Lecture 2: Sampling-based Approximations
And
Function Fitting

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Many slides made with John Schulman, Xi (Peter) Chen and Pieter Abbeel

Quick One-Slide Recap

- Optimal Control

  =

  given an MDP (S, A, P, R, γ, H)

  find the optimal policy π^*

- Exact Methods:
  - ✓ ***Value Iteration***
  - ✓ ***Policy Iteration***

<u>Limitations:</u>

- Update equations require access to dynamics model
- Iteration over / Storage for all states and actions: requires small, discrete state-action space

-> sampling-based approximations

-> Q/V function fitting

Sampling-Based Approximation

- Q Value Iteration
- Value Iteration?
- Policy Iteration
  - Policy Evaluation
  - Policy Improvement?

Recap Q-Values

$Q^*(s, a)$ = expected utility starting in s, taking action a, and (thereafter) acting optimally

Bellman Equation:

$$Q^*(s,a) = \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q^*(s',a'))$$

Q-Value Iteration:

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q_k(s',a'))$$

要知道P（機率）是多少

(Tabular) Q-Learning

- Q-value iteration: $Q_{k+1}(s,a) \leftarrow \sum_{s'} P(s'|s,a)(R(s,a,s') + \gamma \max_{a'} Q_k(s',a'))$
- Rewrite as expectation: $Q_{k+1} \leftarrow \mathbb{E}_{s' \sim P(s'|s,a)} \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$
- (Tabular) Q-Learning: replace expectation by samples
  - For an state-action pair (s,a), receive: $s' \sim P(s'|s,a)$
  - Consider your old estimate: $Q_k(s,a)$
  - Consider your new sample estimate:

    $$\text{target}(s') = R(s,a,s') + \gamma \max_{a'} Q_k(s',a')$$

  - Incorporate the new estimate into a running average:

    $$Q_{k+1}(s,a) \leftarrow (1-\alpha) Q_k(s,a) + \alpha \left[\text{target}(s') \right]$$

(Tabular) Q-Learning

Algorithm:

Start with $Q_0(s, a)$ for all s, a.

Get initial state s

For k = 1, 2, … till convergence

Sample action a, get next state s'

If s' is terminal:

$$\text{target} = R(s, a, s')$$

Sample new initial state s'

else:

$$\text{target} = R(s, a, s') + \gamma \max_{a'} Q_k(s', a')$$

$$Q_{k+1}(s, a) \leftarrow (1 - \alpha) Q_k(s, a) + \alpha \left[\text{target}\right]$$

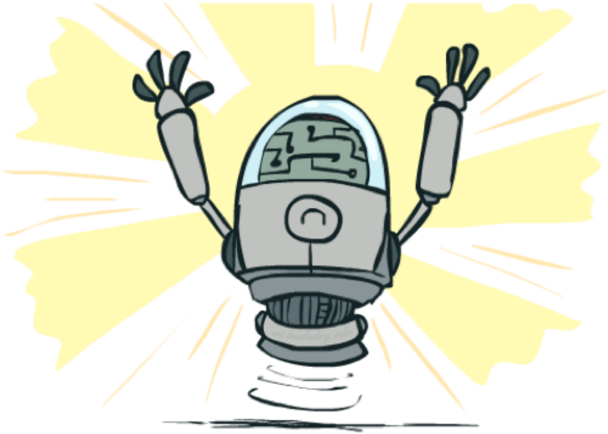
$$s \leftarrow s'$$

How to sample actions?

