bargaining equilibrium. In this context we determine the conditions under which agents have similar preferences over the implementation scheme and the conditions under which they have conflicting preferences. Our analysis shows the existence of equilibrium even when both players have uncertain information about each other, and each agent's information is its private knowledge. We also study the properties of the equilibrium solution and determine conditions under which it is unique, symmetric, and Pareto-optimal.

Keywords: Multi-issue negotiation; Game theory; Agendas; Intelligent agents

1. Introduction

Negotiation is a means for agents to communicate and compromise to reach mutually beneficial agreements [11,14,18,29,30,40]. In such situations, agents have a common interest to cooperate, but have conflicting interests over exactly how to cooperate. Put differently, agents can mutually benefit from reaching agreement on an outcome from a

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set of possible outcomes, but have conflicting interests over the set of outcomes. In this context, the main problem that confronts agents is to decide how to cooperate—before they actually enact the cooperation and obtain the associated benefits. On the one hand, each agent would like to reach some agreement rather than disagree and not reach any agreement. But, on the other hand, each agent would like to reach an agreement that is as favourable to it as possible.

To this end, a number of negotiation models that address this problem have been developed and applied to data allocation in information servers, resource allocation and task distribution [18,19,27,31,32]. Apart from these, another application area in which agent-mediated negotiation has received considerable attention is in the field of electronic commerce [22–24,35,36]. In this domain, which is the main focus of this paper, the aim is to build software agents that will optimally negotiate with other agents on behalf of users for buying and selling goods/services. Here we look at one-to-one negotiation between a buyer and a seller. In order to develop software agents for such bilateral encounters, we first examine the important features of real-life bargaining situations that need to be incorporated in the software agents. To this end, the three crucial features of most practical bargaining processes are as follows [28]:

(1) The time constraints of the bargainers.
(2) The information state of the bargainers.
(3) The number of issues to be bargained over.

We first explain the role of time in negotiation. Consider an e-commerce scenario in which a buyer agent and a seller agent negotiate over the price of a good or service. The buyer clearly prefers a low price, while the seller prefers a high one (hence the competitive nature of the encounter). In addition to attempting to obtain the best price, agents also usually need to ensure that negotiation ends before a certain deadline. However, the end point may not be the only way in which time influences negotiation behaviour. Consider the case in which the service is provided immediately after negotiation ends successfully (say at price P and time T). In some situations, it is not sufficient merely for an agent to ensure that T is any time less than its deadline. This may be the case, for instance, because one of the agents, say the buyer, could be losing utility with time as a result of not getting the service. On the other hand, the seller may perhaps gain more utility by providing the service as late as possible. Thus, in this case, the seller tries to maximize T (within the limit of its deadline) and the buyer tries to minimize T. In short, it is clear that agents can have different attitudes toward time. Generally speaking, the most common time effects in bargaining situations are *time discounting* and *deadlines* [10,21]. An agent that gains utility with time and has the incentive to reach a late agreement (within its deadline) is said to be a *strong* or *patient* player. An agent that loses utility with time and tries to reach an early agreement is said to be a *weak* or *impatient* player. As we will show, this disposition and the actual deadline itself strongly influence the negotiation outcome.

The second crucial feature of a bargaining process is information. During negotiation, each agent has to make decisions about generating offers and counter-offers in such a way that its own utility from the final agreement is maximized. An essential input to this decision making process is information; here defined as knowledge about all factors which affect the ability of an individual to make choices in a given situation. For instance, in bargaining between a buyer and a seller, information includes not only information about an agent's own parameters (such as its reservation price or its preferences over possible outcomes), but also those of its opponent. In most realistic cases agents have only *incomplete information* about their opponent.

To this end, game theoretic models have already been proposed for bargaining with incomplete information. For instance, Rubinstein [34] developed a model in which agents have incomplete information about time preferences. Fudenberg et al. [12] analyse buyer-seller negotiation in which reservation prices are uncertain. Sandholm and Vulkan [37] consider uncertainty over agent deadlines. All these models are built on the assumption that information about the uncertain parameter (in the form of possible values and a probability distribution over them) is the agents' *common knowledge*. However, in practice, perhaps the main way of acquiring information about the opponent is through learning from previous encounters. In such cases, an agent's beliefs about its opponent will not be known to its opponent. We therefore study the strategic behaviour of agents by treating each agent's information as its *private knowledge*.

The third key feature is the number of issues that have to be negotiated. In many of the applications that are conceived in the domain of e-commerce, it is important that the agents should not only bargain over the price of a product, but also take into account issues such as the delivery time, quality, payment methods, and other product specific properties. In such multi-issue negotiations, one approach is to bundle all the issues and discuss them simultaneously as a complete package. This allows the players to exploit trade-offs among different issues, but requires complex computations to be carried out [4,17]. The other approach, which is computationally simpler, is to negotiate the issues sequentially. A second and more important reason why parties may choose to settle issues one by one is the strategic implications of the choice of the negotiation procedure (i.e., issue-by-issue vs. complete package). When there are two objects to negotiate, the decision to negotiate them simultaneously or one by one is by no means neutral to the outcome [2,38]. Although issue-by-issue negotiation minimizes the complexity of the negotiation procedure, an important question that arises is the order in which the issues are bargained over. This ordering is called the *negotiation agenda* and it has been shown to be one of the factors that determines the outcome of negotiation [9]. For instance, if there are two issues, X and Y, the two agendas XY and YX can lead to two different outcomes. The agents need not have identical preferences over these outcomes and one of them may prefer the agenda XY to YX, while the other may prefer YX to XY. Given this fact, exploring the role of the bargaining agenda, and how players might manipulate it, is timely, especially given that many real-life negotiations involve multiple issues.

There are two ways of incorporating agendas in the negotiation model. One is to fix the agenda exogenously as part of the negotiation procedure. Considering the above example, one of the agendas, say XY is imposed exogenously. Then the bargainers have to settle X first, and will be allowed to negotiate Y only after X is settled. The other way, which is more flexible, is to allow the bargainers to decide which issue they will negotiate next during the process of negotiation. This is called an endogenous agenda [16] and is the approach we explore in this paper.

Existing game theoretic models for issue-by-issue negotiation [1,9,16], which are basically extensions of [33,34], have two main shortcomings. Firstly, they study the strategic behaviour of agents by treating the information they have as *common knowledge*. In practice however, the information that a player has about its opponent is mostly acquired through learning from previous encounters. An agent's beliefs about its opponent will therefore not be known to its opponent. Secondly, these models do not consider agent deadlines. We overcome these problems by considering each agent to have its own deadline and by treating each agent's information state as its *private knowledge*. In this case we obtain the connection between this private knowledge and the existence of equilibrium for single issue negotiation. We then extend this model for multi-issue negotiation and study the properties of the equilibrium solution.

To provide a setting for our negotiation model, we consider the case in which negotiation needs to be completed by a specified time, which may be different for different parties (since this is the most realistic case). Apart from the agents' respective deadlines, the time at which agreement is reached can affect the agents (patient or impatient) in different ways [7]. To this end, Fatima et al. [7] presented a single-issue model for negotiation between two agents under time constraints and in an incomplete information setting by considering the agents' information as its private knowledge. Within this context, they determined optimal strategies for agents but did not address the issue of the existence of equilibrium. Here we adopt this framework and prove that mutual strategic behavior of agents, where both use their respective optimal strategies, results in equilibrium. We then extend this framework for multi-issue negotiation. The order in which issues are bargained over and agreements are reached is determined by the equilibrium strategies. The time of equilibrium agreement may not be equal for all the issues. Consequently, the outcome of multi-issue negotiation can be implemented in two ways: sequentially or simultaneously. We then determine conditions under which agents have similar, as well as conflicting, preferences over the implementation scheme. Finally, we study the properties of the equilibrium solution.

This work extends the state of the art in multi-issue negotiation by presenting a more realistic negotiation model that captures the three aspects, mentioned above, that are associated with many real-life bargaining situations. Firstly, it is a model for negotiating multiple issues. Secondly, it takes the time constraints of bargainers into consideration, both in the form of agent deadlines and their discounting factors. Thirdly, it allows agents to have incomplete information about each other, and each agent's information is treated as its private knowledge. Although we study bargaining in which agents have one specific information state and the agenda is endogenous, our negotiation framework is general and can be used for exploring a wide range of negotiation environments by changing the agents' information states or the way in which the players manipulate the agenda.

The paper is organised as follows. Section 2 describes the components that make up a negotiation model. In Section 3, we describe the single issue negotiation model. Section 4 extends this for negotiating multiple issues. Section 5 discusses related work. Finally, Section 6 gives some conclusions and suggests some topics for future work. Appendix A provides a summary of notation employed throughout the paper.

2. Components of a negotiation model

The four components of a negotiation model are as follows [31]:

(1) The negotiation protocol.
(2) The negotiation strategies.
(3) The information state of agents.
(4) The negotiation equilibrium.

The protocol specifies the rules of encounter between the negotiation participants. That is, it defines the circumstances under which the interaction between agents takes place, what deals can be made and what sequences of offers are allowed. In general, agents must reach agreement on the negotiation protocol to use before negotiation proper begins. A negotiation protocol can be designed for handling a single issue or multiple issues. Within the class of multi-issue negotiations, we can have protocols that negotiate on all the issues together or one by one.

An agent's negotiation strategy is a specification of the sequence of actions (usually offers or responses) the agent plans to make during negotiation. There will usually be many strategies that are compatible with a particular protocol, each of which may produce a different outcome. For example, an agent could concede in the first round or bargain very hard throughout negotiation until its timeout is reached. It follows that the negotiation strategy that an agent employs is crucial with respect to the outcome of negotiation. It should also be clear that the strategies which perform well with certain protocols will not necessarily do so with others. The choice of a strategy to use is thus a function not just of the specifics of the negotiation scenario, but also the protocol in use.

An agent's information state describes the information it has about the negotiation game. Von Neumann and Morgenstern [26] introduced the fundamental classification of games into those of *complete information* and those of *incomplete information*. The former category is basic. In these games the players are assumed to know all relevant information about the rules of the game and players' preferences that are represented by utility functions. In the latter category, information may be lacking about a variety of factors in the bargaining problem. Thus each player may have some private information about his own situation that is unavailable to the other players, while having only probabilistic information about the private information of other players. Following Harsanyi [14,15], models of games of incomplete information proceed from the assumption that all players start with the same probability distribution on this private information and that these priors are *common knowledge*. This is modelled by having the game begin with a probability distribution, known to all players. Thus players not only have priors over other players' private information, they also know what priors the other players have over their own private information. Strategic models of incomplete information thus include an extra level of detail, since they specify not only the actions and information available to the other players in the course of the game, but also their probability distributions and information prior to the start of the game.

A negotiation mechanism consists of a negotiation protocol together with the negotiation strategies for the agents involved. A negotiation mechanism has to be stable (i.e., a strategy profile must constitute an equilibrium), the earliest concept of which was the Nash equilibrium for games of simultaneous offers [25]. Two strategies are in Nash equilibrium if each agent's strategy is a best response to its opponent's strategy. This is a necessary condition for system stability where both agents act strategically. For sequential offer protocols, the Nash equilibrium concept was strengthened in several ways by requiring that the strategies stay in equilibrium at every step of the game [39]. In summary, rationality, as understood in game theory, requires that each agent will select an equilibrium strategy when choosing independently. Given this, game theory prescribes the following main criteria [28] for evaluating the equilibrium outcome:

(1) *Uniqueness*. If the solution of the negotiation game is unique, then it can be identified unequivocally.
(2) *Efficiency*. An agreement is efficient if there is no wasted utility (i.e., the agreement satisfies Pareto-optimality). An outcome is Pareto-efficient if there is no other outcome that improves the payoff of one agent without making another agent worse off. All other things being equal, Pareto-efficient solutions are preferred over those that are not.
(3) *Symmetry*. A bargaining mechanism is said to be symmetric if it does not treat the players differently on the basis of inappropriate criteria. Exactly what constitutes inappropriate criteria depends on the specific domain. For example, if the bargaining outcome remains the same irrespective of which player starts the process of bargaining, then it is said to be symmetric with respect to the identity of the first player.
(4) *Distribution*. This property relates to the issue of how the gains from trade are split between the players; does the outcome split the gains equally between the traders or does it favour one agent more than the other? In this paper, our aim is not to design a negotiation mechanism that divides the gains fairly among players but to study the outcome that results when both agents are self-interested.

With these broad guidelines in mind, many different models can be designed. Below, we report on the development of a new model based on negotiation decision functions (see Section 3.2) for bargaining over multiple issues. We first describe the single issue model and study its equilibrium strategies and outcomes. We then extend this model for multi-issue negotiation and study its equilibrium properties.

3. The single-issue negotiation model

We first describe the single issue negotiation protocol and obtain the agents' optimal strategies. We then prove that the optimal strategy profiles form sequential equilibrium.

3.1. The negotiation protocol

Here we adopt what is basically an alternating offers protocol [28]. Let b denote the buyer, s the seller and let $[IP^a, RP^a]$ denote the range of values for price that are acceptable to agent a, where $a \in \{b, s\}$. We let $\hat{a}$ denote agent a's opponent. A value for price that is acceptable to both b and s (i.e., the zone of agreement) is the interval $[RP^s, RP^b]$ and $(RP^b - RP^s)$ is known as the *price-surplus*. The buyer's initial price, IP^b, has a value less than the seller's reservation price. Similarly, the seller's initial price has a value greater than the buyer's reservation price. In other words, both IP^b and IP^s lie outside the zone of agreement.

The agents alternately propose offers at times in $\mathcal{T} = \{0, 1, \ldots\}$. Each agent has a deadline. T^a denotes agent a's deadline where $T^a \in \mathcal{T}$. Let $p^t_{b\to s}$ denote the price offered by agent b at time t. Negotiation starts when the first offer is made by an agent. The agent who makes the initial offer is selected randomly at the beginning of negotiation. When an agent, say s, receives an offer from agent b at time t, i.e., $p^t_{b\to s}$, it rates the offer using its utility function U^s. If the value of U^s for $p^t_{b\to s}$ at time t is greater than the value of the counter-offer agent s is ready to send in the next time period, t', i.e., $U^s(p^t_{b\to s}, t) \geqslant U^s(p^{t'}_{s\to b}, t')$ for $t' = t + 1$, then agent s accepts the offer at time t and negotiation ends successfully in an agreement. Otherwise a counter-offer is made in the next time period, t'. Thus the action, A^s, that agent s takes at time t, in response to the offer $p^t_{b\to s}$, is defined as:

$$A^s\big(t, p^t_{b\to s}\big) = \begin{cases} \text{Quit} & \text{if } t > T^s, \\ \text{Accept} & \text{if } U^s(p^t_{b\to s}) \geqslant U^s(p^{t'}_{s\to b}), \\ \text{Offer} p^{t'}_{s\to b} \text{ at } t' & \text{otherwise.} \end{cases}$$

Agents' utilities are defined with the following two von Neumann–Morgenstern utility functions [17] that incorporate the effect of time discounting

$$U^a(p, t) = U^a_p(p)U^a_t(t). \tag{1}$$

U^a_p and U^a_t are unidimensional utility functions. Here, preferences for attribute p, given the other attribute t, do not depend on the level of t. U^a_p is defined as:

$$U^a(p) = \begin{cases} RP^b - p & \text{for the buyer,} \\ p - RP^s & \text{for the seller.} \end{cases}$$

U^a_t is defined as $U^a_t(t) = (\delta^a)^t$, where δ^a is the discounting factor. Thus when $(\delta^a > 1)$ the agent is *patient* and gains utility with time and when $(\delta^a < 1)$ the agent is *impatient* and loses utility with time. The utility from conflict is lower than the utility from any of the possible agreements for both agents. Each agent prefers to reach an agreement, rather than disagree and not reach any agreement at all, since the utility from an agreement is always higher than conflict utility. Consequently, it is optimal for agent a to offer RP^a at the latest by its deadline, if it has not done so earlier, and avoid a conflict (see Section 3.5 for details on an agent's optimal strategy). Agents are said to have *similar* time preferences if both gain on time or both lose on time. Otherwise they have *conflicting* time preferences.

