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Varying GARCH option pricing models and their behavior during financial turmoil.

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Statement of originality

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Gifhorn, 15.07.2020

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Abstract

In option pricing, assumptions about future price progression have to be made. Popular models such as the Black Scholes model show significant shortcomings. In this thesis, three different stochastic processes are used for stock price simulation to price stock options. Three different GARCH model variations are used to describe each stochastic process' future volatility and thus simulate stock and option prices. Future volatility is assumed to be non-constant in these model combinations while Black Scholes assumes constant future volatility. Resulting option prices of these nine model combinations are compared to exchange-traded S&P 500 option prices. From that, pricing errors are obtained and compared across each model combination to show differences in pricing accuracy and how model specifics drive them. The results show that GARCH option pricing models value options more accurately than models that assume constant future volatility. Results indicate that GARCH models with less parameters perform better than estimation-heavy GARCH option pricing models before the COVID-19 induced financial turmoil. Performance supremacies of GARCH models compared to constant volatility models persisted throughout recent financial turmoil. During the COVID-19 downturn, models that emphasize the influence of future negative returns ranked above other model combinations.

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Chapter 1

Introduction

Equity derivatives constitute an alternative way for investors to gain or hedge exposure to a company's or index' future development. A stock option gives the buyer the right to buy (Call option) or sell (Put option) the underlying product for a fixed price (strike). He will exercise this right if the stock price is higher (Call) or lower (Put) than the strike price. Given that, he will earn the difference between the stock price and the strike price at maturity of the Call option if the stock price is higher than the strike price or nothing if the stock price is below the strike price. The fair price of such an option is a discounted risk-neutral expected value of the future payoff at maturity. Thus, one needs to make assumptions about future price progressions of the option's underlying security to value the option today. To explain these progressions stochastic processes that depend on a random variable and the process' volatility are used. I use three different stochastic processes in this thesis. Additionally, for each stochastic process I employ three different GARCH models to explain each process' non-constant volatility progression. This results in nine model combinations to explain future stock progression and value options. The GARCH model implicates that future volatility is not constant and changes with the magnitude of changes in the index price.

This thesis is concerned with the mispricing of options from different pricing models over time - with special emphasis on pricing errors during times of extreme market turmoil. The pricing errors also provide information on the degree to which market conditions and imperfections drive these errors. Many option pricing models have been established, with the most famous option pricing model - the Black Scholes (BS) model - showing significant shortcomings when compared to reality. The model inherently postulates that the underlying's price follows a stochastic process that has a constant volatility component. It implies that the underlying's future returns are log-normally distributed and follow a Geometric Brownian Motion with constant volatility (see Black and Scholes (1973)). Together, this stochastic stock price process and the volatility drive the option price. This inherent structure of one component modeling the volatility which is used in a stochastic process to model future stock prices is upheld throughout all nine model combinations which are covered in this thesis. The underlying of the options used in this thesis are different stock price indices on which options are traded on public exchanges.

That way, combinations of stock price processes and volatility models are established to provide a framework to price equity derivatives. After that, they can be compared for their pricing accuracy. Estimated prices are based on the forecasted future price progression and current market conditions. The future price progression is estimated by using Monte Carlo simulation. Based on generated random numbers and each model's estimated parameters future stock prices are simulated. The simulated stock prices at maturity can be used to get simulated payoffs of each option. These payoffs, discounted back to the option's valuation date, are averaged and constitute the option price of the given model combination. To ensure comparability, the same parameter estimation methodology (i.e. maximum likelihood estimation) is used for all model combinations between stock price processes and GARCH volatility models.

After showing the process and its validity, empirical tests of the pricing performance of these model combinations are conducted to compare different combinations and establish supremacy among them. The parameters of each model combination have inherent assumptions about future price progression of the underlying index. Given each model's estimated parameters, the comparison among them may yield insights in option market behavior. If a model performs better that assumes a higher influence of future negative returns on future volatility and thus today's option price than a model that does not assume this we can deduce market's assumptions about future index price progression from this fact.

The empirical analysis is divided into four stages. First, each model combination is used to price a given set of options. The obtained prices are then compared to historical prices in order to calculate their pricing errors. After that, these pricing errors are aggregated and compared to the other combination's pricing error. They are used to test whether the models price options more precisely than pricing errors of models that assume constant future volatility such as the Black Scholes model. Third, I examine the pricing errors by running a linear regressions to see if they are correlated with market dynamics. Finally, the error magnitude and progression throughout the recent financial turmoil is examined. That way, this thesis investigates pricing error behavior during a crisis and whether pricing supremacies are robust during this crisis.