- Choose random actions?
- Choose action that maximizes $Q_k(s, a)$ (i.e. greedily)?
- ε-Greedy: choose random action with prob. ε, otherwise choose action greedily

Q-Learning Properties

- Amazing result: Q-learning converges to optimal policy -- even if you're acting suboptimally!
- This is called off-policy learning
- Caveats:
  - You have to explore enough
  - You have to eventually make the learning rate small enough
  - … but not decrease it too quickly



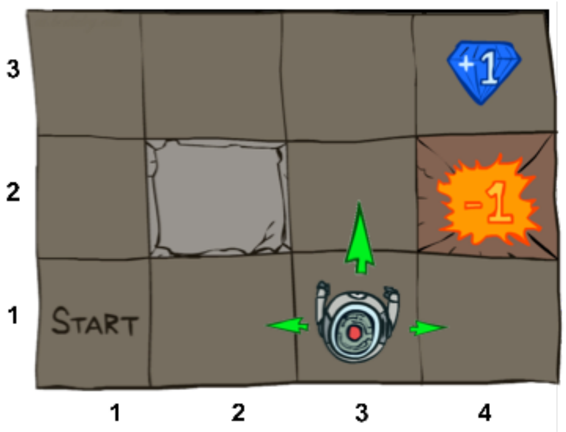
Q-Learning Properties

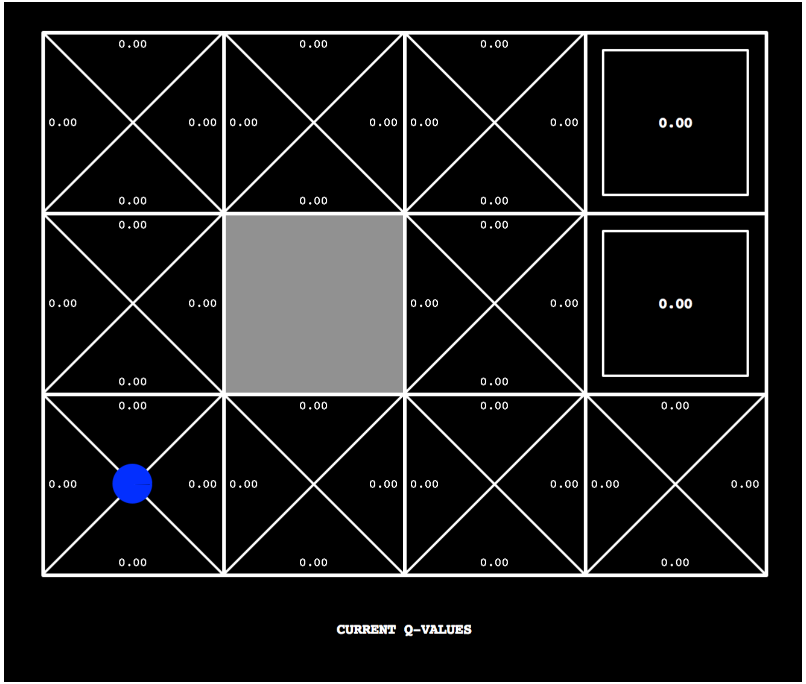
- Technical requirements.
  - All states and actions are visited infinitely often
    - Basically, in the limit, it doesn't matter how you select actions (!)
  - Learning rate schedule such that for all state and action pairs (s,a):

$$\sum_{t=0}^{\infty} \alpha_t(s,a) = \infty \qquad \sum_{t=0}^{\infty} \alpha_t^2(s,a) < \infty$$

For details, see Tommi Jaakkola, Michael I. Jordan, and Satinder P. Singh. On the convergence of stochastic iterative dynamic programming algorithms. Neural Computation, 6(6), November 1994.

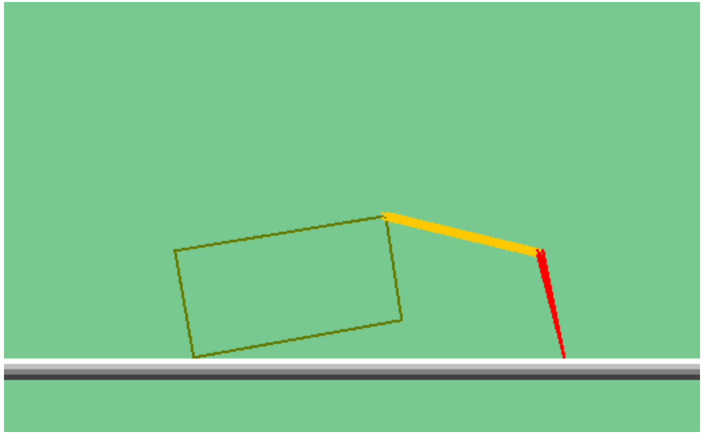
Q-Learning Demo: Gridworld





- States: 11 cells
- Actions: {up, down, left, right}
- Deterministic transition function
- Learning rate: 0.5
- Discount: 1
- Reward: +1 for getting diamond, -1 for falling into trap