3.2. Counter-offer generation

The tactics for generating offers and counter-offers are defined as follows. Since both agents have a deadline, we assume that they use a time-dependent tactic (e.g., linear (L), Boulware (B) or Conceder (C) [3]) for generating offers. In these tactics, the predominant factor used to decide which value to offer next is time t. These tactics vary the value of price depending on t and T^a. The initial offer is a point in the interval [IP^a, RP^a]. The constant k^a multiplied by the size of interval determines the price to be offered in the first proposal by agent a. The offer made by agent a to agent $\hat{a}$ at time t ($0 \leqslant t \leqslant T^a$) is modelled as a function ϕ^a depending on time as follows:

$$p^t_{a\to\hat{a}} = \begin{cases} IP^a + \phi^a(t)(RP^a - IP^a) & \text{for } a = b, \\ RP^a + (1 - \phi^a(t))(IP^a - RP^a) & \text{for } a = s. \end{cases}$$

A wide range of time-dependent functions can be defined by varying the way in which $\phi^a(t)$ is computed (see [3] for more details). However, functions must ensure that $0 \leqslant \phi^a(t) \leqslant 1$, $\phi^a(0) = k^a$ (where k^a lies in the interval $[0, 1]$), and $\phi^a(T^a) = 1$. That is, the offer will always be between the range [IP^a, RP^a], at the beginning it will give the initial constant and when the deadline is reached it will offer the reservation value. The initial offer is IP^a if $k^a = 0$, lies between IP^a and RP^a for $0 < k^a < 1$, and is RP^a for $k^a = 1$. Thus by varying k^a between 0 and 1, the initial price that is offered can be varied between IP^a and RP^a. Since we want IP^a to be the initial offer, we set k^a to 0. Function $\phi^a(t)$ is called the negotiation decision function (NDF) and is defined as follows:

$$\phi^a(t) = k^a + \left(1 - k^a\right)\left(\frac{t}{T^a}\right)^{1/\psi}. \tag{2}$$

These NDFs represent an infinite number of possible tactics, one for each value of ψ (see [3] for more details). However, depending on the value of ψ, three extreme sets show clearly different patterns of behaviour (see Fig. 1).

(1) *Boulware* (B) [30]. For this tactic, $\psi < 1$ and close to zero. The initial offer is maintained till time is almost exhausted, when the agent concedes up to its reservation value. Fig. 1 shows the Boulware function for $\psi = 0.02$.

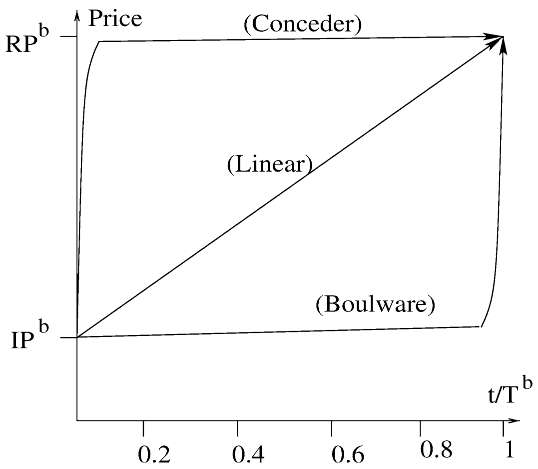

Fig. 1. Negotiation decision functions for the buyer.

(2) *Conceder* (C) [29]. For this tactic, $\psi > 1$. The agent goes to its reservation value very quickly[1] and maintains the same offer till the deadline. Fig. 1 shows the Conceder function for $\psi = 50$.
(3) *Linear* (L) Finally, when $\psi = 1$, price is increased linearly.

The value of a counter offer depends on the initial price (*IP*) at which the agent starts negotiation, the final price (*FP*) beyond which the agent does not concede, the time t at which it offers the final price, and ψ. These four variables form an agent's strategy.

Definition 1. An agent's strategy S^a is defined as a quadruple whose elements are the initial price (IP^a) at which the agent starts negotiation, the final price (FP^a) beyond which the agent does not concede, time (t^a) at which the final price is offered, and ψ^a. Thus

$$S^a = \langle IP^a, FP^a, t^a, \psi^a \rangle.$$

Agent a uses its strategy, S^a, to generate an offer, $p^t_{a \to \hat{a}}$, for $t \leqslant t^a$. Different strategies can be defined for different values of these four elements. For example, when b starts making offers at s's reservation price, and offers its own reservation price at a time, say T, and uses an extreme Boulware NDF, then S^b is defined as $S^b = \langle RP^s, RP^b, T, B \rangle$. Note that the B in S^b is a value for ψ that gives the Boulware function. In general, we use B, C, and L to indicate a value for ψ that gives the Boulware, Conceder, and Linear NDFs respectively. When both agents use strategies of this form, negotiation can end either in an agreement or a conflict, depending on the four elements that constitute each agent's strategy.

Definition 2. The negotiation outcome (O) is an element of $\langle (p, t), \widehat{C} \rangle$. The pair (p, t) denotes the price and time of agreement where $p \in [RP^s, RP^b]$ and $t \in [0, \min(T^b, T^s)]$. $\widehat{C}$ denotes the conflict outcome.

As an illustration, when agent b's strategy is defined as $S_1^b = \langle IP^b, RP^b, T^s, B \rangle$ and agent s's strategy is defined as $S_1^s = \langle IP^s, RP^s, T^s, B \rangle$, the outcome ($O_1$) that results is shown in Fig. 2(a) (i.e., the point where S_1^b and S_1^s converge). As shown in the figure, agreement is reached at a price $RP^s + (\textit{price-surplus}/2)$ and at a time close to T^s. Similarly when the NDF in both strategies is replaced with C, then agreement (O_2) is reached at the same price but towards the beginning of negotiation. If the agents' strategies do not converge before the deadline, then negotiation results in a conflict. This is illustrated in Fig. 2(b), where both agents use the extreme Boulware NDF, but offer the final price at different times, thereby resulting in conflict.

Since agents are assumed to be Von Neumann and Morgenstern [26] expected utility maximizers, we need to determine the four elements of each agent's strategy that will give

[1] As ψ increases (decreases) ϕ becomes more Conceder (Boulware). At very high (low) values of ψ, ϕ is an extreme Boulware (Conceder). In our discussion, Boulware always refers to the extreme Boulware for which the function generates the initial price from the beginning until the time point just prior to T, and the final price at time T. Similarly Conceder always refers to the extreme Conceder.

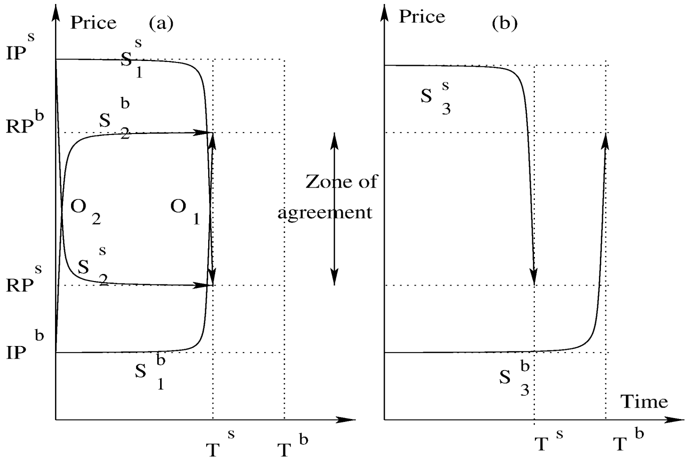

Fig. 2. Negotiation outcome for Boulware and Conceder functions. (a) Agreement. (b) Conflict.

it maximum possible utility. An agent's optimal strategy depends on the information it has about the negotiation parameters. We therefore define the information state for each agent and then show how the optimal strategies are determined.

3.3. Agents' information states

Each agent has a reservation limit, a deadline, a utility function and a strategy. Thus the buyer and seller each have four parameters denoted $\langle RP^b, T^b, U^b, S^b\rangle$ and $\langle RP^s, T^s, U^s, S^s\rangle$ respectively. The outcome of negotiation depends on all these eight parameters. The information state, I^a, of an agent a is the information it has about the negotiation parameters. An agent's own parameters are known to it, but the information it has about the opponent is not complete. We define I^b and I^s as:

$$I^b = \langle RP^b, T^b, U^b, S^b, L^s_p, L^s_t\rangle$$

and

$$I^s = \langle RP^s, T^s, U^s, S^s, L^b_p, L^b_t\rangle,$$

where RP^b, T^b, U^b and S^b are the information about the buyer's own parameters and L^s_p and L^s_t are its beliefs about the seller. Similarly, RP^s, T^s, U^s and S^s are the seller's own parameters and L^b_p and L^b_t are its beliefs about the buyer. L^s_t and L^s_p are two probability distributions[2] that denote agent b's beliefs about agent s's deadline and reservation price. L^s_t is an n-tuple of ordered pairs of the form $\langle T^s_i, \alpha^s_i\rangle$, where $1 \leqslant i \leqslant n$. The first element in a pair, T^s_i, (where $T^s_i \in \mathcal{T}$ for $1 \leqslant i \leqslant n$) denotes a possible value for the seller's deadline and the second element, α^s_i, denotes the probability with which the seller's deadline is T^s_i.

[2] The difference between this model and [6,7] is that in the latter, agents have a binary probability distribution over their opponent's reservation value and deadline whereas here we consider the more general case by taking a probability distribution over n values.

In other words, the pairs are possible time values for agent s's deadline and the associated probabilities. One of the n possible values is agent s's actual deadline. The pairs are assumed to be arranged in ascending order of time values, i.e., $T_i^s < T_{i+1}^s$ for $1 \leqslant i \leqslant n-1$.

L_p^s is analogous to L_t^s and denotes the buyer's beliefs about the seller's reservation price. The elements of L_p^s are pairs are denoted $\langle RP_i^s, \beta_i^s \rangle$ where $1 \leqslant i \leqslant m$. The first element is a possible value for the seller's reservation price and β_i^s is the associated probability. Similarly L_t^b and L_p^b are two probability distributions that denote the seller's beliefs about the buyer's deadline and reservation price. The elements of L_t^b are of the form $\langle T_i^b, \alpha_i^b \rangle$ (where $T_i^b \in \mathcal{T}$ for $1 \leqslant i \leqslant n$) and the elements of L_p^b are of the form $\langle RP_i^b, \beta_i^b \rangle$. For our present analysis we consider the case where $RP_1^s < RP_m^b$, i.e., the highest possible value for the seller's reservation price is less than the lowest possible value for the buyer's reservation price.[3] We treat the agents' beliefs as being static[4] and not changing during negotiation.

Thus agents have uncertain information about each other's deadline and reservation value. Moreover, agents do not know their opponent's utility function or strategy. In other words, an agent's information state models two[5] parameters of the opponent: the opponent's reservation price and its deadline. Each agent's information state is its *private* information that is not known to its opponent.

3.4. Negotiation scenarios

On the basis of the relationship between agent deadlines and their discounting factors, we define six negotiation scenarios. An agent negotiates in one of these six scenarios. The buyer believes that with probability α_i^s, the seller's deadline is T_i^s. This gives rise to three relations between agent deadlines. All the n possible seller deadlines could be less than the buyer's deadline, some of them could be less and the others greater, and finally all of them could be greater than the buyer's deadline. For each of the two possible realizations of the buyer's discounting factor, these three relations can hold between agent deadlines. In other words, negotiation can take place in any one of the six scenarios, $N_1, \ldots, N_6$, listed in Table 1. The set of negotiation scenarios for the seller can be defined in the same way.

The scenario combinations that are possible for the two agents to interact are listed in Table 2. For instance, when agent b is in scenario N_1, T^s is less than T^b. In such a situation, agent s can only be in one of the four scenarios N_2, N_3, N_5 or N_6, since T^s can be less then T^b in only these four scenarios. Recall that one of the possible values is the opponent's actual deadline. This implies that when agent b is in scenario N_1, agent s can neither be in scenario N_1 nor in N_4. Thus in general when agent a is in scenario N_1, agent $\hat{a}$ may be in any one of the four scenarios—N_2, N_3, N_5, or N_6. The remaining scenario combinations, listed in Table 2 can be obtained using similar reasoning. Note that it is possible for the agents to have equal deadlines in the following cases: when both agents are in scenario N_2,

[3] Future work will deal with the situation where $RP_1^s > RP_m^b$.

[4] Future work will deal with the situation where an agent learns and changes its beliefs during negotiation.

[5] An agent's information state may be different for different negotiations. Also, as shown in [5], the information states of agents strongly influence the negotiation outcome. It would therefore be interesting to study the negotiation process by modelling other parameters of the opponent. Future work will deal with such a study.

Table 1
Possible negotiation scenarios for agent b

Negotiation scenario	Relationship between agent deadlines	Discounting factor
N_1	$T_n^s < T^b$	$\delta^b > 1$
N_2	$T_k^s < T^b \leqslant T_{k+1}^s$ for $k+1 < n$	$\delta^b > 1$
N_3	$T^b < T_1^s$	$\delta^b > 1$
N_4	$T_n^s < T^b$	$\delta^b < 1$
N_5	$T_k^s < T^b \leqslant T_{k+1}^s$ for $k+1 < n$	$\delta^b < 1$
N_6	$T^b < T_1^s$	$\delta^b < 1$

Table 2
Possible negotiation scenarios for buyer-seller interactions

Agent a	Agent $\hat{a}$
N_1	N_2, N_3, N_5, N_6
N_2	$N_1, N_2, N_3, N_4, N_5, N_6$
N_3	N_1, N_2, N_4, N_5
N_4	N_2, N_3, N_5, N_6
N_5	$N_1, N_2, N_3, N_4, N_5, N_6$
N_6	N_1, N_2, N_4, N_5

or when both agents are in scenario N_5, or when agent a is in scenario N_2 and agent $\hat{a}$ is in scenario N_5. For all the other combinations, the agents have different deadlines.

3.5. *Optimal strategies*

We describe how optimal strategies are obtained for players that are von Neumann–Morgenstern expected utility maximizers. The discussion is from the perspective of the buyer (although the same analysis can be taken from the perspective of the seller). In order to simplify the discussion we first assume that L_p^s contains a single element, which is the seller's reservation price, and obtain the optimal strategy. We then extend the analysis to the more general case where L_p^s contains m elements.

Each agent's optimal strategy is determined on the basis of its own information state, i.e., the buyer's optimal strategy is determined on the basis of I^b and the seller's optimal strategy is determined on the basis of I^s. We then determine if this mutual strategic behavior of agents results in equilibrium.

3.5.1. *Optimal strategies for the buyer when L_p^s contains a single element*

In all the six scenarios, the strategies should ensure agreement by the earlier deadline (i.e., T^s if $T^s < T^b$ and T^b if $T^b < T^s$). Otherwise the agent with the earlier deadline quits and negotiation ends in a conflict, a situation which both agents prefer to avoid. We begin with scenario N_1 where all the n possible values for the seller's deadline are less than T^b. Since $\delta^b > 1$ in scenario N_1, the buyer prefers to reach agreement at the latest possible time and at the lowest possible price. As $T^s < T^b$ in scenario N_1, the latest possible time for reaching an agreement is T_n^s.

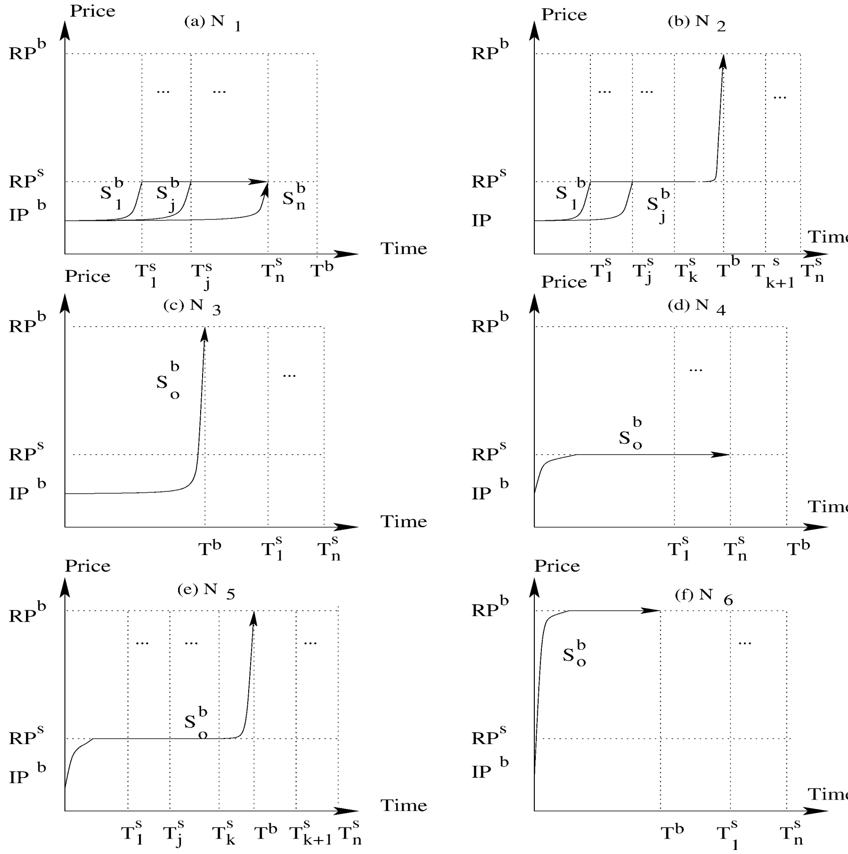

Fig. 3. Buyer strategies in different scenarios when L^s_p contains a single element.