Research question

Equity derivatives are liquid markets with high importance for institutional investors as well as banks that want to take or reduce risk. To do this efficiently, one needs to have models that accurately price options. As options are very specific and show structural differences across properties such as maturity or moneyness one needs to examine pricing models in great depth. Furthermore, an understanding of option markets in general is important for investors to accurately assess risks from investing in options on the basis of a given model. For that reason, pricing precision across the option landscape and different market regimes need to be examined. I aim at providing information on pricing capabilities, their behavior during financial turmoil and what those empirical results imply about option markets in general by examining pricing errors of the above proposed model combinations. It is also answered whether the inclusion of stochastic processes with more realistic statistical assumptions improve option pricing.

This thesis fits into existing literature by focusing on option pricing methods based on GARCH volatility models and different stock price processes. I use a similar framework as Christoffersen et. al. (2004) by applying Monte Carlo simulations and (asymmetric) GARCH models to option pricing. The thesis will, however, focus more on out-of-sample pricing errors of this setup, their sources, and their behavior during the recent COVID-19 induced crisis of financial markets. Thus, the setup gets empirically investigated with a very recent context to a degree most literature did for stochastic volatility models in option pricing. Furthermore, most literature does not differentiate between different market regimes (see Bakshi et. al. (1997)). Empirical investigation of option prices will be further expanded by this thesis. Also, there is little literature on if the stock prices process combination with changing volatility models influence differences in pricing supremacies.

The next section continues with an overview of published literature on volatility modeling and option pricing as well as empirical comparisons of different option pricing models and derives this thesis' hypotheses. After that, the methodology applied in this thesis is detailed. The needed data for this thesis is outlines and described. Afterward, the methodology is detailed and results to test the aforementioned hypotheses are shown. For validation purposes the robustness of the results are checked by employing the methodology to another set of options. This test for robustness with options on a different index from a different continent ought to show that results obtained before are generally also true for these other options. The thesis is concluded by giving an overview of the results' implications, this thesis' limitation and possible future research. Explanations of financial and statistical terms used in this thesis can be found in Table 7.1 in the Appendix.

Chapter 2

Literature review

2.1 Volatility modeling with (asymmetric) GARCH models

In his thesis I use volatilities obtained with time series models with conditional heteroscedasticity. In contrast to constant or equally weighted volatility measures Engle (1982) developed a model that assumes a constant unconditional variance and changing conditional variance based on past realizations and termed it autoregressive conditional heteroscedastic (ARCH) model. Furthermore, he introduced parameter estimation of this new model class via Maximum Likelihood estimation and developed a test for ARCH effects in time series which found that volatilities of financial time series are not constant. The ARCH process introduced by Engle was only dependent on past innovations of the stochastic process z which are assumed to be normally distributed. This was extended by Bollerslev (1986) who expanded the model class by allowing it to be dependent on past conditional variances as well (so-called GARCH - see below). The main idea behind the model class is that the volatility of the time series is continously changing on past shocks but converging in the long run when the forecast window is expanded.

$$\sigma_t^2 = \omega + \alpha z_{t-1}^2 + \beta \sigma_{t-1}^2, \quad z \sim N(0,1)$$

There have been numerous extensions of the GARCH model class. Nelson (1991) based his work upon multiple observations in financial time series data. Mainly, he stated that financial time series show negative correlations between current returns and future return volatility (leverage effect) which was not included in current GARCH models. As seen in the equation above the sign of z doesn't have impact on the magnitude of σ_t^2. The leverage effect implies that negative z ought to have a bigger influence on σ_t^2 than a positive innovation z. The model introduced by Nelson treats the shocks z asymmetrically. A negative return has a higher influence on future conditional variance than a positive shock. In his exponential GARCH model (EGARCH), the logarithm of the conditional variance of a time series is estimated. He employed this model to stock market indices and found that the asymmetric return and volatility relations are highly significant. Additional research in the field of asymmetric GARCH models was published in Glosten et. al. (1993) where a GARCH process was proposed that incorporates this leverage effect too but models conditional variance rather than its logarithm. In general, it is possible to include the leverage effect into a GARCH model by changing the influence of z on σ_t^2. This influence can be shown by the News Information Curve (NIC) $f(z_t)$ which is given in the equation above by $f(z_t) = z_t^2$.

