

ESS 326 Atmospheric Physics

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Topics to be covered:

1. Review of dry and moist atmospheric thermodynamics
2. Warm cloud physics
 - ◆ Supersaturation in updrafts
 - ◆ Cloud droplet surface tension, vapor pressure over flat and curved surfaces
 - ◆ Homogeneous nucleation
 - ◆ Heterogeneous nucleation, solute effect, Kohler curves, etc.
 - ◆ Droplet growth - condensation and collision/coalescence
 - ◆ Warm rain initiation
3. Cold and mixed-phase cloud physics
 - ◆ Homogeneous and heterogeneous nucleation of ice
 - ◆ Ice crystal depositional growth, Bergeron process
 - ◆ Ice crystal growth by aggregation and riming
 - ◆ Dry and wet growth of hail
 - ◆ Thunderstorm electrification
4. Precipitation physics
 - ◆ Mesoscale structure of precipitating systems
 - ◆ Convective vs. stratiform precipitation
 - ◆ Microphysical structure of convective storms and feedback between dynamics and thermodynamics
5. Aerosol-cloud-precipitation interaction
 - ◆ Aerosol sources and properties in the atmosphere
 - ◆ Cloud condensation nuclei and their impacts
 - ◆ Ice nucleating particles and their impacts
6. Observations and modeling of clouds and precipitation
 - ◆ Intro to remote sensing of cloud and aerosols
 - ◆ Cloud/aerosol microphysics parameterizations in weather forecast and climate models

Textbooks:

- Rogers and Yau, *A Short Course in Cloud Physics*, 3rd edition
- Wallace and Hobbs, *Atmospheric Science: An Introductory Survey*, 2nd edition
- Pruppacher and Klett, *Microphysics of Clouds and Precipitation*

Thermodynamics of moist air

- Fundamentals of thermodynamics

Ideal gas law

$$pV = nRT, \quad p = \rho R_d T, \quad \alpha = \frac{R_d T}{p}, \quad R_d = 287.0 \text{ J/(kg} \cdot \text{K)}$$

The specific volume is defined as $\alpha = \rho^{-1}$

First law of thermodynamics

For an air parcel, we have

$$\delta q = du + \delta w = c_V dT + pd\alpha = c_p dT - \alpha dp$$

The specific heat constants are

$$c_V = \left(\frac{\delta q}{dT} \right)_\alpha, \quad c_p = c_V + R_d$$

Dry adiabatic process

$$\delta q = c_p dT - \alpha dp = c_p dT - \frac{R_d T}{p} dp = 0$$

Integrating from the initial state to an arbitrary state leads to

$$\ln \frac{T}{T_0} = \frac{R_d}{c_p} \ln \frac{p}{p_0}, \quad \kappa = \frac{R_d}{c_p} = 0.286, \quad \frac{T}{T_0} = \left(\frac{p}{p_0} \right)^\kappa$$

Dry adiabatic lapse rate

Using the hydrostatic balance $dp = -\rho g dz$, we have

$$\delta q = c_p dT - \alpha dp = c_p dT + g dz = 0, \quad \Gamma_d = -\frac{dT}{dz} = \frac{g}{c_p} \approx 9.8 \text{ K/km}$$

Potential temperature

$$\theta = T \left(\frac{p_0}{p} \right)^\kappa, \quad p_0 = 1000 \text{ mb}$$

Potential temperature is conserved in any dry adiabatic process. The line of constant θ is called a dry adiabat or an isentrope

- Thermodynamics of moist air

Water vapor equation of state

$$e = \rho_v R_v T, \quad R_v = 461.5 \text{ J/(kg} \cdot \text{K}), \quad \varepsilon = \frac{R_d}{R_v} = 0.622$$

Latent heat

Fusion	Vaporization	Sublimation
$L_f = 3.34 \times 10^5 \text{ J/kg}$	$L_v = 2.5 \times 10^6 \text{ J/kg}$	$L_s = 2.83 \times 10^6 \text{ J/kg}$

Clausius-Clapeyron equation

The saturation water vapor pressure e_s (over liquid surface) or e_{si} (over ice surface) depends only on the temperature

$$\frac{de_s}{dT} = \frac{L_v e_s}{R_v T^2}, \quad \frac{de_{si}}{dT} = \frac{L_s e_{si}}{R_v T^2}$$

Assume the latent heat does not change, and we have the approximated equation

$$e_s = e_{s0} \cdot \exp \left[\frac{L_v}{R_v} \left(\frac{1}{T_0} - \frac{1}{T} \right) \right], \quad e_{s0} = 611 \text{ Pa}, \quad T_0 = 273.16 \text{ K}$$

Below 0 °C, we always have $e_{si}(T) < e_s(T)$

Moisture variables

Mixing ratio w

Mass of water vapor per unit mass of dry air

$$w = \frac{M_v}{M_d} = \frac{\rho_v}{\rho_d} = \frac{e/R_v T}{(p-e)/R_d T} = \frac{\varepsilon e}{p-e}$$

Specific humidity q

Mass of water vapor per unit mass of moist air

$$q = \frac{M_v}{M_v + M_d} = \frac{w}{1+w} \approx w$$

Relative humidity

$$\text{RH} = \frac{e}{e_s} \times 100 \%$$

Dew point T_d

Saturation occurs at T_d when air is cooled isobarically

Wet bulb T_w

Saturation occurs at T_w when air is cooled isobarically and by evaporating liquid water into it

$$T_d < T_w < T, \quad T_w \approx \frac{T_d + T}{2}$$

Example: Suppose we have a sub-saturated parcel at the surface, and we keep lifting the air parcel dry adiabatically

$$\begin{array}{llll} T \downarrow & \theta = & w = & p \downarrow \\ e \downarrow & e_s \downarrow & \text{RH} \uparrow & \end{array}$$

Saturated adiabatic process

Once the parcel becomes saturated, if all the water droplets (or ice) remain in the rising air parcel, the process may still be considered adiabatic and reversible, provided the heat does not pass through the boundaries of the parcel

Moist adiabatic lapse rate

Considering the latent heat of vaporization, and that c_p is the same for both dry and saturated air when $q \ll 1$, we have

$$\delta q = -L_v dw_s = c_p dT + gdz, \quad \Gamma_s = \Gamma_d + \frac{L_v}{c_p} \frac{dw_s}{dz} < \Gamma_d$$

Pseudo-adiabatic process

Once the parcel becomes saturated, if we assume that all of the suspended water droplets (or ice) fall out of the parcel immediately, the process is irreversible and not strictly adiabatic. It is called pseudo-adiabatic, and its lapse rate is similar to Γ_s

The real atmosphere lies between the saturated adiabatic and the pseudo-adiabatic process. Cloud water is carried along with the parcel, and precipitation falls out