The outcome of negotiation depends on both agents' strategies. Since both agents use a time-dependent strategy, an agent always plays a strategy that offers its own reservation price at its deadline. The buyer does not know the seller's deadline, but it has a lottery (L^s_t) over n possible values for the seller's deadline. So the buyer knows that if the seller's deadline is T^s_i, then the seller will play a strategy, S^s_i, that offers RP^s at T^s_i. The probability that the seller's deadline is T^s_i is α^s_i, i.e., the seller will play strategy S^s_i with probability α^s_i. From its lottery (L^s_t) the buyer knows that the seller can play n different strategies, and will play strategy S^s_i with probability α^s_i. In other words, although the buyer does not know the seller's actual strategy, it knows[6] the possible strategies the seller can play and the associated probabilities.

Since the maximum possible value for the seller's deadline is less than T^b, the buyer can minimize the price of agreement by waiting for the seller to offer RP^s. Thus the

[6] Note that the buyer does not know the seller's complete strategy. It only knows the seller's final price and the time at which the price is offered.

optimal price of agreement, denoted P_o^b, is RP^s. As an agent's utility also depends on time, and $\delta^b > 1$, the buyer tries to maximize the time of agreement. Since the buyer has n possible values for the seller's deadline, it has n strategies to choose from. At time t during negotiation, strategy S_j^b is defined as $\langle IP^b, RP^s, T_j^s, B\rangle$ for all $t \leqslant T_j^s$. At all later times, (i.e., between T_j^s and T_n^s) the strategy offers the price RP^s. Thus the earliest time at which agreement can be reached using strategy S_j^b is T_j^s and the latest time is T_n^s. If the seller's actual deadline is less than T_j^s, then S_j^b results in conflict. These strategies are depicted in Fig. 3(a). Out of these n strategies, the one that gives the buyer the maximum expected utility (EU_o^b) is its optimal strategy (S_o^b). Agent b's expected utility from strategy S_j^b, is:

$$EU_j^b = \sum_{x=1}^{j-1} \alpha_x^s U^b(\widehat{C}) + \alpha_j^s U^b\big(RP^s, T_j^s\big) + \sum_{y=j+1}^{n} \alpha_y^s U^b\big(RP^s, t\big) \tag{3}$$

where $T_j^s \leqslant t \leqslant T_n^s$.

This is the general expression for the buyer's EU from different strategies. Here, the value of t depends on the opponent's strategy. In Section 3.5.2 we will explain how to obtain the value of t. For the present assume that this value is known to us. For this given value of t, the expected utility depends on the probability distribution (α^s), the utility function (U^b), and j. For example, the EU for different values of j between 1 and 15 is illustrated in Fig. 4. In this example, α^s was defined as a Poisson distribution and δ^b was 1.6 (a value greater than 1). As seen in the figure, EU_j^b is maximum at $j = 7$, indicating that the optimal time for entering the zone of agreement, denoted T_J^s, is T_7^s. The optimal strategy is therefore S_7^b. In this figure, the time points are at uniform discrete intervals. However, this is not necessary as long as the conditions for convergence of agents' strategies (listed

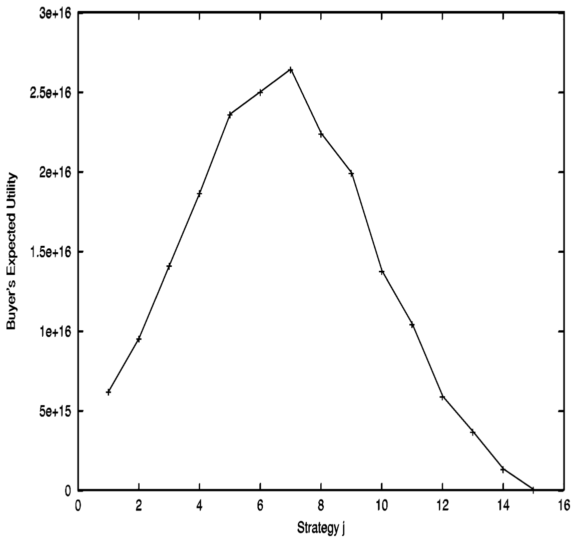

Fig. 4. Buyer's EU for the possible strategies in scenario N_1.

Table 3
Optimal buyer strategies in different negotiation scenarios when L_p^s contains a single element. T' denotes the second time period, i.e., if negotiation begins at time t, $T' = t + 1$

Negotiation scenario	Time t during negotiation	Optimal strategy
N_1	$t \leqslant T_J^s$	$\langle IP^b, RP^s, T_J^s, B\rangle$
	$t > T_J^s$	$\langle RP^s, RP^s, T_n^s, L\rangle$
N_2	$t \leqslant T_J^s$	$\langle IP^b, RP^s, T_J^s, B\rangle$
	$T_J^s < t \leqslant T_k^s$	$\langle RP^s, RP^s, T_k^s, L\rangle$
	$t > T_k^s$	$\langle RP^s, RP^b, T^b, B\rangle$
N_3	$t \leqslant T^b$	$\langle IP^b, RP^b, T^b, B\rangle$
N_4	$t \leqslant T'$	$\langle IP^b, RP^s, T', C\rangle$
	$t > T'$	$\langle RP^s, RP^s, T_n^s, L\rangle$
N_5	$t \leqslant T'$	$\langle IP^b, RP^s, T', C\rangle$
	$T' < t \leqslant T_k^s$	$\langle RP^s, RP^s, T_k^s, L\rangle$
	$t > T_k^s$	$\langle RP^s, RP^b, T^b, B\rangle$
N_6	$t \leqslant T'$	$\langle IP^b, RP^b, T', C\rangle$
	$t > T'$	$\langle RP^b, RP^b, T^b, L\rangle$

in Section 3.5.3) are satisfied. For a higher value of δ^b, EU_j^b is maximum at a higher value of j. Lowering the value of δ^b causes the peak of the curve to shift left. In other words T_J^s increases as δ^b increases and T_J^s decreases as δ^b decreases. For $\delta^b = 1$, EU^b is at a maximum for $j = 1$. This happens because the agent is indifferent to time. Higher values of j result in some conflict situations and thus give a lower utility. But when $\delta^b > 1$, the agent gains utility with time and the maximum utility is obtained for $j > 1$.

The buyer's optimal strategy for scenario N_1 is listed in Table 3. Let $S_o^b(t)$ denote the price generated by the buyer's optimal strategy at time t. The buyer's action function for scenario N_1 is defined as follows:

$$A^b\big(t, p_{s\to b}^t\big) = \begin{cases} \text{Quit} & \text{if } t > T^b, \\ \text{Accept} & \text{if } p_{s\to b}^t \leqslant S_o^b(t), \\ \text{Offer } S_o^b(t') \text{ in the next time period } t' & \text{otherwise.} \end{cases}$$

In the definition of an agent's action given in Section 3.1, the opponent's offer is accepted if the utility from the opponent's offer at time t is greater than or equal to the utility of the offer the agent is willing to generate at time t'. But here, in order to decide when to accept the seller's offer, the price offered by the seller at time t ($p_{s\to b}^t$) is compared with the price generated by the buyer's optimal strategy (S_o^b) at time t. This is because the seller's actual deadline is not known to the buyer, and t could be the seller's deadline, in which case the seller quits and negotiation ends in a conflict if the buyer does not accept the offer at time t. So even though the buyer's utility increases with time, it has to accept the seller's offer if $p_{s\to b}^t \leqslant S_o^b(t)$ and thereby avoid the chance of a conflict.

In scenario N_2, the seller's deadline can be either less than or greater than T^b. Since some of the possible values for the seller's deadline are less than T^b, the buyer's optimal strategy would be to wait for the opponent to offer RP^s. If $T^s < T^b$, the latest time by which the seller will offer RP^s is T_k^s. Thus until T_k^s, the buyer need not offer a price greater than RP^s. If an agreement is not reached by T_k^s, it implies that the seller's deadline is greater than T^b and to avoid conflict, the buyer needs to offer its reservation price RP^b at T^b. Thus agent b should enter the zone of agreement at the latest possible time (to ensure that agreement is not reached earlier than that), remain at RP^s until T_k^s and then offer/accept its own reservation price, RP^b, at T^b. The possible times for entering the zone of agreement are $T_1^s, \ldots, T_k^s$. These strategies are depicted in Fig. 3(b), where strategy S_j^b enters the zone of agreement at T_j^s. The expected utility for strategy S_j^b is:

$$\begin{aligned} EU_j^b = {} & \sum_{x=1}^{j-1} \alpha_x^s U^b(\widehat{C}) + \alpha_j^s U^b\bigl(RP^s, T_j^s\bigr) \\ & + \sum_{y=j+1}^{k} \alpha_y^s U^b\bigl(RP^s, t_1\bigr) + \sum_{z=k+1}^{n} \alpha_z^s U^b(p, t_2) \end{aligned} \tag{4}$$

where $RP^s \leqslant p \leqslant RP^b$ and $T_j^s \leqslant t_1 \leqslant T_y^s$ and $T_j^s \leqslant t_2 \leqslant T^b$.

As for scenario N_1, assume that the values of p, t_1, and t_2, are known. In Section 3.5.2 we will explain how to obtain these values. For the given values of p, t_1, and t_2, the values of EU_j^b for different values of j between 1 and 15 and $\delta^b = 1.6$ (where α^s is a Poisson distribution) are depicted in Fig. 5. As seen in the figure, the value of j for which EU_j^b is maximum depends on the value of k. For higher values of δ^b, we get the same pattern as in Fig. 5 but the peak of the curve shifts to the right. Lowering the value of δ^b shifts the peak to the left. In other words, the optimal time (T_J^s) for entering the zone of agreement increases as δ^b increases and decreases as δ^b decreases. Fig. 6 shows EU^b for $\delta^b = 1$. As seen from the figure, EU^b is maximum at $j = 1$. This happens because, for $\delta^b = 1$, the agent is

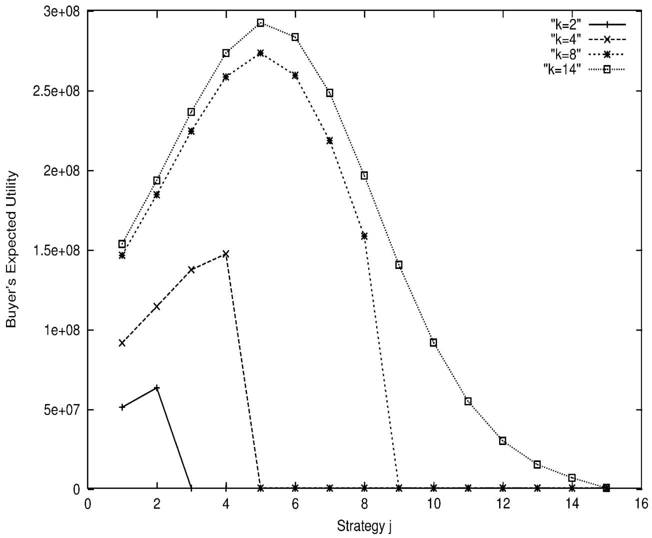

Fig. 5. Buyer's EU for different strategies in scenario N_2 when $\delta^b > 1$.

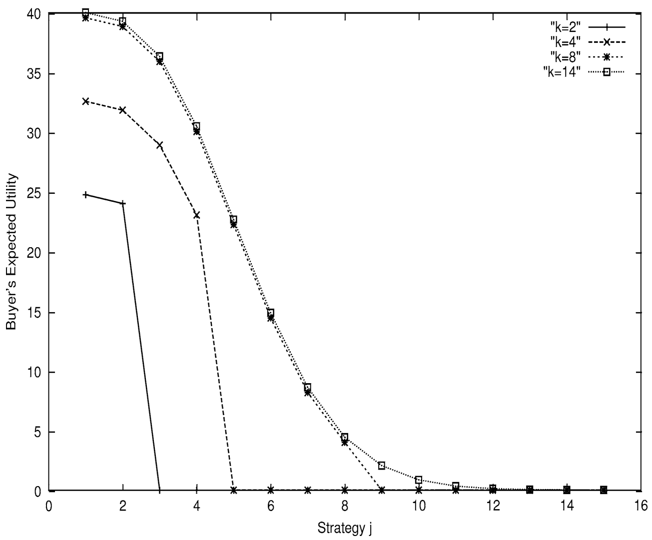

Fig. 6. Buyer's EU for different strategies in scenario N_2 when $\delta^b = 1$.

indifferent to time. Higher values of j result in some conflict situations and thus give a lower utility. But when $\delta^b > 1$, the agent gains utility with time and the maximum utility is obtained for $j > 1$. The buyer's optimal strategy for scenario N_2 is listed in Table 3. The buyer's action function for scenario N_2 is the same as that for scenario N_1.

In scenario N_3, the buyer gains utility with time (i.e., $\delta^b > 1$) and $T^b < T_1^s$. The buyer's optimal strategy here is $S_o^b = \langle IP^b, RP^b, T^b, B\rangle$. This strategy (shown in Fig. 3(c)) enters the zone of agreement at the latest possible time, which is close to the earlier deadline T^b, and thereby maximizes the time of agreement. It also optimizes the price of agreement by offering RP^b only at T^b.

In the remaining three scenarios, N_4 to N_6, $\delta^b < 1$ and the buyer loses utility with time. In scenario N_4 (shown in Fig. 3(d)), it is clear that the buyer can optimize both the price and the time of agreement by offering RP^s right from the beginning of negotiation, until T_n^s (see Table 3). Contrast this with S_o^b of scenario N_1, in which the zone of agreement is entered at T_J^s, whereas here it is entered at the beginning of negotiation using the Conceder function (since $\delta^b < 1$).

In scenario N_5, the buyer's optimal strategy is to offer RP^s from the beginning of negotiation until T_k^s. If $T^s \leqslant T_k^s$, then negotiation ends at the latest by T_k^s. Otherwise it continues beyond T_k^s. The buyer then has to concede up to RP^b in order to ensure agreement (see Fig. 3(e)). This strategy is listed in Table 3.

Finally, in scenario N_6, the buyer's optimal strategy is to offer RP^b right from the beginning of negotiation until T^b (see Fig. 3(f)). This is because when the buyer is in scenario N_6, the possible scenarios for the opponent are N_1, N_2, N_4 or N_5. Since the seller also behaves strategically, in none of these scenarios will agent s concede beyond RP^b, until time T^b, using its optimal strategy. Thus for the price RP^b, the time of agreement is optimized in strategy S_o^b using the Conceder function. Table 3 lists the buyer's optimal strategies in all the six negotiation scenarios. The buyer's action function in all the scenarios is the same as that for scenario N_1.

3.5.2. Optimal strategies for the buyer when L_p^s contains more than one element

Optimal strategies for the buyer when L_p^s contains more than one element remain the same as those obtained in Section 3.5.1 in some, but not all, negotiation scenarios. Only those optimal strategies (listed in Table 3) that depend on the opponent's reservation price change, while the others remain the same. More specifically, the optimal strategies in scenarios N_3 and N_6 remain the same, while those in scenarios N_1, N_2, N_4, and N_5 change when L_p^s contains more than one element. We analyze each of these four scenarios below. The buyer's action function, A^b, does not depend on the number of elements in L_p^s and therefore remains the same as defined in Section 3.5.1 for all the scenarios.

As mentioned in Section 3.3, the information state of the buyer, I^b, has n possible values for the seller's deadline and m possible values for its reservation price. Also, recall that agent b believes that β_i^s is the probability that the opponent's reservation price is RP_i^s and that α_j^s is the probability that the opponent's deadline is T_j^s. The probability that the seller has the reservation price RP_i^s and deadline T_j^s is thus the product of β_i^s and α_j^s, and is denoted $\gamma_{i,j}^s$.

Consider scenario N_1 first. The possible buyer strategies for this scenario are depicted in Fig. 7. The number of possible strategies here is $m \times n$. We use $S_{i,j}^b$ to denote the strategy that starts making offers at IP^b, offers RP_i^s at T_j^s using the Boulware function, and does not change the price thereafter. The strategy that yields the highest EU is the buyer's optimal strategy. Let I and J denote the values of i and j that give agent b the highest utility. Here we need to find these two values. Contrast this with the case where L_p^s had a single element which required finding only the optimal value of j, i.e., J.