2.2 Option pricing methods

The central papers in option pricing were written by Fischer Black and Myron S. Scholes as well as Robert Merton in 1973. The later Nobel laureates developed a closed-form solution to price stock options that expire in the future based on today's available data. Their underlying assumptions were that stock prices follow a Geometric Brownian Motion (GBM) as their stock price process with a constant volatility parameter. They solved the resulting stochastic differential equations to get a closed-form solution to calculate prices of options. Black and Scholes expanded the model to pricing corporate liabilities in their paper as they can be seen as a combination of options (e.g. Equity resembles a Call option) while Merton used the results to price "down-and-out" barrier options. An extension of this approach was published by Dumas et. al (1998) which applied deterministic volatility function based on time to maturity and price of the underlying to fit implied volatilities as volatility input for the Black Scholes (BS) model.

One of the first papers to invoke Monte Carlo simulations to price options was published by Phelim P. Boyle in 1976. He introduced the usage of simulated stock paths and risk neutrality assumptions to derive option values. The approach also assumed that stock returns were driven by GBM with constant volatility. The future stock prices were simulated with the GBM stock price process and used constant mean and volatility inputs to generate stock price paths of the future. The process employed risk-neutral probability measures proposed by Cox and Ross (1976) from which he deviated by not using binomial trees but the generated stock paths from Monte Carlo simulation. The option price of today is obtained with the before simulated stock price paths. At maturity, each path results in a different payoff whose worth today can be estimated by discounting it with a risk free rate. The mean of these discounted payoff simulations present the option price. He also introduced market realities such as dividends. To price options, he used 5000 stock price paths and antithetic variates (explained later) as a method to decrease standard deviations of simulated option prices. This approach is widely used in practice and theory due to its flexibility and customization possibilities. In following papers such as Kemna and Vorst (1988), the Monte Carlo framework was used to price exotic derivatives as it simplifies their valuation as opposed to closed-form solutions which often don't even exist. Future price movements of the underlying are simulated and no future distribution of the underlying and the resulting future payoff must be assumed.

After his work on pricing options and corporate liabilities, Robert Merton published a paper in 1976 in which he discussed the importance of the underlying stock price process of the option Black-Scholes pricing model. As opposed to the before used Geometric Brownian Motion Merton, he derived an option pricing model that is based on the assumption that stock prices not only follow a Brownian Motion but have a jump component. The process includes Brownian Motion and a jump component of log-normally distributed jumps that are driven by a Poisson process. This allows the return distribution to have a non-normal Kurtosis. Every time the Poisson process results in a non-zero number the stock price is influenced by the jump component beyond the movement from the Brownian Motion.

The Achilles heel of those developed models is that they did not fit empirical option prices very well in magnitude and structure. Option prices are more expensive and thus have higher implied volatilities when they are further in and out of the money. An early attempt to include time-varying volatilities of the underlying in option pricing was by Hull and White (1987) who included a stochastic volatility which was correlated with the stock price Geometric Brownian Motion. They derived an approximate solution as well as used Monte Carlo simulation to price options with changing future volatility based on the stochastic volatility process and found that Black-Scholes does not accurately describe prices on publicly traded option markets.

Based on this setup of the combination of a GBM stock price process which is correlated with a stochastic volatility process, Heston (1993) derived a closed-form solution to value Call options. He used unreal numbers to characterize the terminal condition the two process' differential equations must satisfy in order to derive risk-neutral pricing probabilities. Later, he extended his work on Bond as well as Currency options and shows the impact of the method on return Kurtosis and it's correlation with return Skewness. Another extension of this framework was given by Amin and Ng (1993) who modeled stock prices as well as market returns and their correlations while integrating a stochastic process for interest rates. The pricing errors in Black Scholes pricing were found to be biggest in options of stocks that have a strong idiosyncratic variance (less market dependency).

Later, Bates (1996) combined the jump-diffusion process and the stochastic volatility processes in a holistic option pricing model. Subsequently, the model was applied to pricing Deutsche Mark (DM) exchange rate options to test the approach by comparing the

DM options' implicit distributions. Most notably, implicit volatility distributions matched the model quite well, yet the model failed to rationally explain Kurtosis measures.