Equivalent potential temperature

One approximated form is

$$\theta_e = \theta \exp\left(\frac{L_v w_s}{c_p T}\right) \geq \theta$$

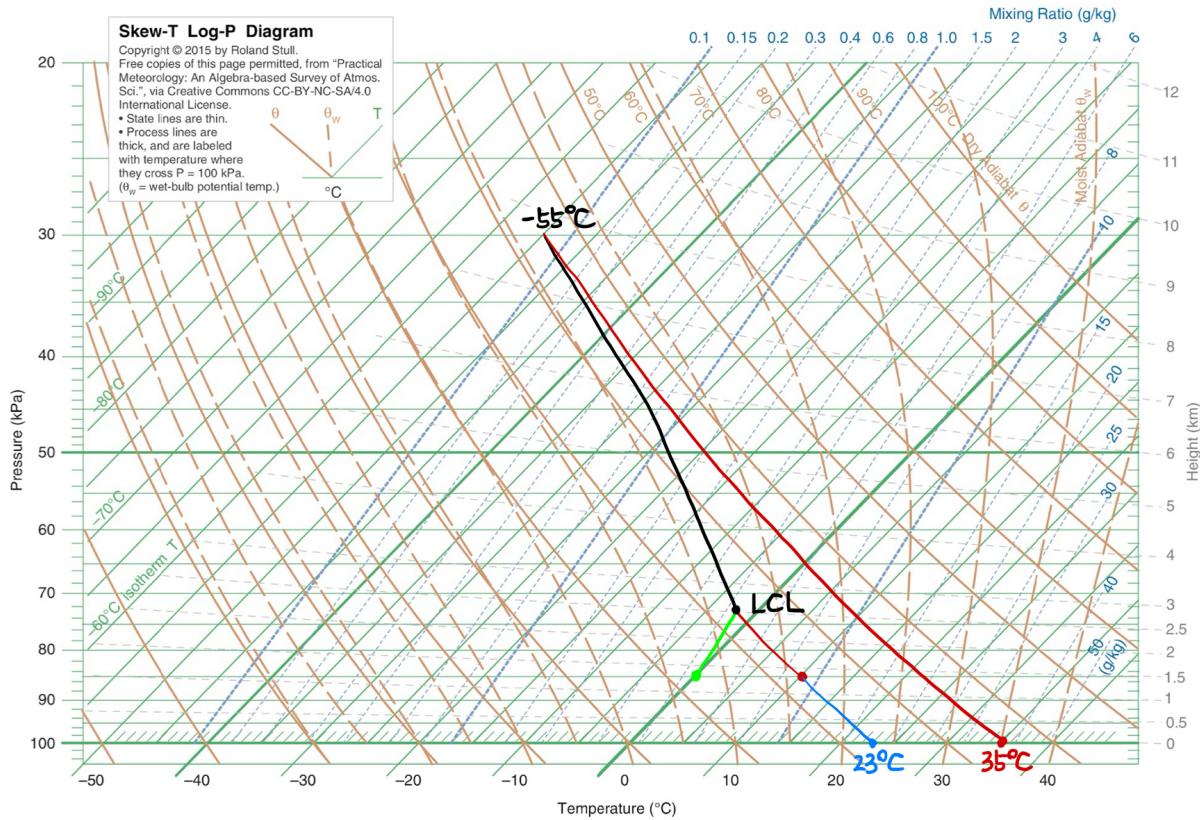
θ_e corresponds to that of a parcel of air after it has been lifted pseudo-adiabatically until all water vapor has condensed, released its latent heat, and fallen out of the parcel. The values w_s and T at the **Lifting Condensation Level (LCL)** should be used. θ_e is conserved for dry adiabatic, saturated adiabatic, and pseudo-adiabatic processes

Example: Suppose we keep lifting the air parcel adiabatically from the LCL

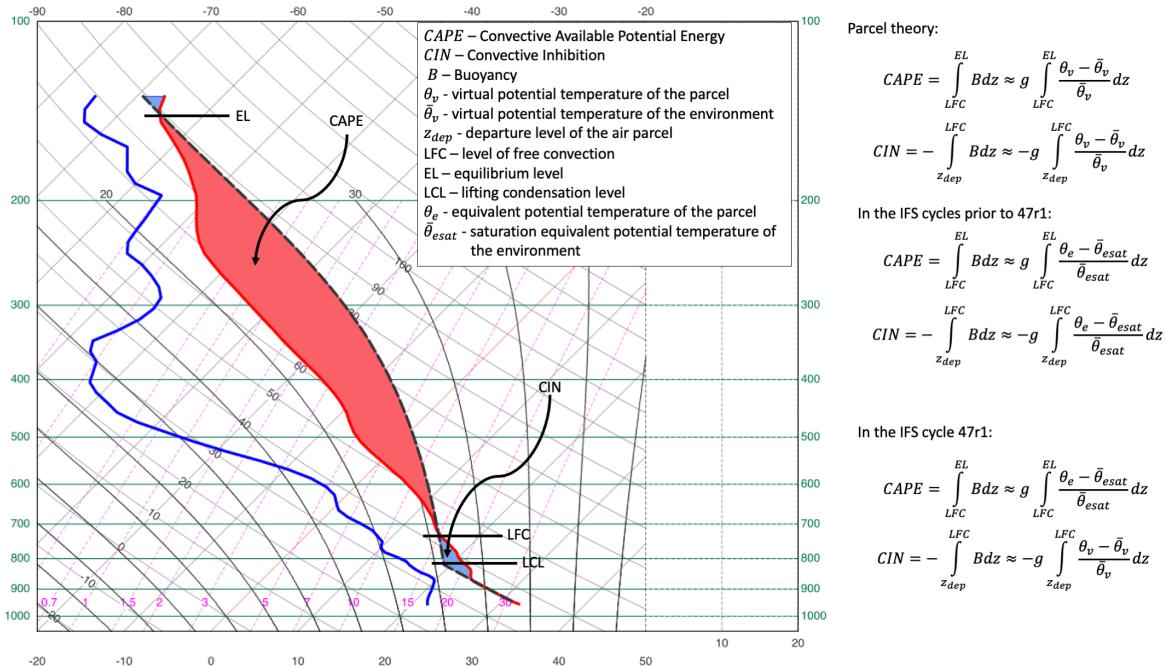
$$\begin{array}{l} T \downarrow \\ e = e_s \downarrow \end{array} \quad \begin{array}{l} \theta \uparrow \\ \text{RH} = 100 \% \end{array} \quad \theta_e = \quad w = w_s \downarrow$$

➤ Thermodynamic diagrams

Skew-T log-P chart



Convective Available Potential Energy (CAPE) & Convective Inhibition (CIN)



➤ Thermodynamics of phase change

Enthalpy

The enthalpy H and the specific enthalpy h (per unit mass) are defined as

$$H = U + pV, \quad h = u + p\alpha$$

The change in specific enthalpy is

$$dh = du + pd\alpha + \alpha dp = c_V dT + R_d dT = c_p dT$$

For an isobaric process

$$\delta q = c_p dT - \alpha dp = dh$$

For an adiabatic and isobaric process, we have $dh = 0$

Entropy

For a reversible process, we have

$$\frac{\delta q}{T} = c_p \frac{dT}{T} - \alpha \frac{dp}{T} = c_p \frac{dT}{T} - R_d \frac{dp}{p}$$

The definition of specific entropy is

$$ds = \left(\frac{\delta q}{T} \right)_{rev} = c_p d(\ln T) - R_d d(\ln p)$$

For an irreversible process, we have

$$\Delta s > \int \left(\frac{\delta q}{T} \right)_{irrev}$$

This shows that a given change of state requires more heating through an irreversible process.