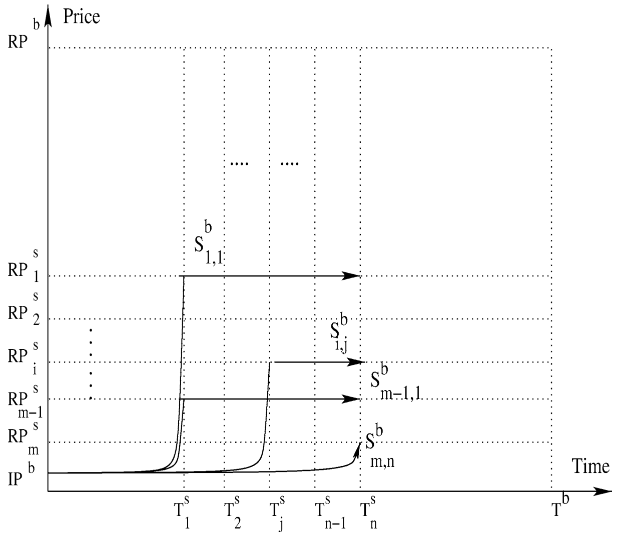

Fig. 7. Possible buyer strategies in scenario N_1 when L_p^s contains more than one element.

The outcome of negotiation depends on both the buyer's and the seller's strategy. The buyer does not know the seller's strategy, but it has two lotteries, L^s_p and L^s_t, over the seller's reservation price and deadline. So if the seller's reservation price and deadline are RP^s_i and T^s_j, then it plays strategy $S^s_{i,j}$ that offers RP^s_i at T^s_j. The probability with which the seller plays strategy $S^s_{i,j}$ is $\gamma^s_{i,j}$. Thus although the buyer does not know the seller's actual strategy, it knows that the seller can play $m \times n$ different strategies and the associated probabilities.

Consider the strategy $S^b_{m,n}$. This strategy results in an agreement only if the seller's actual reservation price is RP^s_m and its deadline is T^s_n. All the other values for the seller's reservation price or deadline result in a conflict. Thus the EU from strategy $S^b_{m,n}$ is:

$$EU^b_{m,n} = \sum_{x=1}^{m-1}\sum_{y=1}^{n} \gamma^s_{x,y} U^b(\widehat{C}) + \sum_{x=1}^{n-1} \gamma^s_{m,x} U^b(\widehat{C}) + \gamma^s_{m,n} U^b\big(RP^s_m, T^s_n\big). \tag{5}$$

In general, strategy $S^b_{i,j}$ results in conflict if either the seller's reservation price is higher than RP^s_i, or its deadline is less than T^s_j. The utility from $S^b_{i,j}$ is therefore:

$$\begin{aligned} EU^b_{i,j} &= \sum_{c=1}^{i-1}\sum_{d=1}^{n} \gamma^s_{c,d} U^b(\widehat{C}) + \sum_{c=1}^{j-1} \gamma^s_{i,c} U^b(\widehat{C}) \\ &\quad + \gamma^s_{i,j} U^b\big(RP^s_i, T^s_j\big) + \sum_{x=j+1}^{n} \gamma^s_{i,x} U^b\big(RP^s_i, t_1\big) \\ &\quad + \sum_{y=i+1}^{m} \left(\sum_{z=1}^{j-1} \gamma^s_{y,z} U^b(\widehat{C}) + \gamma^s_{y,j} U^b\big(p_1, T^s_j\big) + \sum_{z=j+1}^{n} \gamma^s_{y,z} U^b(p_2, t_2) \right) \end{aligned} \tag{6}$$

where $RP^s_y \leqslant p_1 \leqslant RP^s_i$ and $RP^s_y \leqslant p_2 \leqslant RP^s_i$
and $T^s_j \leqslant t_1 \leqslant T^s_x$ and $T^s_j \leqslant t_2 \leqslant T^s_z$.

In the above expression, the values of p_1 and p_2 depend on two factors: the opponent's strategy and the identity of the player that makes a move at the earlier deadline. The values of t_1 and t_2 depend only on the opponent's strategy. Although the buyer does not know the opponent's actual strategy, it does know that the opponent will also behave strategically. This strategic behavior depends on the opponent's scenario. Recall that when the buyer's scenario is N_1, the seller can be in any of the four scenarios: N_2, N_3, N_5, or N_6. We know from Section 3.5.1 that in scenario N_6, an agent's optimal strategy is to offer its reservation price using the Conceder function. Thus if agent s is in scenario N_6, its optimal strategy is to offer RP^s using the Conceder function. In addition to the seller's strategy, the values of p_1 and p_2 also depend on who makes an offer at the earlier deadline. The player that makes an offer at the earlier deadline could be the buyer or the seller, depending on who made the initial offer. Consider the case where it is the seller's turn to make a move at the earlier deadline. The seller's optimal strategy in scenario N_6 is to offer RP^s using the Conceder function. As per the buyer's action function, the buyer accepts the seller's offer at T^s_j. We therefore get $p_1 = p_2 = RP^s_y$ and $t_1 = t_2 = T^s_j$. On the other hand, if it is the buyer's turn to make a move at the earlier deadline, it offers RP^s_i. For $z \geqslant j$, $RP^s_i \geqslant RP^s$, and the seller accepts the buyer's offer at time T_j^s. This makes $p_1 = p_2 = RP_i^s$ and $t_1 = t_2 = T_j^s$. Using similar analysis, it can be seen that when agent s is in any of the remaining three scenarios (N_2, N_3, or N_5), we get $p_1 = p_2 = RP_y^s$, $t_1 = T_x^s$, and $t_2 = T_z^s$ if the seller makes an offer at the earlier deadline; and $p_1 = p_2 = RP_i^s$, $t_1 = T_x^s$, and $t_2 = T_z^s$ if the buyer makes an offer at the earlier deadline. The buyer knows who will make an offer at the earlier deadline, since the decision about which player will make the initial offer is made at the beginning of negotiation and thereafter players take turns alternately at each successive time period. Since the buyer does not know the seller's scenario, we associate equal probabilities with each of the four possible seller's scenarios, N_1, N_3, N_5, and N_6. Let eu_1^b denote the value of Eq. (6) when the seller's scenario is N_2, N_3, or N_5. Also, let eu_2^b denote the value of Eq. (6) when the seller's scenario is N_6. The buyer's EU therefore becomes:

$$EU_{i,j}^b = \tfrac{3}{4} eu_1^b + \tfrac{1}{4} eu_2^b. \tag{7}$$

The values of i and j for which Eq. (7) is at a maximum are denoted I and J. The buyer's optimal strategy for scenario N_1, in terms of I and J, is listed in Table 4.

In scenario N_2, the buyer uses a strategy $S_{i,j}^b$ of the form depicted in Fig. 8. This strategy starts at IP^b, offers RP_i^s at T_j^s using the Boulware function, keeps the price constant at RP_i^s until T_k^s, and thereafter uses the Boulware function again to offer RP^b at T^b. It is clear from Fig. 8 that i can vary between 1 and m and j can vary between 1 and k. Thus there are $m \times k$ possible strategies and the one that yields the maximum EU is the buyer's optimal strategy. Let I and J denote the values of i and j respectively that give the highest utility. Here we need to find these two values. Contrast this with the case where L_p^s had a single element, which required finding only J. The buyer's EU from strategy $S_{i,j}^b$ is:

$$EU_{i,j}^b = EU_1^b + EU_2^b + EU_3^b. \tag{8}$$

Table 4
Optimal buyer strategies in different scenarios when L_p^s contains more than one element

Negotiation scenario	Time t during negotiation	Optimal strategy
N_1	$t \leqslant T_J^s$	$\langle IP^b, RP_I^s, T_J^s, B\rangle$
	$t > T_J^s$	$\langle RP_I^s, RP_I^s, T_n^s, L\rangle$
N_2	$t \leqslant T_J^s$	$\langle IP^b, RP_I^s, T_J^s, B\rangle$
	$T_J^s < t \leqslant T_k^s$	$\langle RP_I^s, RP_I^s, T_k^s, L\rangle$
	$t > T_k^s$	$\langle RP_I^s, RP^b, T^b, B\rangle$
N_3	$t \leqslant T^b$	$\langle IP^b, RP^b, T^b, B\rangle$
N_4	$t \leqslant T'$	$\langle IP^b, RP_I^s, T', C\rangle$
	$t > T'$	$\langle RP_I^s, RP_I^s, T_n^s, L\rangle$
N_5	$t \leqslant T'$	$\langle IP^b, RP_I^s, T', C\rangle$
	$T' < t \leqslant T_k^s$	$\langle RP_I^s, RP_I^s, T_k^s, L\rangle$
	$t > T_k^s$	$\langle RP_I^s, RP^b, T^b, B\rangle$
N_6	$t \leqslant T'$	$\langle IP^b, RP^b, T', C\rangle$
	$t > T'$	$\langle RP^b, RP^b, T^b, L\rangle$

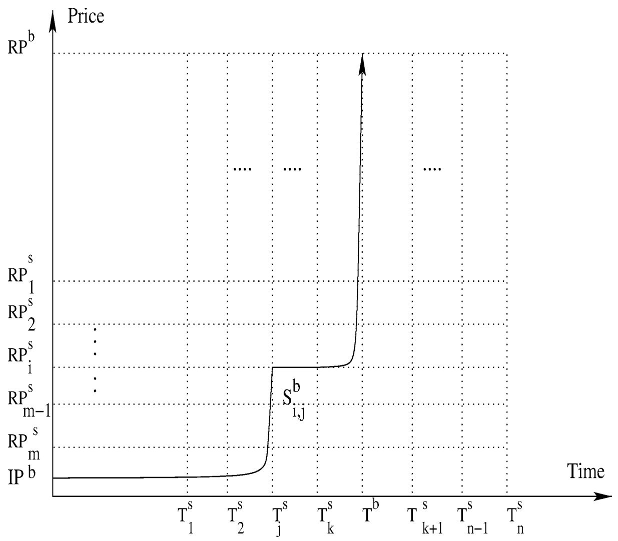

Fig. 8. The buyer's strategy $S^b_{i,j}$ in scenario N_2 where L^s_p contains more than one element.

Here, the term EU^b_1 denotes agent b's EU if the seller's actual reservation price is higher than RP^s_i, EU^b_2 denotes its EU if the seller's actual reservation price is equal to RP^s_i, and EU^b_3 denotes its EU if the seller's actual reservation price is lower than RP^s_i. We obtain each of these three terms below.

For EU^b_1 (i.e., for $RP^s > RP^s_i$), the seller's deadline can be either less than or equal to T^s_k, or it can be greater than or equal to T^s_{k+1} (see Fig. 8). If $T^s \leqslant T^s_k$, then negotiation ends in a conflict. EU^b_1 is therefore given by:

$$EU^b_1 = \sum_{x=1}^{i-1}\left(\sum_{y=1}^{k}\gamma^s_{x,y}U^b(\widehat{C}) + \sum_{y=k+1}^{n}\gamma^s_{x,y}U^b\bigl(p_1, T^b\bigr)\right) \tag{9}$$

where $(RP^s_x \leqslant p_1 \leqslant RP^b)$.

Note that the value of p_1 depends on the opponent's strategy and the identity of the player that makes an offer at the earlier deadline. The four possible seller scenarios for the second term of Eq. (9) (i.e., $T^s > T^b$) are N_1, N_2, N_4, or N_5. For each of these scenarios, the seller's strategic behavior gives $p_1 = RP^b$ if the buyer makes a move at the earlier deadline, and $p_1 = RP^b_I$ if the seller makes a move at the earlier deadline. Note that in order to get these values for p_1, the buyer and seller strategies need to converge before the earlier deadline. The conditions for convergence of agents' strategies are listed in Section 3.5.3. Also note that the value of RP^b_I is present in the seller's information state and is not known to the buyer. The buyer can therefore only take $p_1 = RP^b$ as the closest approximation.

The next term, EU^b_2, is the buyers EU when RP^s is equal to RP^s_i and is:

$$EU_2^b = \sum_{x=1}^{j-1} \gamma_{i,x}^s U^b(\widehat{C}) + \gamma_{i,j}^s U^b(RP_i^s, T_j^s) + \sum_{x=j+1}^{k} \gamma_{i,x}^s U^b(RP_i^s, t_1) + \sum_{x=k+1}^{n} \gamma_{i,x}^s U^b(p_2, t_2) \tag{10}$$

where $(RP_i^s \leqslant p_2 \leqslant RP^b)$ and $(T_j^s \leqslant t_1 \leqslant T_x^s)$ and $(T_j^s \leqslant t_2 \leqslant T^b)$.

The possible scenarios for the seller for the third term of Eq. (10) are N_2, N_3, N_5, or N_6. Considering the seller's strategic behavior, we get $t_1 = T_j^s$ if the seller's scenario is N_6 and $t_1 = T_x^s$ otherwise. The possible scenarios for the seller, for the fourth term of Eq. (10), are N_1, N_2, N_4, or N_5. Considering the seller's strategic behavior, we get $t_2 = T^b$ for all the four scenarios. The value of p_2 depends on the player that makes a move at the earlier deadline. If the buyer makes a move at the earlier deadline, we get $p_2 = RP^b$. On the other hand, if the seller makes a move at the earlier deadline we get $p_2 = RP_I^b$. As for p_1, since the buyer does not know RP_I^b, it can only take $p_2 = RP^b$ as the closest approximation for all possible seller scenarios.

The last term, EU_3^b (i.e., for the case $RP^s > RP_i^s$) is as follows:

$$EU_3^b = \sum_{x=i+1}^{m} \left(\sum_{y=1}^{j-1} \gamma_{x,y}^s U^b(\widehat{C}) + \gamma_{x,j}^s U^b(p_3, T_j^s) + \sum_{y=j+1}^{k} \gamma_{x,y}^s U^b(p_4, t_3) + \sum_{y=k+1}^{n} \gamma_{x,y}^s U^b(p_5, t_4) \right) \tag{11}$$

where $(RP_x^s \leqslant p_3 \leqslant RP_i^s)$ and $(RP_x^s \leqslant p_4 \leqslant RP_i^s)$ and $(RP_x^s \leqslant p_5 \leqslant RP^b)$ and $(T_j^s \leqslant t_3 \leqslant T_y^s)$ and $(T_j^s \leqslant t_4 \leqslant T^b)$.

The possible scenarios for the seller for the second and third terms of Eq. (11) are N_2, N_3, N_5, or N_6, while for the fourth term they are N_1, N_2, N_4, or N_5. Considering the seller's strategic behavior we get $t_3 = T_j^s$ if the seller's scenario is N_6, and $t_3 = T_y^s$ otherwise. For all the possible seller's scenarios $t_4 = T^b$. The values of p_3, p_4, and p_5 depend on the identity of the player that makes a move at the earlier deadline. $p_3 = p_4 = RP_x^s$ if the seller makes a move at the earlier deadline, and $p_3 = p_4 = RP_i^s$ if the buyer makes a move at the earlier deadline. Finally, $p_5 = RP_I^b$ if the seller makes a move at the earlier deadline and $p_5 = RP^b$ if the buyer makes a move at the earlier deadline. Again, as for p_1, the buyer can only take $p_5 = RP^b$ as an approximation.

The buyer's utility from strategy $S_{i,j}^b$, is the sum of EU_1^b, EU_2^b, and EU_3^b. Let eu_1^b denote the value of Eq. (8) if the seller's scenario is N_6, and eu_2^b denote its value otherwise. As each of the four possible scenarios for the seller is equally probable, $EU_{i,j}^b$ becomes:

$$EU_{i,j}^b = \tfrac{1}{4} eu_1^b + \tfrac{3}{4} eu_2^b. \tag{12}$$

The values of i and j that give the buyer the maximum EU are denoted I and J. The buyer's optimal strategy for scenario N_2, in terms of I and J, is listed in Table 4.

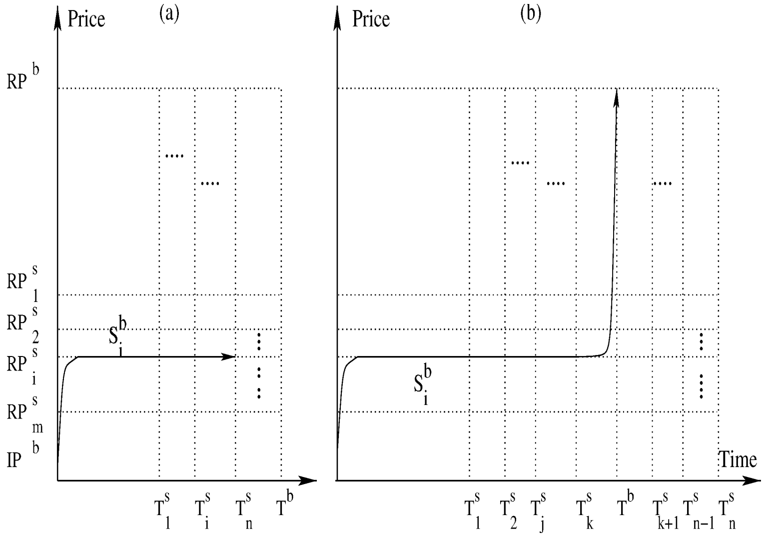

Fig. 9. The buyer's strategy, S_i^b, when L_p^s contains more than one element. (a) Scenario N_4. (b) Scenario N_5.