Q-Learning Demo: Crawler



- States: discretized value of 2d state: (arm angle, hand angle)
- Actions: Cartesian product of {arm up, arm down} and {hand up, hand down}
- Reward: speed in the forward direction

Sampling-Based Approximation

- Q Value Iteration → (Tabular) Q-learning
- Value Iteration?
- Policy Iteration
  - Policy Evaluation
  - Policy Improvement?

Value Iteration w/ Samples?

- Value Iteration

$$V_{i+1}^*(s) \leftarrow \max_a \mathbb{E}_{s' \sim P(s'|s,a)} \left[R(s,a,s') + \gamma V_i^*(s')\right]$$

- unclear how to draw samples through max......

Sampling-Based Approximation

- Q Value Iteration → (Tabular) Q-learning
- ~~Value Iteration?~~
- Policy Iteration
  - Policy Evaluation
  - Policy Improvement?

Recap: Policy Iteration

One iteration of policy iteration:

- Policy evaluation for current policy π_k :
  - Iterate until convergence

$$V^{\pi_k}_{i+1}(s) \leftarrow \mathbb{E}_{s' \sim P(s'|s,\pi_k(s))}[R(s,\pi_k(s),s') + \gamma V^{\pi_k}_i(s')]$$

Can be approximated by samples
This is called Temporal Difference (TD) Learning

- Policy improvement: find the best action according to one-step look-ahead

$$\pi_{k+1}(s) \leftarrow \arg\max_a \mathbb{E}_{s' \sim P(s'|s,a)}[R(s,a,s') + \gamma V^{\pi_k}(s')]$$

Unclear what to do with the max (for now)

Sampling-Based Approximation

- Q Value Iteration → (Tabular) Q-learning
- ~~Value Iteration?~~
- Policy Iteration
  - Policy Evaluation → (Tabular) TD-learning
  - ~~Policy Improvement (for now)~~

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✓ **sampling-based approximations**

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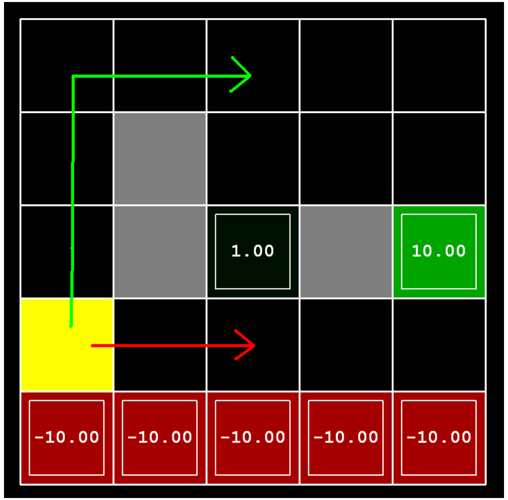
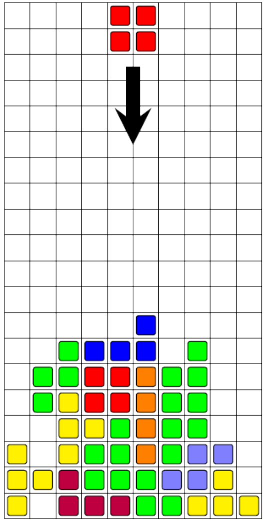
Can tabular methods scale?