Hence, reversible processes are more efficient

Example: Free expansion (irreversible process)

This process is adiabatic ($\Delta q = 0$) and no work done is involved ($\Delta w = 0$). Therefore, the internal energy and enthalpy are not changed

$$\Delta u = \Delta q - \Delta w = 0, \quad \Delta T = 0, \quad \Delta h = c_p \Delta T = 0$$

The entropy change needs to be calculated from a reversible isothermal expansion

$$\Delta s = \int_{\alpha_1}^{\alpha_2} \left(\frac{\delta q}{T} \right)_{rev} = \int_{\alpha_1}^{\alpha_2} \frac{pd\alpha}{T} = R_d \int_{\alpha_1}^{\alpha_2} \frac{d\alpha}{\alpha} = R_d \ln \left(\frac{V_2}{V_1} \right) > 0 = \int_{\alpha_1}^{\alpha_2} \left(\frac{\delta q}{T} \right)_{irrev}$$

Entropy & Potential temperature

From the definition of the potential temperature, we have

$$\theta = T \left(\frac{p_0}{p} \right)^\kappa, \quad c_p d(\ln \theta) = c_p d(\ln T) - R_d d(\ln p) = ds$$

Therefore, the entropy is related to potential temperature as

$$S = c_p \ln \theta + \text{const.}$$

A reversible adiabatic process is isentropic, and dry adiabats are also called isentropes

Second law of thermodynamics

For a natural process, the total entropy change for the system and environment satisfies

$$\Delta s_{\text{total}} = \Delta s_{\text{sys}} + \Delta s_{\text{env}} \geq 0$$

The equality is reached for a reversible process. A state of equilibrium is reached when the total entropy of the system has achieved its maximum possible value

Gibbs free energy

Consider a phase change of water evaporation at constant T and $p = e_s$. The latent heat can be expressed as

$$L_v = \int \delta q = \int_{u_1}^{u_2} du + \int_{\alpha_1}^{\alpha_2} p d\alpha = (u_2 - u_1) + e_s(\alpha_2 - \alpha_1)$$

Meanwhile, we also have

$$L_v = \int \delta q = T \int_{s_1}^{s_2} ds = T(s_2 - s_1)$$

These two equations lead to

$$u_1 + e_s \alpha_1 - Ts_1 = u_2 + e_s \alpha_2 - Ts_2$$

The Gibbs free energy G is constant during an isothermal and isobaric phase change

$$G = h - Ts = u + p\alpha - Ts$$

At constant T and $p = e_s$, we have

$$dG = dh - Tds = e_s d\alpha - Tds = \delta q - Tds$$

Gibbs free energy measures the amount of energy in a system that is available to do work not associated with expansion or compression (such as a change of phase or a chemical reaction)

For a reversible process at constant T and p , we have

$$dG = \delta q - Tds = 0$$

For an irreversible natural process at constant T and p , we have

$$dG = \delta q - Tds < 0$$

G decreases during a non-equilibrium (irreversible) phase change, and a minimum in G is obtained when the system reaches equilibrium. Gibbs free energy can tell us whether a given phase change (or chemical reaction) will occur spontaneously

At constant T and p , $\Delta s_{\text{total}} > 0$ is equivalent to $\Delta G < 0$

Derivation of Clausius-Clapeyron equation

At equilibrium, the Gibbs free energy for the two-phase system should satisfy

$$dG = G_v dm_v + G_l dm_l = 0, \quad dm_v = -dm_l \Leftrightarrow G_v = G_l$$

Now consider a perturbation in T and corresponding e_s . For a reversible process, we have $\delta q = Tds$ and thus for each phase, we have

$$dG_{v,l} = \delta q + \alpha dp - Tds - sdT = \alpha dp - sdT$$

Using $G_v = G_l$ at saturation vapor pressure e_s , we can obtain

$$dG_v - dG_l = (\alpha_v - \alpha_l)de_s - (s_v - s_l)dT = 0$$

Because $\rho_v \ll \rho_l$, we have $\alpha_v \gg \alpha_l$ and then we obtain

$$\alpha_v de_s = (s_v - s_l)dT, \quad \frac{R_v T}{e_s} de_s = \frac{L_v}{T} dT$$

Therefore, the Clausius-Clapeyron equation is derived as

$$\frac{de_s}{e_s} = \frac{L_v dT}{R_v T^2}, \quad e_s(T) = e_{s0} \cdot \exp \left[\frac{L_v}{R_v} \left(\frac{1}{T_0} - \frac{1}{T} \right) \right]$$

Formation of cloud droplets

Water vapor molecule	CCN	Cloud Droplet	Raindrop
0.27 nm	~ 200 nm ~ 200 #/cm ³	~ 10 μm ~ 50 #/cm ³	~ 2 mm ~ 100 #/m ³

➤ Homogeneous nucleation

From the definition of (specific) Gibbs free energy

$$G = u + p\alpha - Ts, \quad dG = du + pd\alpha + \alpha dp - Tds - sdT$$

For a reversible process, we have

$$Tds = du + pd\alpha, \quad dG = \alpha dp - sdT$$

Multiply the expression by mass of a substance, we have

$$dG_m = Vdp - SdT$$

Now at the saturation pressure $p = e_s$, we consider a small pressure perturbation de while the temperature T is held constant. For a single water vapor of liquid molecule, its Gibbs free energy is also denoted as the chemical potential, and we have

$$d\mu_v = V_v de, \quad d\mu_l = V_l de, \quad d(\mu_v - \mu_l) \approx V_v de = kT \frac{de}{e}$$

We already consider that the water vapor molecule takes up much larger volume $V_v \gg V_l$ and use the ideal gas law of the following form (N is the number of molecules)

$$pV = NkT, \quad V_v = \frac{kT}{e}, \quad k = 1.38 \times 10^{-23} \text{ J/K}$$

At equilibrium, we have $\mu_v = \mu_l$ and $e = e_s$. Integrating from the equilibrium to an arbitrary state leads to the result

$$\mu_l - \mu_v = -kT \ln(e/e_s)$$

Now consider we have n molecules per unit volume, the change in Gibbs free energy for the condensation of water droplets becomes

$$\Delta G = -nVkT \ln(e/e_s)$$

Surface tension effect

In reality, the condensation also creates a new interface between liquid and vapor. Therefore, an energy barrier needs to be overcome due to the surface tension

$$\Delta G = A\sigma - nVkT \ln(e/e_s), \quad \sigma = 0.075 \text{ N/m}$$

The critical radius R_c is the drop radius corresponding to an unstable equilibrium for a given vapor pressure e

$$R_c = \frac{2\sigma}{nkT \ln(e/e_s)} = \frac{2\sigma}{\rho_l R_v T \ln(e/e_s)}$$