In the next scenario, i.e., N_3, the buyer's optimal strategy does not depend on the opponent's reservation price. Thus the buyer's optimal strategy when L_p^s contains more than one element is the same as its optimal strategy when L_p^s contains a single element. This is also true for scenario N_6.

In negotiation scenario N_4, the buyer's optimal strategy is to offer the opponent's reservation price, RP^s, immediately after negotiation starts and continue to offer the same price until negotiation ends. The possible strategies that the buyer can use when L_p^s has more than one element are of the form S_i^b, where $1 \leqslant i \leqslant m$. This is shown in Fig. 9(a). The buyer's EU from strategy S_i^b is:

$$EU_i^b = \sum_{x=1}^{i-1}\sum_{y=1}^{n} \gamma_{x,y}^s U^b(\widehat{C}) + \sum_{x=1}^{n} \gamma_{i,x}^s U^b\big(RP_i^s, t_1\big) + \sum_{x=i+1}^{m}\sum_{y=1}^{n} \gamma_{x,y}^s U^b(p_1, t_2) \qquad (13)$$

where $RP_x^s \leqslant p_1 \leqslant RP_i^s$ and $T' \leqslant t_1 \leqslant T_x^s$ and $T' \leqslant t_2 \leqslant T_y^s$.

The values of t_1 and t_2 depend on the opponent's scenario, while p_1 depends on the opponent's scenario and the identity of the player that makes a move at T' or the earlier deadline. If the opponent is in scenario N_6, then $t_1 = t_2 = T'$. On the other hand, if the seller's scenario is N_2, N_3, or N_5, then $t_1 = T_x^s$ and $t_2 = T_y^s$. If the seller's scenario is N_6, $p_1 = RP_x^s$ if the seller makes a move at time T' and $p_1 = RP_i^s$ if the buyer makes a move at time T'. On the other hand, if the seller's scenario is N_2, N_3, or N_5, $p_1 = RP_x^s$ if the seller makes a move at the earlier deadline and $p_1 = RP_i^s$ if the buyer makes a move at the earlier deadline. Let eu_1^b denote the value of Eq. (13) if the seller's scenario is N_6 and let

eu_2^b denote its value if the seller's scenario is N_2, N_3, or N_5. All the four possible seller's scenarios being equally probable, EU_i^b becomes:

$$EU_i^b = \tfrac{1}{4}eu_1^b + \tfrac{3}{4}eu_2^b. \tag{14}$$

The buyer's optimal strategy for scenario N_4 is listed in Table 4.

Finally, in scenario N_5 the buyer's possible strategies are of the form S_i^b shown in Fig. 9(b). The expected utility from S_i^b is:

$$\begin{aligned} EU_i^b = {} & \sum_{x=1}^{i-1}\left(\sum_{y=1}^{k}\gamma_{x,y}^s U^b(\widehat{C}) + \sum_{y=k+1}^{n}\gamma_{x,y}^s U^b\big(p_1, T^b\big)\right) \\ & + \sum_{x=1}^{k}\gamma_{i,x}^s U^b\big(RP_i^s, t_1\big) + \sum_{x=k+1}^{n}\gamma_{i,x} U^b(p_2, t_2) \\ & + \sum_{x=i+1}^{m}\left(\sum_{y=1}^{k}\gamma_{x,y}^s U^b(p_3, t_3) + \sum_{y=k+1}^{n}\gamma_{x,y}^s U^b(p_4, t_4)\right) \end{aligned} \tag{15}$$

where $(T' \leqslant t_1 \leqslant T_x^s)$ and $(T' \leqslant t_2 \leqslant T^b)$ and $(T' \leqslant t_3 \leqslant T_y^s)$

and $(T' \leqslant t_4 \leqslant T^b)$ and $(RP_x^s \leqslant p_1 \leqslant RP^b)$ and $(RP_i^s \leqslant p_2 \leqslant RP^b)$

and $(RP_x^s \leqslant p_3 \leqslant RP_i^s)$ and $(RP_i^s \leqslant p_4 \leqslant RP^b)$.

Using similar analysis, as for scenario N_2 for the buyer, we get the following values. The values of t_1, t_2, t_3 and t_4 depend on the seller's scenario. We get $t_1 = T'$ if the seller's scenario is N_6, and $t_1 = T_x^s$ otherwise. We get $t_3 = T'$ if the seller's scenario is N_6, and $t_3 = T_y^s$ otherwise. Similarly, $t_2 = t_4 = T^b$ for all possible seller scenarios. The values of p_1, p_2, and p_4 depend on the identity of the player that makes a move at the earlier deadline. The value of p_3 depends on the identity of the player that makes a move at T' or the earlier deadline. We get $p_1 = p_2 = p_4 = RP^b$ if the buyer makes a move at the earlier deadline, and $p_1 = p_2 = p_4 = RP_I^b$ if the seller makes a move at the earlier deadline. Finally, $p_3 = RP_x^s$ if the seller's scenario is N_6 and the seller makes a move at T'. But $p_3 = RP_i^s$ if the seller's scenario is N_6 and the buyer makes a move at T'. For the remaining seller's scenarios, $p_3 = RP_x^s$ if the seller makes a move at the earlier deadline and $p_3 = RP_i^s$ if the buyer makes a move at the earlier deadline. Since RP_I^b is not known to the buyer, it can only take RP^b as the values of p_1, p_2 and p_4. Let eu_1^b denote the value of Eq. (15) if the seller's scenario is N_6, and eu_2^b denote its value otherwise. The expression for EU_i^b therefore becomes

$$EU_i^b = \tfrac{1}{4}eu_1^b + \tfrac{3}{4}eu_2^b. \tag{16}$$

The buyer's optimal strategy for scenario N_5 is listed in Table 4. Optimal strategies for the seller, S_o^s, can be obtained in the same way.

3.5.3. Conditions for convergence of optimal strategies

It is clear from Section 3.5.2, that when both agents use their respective optimal strategies, the outcome of negotiation depends on RP^s_I, T^s_J, RP^b_I, and T^b_J. For instance, consider the case where the buyer's scenario is N_1, and T^s_J has a value greater than T^s and RP^s_I has a value less than RP^s. Here, the buyer starts at IP^b and uses the Boulware function to offer RP^s_I at time T^s_J. Agent s quits at T^s and since $T^s < T^s_J$, negotiation ends in a conflict. Thus in scenario N_1, RP^s_I in the buyer's optimal strategy should be greater than or equal to the seller's actual reservation price (RP^s) and T^s_J should be less than or equal to the seller's actual deadline (T^s) for the buyer and seller strategies to converge. Likewise, when the seller's scenario is N_1, RP^b_I in the seller's optimal strategy should be less than or equal to the buyer's actual reservation price and T^b_J in the seller's optimal strategy should be less than or equal to the buyer's actual deadline. The outcomes given in Table 6 will result only if the agents' beliefs about each other satisfy the conditions for convergence of optimal strategies listed in Table 5. If these conditions are not satisfied, bargaining will end in a conflict. Furthermore, the more accurate agent a's beliefs about agent $\hat{a}$ are, the closer T^a_J and RP^a_I are to T^a and RP^a respectively.

The outcomes of negotiation, i.e., the price and time of agreement for all possible scenarios, when the conditions for convergence of optimal strategies are satisfied, are summarised in Table 6. For instance, consider row 1, where the buyer's scenario is N_1 and the seller's scenario is N_2. Here $T^s < T^b$ since the buyer's scenario is N_1. The buyer's optimal strategy in scenario N_1 is to offer a price lower than RP^s_m (whenever it is the buyer's turn) at all times t less than T^s_J. At any time t greater than or equal to T^s_J, the buyer accepts the seller's offer if the seller offers a price lower than or equal to RP^s_I; otherwise it offers RP^s_I. Recall that in scenario N_2, the seller will always offer a price higher than RP^s_I before T^s and offer RP^s at T^s. When the conditions for convergence of optimal strategies are satisfied, the possible values for the seller's reservation price and deadline are shown in Fig. 10 as circles. One of the circles is the seller's actual reservation price and deadline. Let (RP^s_c, T^s_d) (shown as the shaded circle) be the seller's actual reservation price and deadline. At time T^s_d it could be the buyer's or the seller's turn to make a move. Consider the case where it is the buyer's turn at T^s_d. As per its optimal strategy, the buyer offers RP^s_I at T^s_d if the offer it receives in the previous time period is higher than IP^b. In scenario N_2, the price that the seller offers in the previous time period lies in the range $[RP^b_1, RP^b_m]$, i.e.,

Table 5
Conditions for convergence of optimal strategies

Negotiation scenario	Condition for convergence	
	Buyer's strategy	Seller's strategy
N_1	$RP^s_I \geqslant RP^s$ and $T^s_J \leqslant T^s$	$RP^b_I \leqslant RP^b$ and $T^b_J \leqslant T^b$
N_2	$RP^s_I \geqslant RP^s$ and $T^s_J \leqslant T^s$	$RP^b_I \leqslant RP^b$ and $T^b_J \leqslant T^b$
N_3	None	None
N_4	$RP^s_I \geqslant RP^s$	$RP^b_I \leqslant RP^b$
N_5	$RP^s_I \geqslant RP^s$	$RP^b_I \leqslant RP^b$
N_6	None	None

Table 6
Negotiation outcome for different scenarios. The symbol $\triangledown$ denotes the outcome if $T^s < T^b$, $\triangle$ denotes the outcome if $T^b < T^s$, and $\Diamond$ denotes the outcome if $T^s = T^b$

	Negotiation scenario		Negotiation outcome (price, time)	Negotiation scenario		Negotiation outcome (price, time)
	Buyer	Seller		Buyer	Seller	
1	N_1	N_2	(RP^s, T^s) or (RP^s_I, T^s)	N_4	N_2	(RP^s, T^s) or (RP^s_I, T^s)
2	N_1	N_3	(RP^s, T^s) or (RP^s_I, T^s)	N_4	N_3	(RP^s, T^s) or (RP^s_I, T^s)
3	N_1	N_5	(RP^s, T^s) or (RP^s_I, T^s)	N_4	N_5	(RP^s, T^s) or (RP^s_I, T^s)
4	N_1	N_6	(RP^s, T^s_J) or (RP^s_I, T^s_J)	N_4	N_6	(RP^s, T') or (RP^s_I, T')
5	N_2	N_1	(RP^b, T^b) or (RP^b_I, T^b)	N_5	N_1	(RP^b, T^b) or (RP^b_I, T^b)
6	N_2	N_2	$((RP^s, T^s)$ or $(RP^s_I, T^s))\triangledown$ $((RP^b, T^b)$ or $(RP^b_I, T^b))\ \triangle$ $((RP^b, T^b)$ or $(RP^s, T^b))\Diamond$	N_5	N_2	$((RP^s, T^s)$ or $(RP^s_I, T^s))\triangledown$ $((RP^b, T^b)$ or $(RP^b_I, T^b))\ \triangle$ $((RP^b, T^b)$ or $(RP^s, T^b))\Diamond$
7	N_2	N_3	(RP^s, T^s) or (RP^s_I, T^s)	N_5	N_3	(RP^s, T^s) or (RP^s_I, T^s)
8	N_2	N_4	(RP^b, T^b) or (RP^b_I, T^b)	N_5	N_4	(RP^b, T^b) or (RP^b_I, T^b)
9	N_2	N_5	$((RP^s, T^s)$ or $(RP^s_I, T^s))\triangledown$ $((RP^b, T^b)$ or $(RP^b_I, T^b))\ \triangle$ $((RP^s, T^b)$ or $(RP^b, T^b))\Diamond$	N_5	N_5	$((RP^s, T^s)$ or $(RP^s_I, T^s))\triangledown$ $((RP^b, T^b)$ or $(RP^b_I, T^b))\ \triangle$ $((RP^s, T^b)$ or $(RP^b, T^b))\Diamond$
10	N_2	N_6	(RP^s, T^s_J) or (RP^s_I, T^s_J)	N_5	N_6	(RP^s, T') or (RP^s_I, T')
11	N_3	N_1	(RP^b, T^b) or (RP^b_I, T^b)	N_6	N_1	(RP^b, T^b_J) or (RP^b_I, T^b_J)
12	N_3	N_2	(RP^b, T^b) or (RP^b_I, T^b)	N_6	N_2	(RP^b, T^b_J) or (RP^b_I, T^b_J)
13	N_3	N_4	(RP^b, T^b) or (RP^b_I, T^b)	N_6	N_4	(RP^b, T') or (RP^b_I, T')
14	N_3	N_5	(RP^b, T^b) or (RP^b_I, T^b)	N_6	N_5	(RP^b, T') or (RP^b_I, T')

a value greater than IP^b. Thus the buyer offers RP^s_I at T^s_d. The seller accepts RP^s_I since RP^s_I is greater than the seller's actual reservation price, RP^s_c, and T^s_d is the seller's actual deadline. In other words an agreement takes place at price RP^s_I and at time T^s_d if it is the buyer's turn to make a move at T^s_d. In the same way it can be seen that for all the circles shown in Fig. 10 an agreement occurs at (RP^s_I, T^s) if it is the buyer's turn to make an offer at T^s.

On the other hand, if it is the seller's turn to make an offer at T^s_d it offers RP^s_c because $RP^s = RP^s_c$. Since $RP^s_c < RP^s_I$, the buyer accepts the seller's offer at T^s_d. So an agreement takes place at price RP^s_c and at time T^s_d if it is the seller's turn to make a move at T^s_d. In the same way it can be seen that for all the circles shown in Fig. 10, an agreement occurs at (RP^s, T^s) if it is the seller's turn to make an offer at T^s. Thus when the buyer's scenario is N_1 and the seller's scenario is N_2, the outcome is (RP^s_I, T^s) if the buyer has to make a move at T^s and the outcome is (RP^s, T^s) if the seller has to make a move at T^s. The remaining entries in Table 6 can be obtained using similar analysis.

The similarity between these results and those of Sandholm and Vulkan [37] on bargaining with deadlines is that, in both cases, the price-surplus always goes to the agent with the longer deadline. However, the key difference is that in [37] the deadline effect overrides time discounting, whereas here the deadline effect does not override time

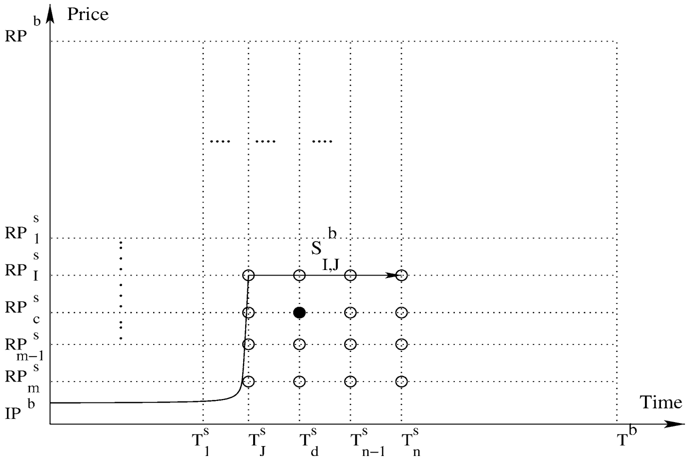

Fig. 10. Possible values for the seller's reservation price and deadline when the buyer's scenario is N_1. Strategy $S^b_{I,J}$ is the buyer's optimal strategy for scenario N_1.

discounting. This happens because in [37] the agents always make offers that lie within the zone of agreement. In our model, agents initially make offers that lie outside this zone, and thereby delay the time of agreement. Thus when agents have conflicting time preferences, in our case, agreement is reached near the earlier deadline, but in [37] agreement is reached towards the beginning of negotiation.

The outcomes listed Table 6 are possible only if this mutual strategic behavior of agents leads to equilibrium (i.e., neither agent has the motivation to deviate from its optimal strategy). In the following subsection we prove this with respect to the standard game theoretic solution concept of *sequential equilibrium* [20,28].