Another option pricing approach was developed by Duan (1995) which used the context of the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model to describe the underlying volatility process. The assumed stock price process was an extension of the GBM which included a measure of market risk and Monte Carlo simulation was used to calculate numerical results. The approach was extended to option hedging and showed improvements in systematic Black Scholes forecast errors. Following those results, Heston and Nandi (2000) derived a closed-form solution for options which underlying's conditional variance is following a GARCH model. First empirical tests showed that the model was able to price options more precisely than the BS model.

In Duan and Simonato (1998) the Empirical Martingale Simulation (EMS) was introduced and applied to price options based on Monte Carlo simulation of stock prices with the inclusion of GARCH models. The EMS aims at reducing Monte Carlo price variance by adjusting the simulated stock prices to ensure that those satisfy the martingale property. The application on the GARCH based option pricing framework found that substantial variance reduction in their pricing errors can be obtained by employing EMS. Using this framework and focusing on volatility modeling Christoffersen and Jacobs (2004) published a study investigating the Monte Carlo option pricing based on a market risk stock price model and different GARCH volatility models. They focused on how different innovation curves of GARCH models influence option pricing compared to the Black Scholes model. They found that option pricing with GARCH volatility models favors models allow for the standard leverage effect for in-sample data. Without continuous estimation updates for the GARCH parameters the simpler GARCH specifications performed better. Additionally, they also used loss functions for parameter estimation and found that this results in more parsimonious model parameters.

2.3 Empirical evaluation of option pricing models

To compare some of the aforementioned option pricing models, Bakshi et. al. (1997) studied option pricing models that relaxed the restrictive assumptions of constant volatility and interest rates. They used option pricing models build on stochastic volatility, stochastic interest rates and jumps as explained above. Parameter estimation of each model was achieved by minimizing the sum of squared pricing error of the given model with an in-sample of option prices from June 1988 to May 1991. Their testing on S&P 500 index options out-of-sample found that employing stochastic volatilities leads to a significant pricing improvement compared to Black Scholes. Furthermore, the employment of random jumps improves option pricing more, especially for short term options. For all models and option sets, they could show illiquidity to be a significant contributor to option pricing errors. They also tested hedging performance and found that the addition of jumps or stochastic interest rates did not improve it while assuming future non-constant volatility does.

Following similar frameworks multiple papers were published subsequently to test in-sample parameter estimation and out-of-sample pricing performance. One example was published by Lehar et. al. (2002) who tested Black Scholes as well as models based on stochastic volatility (SV) and interest rates processes and the GARCH model developed by Duan (1995). Their option pricing errors showed that for the FTSE 100 index the GARCH model dominates the BS and SV model. Those results were solidified by Su et. al. (2010) that verified for FTSE 100 options that GARCH models improve Black Scholes option pricing. Apart from the shown work, there have been numerous studies using different option pricing models and employing them to specific option markets. Examples are Ferreira et. al. (2005) that used IEBX-35 option prices to compare models or Jang et. al. (2014) who compared stochastic volatility option pricing models with more recent S&P 500 data and confirmed pricing supremacy of stochastic volatility models.

2.4 Option pricing during market turbulence

The financial crash of 1987 also triggered research about option pricing during this period and the observable parameters implied by common option pricing models before the crisis. In Bates (2000) an option pricing model based on jump-diffusion processes for the stock price model inspired by Merton (1976) was derived and applied to empirical option prices prior to the collapse of the stock market in the fall of 1987. In his paper, he researched the estimated jump parameters implied by option prices before 1987. Those parameters as well as out-of-money option prices increased during August 1987 before the late-1987 crisis showing weak evidence of a crash fear in the market. This information was extracted from option prices since they are containing information about a market's perception of the future underlying price progression/distribution.

In Brownlees et. al. (2012) a comprehensive comparison of GARCH model volatility forecasts was published which included the option pricing precision in times of crises. Their findings indicated that pricing errors decreased when moving from a classical GARCH to asymmetric GARCH models for a specific set of short maturity options during the crisis of 2008. In their work, Lento et. al (2012) compared the Black Scholes model with a non-parametric option pricing model during the turmoil of the '87 and '08 financial crises. Their findings indicated that pricing error reductions of the non-parametric model found during less volatile market conditions disappear in crises. Their stated hypothesis is that structural changes and regime-switching cause the advantages to disappear and make the less estimation driven Black Scholes model perform relatively better. In Duan et. al. (2001) option pricing of an Asian stock index with GARCH option pricing models were research. They found that the simple GARCH model outperforms Black Scholes before, during and after the Asian financial crisis of 1999. Also, Moyaert et. al. (2011) compared pricing and hedging performances of different option pricing models throughout the subprime crisis of 2007/08 and showed that those models outperform the Black Scholes model.