However, the supersaturation required by homogeneous nucleation is extremely high and cannot happen in the real atmosphere

Kelvin's equation

$$\frac{e}{e_s} = \exp\left(\frac{2\sigma}{\rho_l R_v T r}\right) \approx 1 + \frac{a}{r}, \quad a = \frac{2\sigma}{\rho_l R_v T}$$

➤ Heterogeneous nucleation

In reality, cloud droplets form on atmospheric aerosols via heterogeneous nucleation

Aerosols

Primary sources (natural or anthropogenic)

- Wind-generated dust, sea spray, forest fires, combustion / industry

Secondary sources (gas-to-particle conversion and chemical reactions)

- SO₂ to sulfate (SO₄), NO₂ to nitrate (NO₃), NH₃ to ammonium (NH₄), volatile organic compounds (VOC) to SOA

Aerosol lifetime

Troposphere
~ days to weeks

Stratosphere
~ years

Aerosol size

$D < 0.1 \mu\text{m}$	$0.1 \mu\text{m} < D < 1 \mu\text{m}$	$D > 1 \mu\text{m}$
Aitken-mode particles	Fine-mode / accumulation-mode aerosols	Coarse-mode aerosols

To become Cloud Condensation Nuclei (CCN), aerosols need to be **hygroscopic**: soluble, or insoluble but wettable. As an example, an insoluble hygroscopic aerosol larger than 0.1 μm can allow for a droplet to grow on it at supersaturation of $\sim 1\%$

Solute effect & Raoult's law

$$f = \frac{e'_s(r)}{e_s(r)} = \frac{(4/3\pi r^3 \rho' - m)/M_w}{(4/3\pi r^3 \rho' - m)/M_w + im/M_s} \approx 1 - \frac{b}{r^3}, \quad b = \frac{3imM_w}{4\pi M_s \rho_l}$$

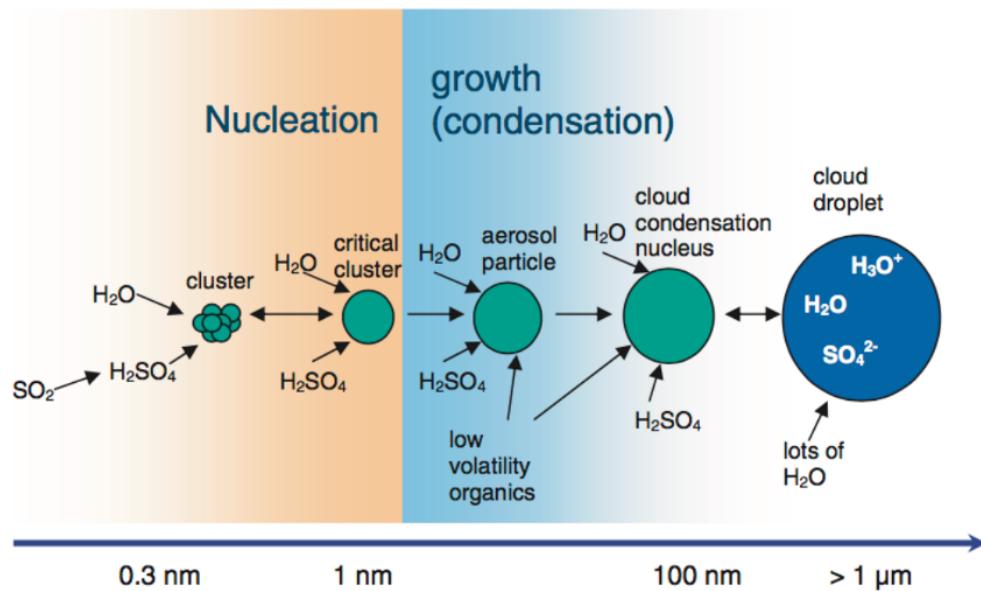
➤ Kohler curve

$$\frac{e'_s(r)}{e_s(\infty)} \approx 1 + \frac{a}{r} - \frac{b}{r^3}$$

The critical radius and supersaturation are

$$r^* = \sqrt{\frac{3b}{a}}, \quad S^* - 1 = \sqrt{\frac{4a^3}{27b}}$$

At high enough supersaturation, essentially all aerosol particles will be activated. Therefore, the total aerosol concentration is often called the condensation nuclei (CN) concentration



Warm cloud physics

➤ Warm clouds

Warm clouds lie completely below the 0 °C level and contain no ice. It is less relevant in the hydrological cycle (produces little precipitation), but is more relevant in the radiation balance (contribute predominantly to radiative cooling effect)

Key measures of warm cloud structure

Liquid water content (LWC)	Liquid water path (LWP)	Number concentration
Mass of liquid water in droplets per unit volume of air g/m ³	$\text{LWP} = \int \text{LWC } dz$	Number of droplets per unit volume of air #/cm ³

LWC is highly correlated with updrafts. Downdrafts have less or even zero LWC

➤ Droplet spectrum: **Gamma distribution**

$$N(D) = N_0 D^\alpha \exp(-\lambda D)$$

N_0 is intercept, α is spectral shape and λ is slope

Spectral moments

$$M(p) = \int_0^{\infty} D^p N(D) dD$$

$M(0)$	$M(2)$	$M(3)$	$M(6)$
number concentration	~ droplets surface area	~ droplets volume	~ reflectivity

Effective radius of cloud droplets: Cross-section weighted mean radius

$$r_{\text{eff}} = \frac{M(3)}{M(2)} = \frac{3}{4} \cdot \frac{\text{Volume}}{\text{Area}}$$

For non-spherical droplets, the mean volume and area are applied

It is highly relevant to atmospheric radiation $\tau_{\text{cloud}} = f(\text{LWP}, r_{\text{eff}})$

➤ Adiabatic warm cloud

From parcel theory, LWC at any height is estimated from the difference in w_s between the cloud base and that height. This is the amount that must have condensed out of the air parcel as it rises upward. However, actual cloud LWC measurements are always lower than the adiabatic value

➤ Droplet diffusion growth (**adiabatic condensation**)

Fick's law of diffusion

$$J = -D \frac{\partial n}{\partial x}$$

For number concentration $n(x, t)$, the corresponding flux J has unit of #/(m²·s)

Diffusion equation

$$\frac{\partial n}{\partial t} = -\frac{\partial J}{\partial x} = D \frac{\partial^2 n}{\partial x^2}$$