3.6. Equilibrium agreements

Recall that an agent's information state does not contain the opponent's strategy or its utility function. This makes negotiation a game, $\mathcal{G}$, of incomplete information. Furthermore, agents have uncertain information about each other's reservation price and deadline. The extensive game, $\mathcal{G}$, is formally defined as a 5-tuple $\langle \mathcal{N}, \mathcal{H}, \mathcal{P}, \mathfrak{I}^b, \mathfrak{I}^s \rangle$. The set $\mathcal{N} = \{b, s\}$ denotes the set of players, each member of the set $\mathcal{H}$ is a history, $\mathcal{P}$ is the player function that assigns a member of $\mathcal{N}$ to each history. The player that initiates negotiation is chosen randomly, the players then take turns as defined in the negotiation protocol. The set $\mathfrak{I}^a$ denotes the set of agent a's *information sets*. Let $\mathcal{I}^a_i$ denote the ith element of $\mathfrak{I}^a$. The first three levels of the extensive form of game, $\mathcal{G}$, are shown in Fig. 11. One of the players, say agent a, starts negotiation. The EUs that agents get from the terminal histories depend on their negotiation scenarios, and are as determined in Section 3.5.2. For instance, if agent a's scenario is N_1, then its utility from the terminal histories would be one of $m \times n$ possible values.

We first introduce the notion of *information set*. In this game $\mathcal{G}$, an agent may not know which of the nodes it is actually at. Agent a's *information set* [20,28], $\mathcal{I}^a_i$, is defined as

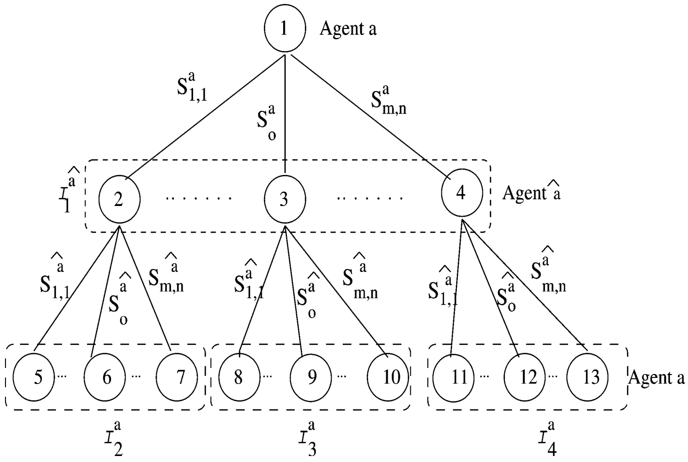

Fig. 11. Extensive form of the negotiation game.

a subset of its decision nodes such that when play reaches one of the decision nodes in the information set, and it is the agent's turn to make a move, it does not know which of these nodes it is actually at. This is because although an agent knows the offer made by the opponent, it does not know the actual strategy that was used to make the offer. For instance, in Fig. 11, when it is agent $\hat{a}$'s turn to make a move (at level 2), it does not know agent a's actual strategy. The nodes labelled $2, \ldots, 3, \ldots, 4$ thus form agent $\hat{a}$'s information set, $\mathcal{I}^{\hat{a}}_1$.

Since agents have uncertain information about the opponent, we use the solution concept of sequential equilibrium for the game $\mathcal{G}$. There are three key notions related to sequential equilibrium [20,28] of an extensive game: *assessment*, *sequential rationality*, and *consistency*. An *assessment* in an extensive game is a pair (σ, μ), where σ is a strategy profile and μ is a function that assigns to every information set a probability measure on the set of histories in the information set: μ is referred to as the belief system. In Fig. 11, agent $\hat{a}$ believes that agent a plays strategy S^a_{ij} with probability γ^a_{ij}, i.e., $\mu(\{S^a_{11}, S^a_{12}, \ldots, S^a_{mn}\})(S^a_{ij}) = \gamma^a_{ij}$. Recall that γ^a_{ij} is obtained from agent $\hat{a}$'s lotteries, L^a_p and L^a_t, and is equal to $\alpha^a_i \times \beta^a_j$. An assessment is sequentially rational if for each information set of each player $a \in \mathcal{N}$, the strategy of player a is a best response to the other player's strategies, given a's beliefs at that information set. An assessment is consistent if there is a sequence $((\sigma^n, \mu^n))^{\infty}_{n=1}$ of assessments that converges to (σ, μ) and has the properties that each strategy profile σ^n is completely mixed and that each belief system μ^n is derived from σ^n using Bayes' rule. An assessment is a sequential equilibrium of an extensive game if it is sequentially rational and consistent [20,28].

Theorem 1. *The assessment $(\sigma, \mu)_{x,y}$ in which $\sigma_a = S^a_o$ for scenario x, $\sigma_{\hat{a}} = S^{\hat{a}}_o$ for scenario y, and $\mu(\{S^a_{11}, S^a_{12}, \ldots, S^a_{mn}\})(S^a_{ij}) = \gamma^a_{ij}$ for $1 \leqslant i \leqslant m$ and $1 \leqslant j \leqslant n$ forms a sequential equilibrium of the game $\mathcal{G}$, for $1 \leqslant x \leqslant 6$ and $1 \leqslant y \leqslant 6$.*

Proof. Let the negotiation scenario for one of the agents, say agent a, be N_1^a and let the opponent's scenario be $N_2^{\hat{a}}$, i.e., $x = 1$ and $y = 2$. The first three levels of the extensive form of this game are shown in Fig. 11. At node 1, one of the players, say agent a, starts negotiation. Agent a has $m \times n$ possible strategies, and it selects a strategy at node 1. Once it selects a strategy at node 1, it generates offers using that strategy every time it has to make a move. Agent a's strategy $S_{i,j}^a$ is as defined in Section 3.5.2. Agent a's utility from any of these $m \times n$ strategies depends on the opponent's strategy. Although agent a does not know the opponent's strategy, it has beliefs about the opponent's reservation price and deadline. Agent a believes that there are $m \times n$ different (reservation price, deadline) pairs and also has the associated probabilities in the two lotteries $L_p^{\hat{a}}$ and $L_t^{\hat{a}}$. Recall that an agent always plays a strategy that offers its own reservation price at its deadline. Thus agent a believes (on the basis of its lotteries $L_p^{\hat{a}}$ and $L_t^{\hat{a}}$) that $\gamma_{i,j}^{\hat{a}}$ is the probability with which the opponent will play the strategy $S_{i,j}^{\hat{a}}$ that offers $RP_i^{\hat{a}}$ at time $T_i^{\hat{a}}$. The different strategies that agent a can play and the expressions for computing agent a's utility for different strategies are as given in Section 3.5.2. Agent a gets maximum EU from strategy S_o^a defined in terms of $RP_I^{\hat{a}}$ and $T_J^{\hat{a}}$. Thus as per agent a's beliefs about the opponent, strategy S_o^a is agent a's optimal strategy. Once this strategy is selected, agent a uses it, from the beginning to the end of negotiation, to generate an offer whenever it is its turn.

At level 2 of the tree, it is agent $\hat{a}$'s turn. From agent $\hat{a}$'s perspective of the game tree, $\mathcal{I}_1^{\hat{a}}$ forms its information set since it does not know the strategy used by agent a. However, agent $\hat{a}$ too has beliefs about agent a's strategy in the form of lotteries L_p^a and L_t^a. Agent $\hat{a}$ believes that agent a will play strategy $S_{i,j}^a$ with probability $\gamma_{i,j}^a$ where strategy $S_{i,j}^a$ is a strategy that offers the final price RP_i^a at time T_j^a. Agent $\hat{a}$'s EU if it plays strategy $S_{p,q}^{\hat{a}}$ for $1 \leqslant p \leqslant m$ and $1 \leqslant q \leqslant n$ depends on agent a's strategy and is given by the expression:

$$EU_{p,q}^{\hat{a}} = \sum_{i=1}^{m} \sum_{j=1}^{n} \gamma_{i,j}^a EU^{\hat{a}}\big(S_{p,q}^{\hat{a}}, S_{i,j}^a\big). \tag{17}$$

The values of p and q that give agent $\hat{a}$ the maximum EU form its optimal strategy. We know from Section 3.5.2 that agent $\hat{a}$'s optimal strategy is $S_o^{\hat{a}}$ for $p = I$ and $q = J$. No matter which node in the information set ($\mathcal{I}_1^{\hat{a}}$) agent $\hat{a}$ is at, strategy $S_o^{\hat{a}}$ is better than all the other strategies. The strategy $S_o^{\hat{a}}$ is agent $\hat{a}$'s optimal strategy which agent $\hat{a}$ uses, from the beginning to the end of negotiation, to make an offer whenever it is its turn.

Thus strategy S_o^a is agent a's optimal strategy whenever it is agent a's turn to make an offer, for $a = b$ and $a = s$. The assessment $(\sigma, \mu)_{x,y}$ is therefore sequentially rational. This holds good for all other scenario combinations. We know from Section 3.5.2 that the number of possible strategies may be different for different scenarios but the condition for sequential rationality holds good for all possible scenario combinations. Thus the assessment $(\sigma, \mu)_{x,y}$ is sequentially rational for $1 \leqslant x \leqslant 6$ and $1 \leqslant y \leqslant 6$.

The second condition for sequential equilibrium is consistency of the strategy profile and the beliefs. The assessment $(\sigma, \mu)_{x,y}$ in which $\sigma_a = S_o^a$ for scenario x, $\sigma_{\hat{a}} = S_o^{\hat{a}}$ for scenario y, and $\mu(\{S_{11}^a, S_{12}^a, \ldots, S_{mn}^a\})(S_{ij}^a) = \gamma_{ij}^a$ for $1 \leqslant i \leqslant m$ and $1 \leqslant j \leqslant n$ is consistent since it is the limit as $\varepsilon \to 0$ of assessments $(\sigma^\varepsilon, \mu^\varepsilon)$ where

$$\sigma_a^\varepsilon = \big(\varepsilon\gamma_{11}^a, \varepsilon\gamma_{12}^a, \ldots, (1-\varepsilon)\gamma_{I,J}^a, \ldots, \varepsilon\gamma_{mn}^a\big), \tag{18}$$

$$\sigma^{\varepsilon}_{\hat{a}} = \big(\varepsilon, \varepsilon, \ldots, (1-\varepsilon), \ldots, \varepsilon\big), \quad \text{and} \tag{19}$$

$$\mu^{\varepsilon}\big(\{S^a_{11}, S^a_{12}, \ldots, S^a_{mn}\}\big)\big(S^a_{ij}\big) = \gamma^a_{ij} \tag{20}$$

$$\text{for } 1 \leqslant i \leqslant m,\ 1 \leqslant j \leqslant n,\ \text{and for every } \varepsilon.$$

The entry $(1-\varepsilon)$ in $\sigma^{\varepsilon}_{\hat{a}}$ is for agent $\hat{a}$'s optimal strategy.

The assessment $(\sigma, \mu)_{x,y}$ in which $\sigma_a = S^a_o$ for scenario x, $\sigma_{\hat{a}} = S^{\hat{a}}_o$ for scenario y, and $\mu(\{S^a_{11}, S^a_{12}, \ldots, S^a_{mn}\})(S^a_{ij}) = \gamma^a_{ij}$ for $1 \leqslant i \leqslant m$ and $1 \leqslant j \leqslant n$ is therefore a sequential equilibrium of the game $\mathcal{G}$, for $1 \leqslant x \leqslant 6$ and $1 \leqslant y \leqslant 6$. □

Theorem 2. *If the conditions for convergence of optimal strategies are true, the time of agreement is unique for each possible scenario combination. The price of equilibrium agreement is unique if the agents have different deadlines, and $RP^a_I = RP^a$ for $T^a < T^{\hat{a}}$.*

Proof. It is straightforward to verify the uniqueness of the time of equilibrium agreement from Table 6. In Table 6, the price of agreement is either RP^s or RP^s_I for $T^s < T^b$, i.e., in rows 1, 2, 3, 4, 6, 7, 9, and 10. On the other hand the price of agreement is either RP^b or RP^b_I for $T^s > T^b$, i.e., rows 5, 6, 8, 9, 11, 12, 13, and 14. Thus for each scenario combination, there are two possible values for the price of agreement. When the agents have different deadlines, the price of agreement is either RP^a or RP^a_I for $T^a < T^{\hat{a}}$. Recall from Section 3.3 that $RP^b_m > RP^s_1$. The price of agreement for $T^s = T^b$ is either RP^s or RP^b. This means that the equilibrium solution cannot be unique when $T^s = T^b$. But when the agents have different deadlines, the equilibrium solution is unique if $RP^a_I = RP^a$. □

Theorem 3. *The equilibrium agreement is Pareto-optimal if*

(1) *both agents gain utility with time, or*
(2) *both agents lose utility with time and one of them is in scenario N_6.*

Proof. Consider the case where both agents gain utility with time. This happens in rows 1, 2, 5, 6, 7, 11, and 12 of Table 6. The equilibrium outcome in these cases is either (RP^s, T^s) or (RP^s_I, T^s) if $T^s < T^b$, either (RP^b, T^b) or (RP^b_I, T^b) if $T^b < T^s$, and either (RP^s, T^b) or (RP^b, T^b) if $T^b = T^s$. In other words, the time of agreement is always the earlier deadline. The utility of an agent can be changed by changing the price, or the time of agreement, or both. When both agents gain utility with time, the time of agreement can only be decreased, since the agent with the earlier deadline quits if agreement is not reached by its deadline. Consider the case where $T^s < T^b$. Since the time of agreement can only be decreased and both agents gain utility with time, a change in time decreases the utility of both agents. The price of agreement here is either RP^s or RP^s_I. If the price of agreement is RP^s, it can only be increased, since a price below RP^s will never be acceptable to the seller. So there are three possible changes to the equilibrium agreement (RP^s, T^s): a decrease in time, a decrease in price, or both. The first change decreases the utility of both agents. The second change increases the seller's utility but decreases the buyer's utility. Finally, the third change decreases the buyer's utility and can either increase or decrease the seller's utility. In other words, in none of the three possible changes to the equilibrium agreement is it possible to improve the utility of both agents. Likewise, it is not possible to increase the utility of both agents when the outcome is (RP^s_I, T^s). The equilibrium agreements (RP^s, T^s) and (RP^s_I, T^s) for $T^s < T^b$ are thus Pareto-optimal. In the same way it can be seen that the equilibrium agreements (RP^b, T^b) and (RP^b_I, T^b) for $T^s > T^b$ are Pareto-optimal; and the agreements (RP^s, T^b) and (RP^b, T^b) for $T^s = T^b$ are also Pareto-optimal.

When both agents lose utility with time and one of them is in scenario N_6, the equilibrium agreement is either (RP^s, T') or (RP^s_I, T') for $T^s < T^b$, and either (RP^b, T') or (RP^b_I, T') for $T^b < T^s$. This corresponds to rows 4, 10, 13, and 14 of Table 6. Here, the time of agreement can only be increased and since both agents lose utility with time, a change in time decreases the utility of both agents. If the price of agreement is RP^s, then price can only be increased. This decreases the buyer's utility and increases the seller's utility. If the price of agreement is RP^s_I, then price can either be increased or decreased. An increase in price decreases the buyer's utility and increases the seller's utility, while a decrease in price increases the buyer's utility and decreases the seller's utility. In other words, it is not possible to improve the utility of both agents simultaneously when both agents lose utility with time and one of them is in scenario N_6. $\square$

Our analysis therefore shows that even when players have incomplete and uncertain information about each other, and each agent's information is its private knowledge, a unique equilibrium agreement exists for $T^s \neq T^b$ under the conditions listed in Theorem 2. When these conditions are not satisfied, there are two possible equilibrium solutions for each possible scenario combination.

4. The multi-issue negotiation model

We extend the above model for multi-issue bargaining. The buyer, b, and the seller, s, that each have deadlines, bargain over the price of two distinct goods/services, X and Y. Here, T^a denotes agent a's deadline for reaching agreement on both the issues. Negotiation on all the issues must end by the earlier of the two deadlines. We consider two goods/services in order to simplify the discussion but this is a general framework that works for more than two goods/services. As we will show in Section 4.6, this framework can in fact be used for negotiating multiple issues associated with a single good/service and multiple goods/services.

4.1. Agents' information states

Let the buyer's reservation values for X and Y be RP^b_X and RP^b_Y and the seller's reservation prices be RP^s_X and RP^s_Y respectively. Also, let S^a_X denote agent a's strategy for issue X and S^a_Y denote agent a's strategy for issue Y. The buyer's information state is:

$$I^b = \langle RP^b_X, RP^b_Y, T^b, U^b, S^b_X, S^b_Y, L^s_t, L^s_X, L^s_Y \rangle$$

where RP^b_X, RP^b_Y, T^b, U^b, S^b_X, and S^b_Y are the information about its own parameters and L^s_t, L^s_X and L^s_Y are three probability distributions that denote its beliefs about the opponent's parameters. As described in Section 3.3, L_t^s, L_X^s and L_Y^s denote the buyer's beliefs about the seller's deadline, its reservation value for X, and its reservation value for Y respectively. Analogously, the seller's information state is defined as:

$$I^s = \langle RP_X^s, RP_Y^s, T^s, U^s, S_X^s, S_Y^s, L_t^b, L_X^b, L_Y^b \rangle.$$

Each agent's information state is its private knowledge.

