Reflection - Reversible jump MCMC - Green

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Given a set of parameters (θ_i for $i = 1, 2, \ldots n$) that describe an unknown distribution, Gibbs sampling systematically visits subsets of these parameters and updates them based on the *full conditionals*. The *full conditionals* are the conditional distributions of a subset $T \subset \{1, 2, \ldots n\}$ of the parameters given all other paramters not in T. In other words, we use the conditional distributions $P(\theta_T \mid \theta_{-T})$ where θ_{-T} denotes all the parameters that are not in T. Note that the paper introduces the concept of 'jumping' between subspaces of differing dimensionality, in this case, we are only concerned with one of those subspaces. An example of a full conditional is given in 6.1 where $\theta_i \mid \ldots \sim Beta(\alpha, \beta)$ where $\ldots$ refers to all other parameters associated with that dimension. A (not full) conditional is any other conditional distribution that is not a full conditional (an example can be seen in section 3 where the probability of parameters $\theta^{(1)}$ associated with subspace 1 are conditioned on that subspace being chosen ($P(\theta^{(1)} \mid k = 1)$).

This technique is called 'reversible jump' as the sampler needs to evaluate proposals that include parameters that might exist in a different subspace. The sampler must therefore allow 'jumps' to the parameters associated with a different subspace (the jump includes a change in parameter dimension). The jumps must be 'reversible' to maintain detailed balance between subspaces.

We could not directly use Variational Bayes to solve the problem as it is formulated here. In theory, we could ask for a family of approximating functions Q and select $q^* \in Q$ that minimizes the KL divergence to the joint posterior $P(k, \theta^{(k)} \mid y)$. This is minimization objective would change with different k and $\theta^{(k)}$ (due to changes in dimension of $\theta^{(k)}$) making this technique invalid.