Droplet diffusion growth

Consider the droplet with radius r is surrounded by water vapor molecules with concentration $n(R)$ as a function of radial distance R from the droplet center. The flux normal to the droplet surface is evaluated as

$$J(r) = -D \frac{\partial n}{\partial R} \Big|_r$$

For vapor density $\rho_v = nm_0$ with m_0 being the mass of one vapor molecule, the mass flux is

$$J_m(r) = -D \frac{\partial \rho_v}{\partial R} \Big|_r$$

The rate of total mass change of the droplet is thus

$$\frac{dM}{dt} = 4\pi R^2 D \frac{\partial \rho_v}{\partial R}$$

Now assume that this rate is independent of distance R . Integrating from the droplet surface to infinity leads to

$$\frac{dM}{dt} \int_r^\infty \frac{dR}{R^2} = 4\pi D \int_r^\infty \frac{\partial \rho_v}{\partial R} dR, \quad \frac{dM}{dt} = 4\pi r D [\rho_v(\infty) - \rho_v(r)]$$

The rate of mass increase of a droplet in supersaturated conditions depends on the difference between the ambient vapor density $\rho_v(\infty)$ and that at the droplet surface $\rho_v(r)$. In terms of droplet radius r , we have

$$\frac{dr}{dt} = \frac{D}{r\rho_l} [\rho_v(\infty) - \rho_v(r)], \quad M = \frac{4}{3}\pi\rho_l r^3$$

To estimate $\rho_v(r)$, we can assume saturation near the droplet surface, and assume that the temperature near the droplet surface is equal to the ambient, which is $T_r = T_\infty$

$$\rho_v(r) = \frac{e'_s(r)}{R_v T} = \left(1 + \frac{a}{r} - \frac{b}{r^3}\right) \cdot \frac{e_s(T)}{R_v T}$$

However, condensation near the droplet surface releases latent heat. We also need to consider the heat diffusion equation

$$\frac{dQ}{dt} = 4\pi r K (T_r - T_\infty)$$

The latent heat is related to the mass change as $dQ = L_v dM$, therefore we obtain

$$\frac{dM}{dt} = \frac{4\pi r K}{L_v} (T_r - T_\infty)$$

Eventually, we solve the following equations to obtain $T(r)$ and $\rho_v(r)$

$$\frac{K}{L_v D} = \frac{\rho_v(\infty) - \rho_v(r)}{T_r - T_\infty}, \quad \rho_v(r) = \left(1 + \frac{a}{r} - \frac{b}{r^3}\right) \cdot \frac{e_s[T(r)]}{R_v T(r)}$$

Diffusion growth equation

An analytical approximation is (Mason, 1971)

$$r \frac{dr}{dt} = \frac{(S - 1) - a/r + b/r^3}{F_k + F_d}, \quad S = \frac{e}{e_s(T)}$$

The thermodynamic and vapor diffusion terms are given as

$$F_k = \left(\frac{L_v}{R_v T} - 1\right) \frac{L_v \rho_l}{K T}, \quad F_d = \frac{\rho_l R_v T}{e_s(T) D}$$

For large enough radius r , the solute and curvature terms are negligible

$$r \frac{dr}{dt} = \frac{S - 1}{F_k + F_d}, \quad r(t) = \sqrt{r^2(t_0) + \frac{2(S - 1)}{F_k + F_d} t}$$

Diffusion growth is proportional to \sqrt{t} . Because the slowing of growth rate with time, cloud droplets tend toward a uniform (monodisperse) size distribution due to diffusion growth.

However, narrowing of droplet size distribution is not observed in reality. Other [diffusional factors](#) can be giant CCN, SS fluctuation and entrainment

Time scales for vapor diffusional growth of drops

The rate of growth by condensation (diffusion growth) is too slow to explain the raindrops in warm clouds on the short time scales. We need another mechanism

Typical cloud droplet	Large cloud droplet / Small raindrop	Typical raindrop
$\sim 10 \mu\text{m}$	$\sim 100 \mu\text{m}$	$\sim 1 \text{ mm}$
$\sim 1 \text{ hour}$	$\sim 1 \text{ day}$	$\sim 2 \text{ weeks}$

Evaporation rate in subsaturated conditions

The diffusion growth equation can also be applied to calculate evaporation rate when $S < 1$

Initial radius	1 μm	3 μm	10 μm	30 μm	0.1 mm	0.15 mm
Distance fallen	2.0 μm	0.17 mm	2.1 cm	1.69 m	208 m	1.05 km

The last two scenarios correspond to drizzle / precipitation

➤ Growth by **collision and coalescence**

Terminal velocity of drops

$$\frac{4}{3}\pi r^3(\rho_l - \rho_a)g = F_{\text{drag}}$$

$r < 30 \mu\text{m}$	Stoke's drag	$v_t = \frac{2\rho'gr^2}{9\eta}$	$\rho' = \rho_l - \rho_a$
$40 \mu\text{m} < r < 0.6 \text{ mm}$	Turbulent drag	$v_t = k_3 r$	$k_3 = 8 \times 10^3 \text{ s}^{-1}$
$r > 0.6 \text{ mm}$	Drop flattening	$v_t = k_2 \sqrt{r}$	$k_2 = 2.2 \times 10^3 \sqrt{\rho_0/\rho} \text{ cm}^{1/2} \cdot \text{s}^{-1}$

Collision, coalescence and collection efficiencies

The collision efficiency E is defined as

$$E = \frac{y^2}{(R + r)^2}$$

It represents the fraction of droplets of radius r that will collide (by critical radius y) with a collector drop of radius R . The coalescence efficiency is denoted as E' and the collection efficiency E_c is $E_c = E \times E'$

Continuous collection equation

Consider a collector drop of radius R falling through a **uniform** field of smaller droplets of radius r . The rate of mass increase is

$$\frac{dM}{dt} = \pi R^2(v_R - v_r)w_l E_c$$

The terminal velocities are v_R and v_r , and w_l is the liquid water content of the field of the collected drops. The rate of radius increase is

$$\frac{dR}{dt} = \frac{(v_R - v_r)w_l E_c}{4\rho_l}$$

When $R \gg r$ we can simplify the expression to

$$\frac{dR}{dt} \approx \frac{v_R w_l E}{4\rho_l}$$

Note that $v_R(R)$ and $E(R)$ both increase with R , leading to exponential growth of collector

Broadening the cloud droplet spectrum

This corresponds to **stochastic effects** that overcome the growth gap between condensation and collection, and provide initial large drops to begin collision/coalescence

- a. Giant CCN ($> 3 \mu\text{m}$): Embryos for rare collector drops, such as sea salt, mineral dust
- b. Turbulence & entrainment: Entraining dry air into growing clouds can evaporate small cloud droplets, and the survived droplets will have less competition for water vapor and will grow faster (inhomogeneous entrainment). Repeated entrainment episodes will naturally lead to a broader size distribution
- c. Radiative broadening
- d. Stochastic collection