4.2. *The negotiation protocol*

Again we use an alternating offers negotiation protocol. There are two types of offers. An offer on just one good is referred to as a *single offer* and an offer on two goods is referred to as a *combined offer*. One of the agents starts by making a combined offer. The other agent can accept/reject part of the offer (single issue) or the complete offer. If it rejects the complete offer, then it sends a combined counter-offer. This process of making combined offers continues until agreement is reached on one of the issues. Thereafter agents make offers only on the remaining issue (i.e., once agreement is reached on an issue, it cannot be renegotiated). Negotiation ends when agreement is reached on both the issues or a deadline is reached. Let $S_{oX}^b(t)$ denote the price generated by agent b's optimal strategy for issue X at time t. Thus the action, A^s, that agent s takes at time t on a single offer is as defined in Section 3.5.1. Its action on a combined offer, $A^s(t, X_{b\to s}^t, Y_{b\to s}^t)$, is defined as:

$$A^s\big(t, X_{b\to s}^t, Y_{b\to s}^t\big) = \begin{cases} \text{Quit} & \text{if } t > T^s, \\ \text{Accept } X_{b\to s}^t & \text{if } X_{b\to s}^t \geqslant S_{oX}^s(t), \\ \text{Accept } Y_{b\to s}^t & \text{if } Y_{b\to s}^t \geqslant S_{oY}^s(t), \\ \text{Offer } S_{oX}^s(t') \text{ at } t' & \text{if } X_{b\to s}^t \text{ not accepted}, \\ \text{Offer } S_{oY}^s(t') \text{ at } t' & \text{if } Y_{b\to s}^t \text{ not accepted}. \end{cases}$$

The agents' utility functions are defined as:

$$U^a(p_X, p_Y, t) = \begin{cases} (RP_X^b - p_X)(\delta_X^b)^t + (RP_Y^b - p_Y)(\delta_Y^b)^t & \text{for } b, \\ (p_X - RP_X^s)(\delta_X^s)^t + (p_Y - RP_Y^s)(\delta_Y^s)^t & \text{for } s. \end{cases}$$

Note that the discounting factors are different for different issues. This allows an agent to have a different attitude towards time for different issues.

4.3. *Negotiation agenda*

A negotiation agenda defines the order in which the issues are negotiated. If agents define this order before negotiating the issues, then the agenda is said to be exogenous. On the other hand, if the agents are allowed to decide what issue they will negotiate next during the process of negotiation, then the agenda is said to be endogenous. In the proposed negotiation model, although agents initially make offers on both issues, there is no restriction on the price they offer. Thus by initially offering a price that lies outside the zone of agreement, an agent can effectively delay the time of agreement for that issue. For example, the buyer can offer a very low price which will not be acceptable to the seller and the seller can offer a price which will not be acceptable to the buyer. In this way, the order in which the issues are bargained over and agreements are reached is determined endogenously as part of the bargaining equilibrium rather than imposed exogenously as part of the game tree.

Two implementation rules are possible for this protocol. One is *sequential implementation* in which agreement on an issue is implemented as soon as it is settled; and the other is *simultaneous implementation* in which agreement is implemented only after all the issues are settled. We first show how to obtain equilibrium outcomes for multi-issue negotiation and then compare the outcome that results from the sequential implementation with that of the simultaneous implementation.

4.4. Equilibrium outcomes

As agents negotiate over the price of two distinct goods/services, the equilibrium strategies for the single issue model can be applied to X and Y independently of each other. Since T^a denotes agent a's deadline for reaching agreement on both issues, the relationship between agent deadlines is the same for both issues. However, as mentioned in Section 4.2, an agent can have different discounting factors for the two issues. Thus if agent a's negotiation scenario for issue X is N_1, its scenario for issue Y can be either N_1 or N_4. Likewise, if agent a's scenario for issue X is N_2, its scenario for issue Y can be either N_2 or N_5. Agent a's possible scenarios for two issues are listed in Table 7. For the scenarios listed in Table 7, the equilibrium price and time of agreement for each of the two issues can be obtained from Table 6. For instance, if the buyer's scenario for issues X and Y are N_1 and N_4, and the seller's scenarios for issues X and Y are N_2 and N_5, the price and time of equilibrium agreement for issue X is either (RP^s_X, T^s) or (RP^s_{IX}, T^s) and for issue Y it is either (RP^s_Y, T^s) or (RP^s_{IY}, T^s).

4.5. Implementation schemes

Let (p_X, t) and (p_Y, τ) denote the agreements on issues X and Y respectively. Payoffs for this outcome depend on the rules by which agreements are implemented. Two possible implementation rules are as follows.

Table 7
Agent a's possible scenario combinations for two issues

Issue X	Issue Y
N_1	N_1 or N_4
N_2	N_2 or N_5
N_3	N_3 or N_6
N_4	N_4 or N_1
N_5	N_5 or N_2
N_6	N_6 or N_3

- Sequential implementation. Exchange of a good/service takes place at the time of agreement on price for that good/service. Agents' utilities (U^a_{seq}) from agreements (p_X, t) and (p_Y, τ) are:

$$U^b_{seq}\big((p_X,t),(p_Y,\tau)\big) = \big(RP^b_X - p_X\big)\big(\delta^b_X\big)^t + \big(RP^b_Y - p_Y\big)\big(\delta^b_Y\big)^\tau,$$

$$U^s_{seq}\big((p_X,t),(p_Y,\tau)\big) = \big(p_X - RP^s_X\big)\big(\delta^s_X\big)^t + \big(p_Y - RP^s_Y\big)\big(\delta^s_Y\big)^\tau.$$

- Simultaneous implementation. Exchange of goods/services takes place only after agreement is reached on the prices of all the goods. Agents' utilities (U^a_{sim}) for this rule are:

$$U^b_{sim}\big((p_X,t),(p_Y,\tau)\big) = \big(RP^b_X - p_X\big)\big(\delta^b_X\big)^{\max(t,\tau)} + \big(RP^b_Y - p_Y\big)\big(\delta^b_Y\big)^{\max(t,\tau)},$$

$$U^s_{sim}\big((p_X,t),(p_Y,\tau)\big) = \big(p_X - RP^s_X\big)\big(\delta^s_X\big)^{\max(t,\tau)} + \big(p_Y - RP^s_Y\big)\big(\delta^s_Y\big)^{\max(t,\tau)}.$$

Theorem 4. *If the time of agreement is equal for both issues, each agent gets equal utility from the two implementation schemes. If the time for agreement is different for the two issues and one of them is agreed at T', the outcome generated by sequential implementation is better than that for simultaneous implementation, for both agents. For all other possible values of t and τ, the agents have conflicting preferences over the implementation scheme.*

Proof. From Table 6 we know that there are five possible values for the time of agreement on an issue: T', T^b, T^s, T^b_J, or T^s_J. When there are two issues to be negotiated, the time of agreement may be equal for both issues or it may be different. Consider the case where the time of agreement is equal for both issues, i.e., $t = \tau$. For this case, the agents' utilities from the two implementation schemes are as follows:

$$U^b_{seq} = U^b_{sim} = \big(RP^b_X - p_X\big)\big(\delta^b_X\big)^\tau + \big(RP^b_Y - p_Y\big)\big(\delta^b_Y\big)^\tau,$$

$$U^s_{seq} = U^s_{sim} = \big(p_X - RP^s_X\big)\big(\delta^s_X\big)^\tau + \big(p_Y - RP^s_Y\big)\big(\delta^s_Y\big)^\tau.$$

Each agent gets equal utility from the two different implementation schemes. The agents' preferences for the two implementation schemes, for all possible values of t and τ are shown in Table 8. When $t = T'$ and $\tau \neq T'$, agents' utilities are as follows:

$$U^b_{seq} = \big(RP^b_X - p_X\big)\big(\delta^b_X\big)^{T'} + \big(RP^b_Y - p_Y\big)\big(\delta^b_Y\big)^\tau,$$

$$U^b_{sim} = \big(RP^b_X - p_X\big)\big(\delta^b_X\big)^\tau + \big(RP^b_Y - p_Y\big)\big(\delta^b_Y\big)^\tau,$$

$$U^s_{seq} = \big(p_X - RP^s_X\big)\big(\delta^s_X\big)^{T'} + \big(p_Y - RP^s_Y\big)\big(\delta^s_Y\big)^\tau,$$

$$U^b_{sim} = \big(RP^b_X - p_X\big)\big(\delta^b_X\big)^\tau + \big(RP^b_Y - p_Y\big)\big(\delta^b_Y\big)^\tau.$$

We know from Table 6 that the time of agreement on an issue is T' only when both agents lose utility on time on the issue and one of the agents is in scenario N_6. This corresponds to rows 4, 10, 13, and 14 of Table 6. Since both agents lose utility on time, they both prefer the sequential implementation scheme for issue X. The time of agreement on issue Y is τ. Since $\tau > T'$, an agent's utility for issue Y is equal for the two implementation schemes.

Table 8
Agents' preferences over the implementation schemes for all possible values of time of agreement on two issues

	T'	T^b	T^s	T_J^b	T_J^s
T'	$U_{seq}^a = U_{sim}^s$	$U_{seq}^a > U_{sim}^a$	$U_{seq}^a > U_{sim}^a$	$U_{seq}^a > U_{sim}^a$	$U_{seq}^a > U_{sim}^a$
T^b	$U_{seq}^a > U_{sim}^a$	$U_{seq}^a = U_{sim}^s$	×	$U_{seq}^b > U_{sim}^b$ $U_{seq}^s < U_{sim}^s$	×
T^s	$U_{seq}^a > U_{sim}^a$	×	$U_{seq}^a = U_{sim}^s$	×	$U_{seq}^b < U_{sim}^b$ $U_{seq}^s > U_{sim}^s$
T_J^b	$U_{seq}^a > U_{sim}^a$	$U_{seq}^b > U_{sim}^b$ $U_{seq}^s < U_{sim}^s$	×	$U_{seq}^a = U_{sim}^s$	×
T_J^s	$U_{seq}^a > U_{sim}^a$	×	$U_{seq}^b < U_{sim}^b$ $U_{seq}^s > U_{sim}^s$	×	$U_{seq}^a = U_{sim}^s$

But the combined utility from the two issues is higher for the sequential implementation scheme for both agents. Thus when $t = T'$ and $\tau \neq T'$, both agents prefer the sequential implementation scheme. This corresponds to the first row and the first column of Table 8.

When $t = T^a$ and $\tau = T_J^a$, agents have conflicting preferences over the implementation scheme. Let a represent the buyer. Here, the time of agreement for issue Y is T_J^b. Note that T_J^b is always less than or equal to T^b when the conditions for convergence of optimal strategies are satisfied. We also know from Table 6 that the time of agreement is T_J^b only when agents have conflicting time preferences (see rows 11 and 12). Agents' utilities from the two implementation schemes are as follows:

$$U_{seq}^b = \left(RP_X^b - p_X\right)\left(\delta_X^b\right)^{T^b} + \left(RP_Y^b - p_Y\right)\left(\delta_Y^b\right)^{T_J^b},$$

$$U_{sim}^b = \left(RP_X^b - p_X\right)\left(\delta_X^b\right)^{T^b} + \left(RP_Y^b - p_Y\right)\left(\delta_Y^b\right)^{T^b},$$

$$U_{seq}^s = \left(p_X - RP_X^s\right)\left(\delta_X^s\right)^{T^b} + \left(p_Y - RP_Y^s\right)\left(\delta_Y^s\right)^{T_J^b},$$

$$U_{sim}^b = \left(RP_X^b - p_X\right)\left(\delta_X^b\right)^{T^b} + \left(RP_Y^b - p_Y\right)\left(\delta_Y^b\right)^{T^b}.$$

Each agent gets equal utility from the two schemes for issue X. Since $T_J^b \leqslant T^b$, and the buyer loses utility with time, it prefers sequential implementation for issue Y, while the seller prefers simultaneous implementation because it gains utility with time on issue Y. The buyer's combined utility for the two issues is therefore higher for sequential implementation while the seller's combined utility is higher for the simultaneous implementation scheme. The same result holds good when a represents the seller. Thus when $t = T^a$ and $\tau = T_J^a$, agents have conflicting preferences over the implementation scheme.

The entries marked "×" in Table 8 indicate that agreement cannot be reached at the corresponding times for the two issues. For instance, it is not possible for agreement on issue X to be reached at T^b and issue Y to be reached at T_J^s. This is explained as follows. From Table 6 we know that the time of agreement on an issue is T_J^s when the buyer-seller scenario combination for the issue is (N_1, N_6) or (N_2, N_6) (see rows 4 and 10 of Table 6). Consider the buyer-seller scenario combination (N_1, N_6) for issue Y. Here the buyer-seller scenario combinations that are possible for issue X are (N_1, N_6), (N_1, N_3), (N_4, N_6), or (N_4, N_3). We know from Table 6 that in none of these four combinations the time of agreement is T^b. The same result holds good for the other scenario combination for issue Y, i.e., (N_2, N_6). In other words, when the time of agreement for an issue is T_J^s, the time of agreement for the other issue cannot be T^b. Using similar analysis, it can be seen that the time of agreement on the two issues cannot be T^s and T_J^b. Thus the agents are indifferent to the implementation scheme when $t = \tau$, both agents prefer the sequential scheme when $t = T'$ and $\tau \neq T'$, and have conflicting preferences over the implementation scheme when $t = T^a$ and $\tau = T_J^a$. □

4.6. *Multi-issue negotiation for a single good/service*

The previous subsection described bargaining over the price of more than one good/service. But since this is a general framework it can also be used for negotiating multiple issues associated with a single good/service. Let issue X be the price of a service and issue Y be the quality of service. The utility functions for the buyer and seller are:

$$U^b(p_X, p_Y, t) = \left(RP_X^b - p_X\right)\left(\delta_X^b\right)^t + \left(p_Y - RP_Y^b\right)\left(\delta_Y^b\right)^t$$

and

$$U^s(p_X, p_Y, t) = \left(p_X - RP_X^s\right)\left(\delta_X^s\right)^t + \left(RP_Y^s - p_Y\right)\left(\delta_Y^s\right)^t.$$

Since both issues are associated with a single good/service, only simultaneous implementation applies in this case. The optimal and equilibrium strategies for X and Y still remain the same. Thus the framework can be used for negotiating multiple issues associated with a single good/service and multiple goods/services.

4.7. *Properties of the equilibrium solution*

The main focus in the design of a negotiation model is on the properties of the outcome, since the choice of a model depends on the attributes of the solution it generates. We therefore study some important properties [28] of the equilibrium agreement.

(1) *Uniqueness*. If the solution of the negotiation game is unique, then it can be identified unequivocally.

Theorem 5. *The proposed negotiation model has a unique equilibrium agreement if agents have different deadlines and* $RP_I^a = RP^a$ *for* $T^a < T^{\hat{a}}$, *for each issue.*

Proof. Consider a single issue. When agents have different deadlines, i.e., $T^a < T^{\hat{a}}$, we know from Theorem 2 that if $RP_I^a = RP^a$ then the equilibrium solution for the issue is unique. In general, if there are η different issues to be negotiated, there is a unique solution for all η issues only if there is a unique solution for each individual issue, i.e., when $RP_I^a = RP^a$ for each issue. □

(2) *Symmetry*. A bargaining mechanism is said to be symmetric if it does not treat the players differently on the basis of inappropriate criteria. Exactly what constitutes inappropriate criteria depends on the specific domain.

Theorem 6. *The equilibrium agreement for multiple issues is independent of the identity of the first player if agents have different deadlines, and $RP^a_I = RP^a$ for $T^a < T^{\hat{a}}$, for each issue.*

Proof. The equilibrium price of agreement for a single issue depends on the identity of the player that makes a move at time T^a. If it is agent a's turn to make an offer at T^a, the equilibrium price is RP^a. On the other hand if it is agent $\hat{a}$'s turn, then the equilibrium price is RP^a_I. But if $RP^a_I = RP^a$, the equilibrium solution is unique and does not depend on the identity of the agent that makes an offer at time T^a. When there are η issues to be negotiated, the equilibrium outcome for all the η issues is independent of the identity of the agent that makes an offer at time T^a, if the equilibrium outcome for each of the η issues is unique, i.e., if $RP^a_I = RP^a$ for $T^a < T^{\hat{a}}$, for each issue. □

(3) *Efficiency*. An agreement is efficient if there is no wasted utility, i.e., the agreement satisfies Pareto-optimality. The equilibrium solution in the proposed model is Pareto-optimal under the conditions given in Theorem 7.