- Discrete environments





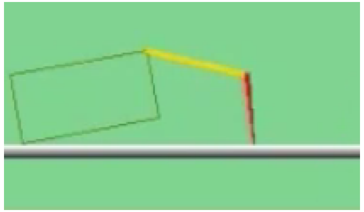
Gridworld
10^1

Tetris
10^60

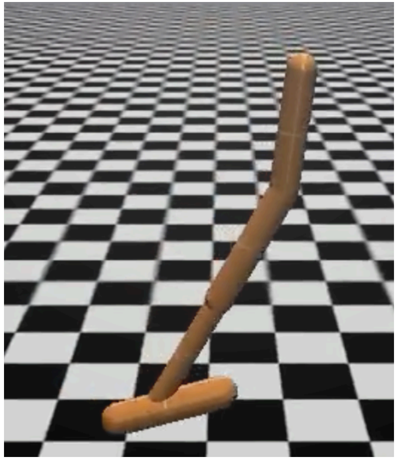
Atari
10^308 (ram) 10^16992 (pixels)

Can tabular methods scale?

- Continuous environments (by crude discretization)



Crawler
10^2

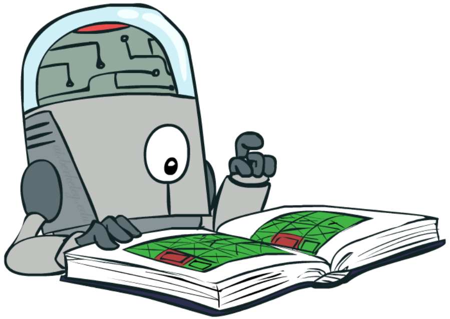


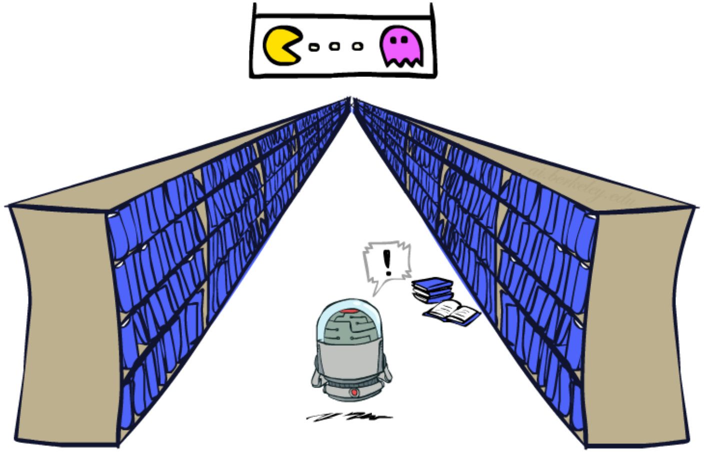
Hopper
10^10

Humanoid
10^100

Generalizing Across States

- Basic Q-Learning keeps a table of all q-values
- In realistic situations, we cannot possibly learn about every single state!
  - Too many states to visit them all in training
  - Too many states to hold the q-tables in memory
- Instead, we want to generalize:
  - Learn about some small number of training states from experience
  - Generalize that experience to new, similar situations
  - This is a fundamental idea in machine learning, and we'll see it over and over again





Approximate Q-Learning

- Instead of a table, we have a parametrized Q function: $Q_\theta(s,a)$
  - Can be a linear function in features:

$$Q_\theta(s,a) = \theta_0 f_0(s,a) + \theta_1 f_1(s,a) + \cdots + \theta_n f_n(s,a)$$

  - Or a complicated neural net

- Learning rule:
  - Remember: $\text{target}(s') = R(s,a,s') + \gamma \max_{a'} Q_{\theta_k}(s',a')$
  - Update:

$$\theta_{k+1} \leftarrow \theta_k - \alpha \nabla_\theta \left[\frac{1}{2} (Q_\theta(s,a) - \text{target}(s'))^2 \right] \bigg|_{\theta=\theta_k}$$

Connection to Tabular Q-Learning

- Suppose $\theta \in \mathbb{R}^{|S| \times |A|}, \quad Q_\theta(s,a) \equiv \theta_{sa}$

$$\nabla_{\theta_{sa}} \left[\frac{1}{2} (Q_\theta(s,a) - \mathrm{target}(s'))^2 \right]$$
$$= \nabla_{\theta_{sa}} \left[\frac{1}{2} (\theta_{sa} - \mathrm{target}(s'))^2 \right]$$
$$= \theta_{sa} - \mathrm{target}(s')$$