Process (b) and (c) are also called **diabatic condensation**

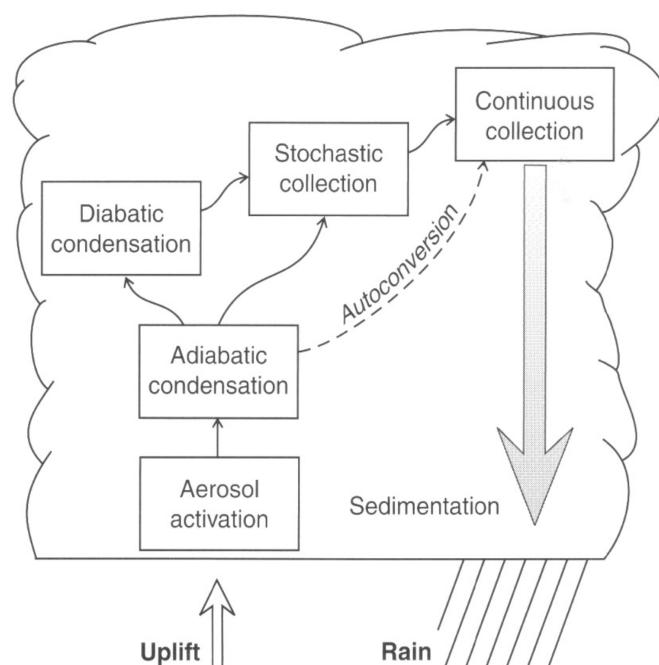
- Favorable conditions for precipitation

Precipitation in warm clouds:

- ✓ High LWC (e.g., deep cumulus)
- ✓ Broad drop spectrum
- ✓ Large drops ($> \sim 20 \mu\text{m}$) / Low CCN
- ✓ Large vertical extent (i.e., long growth / collision time)

Surface precipitation:

- ✓ High in-cloud precipitant (conditions above)
- ✓ Less dry condition under cloud layers (i.e., less evaporation)



Cold cloud physics

- Convective cloud: Liquid and ice distribution

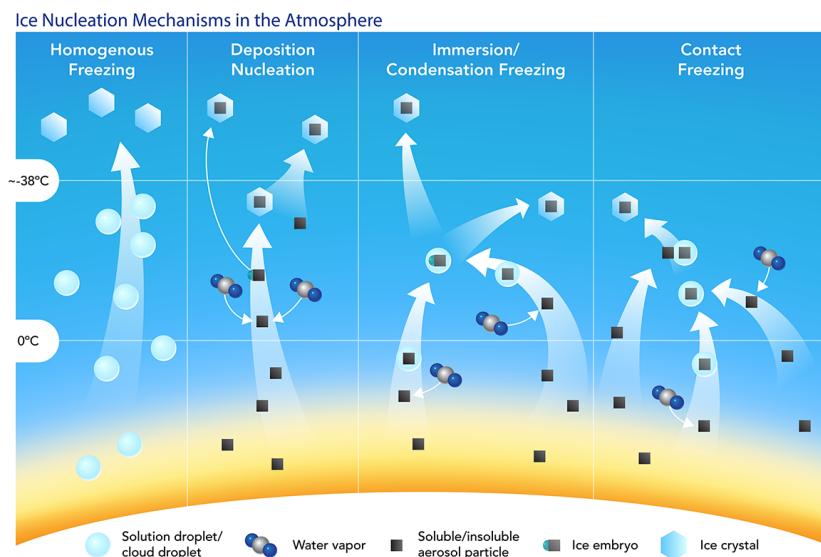
$T < -37^{\circ}\text{C}$	Ice only (glaciated)
$-37^{\circ}\text{C} < T < 0^{\circ}\text{C}$	Mixed ice and water
$T > 0^{\circ}\text{C}$	Liquid water only

- Cold clouds

Cold clouds lying above the 0°C level may contain ice. A cold cloud may still contain liquid droplets, which are called **supercooled** cloud droplets

- Ice nucleation mechanisms (freezing of supercooled droplets)
- Homogeneous nucleation: Spontaneous formation of ice nuclei by “ice-like” arrangements of molecules in the liquid/gas phase
- Heterogeneous nucleation: Formation of ice crystals on **ice nuclei (IN)**, for example, aerosol or a preexisting ice crystal that comes into contact with the supercooled drop

Homogeneous freezing can occur in the atmosphere below around -37°C . It is initialized by water molecule cluster called ice embryo



- Deposition nucleation

This is the direct transfer of vapor to ice under ice supersaturation, and only naturally occurs in the heterogeneous mode, i.e., require IN. A good IN has ice-like crystal structure, and is insoluble, large and hydrophilic. The **best IN is ice itself**.

Ice nuclei (IN) concentration

The number of active ice nuclei per liter of air, denoted by N , is empirically described as

$$\ln N = a(T_1 - T), \quad a = 0.3 \sim 0.8, \quad T_1 \approx -20 \text{ }^{\circ}\text{C}$$
$$N = e^{a+b[100(S_i-1)]}, \quad a = -0.639, \quad b = 0.1296$$

However, observations often show that ice particle concentrations are two or three orders of magnitude higher than that predicted by the IN concentration. Secondary ice production (e.g., ice multiplication) is needed to explain the high ice particle concentration

- Diffusional growth of ice crystals

The capacitance C for a general crystal shape is introduced to replace the radius r in the droplet diffusional growth equation. We thus obtain

$$\frac{dM}{dt} = 4\pi CD[\rho_v(\infty) - \rho_v(r)]$$

Considering latent heating during depositional growth, we have

$$\frac{1}{4\pi C} \frac{dM}{dt} = \frac{S_i - 1}{\left(\frac{L_s}{R_v T} - 1\right) \frac{L_s}{K T} + \frac{R_v T}{e_{si}(T) D}}$$

Bergeron-Findeisen-Wegener process

Because $e_s(T) > e_{si}(T)$, a cloud saturated with respect to water at subfreezing temperatures is supersaturated with respect to ice. In a large region of a typical mixed-phase cloud, ice crystals are far less numerous than supercooled water drops. Therefore, ice crystals will grow rapidly via **vapor deposition** due to high supersaturations

As a crystal grows, it locally depletes the excess water vapor, driving the vapor pressure below saturation with respect to water. This causes the nearby supercooled water droplets to evaporate, and maintaining the excess vapor pressure (with respect to ice) needed for the crystal to continue growing. The net result is **the growth of ice crystals at the expense of supercooled water droplets**

Crystal habits

Ice crystals growth by vapor deposition manifest in a variety of habits, dependent on the ambient temperature and ice supersaturation

$-3 \text{ }^{\circ}\text{C} < T < 0 \text{ }^{\circ}\text{C}$	$-8 \text{ }^{\circ}\text{C} < T < -3 \text{ }^{\circ}\text{C}$	$-40 \text{ }^{\circ}\text{C} < T < -8 \text{ }^{\circ}\text{C}$	$T < -40 \text{ }^{\circ}\text{C}$
Hexagonal plates	Columns	Dendrites, sector plates	Rosettes