2.5 Hypotheses

Model combinations that include future volatilities based on a with GARCH process should perform better than models that postulate constant future volatility because the assumption of constant future volatility is most likely false as empirical investigations of financial time series show that volatilities are continously changing (e.g. Engle (1982), Cont (2001)). This leads to my first hypothesis:

1. **Option pricing models that use non-constant volatility measures improve option pricing precision significantly.**

The comparison of pricing errors across different model combinations may give evidence about the superiority among them. These results should be robust over different evaluation measures and the whole variation of the option markets. Models differ in included properties with the goal too more accurately explain market behavior which requires more parameters to be estimated. This leads to the hypothesis:

2. **Model combinations with more realistic assumptions result in smaller pricing errors.**

This thesis is not only focused on the pricing of options, but also on how resulting pricing errors behave. A natural extension - after looking at pricing performance over the whole data set - is to further investigate how the pricing errors change over time. In most literature (e.g. Lehar et. al. (2002), Moyaert et. al. (2011)) pricing error differences between option pricing models are compared by an aggregation over the whole time frame. As it is known that underlying distributions and statistical properties of financial time series change (see Cont (2001)) over time and structural breaks in markets are observed one needs to also verify pricing supremacy over time. This should verify hypothesis 2 in that shown pricing advantages are not due to aggregation to one measure but persist over the observed sample before the COVID-19 crisis.

3. **Pricing errors differences are robust over time in less volatile markets.**

Afterward, the obtained pricing errors themselves are further examined. I use linear regressions to estimate how mispricings depend on factors such as option properties and liquidity (see Bakshi et. all (1997), Lehar et. al. (2002)). The regressions aim at verifying that well-performing pricing errors are independent of market conditions. Information of the market should be priced into well-performing models. Thus, strengths and weaknesses of model combinations in relation to each other are shown. Through these regressions, the following can be investigated:

4. **Pricing errors of well-performing model combinations are independent of market conditions.**

In connection to these investigations, an interesting extension is to look at the pricing errors in light of the current financial downturn induced by the measures to curb the spread of COVID-19. It is often postulated that underlying distributions driving financial markets change in the face of economic downturns (see Kim et. al. (2011)). This would mean that option pricing models ought to perform worse in times of financial distress in the markets. The current crisis is suitable for this examination as the gradual increase in option prices may yield insights into how the model combinations themselves are able to price options throughout such a crisis.

5. **Pricing errors increased during recent financial turmoil.**

Furthermore, the observed pricing errors throughout the crisis may also show some results on how the model ranking of the combinations behaves during financial turmoil. Brownlees et. al. (2012) showed that GARCH models with incorporated leverage effect perform better during a crisis. The financial turmoil is likely to change a market's perspective on the future distribution of the index. Thus, it is likely that before found pricing supremacies are not robust over this time of market turbulence. This may also give information about option markets' believe during the COVID-19 crisis.

6. **Relative pricing advantages across model combinations are not robust during recent financial turmoil.**

Chapter 3

Methodology

This chapter introduces the methodology used to obtain option prices from different stock price processes and volatility model combinations. The methodology explains how pricing errors are calculated. It is detailed how the pricing errors is compared for model ranking and how linear regression is used to show the mispricings' dependency on market and option conditions. Below, a timeline of the methodology is shown. The start of the pricing error calculations is defined by the first date with options data. The crisis window is chosen based on the fact that in late February and early March stock markets decreased sharply while implied volatilities increased.

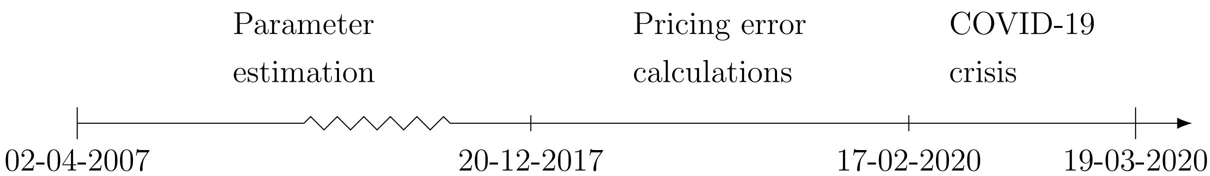

Figure 3.1: Methodology timeline for S&P 500 returns and options.