- Plug into update: $\theta_{sa} \leftarrow \theta_{sa} - \alpha(\theta_{sa} - \mathrm{target}(s'))$

$$= (1-\alpha)\theta_{sa} + \alpha[\mathrm{target}(s')]$$

- Compare with Tabular Q-Learning update:

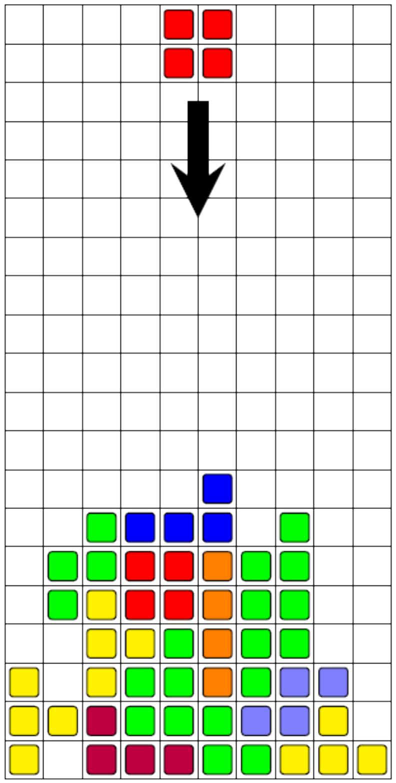
$$Q_{k+1}(s,a) \leftarrow (1-\alpha) Q_k(s,a) + \alpha \left[\mathrm{target}(s') \right]$$

Engineered Approximation Example: Tetris

- state: naïve board configuration + shape of the falling piece ~10^{60} states!
- action: rotation and translation applied to the falling piece

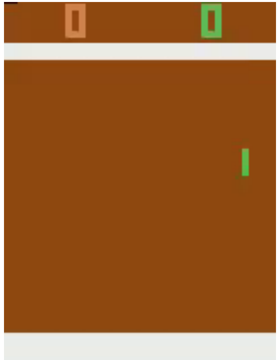
- 22 features aka basis functions ϕ_i
  - Ten basis functions, 0*, . . . , 9, mapping the state to the height h[k] of each* column.
  - Nine basis functions, 10*, . . . , 18, each mapping the state to the absolute difference* between heights of successive columns: $|h[k+1] - h[k]|$, $k = 1, \ldots, 9$.
  - One basis function, 19, that maps state to the maximum column height: $\max_k h[k]$
  - One basis function, 20, that maps state to the number of 'holes' in the board.
  - One basis function, 21, that is equal to 1 in every state.

$$\hat{V}_\theta(s) = \sum_{i=0}^{21} \theta_i \phi_i(s) = \theta^\top \phi(s)$$

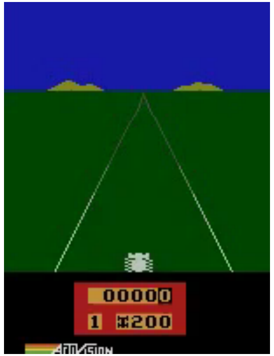


[Bertsekas & Ioffe, 1996 (TD); Bertsekas & Tsitsiklis 1996 (TD); Kakade 2002 (policy gradient); Farias & Van Roy, 2006 (approximate LP)]

Deep Reinforcement Learning

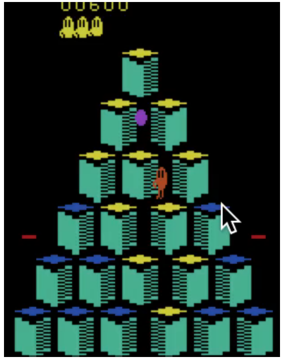


Pong



Enduro

Beamrider



Q*bert

- From pixels to actions
- Same algorithm (with effective tricks)
- CNN function approximator, w/ 3M free parameters