- Growth of ice crystals by **aggregation & accretion**

Aggregational growth

Similar to the continuous collection equation for raindrops, we have

$$\frac{dM}{dt} = \pi R^2 \bar{E} w_i \Delta v, \quad w_i = N \rho_i V$$

The ice water content w_i is used in the aggregational growth. Since the terminal velocities for a single dendrite (~ 0.5 m/s) and aggregates of dendrites (~ 1.0 m/s) are relatively constant with increasing diameters, we can directly use $\Delta v = 0.5$ m/s

Accretion of supercooled drops (riming)

$$\frac{dM}{dt} = \pi R^2 \bar{E} w_l v(R)$$

The liquid water content w_l is used in the riming growth. We also assume the fall speed of the collector $v(R)$ is much faster than the supercooled droplets

As riming continues, the original particle can no longer be recognized, and the rimed particle is called as a **graupel** particle. Terminal velocity of graupel increase rapidly with diameter

- Hailstorm

Hail growth: Continued riming of graupel

Hail represents extreme growth by continued riming of graupel (typically $D > 5$ mm). There are two modes of hail growth:

1. Dry growth: The latent heat released by the freezing drops isn't sufficient to raise the surface temperature of the hailstone to 0 °C. Dry growth results in a milky appearance and low-density spongy texture due to the numerous trapped air bubbles/gaps
2. Wet growth: The latent heat can raise the surface temperature to above 0 °C. Wet growth results in high-density clear ice with few trapped bubbles

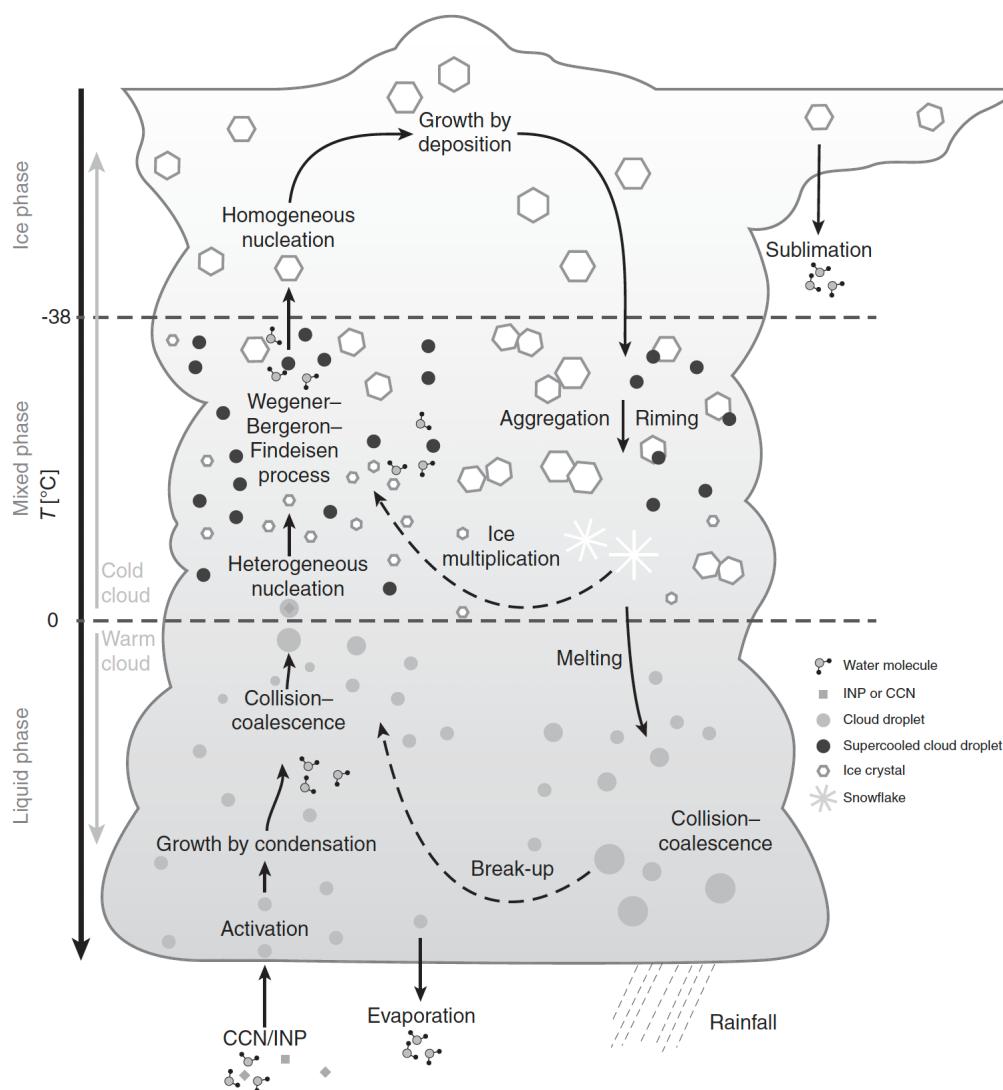
Hail trajectories

1. Embryo zone (EZ): Initial formation of hail embryos
2. Hail growth zone (HGZ): Most formation of hail, characterized by strong updrafts carrying water droplets high enough to freeze
3. Freezing rain zone (FOZ): Formation of freezing rain rather than hail

➤ Lightning

Thermoelectric effect contributes to the **non-inductive charge separation**. The colder center of the graupel is slightly **negatively charged**, while the **graupel surface** is slightly **positively charged**. Charge transfer occurs during collision. Basic thunderstorm has a dipole/tripole structure, with the **cloud top** being **positively charged**

The lightning potential can be predicted based on microphysics. One index is the Lightning Potential Index (LPI)



Aerosol-Cloud Interactions (ACI)

Global aerosol distribution: Dust, biomass burning, urban industrial pollution, sea salt

Direct effects

Scattering & Absorption of radiation

Semi-direct effects

Heating causes cloud burn-off

Cloud albedo effect / 1st indirect effect

Twomey effect

Increased CDNC (constant LWC)

CDNC: Cloud droplet number concentration

Cloud lifetime effect / 2nd indirect effect

Albrecht effect

Drizzle suppression, increased LWC

Increased cloud height, cloud lifetime

- ACI in warm clouds

Twomey effect: Enhanced cloud albedo by CCN

The cloud optical depth (COD), which is dimensionless, is related to the liquid water path LWP (unit: g/m²) and the droplet effective radius r_e

$$\text{COD} = \frac{3\text{LWP}}{2\rho_w r_e} = Nh \cdot 2\pi r_e^2$$

Example: Particles emitted by ships

Ship emission increases CCN concentration, which increases CDNC and reduces the average droplet size. The increased CDNC and smaller droplet size reduce drizzle production, which increases cloud lifetime and LWC. Clouds are optically thicker and brighter along ship track. However, under dry conditions, smaller droplet size leads to evaporation and lower LWC

Nonlinear relationship in ACI

When CCN concentration is large, the increase in CDNC becomes slower, because the drop formation is limited by supersaturation. When CDNC is large, the increase in cloud albedo becomes smaller as well, which means that the albedo susceptibility is lower at high CDNC

- ACI in cold clouds

Glaciation effect: Positive radiative effect at top of atmosphere (TOA), more precipitation.