3.1 Option pricing algorithm

The main method used in this thesis is option pricing with Monte Carlo simulation. Monte Carlo simulated stock price paths are used to obtain option payoffs that can be discounted back to get a fair value of the option at the relevant pricing date. The main input in all stock price processes is the volatility. In the classical Black Scholes model the volatility is assumed to be constant likewise other factors such as the risk free rate. In this thesis this assumption will be relaxed and different GARCH modifications are used to model volatility based on simulated random numbers z. We start by defining the three stock price processes.

$$S_t = S_{t-1} \times \exp\left(\left(r - \frac{\sigma_t^2}{2}\right)dt + (\sigma_t z_t)\sqrt{dt}\right) \tag{1}$$

$$S_t = S_{t-1} \times \exp\left(\left(r - \frac{\sigma_t^2}{2}\right)dt + (\sigma_t z_t + \lambda\sigma_t)\sqrt{dt}\right) \tag{2}$$

$$S_t = S_{t-1} \times \left(\exp\left(\left(r - r_j - \frac{\sigma_t^2}{2}\right)dt + \sigma_t\sqrt{dt}z_{1,t}\right) + \left(\exp\left(\mu_j + \sigma_j z_{2,t}\right) - 1\right) \times y_t\right) \tag{3}$$

In equation (1) the Geometric Brownian motion (GBM) with non-constant volatility is detailed. The second model from equation (2) is a stock price process that takes utility theory into account. It introduces an additional parameter λ which represents that the higher a stock's risk (i.e. volatility) the proportionally higher the return of that stock must be as investors are expected to be risk-averse. This process is used in Christoffersen et. al. (2004) and will be called Market Risk model (MR) in this thesis. The final model is detailed in equation (3) and represents the Euler discretization of a jump-diffusion stock price process (JDP) as introduced for option pricing by Merton (1976). In this model, the stock price has a log-normally distributed component as in the GBM and a jump component which is based on a Poisson process with intensity λ_J. The other parameters σ_j and μ_j represent the jump volatility and the mean of the jumps. Furthermore, the parameter r_j is a drift correction obtained to maintain risk neutrality of the process and is defined as $r_j = \lambda_J \times \left(\exp\left(\mu_j + \frac{\sigma_j^2}{2}\right) - 1\right)$[1]. In all models, dt is assumed to be $\frac{1}{252}$ to represent the discretization of one trading day and r is the 10y Treasury rate of the valuation date of the option.

$$\hat{\sigma}_{t+1}^2 = \omega + \alpha\hat{\sigma_t}^2 f(z_t) + \beta\hat{\sigma_t}^2 \tag{4}$$

The second ingredient in obtaining stock prices to value the option is the underlying volatility model. I focus on GARCH(1,1) models (specified in equation (4)) with varying News Information Curves $f(z_t)$ based on randomly generated numbers z_t. The GARCH model results in a changing conditional variance based on the last random number z_t and last conditional variance σ_t^2. As already mentioned, the leverage effect can be included in the GARCH model by changing the influence of z_t on σ_{t+1}^2. This is done by changing the News Information curve (NIC). The standard NIC is $f(z_t) = z_t^2$. Positive and negative shocks have the same influence on the conditional variance of equation (4). Following the example of Christoffersen et. al. (2004) a tilted "News" NIC $f(z_t) = (|z_t - \theta| - \kappa(z_t - \theta))^2$ (short NGARCH) and also steepened/flattened "News & Power" NIC $f(z_t) = (|z_t - \theta| - \kappa(z_t - \theta))^{2\gamma}$ (short NPGARCH) are used to extend the model and adjust for asymmetric shock and volatility correlation (leverage effect)[2]. For a comprehensive explanation of all parameters of equations (1) to (4) see the Appendix - section H.

[1] See Hilpisch (2019), p. 356-371.