Theorem 7. *The equilibrium agreement for η issues is Pareto-optimal if the agreement on each individual issue is Pareto-optimal and each agent has the same discounting factor for all η issues.*

Proof. From Table 7, we know an agent's possible scenario combinations for multiple issues. Since each agent has the same discounting factor for all the η issues, each agent is in the same scenario for all η issues. We also know from Theorem 3 that the outcome for a single issue is Pareto-optimal either when both agents gain utility with time, or when both lose utility with time and one of them is in scenario N_6. If both agents gain utility with time, we know from Table 6 that for $T^a < T^{\hat{a}}$, the equilibrium agreement is either (RP^a, T^a) or (RP^a_I, T^a). Since both agents gain utility with time, a change in the time of agreement lowers the utility of both agents. A change in the price of agreement has the following effect on agents' utilities. Let agent a be the seller. If P^i_e denotes the equilibrium price on issue i and δ^a denotes agent a's discounting factor for all the issues, the agents' utilities from all η issues are:

$$U^a\big(P^1_e, \ldots, P^\eta_e, T^s\big) = \begin{cases} (\delta^b)^{T^s} \sum_{i=1}^{\eta}(RP^b_i - P^i_e) & \text{for } b, \\ (\delta^s)^{T^s} \sum_{i=1}^{\eta}(P^i_e - RP^s_i) & \text{for } s. \end{cases}$$

Let Δ_i denote the change in price of issue i. Also let ΔU^a denote the overall change in agent a's utility from a change in price of all the η issues. The difference in utilities, ΔU^a, is:

$$\Delta U^a = \begin{cases} -(\delta^b)^{T^s} \sum_{i=1}^{\eta} \Delta_i & \text{for } b, \\ (\delta^s)^{T^s} \sum_{i=1}^{\eta} \Delta_i & \text{for } s. \end{cases}$$

Since $\delta^b > 0$ and $\delta^s > 0$, $\Delta U^s > 0$ if $\Delta U^b < 0$ and $\Delta U^s < 0$ if $\Delta U^b > 0$. In other words, it is not possible to increase the utility of both agents when both gain utility with time. The same result holds good if a represents the buyer. Likewise, the equilibrium solutions (RP^b, T^b) and (RP^s, T^b), for $T^s = T^b$, are Pareto-optimal. In the same way it can be seen that it is not possible to increase the utility of both agents when both lose utility with time and one of them is in scenario N_6. □

(4) *Distribution*. The distribution property of negotiation outcome relates to the issue of how the gains from trade are divided between agents. The equilibrium price (P_e^i for issue i) and the equilibrium time (T_e^i for issue i) of agreement reflect the relationship between the agents' bargaining powers. We say that an agent has more (less) bargaining power over P_e^i, if P_e^i is more (less) favourable to it than its opponent. Similarly, an agent has more (less) bargaining power over T_e^i, if T_e^i is more (less) favourable to it than its opponent.

Theorem 8. *For the equilibrium agreement, the relation between the agents' bargaining powers over price is as follows. If agents have equal deadlines, agent $\hat{a}$ has more bargaining power than agent a on all the issues if agent a makes an offer at T^a. For $T^a < T^{\hat{a}}$, $\hat{a}$ has more bargaining power than agent a on all the issues if agent a makes an offer at T^a. For $T^s < T^b$, the price-surplus is split between b and s in the ratio $(RP^b - RP_I^s){:}(RP_I^s - RP^s)$ if b makes an offer at T^s. For $T^b < T^s$, the price-surplus is split between b and s in the ratio $(RP^b - RP_I^b){:}(RP_I^b - RP^s)$ if s makes an offer at T^b.*

Proof. We know from rows 6 and 9 of Table 6 that there are four buyer-seller scenario combinations in which agents can have equal deadlines: (N_2, N_2), (N_2, N_5), (N_5, N_2), or (N_5, N_5). Consider the case where the buyer-seller scenario for one of the issues is (N_2, N_2). From Table 7, we know that the four possible buyer-seller scenario combinations for each of the remaining issues are (N_2, N_2), (N_2, N_5), (N_5, N_2), or (N_5, N_5). The offer generated by an agent's optimal strategy in scenarios N_2 and N_5 is $RP_I^{\hat{a}}$ in the time interval $[T_J^{\hat{a}}, T_k^{\hat{a}}]$, and it is RP^a at time T^a. Consider the case where the buyer makes an offer (i.e., its reservation price) at its deadline. This is a combined offer since we know from Table 6 that in all the possible scenarios for each issue, the time of agreement for each issue is the earlier deadline. Thus at time T^b the buyer makes a combined offer that includes its reservation price for each of the η issues. Since the conditions for convergence of optimal strategies are satisfied, we know that $RP^b > RP_I^b$ for each issue. The seller's action for each issue at T^s, which is equal to T^b, is to accept an offer greater than or equal to RP^s. Since $RP^b > RP^s$ for each of the η issues, the seller accepts the price of every issue in the buyer's combined offer. Agreement on all the issues therefore takes place at T^b. The price of agreement is the buyer's reservation price for each issue.

On the other hand, if it is the seller's turn to make an offer at time T^b, for each issue it offers its reservation price, which the buyer accepts. Thus if the buyer makes an offer at T^b, the seller has more bargaining power because it gets the entire price-surplus on all the issues and if the seller makes an offer at T^b, the buyer has more bargaining power because it gets the entire price-surplus on all the issues. In the same way the relationship between agents' bargaining power can be verified for the remaining three scenarios for equal deadlines, (N_2, N_5), (N_5, N_2), or (N_5, N_5).

For $T^a < T^{\hat{a}}$, the equilibrium outcome for each issue is (RP^a, T^a) if agent a makes an offer at time T^a and the outcome is (RP^a_I, T^a) if agent $\hat{a}$ makes an offer at time T^a. Thus agent $\hat{a}$ gets the entire price-surplus on all the issues and has more bargaining power if agent a makes an offer at time T^a. The distribution of price-surplus, if agent $\hat{a}$ makes an offer at T^a, can be verified in the same way. □

Theorem 9. *Agents have equal bargaining power over time on an issue if both gain utility with time on the issue, or if both lose utility with time on the issue and one of them is in scenario* N_6.

Proof. We know from Table 6 that when both agents gain utility with time on an issue, the time of equilibrium agreement is the earlier deadline. Since the time of agreement cannot be greater than T^a for $T^a < T^{\hat{a}}$, both agents get the maximum possible utility from time on the issue and thus have equal bargaining power.

Likewise, when both agents lose utility with time on an issue and one of them is in scenario N_6, the time of agreement is T'. This gives the agents equal bargaining power since both of them get the maximum possible utility from time on the issue. □

Theorem 10. *If agents have conflicting time preferences on an issue, and neither agent is in scenario* N_6 *for the issue, the agent that gains utility with time has more bargaining power over time on that issue.*

Proof. We know from Table 6 (see rows 1, 2, 3, 5, 6, 7, 8, 9, 13, and 14) that when agents have conflicting time preferences on an issue and neither agent is in scenario N_6 for the issue, the time of equilibrium agreement is the earlier deadline. In other words, although the agent that loses utility with time prefers an early agreement, an agreement only takes place at the latest possible time. This gives the stronger agent the maximum possible utility from time and it therefore has more bargaining power than the opponent. □

5. Related work

Game theoretic models can be divided into two types; those that deal with complete information and those that deal with incomplete information. In the former setting, agents know each other's characteristics as well as their own. In the latter setting, agents lack information about some specific parameters. For instance there could be uncertainty over player's discounting factors, their reservation values, or their deadlines. These models study the strategic behavior of agents when there is information uncertainty.

Initial game theoretic research typically dealt with coordination and negotiation issues by assuming that agents have complete information about each other and then giving pre-computed solutions to specific problems [25,26]. However this complete information assumption is limiting because uncertainty is endemic in most realistic applications. For this reason, Harsanyi [14,15] originated research in bargaining with incomplete information.

He gave a generalized solution for two person bargaining games with incomplete information. However, there was no notion of timing issues in this model. Another important model of strategic bargaining is Rubenstein's infinite horizon alternating offer game [33]. This model takes the time preferences of bargainers into consideration in the form of their discounting factors but again assumes complete information. It was later extended in [34] for bargaining with incomplete information about time preferences. However, this is an infinite horizon model that considers uncertainty over player's discounting factors. One of the players, say player 2, may be one of two types: weak (for high discounting factor) and strong (for low discounting factor). Player 1 adopts an initial belief about the identity of player 2. Player 1's preference is known to player 2. Agreement is reached in the first or second time period. Its main result is the existence of a unique sequential equilibrium when player 1's belief that player 2 is of type weak, is higher than a certain threshold and another unique equilibrium when this belief is lower than the threshold.

Other models of incomplete information have also been formulated for different environments and the strategic behavior of agents is studied. Fudenberg and Tirole [13] analyse an infinite horizon bargaining game by taking the players' valuations, and a probability distribution over them, as common knowledge. Fudenberg et al. [12] subsequently analysed buyer-seller infinite horizon bargaining games in which reservation prices are uncertain, but time preferences are known. Sandholm and Vulkan [37] consider uncertainty over agent deadlines. However, a common feature of all these models is that they treat the information state of agents as common knowledge.

All the above models deal with single issue negotiation. However, in many real-life bargaining situations, there is more than one issue over which players want to negotiate. As mentioned in the introduction, multiple issues can be negotiated using the bundled approach or the issue-by-issue approach. Although the fact that the negotiation outcome depends on the choice of the negotiation approach[7] was first noted by Schelling [38] in 1956, the literature on issue-by-issue negotiation is small (albeit growing). This includes the work of Fershtman [9] who extends Rubinstein's complete information model [33] for splitting a single pie to multiple pies. This model imposes an agenda exogenously, and studies the relation between the agenda and the outcome of the bargaining game. However, this work is based on the assumption that both players have identical discounting factors and does not consider agent deadlines. Similar work in a complete information setting includes [16], but it considers an endogenous agenda.

Closer to our work is that of Bac and Raff [1] who developed a model that has an endogenous agenda. They extended Rubinstein's model [34] for single pie bargaining with incomplete information by adding a second pie. In this model the price-surplus is known to both agents. For both agents, the discounting factor is assumed to be equal over all the issues. One of the players knows its own discounting factor and that of its opponent. The other player knows its own discounting factor, but is uncertain of the opponent's. This factor can take one of two values, δ_H with probability Π, and δ_L with probability $1-\Pi$. However, these probabilities are again *common knowledge*. Thus agents have asymmetric information about discounting factors. However, they do not associate deadlines with players.

[7] As Theorem 4 shows, issue-by-issue negotiation again is not neutral to the implementation scheme.

In summary, existing models for multi-issue negotiation [1,9,16] are typically extensions of single issue models [33,34] and they tend not consider agent deadlines. In addition, they treat the information state of agents as common knowledge. The main difference between these models and ours is that firstly, our model considers both agent deadlines and discounting factors and uses negotiation decision functions for counter offer generation. Secondly, in our case the players are uncertain about the opponent's reservation value and deadline. Each agent knows its own reservation value and deadline but has a probability distribution over its opponent's reservation value and deadline. Moreover, the discounting factor can be different for different issues and the players have no information about the opponent's discounting factors. Our analysis is thus more comprehensive, since we consider all possible negotiation scenarios (i.e., $\delta^a > 1$ and $\delta^a < 1$). Thirdly, we treat each agent's information state as its *private knowledge* which is not known to its opponent. This is in contrast to the above mentioned models, where the information state of agents is treated as common knowledge. In most realistic cases, an agent's information state is not known to its opponent. We therefore treat each players' beliefs about its opponent as private knowledge and obtain the connection between this private knowledge and the existence of equilibrium. Our model is therefore closer to most real-life bargaining situations than the existing models. The fourth point of difference lies in the attributes of the solution. Comparing the solution properties of multi-issue models, we see that the existing models do not have a unique equilibrium solution. The equilibrium solution in our model depends on the identity of the player that makes a move at T' or the earlier deadline, but is unique and symmetric under certain conditions. Finally, as is the case with our model, the equilibrium solution is not always Pareto-optimal in the other models.

6. Conclusions and future work

This paper presented a new model for multi-issue negotiation under time constraints in an incomplete information setting. The issues to be bargained over can be associated with a single good/service or multiple goods/services. The order in which issues are bargained over and agreements are reached is determined endogenously, as part of the bargaining equilibrium, rather than imposed exogenously, as part of the game tree. Our analysis shows that even when each agent's information is private knowledge, a unique equilibrium exists under certain conditions. Furthermore, we determine conditions under which agents have similar as well as conflicting preferences over the implementation scheme. Finally, we studied the properties of the equilibrium solution and determined conditions under which the equilibrium solution is unique, symmetric, and Pareto-optimal. As highlighted in Section 5, we believe this model is closer to most real-life bargaining situations than others that exist in the literature.

In practice, there is a wide range of environments in which negotiation can take place. For instance, in some applications the buyer may know the seller's reservation price but the seller may not know the buyer's reservation price. Or the seller may know the buyer's deadline, but the buyer may not know the seller's deadline. The information state of agents thus varies from application to application (the influence of the agents' information states on the equilibrium outcome has been explored in [5]). Apart from this, each application will require the players to manipulate the agenda in a different way. For instance, some applications may require bargaining over all the issues to occur simultaneously, while others may be more suited to issue-by-issue negotiation. Within the issue-by-issue negotiation, there can be different agendas. Yet another possibility is for agents to bargain over the agenda prior to the bargaining over the issues. Although we studied bargaining in which agents had one specific information state and the agenda was endogenous, our negotiation framework is general and can be used for exploring a wide range of negotiation environments by changing the agents' information states or the way in which the players manipulate the agenda. In [8], for example, the strategic behavior of agents was studied by allowing the agents to negotiate the agenda before they negotiate the prices of individual issues. The key result of this study is that in some scenarios agents have conflicting preferences over the agenda, while in others they have similar preferences. However, since agents have incomplete information about each other, they do not have the ability to identify scenarios in which they have similar preferences. We therefore presented an extended negotiation protocol that allows agents to identify such scenarios through a mediator.

As it currently stands, our framework treats the agents' beliefs about their opponent as being static. In future, we will introduce learning into the model to allow agents to learn these parameters dynamically during negotiation, and reach a stage where the conditions for convergence are satisfied. Secondly, we studied the process of negotiation for the case where agents' beliefs about each others reservation price do not overlap (i.e., the highest possible value for the seller's reservation price in the buyer's information state was lower than the lowest possible value for the buyer's reservation price in the seller's information state). The model can be made more general by allowing these beliefs to have overlapping values. Thirdly, in our present work we studied the strategic behavior of self-interested agents that use time-dependent strategies to maximize their own benefit. In future, it would be interesting to study the bargaining process by combining time-dependent tactics with tit for tat tactics in order to obtain a fair distribution of gains from trade.

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Appendix A. A summary of notation

b	Buyer
s	Seller
a	An element of the set $\{b, s\}$
$\hat{a}$	Agent a's opponent
IP^b	Buyer's initial price
IP^s	Seller's initial price
RP^b	Buyer's reservation price

RP^s	Seller's reservation price
$p^t_{b\to s}$	Price offered by b to s at time t
A^a	Action taken by agent a
B	Boulware negotiation decision function
C	Conceder negotiation decision function
L	Linear negotiation decision function
I^a	Information state of agent a
$\mathcal{I}^a$	Information set of agent a
T^a	Agent a's deadline
δ^a	Agent a's discounting factor
U^a	Agent a's utility
S^a	Agent a's strategy
S^a_o	Agent a's optimal strategy
L^a_t	A lottery over agent a's deadline
L^a_p	A lottery over agent a's reservation price
β^a_i	Probability that agent a's reservation price is RP^a_i
α^a_j	Probability that agent a's deadline is T^a_j
γ^a_{ij}	Probability that agent a's reservation price is RP^a_i and deadline is T^a_j
N_i	Negotiation scenario i
σ	Strategy profile
μ	Belief system
O	Negotiation outcome
EU^a	Agent a's expected utility
EU^a_o	Agent a's maximum expected utility
RP^a_X	Agent a's reservation price for issue X
RP^a_Y	Agent a's reservation price for issue Y
L^a_X	Agent $\hat{a}$'s beliefs about a's reservation price for issue X
L^a_Y	Agent $\hat{a}$'s beliefs about a's reservation price for issue Y
δ^a_X	Agent a's discount factor for issue X
δ^a_X	Agent a's discount factor for issue Y
U^a_{seq}	Agent a's utility for the sequential implementation scheme
U^a_{sim}	Agent a's utility for the simultaneous implementation scheme
T'	Time at which the second offer is made, i.e., if negotiation starts at time t, $T' = t + 1$
P^i_e	Equilibrium price for issue i
T^i_e	Time of equilibrium agreement for issue i

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