With IN aerosols, liquid droplet freezing is delayed, higher and colder clouds are formed.

Meanwhile, there are more ice crystals and more precipitation

Thermodynamic effect: Not yet known about radiative effect and precipitation change

- ACI in convective clouds

Microphysical cloud invigoration

The ultrafine aerosol particles are numerous but too small to activate at cloud base. The warm rain is delayed, and the mixed phase / ice cloud processes are enhanced. Cloud invigoration eventually leads to more intense precipitation

$$\text{CCN} \uparrow \rightarrow \text{CDNC} \uparrow \rightarrow r_c \downarrow \rightarrow \text{Warm rain} \downarrow \rightarrow \text{Supercool LWC} \uparrow$$

This increased supercool LWC enhances

1. Heterogeneous ice formation
2. Riming and thus the growth of graupel and hailstone

All three enhanced factors contribute to more lightnings

CCN impacts on hurricane

Aerosols can efficiently modulate TC precipitation near rainbands via microphysical cloud invigoration. With increased precipitation in the rainband, wind in the eyewall is weaker

- Weather modification via ACI

Solar radiation management (SRM)

1. Altering reflection of shortwave solar radiation
 - a) Surface albedo enhancement
 - b) Increase reflectivity of marine clouds by injecting aerosol particles (Marine Cloud Brightening, MCB)
 - c) Increase stratospheric aerosol (SAI)
 - d) Space-based methods
2. Altering transmission of longwave radiation
 - e) Decrease the amount of high altitude cirrus clouds (CCT)

Climate system: Forcing & Feedback

- Earth's radiation budget

The effective incoming solar radiation is $S_0/4 = 342 \text{ W/m}^2$

Zero-layer model

The absorbed solar radiation equals the emitted terrestrial radiation. With $\alpha_p = 0.3$ for albedo and $\sigma = 5.67 \times 10^{-8} \text{ W/(m}^2\cdot\text{K}^4)$, the effective Earth's black body temperature is

$$\pi R^2 S_0 (1 - \alpha_p) = 4\pi R^2 \cdot \sigma T_e^4, \quad T_e \approx 255 \text{ K}$$

One-layer model: Greenhouse effect

Consider the atmosphere as a homogenous layer of temperature T_a , which is transparent to the solar radiation and opaque to the infrared radiations emitted by the Earth's surface. The heat balance at the top of the atmosphere (TOA) becomes

$$\pi R^2 S_0 (1 - \alpha_p) = 4\pi R^2 \cdot \sigma T_a^4$$

Similarly, the heat balance at the Earth's surface is

$$\pi R^2 S_0 (1 - \alpha_p) = 4\pi R^2 \cdot (\sigma T_s^4 - \sigma T_a^4), \quad T_s \approx 303 \text{ K}$$

This 30°C surface temperature is a bit too warm

- Radiative forcing

Natural forcing

1. Solar forcing: 11-year solar cycle
2. Volcanic forcing: Explosive volcanic eruptions can transport aerosols directly to the stratosphere where they remain for a few years

Anthropogenic forcing

1. Greenhouse gases (dominant in the future, **positive**)

As an example, for CO₂ concentrations in ppm for the period being investigated and for a reference period, the radiative forcing in W/m² can be approximated as

$$\Delta Q = 5.35 \ln \left(\frac{[\text{CO}_2]}{[\text{CO}_2]_r} \right)$$

2. Aerosols

The net aerosol radiative forcing is **negative**, indicating a cooling effect. The negative forcing includes **direct effect** and **cloud albedo (Twomey) effect**

➤ Feedback

The temporary imbalance ΔR in the radiative budget at TOA can be expressed as the sum of the radiative forcings ΔQ (i.e., sustained break-up of the balance due to sun, human, etc.) and the feedback of the changes in global mean surface temperature ΔT_s

$$\Delta R = \Delta Q + \lambda_f \Delta T_s$$

The climate feedback parameter λ_f has a unit of $(\text{W/m}^2)/\text{K}$. To reach an equilibrium, λ_f must be negative

Equilibrium climate sensitivity

It is the global mean surface temperature change after the climate system has stabilized in a new equilibrium state in response to a **doubling of the CO₂ concentration in the atmosphere**

$$\Delta Q = 5.35 \ln 2 \approx 3.7 \text{ W/m}^2, \quad \Delta T_s = -\frac{\Delta Q}{\lambda_f} = -\frac{3.7}{\lambda_f} (\text{°C})$$

Its value is likely to be in the range 1.5-4.5°C

Direct physical feedback

$$\lambda_f = \lambda_0 + \lambda_w + \lambda_L + \lambda_\alpha + \lambda_c$$

1. Planck / Black body feedback (strongest, **negative** feedback)

$$\lambda_0 = -\frac{\partial}{\partial T_s} (\sigma T_s^4) = -4\sigma T_s^3 \approx -3.2 \text{ (W/m}^2)/\text{K}$$

For a doubling of CO₂, Planck feedback gives $\Delta T_{s,0} \approx 1\text{°C}$

2. Water vapor feedback (strongest **positive** feedback)

Greenhouse gases ↑ → Warming → Atmospheric water vapor ↑ → Water-vapor greenhouse effect ↑ → Warming

Clausius-Clapeyron scaling

$$\frac{de_s}{e_s} = \left(\frac{L_v}{R_v T} \right) \frac{dT}{T} \approx 20 \frac{dT}{T}$$

In present (tropical) climate, a 1°C warming drives roughly a 7% increase in saturation specific humidity. If RH is held as constant, then specific humidity will increase as a rate of 7% per K as well

With water vapor feedback, clear sky outgoing long-wave radiation OLR $\propto T_s$

3. Lapse rate feedback (slightly **negative** feedback)

Tropics: Larger change of T in the upper troposphere, more outgoing radiation, **negative**

Polar: Larger change of T at the surface, **positive**

However, when moist convection occurs, this effect is normally accompanied with more water vapor in the upper troposphere, which may offset the negative feedback

4. Surface albedo feedback (**positive** feedback)

Greenhouse gases $\uparrow \rightarrow$ Warming \rightarrow Melting snow & ice \rightarrow Albedo $\downarrow \rightarrow$ Warming

5. Cloud feedback (slightly **positive** feedback)

a. Altitude mechanism (**positive**): Higher clouds in warming Earth, OLR \downarrow

b. Amount mechanism (**positive**): Fewer low-level clouds, solar reflection \downarrow

c. Composition mechanism (**negative**): More water droplets than ice, solar reflection \uparrow