[2] see Christoffersen (2012), p. 76-80.

$$
\begin{array}{cclllll}
 & & \hat{\sigma}_{t+1,1}, z_{t+1,1} \rightarrow S_{t+1,1} & \hat{\sigma}_{t+2,1}, z_{t+2,1} \rightarrow S_{t+2,1} & \ldots & \hat{\sigma}_{t+h,1}, z_{t+h,1} \rightarrow S_{t+h,1} \\
 & \nearrow & \hat{\sigma}_{t+1,2}, z_{t+1,2} \rightarrow S_{t+1,2} & \hat{\sigma}_{t+2,2}, z_{t+2,2} \rightarrow S_{t+2,2} & \ldots & \hat{\sigma}_{t+h,2}, z_{t+h,2} \rightarrow S_{t+h,2} \\
\hat{\sigma}_t, z_t & \rightarrow & \ldots & \ldots & \ldots & \ldots \\
 & \searrow & \ldots & \ldots & \ldots & \ldots \\
 & & \hat{\sigma}_{t+1,m}, z_{t+1,m} \rightarrow S_{t+1,m} & \hat{\sigma}_{t+2,m}, z_{t+2,m} \rightarrow S_{t+2,m} & \ldots & \hat{\sigma}_{t+h,m}, z_{t+h,m} \rightarrow S_{t+h,m}
\end{array}
$$

Figure 3.2: Monte Carlo simulation procedure.

The stock price processes and volatility models are now combined to generate stock price predictions based on random numbers. Figure 3.1 details this procedure as the starting volatility $\hat{\sigma}_t$ and randomly generated number z_i are used to generate new volatilities and stock prices S_{t+i} for each future day i up until the maturity of the option at time point $t + h$. The starting volatility represents the valuation day's conditional GARCH volatility. This price path generation is used $m = 25,000$ times to get $25,000$ price paths to obtain final option payoffs at maturity $t + h$. Those are discounted back to t with the use of the 10y Treasury rate r. A mean of the discounted payoffs is calculated to get a price estimate of the given model combination for a particular option and its properties (more details & a illustrative example in Appendix - Section E).

To reduce simulation variance for the price result antithetic sampling and moment matching are used. In antithetic sampling one set of random numbers is used and then the numbers with opposite signs added. Additionally, the given set is subtracted from their mean and divided by it's standard deviation (see Central Limit Theorem) to ensure that the resulting set of random numbers follows a standard normal distribution $z \sim N(0, 1)$. That way, model assumptions are met and more precise simulation forecasts obtained. Furthermore, Empirical Martingale Simulation as proposed in Duan et. al. (1998) is employed to make sure that the basic martingale property of the generated stock prices is satisfied[3]. The martingale property states that the risk-neutral expected value of the discounted simulation matches today's price. It ensures that today's price represents the best estimator of tomorrow's risk-neutrally discounted price $E^Q[e^{-rt}S_t|S_0] = S_0$.

3.2 Parameter estimation

The proposed model combinations all depend on various parameters that need to be estimated. An introduction of all model combinations and their needed parameters can be found in the Appendix - section A. For each model combination j, a Maximum Likelihood approach is employed. Each model combination includes a set of standard normally distributed random numbers z. Because of that the resulting returns from the stock

[3]See Chan et. al. (2013), p. 87-89 and Appendix - section D.

price process R_t is normally distributed $R_t \sim N(\hat{\mu}_j, \hat{\sigma_t}^2)$ as well. The $\hat{\mu}_j$ represents the stock price process implied mean and $\hat{\sigma_t}^2$ the specification's conditional variance from the GARCH model. Thus, the probability density function of the normal density function can be used to maximize the likelihood of observable R_t with parameter vector θ which contains each model combination's parameters. The likelihood is maximized at the expected value of a normal distribution. Choosing the expected value of a parameter distribution yields an unbiased estimator of the parameter. By maximizing the likelihood a set of unbiased parameters of the model combination is chosen.

$$
\begin{aligned}
L_j(x|\theta, y_t = 0) &= \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\hat{\sigma_t}^2}} \exp\left[-\frac{(R_t - \hat{\mu}_i)^2}{2\hat{\sigma_t}^2}\right] \\
\text{Max}[ln(L_j(x|\theta))] &= \text{Max} \sum_{t=1}^{T} \frac{1}{2}\left[-ln(2\pi) - ln(\hat{\sigma_t}^2) - \frac{(R_t - \hat{\mu}_i)^2}{\hat{\sigma_t}^2}\right] \\
\hat{\mu}_{GBM} = \left(r - \frac{\hat{\sigma_t}^2}{2}\right); &\qquad \hat{\mu}_{MR} = \left(r + \lambda\hat{\sigma}_t - \frac{\hat{\sigma_t}^2}{2}\right)
\end{aligned}
\tag{5}
$$

For the combinations based on Merton's Jump process, the Likelihood function needs to be adjusted as the discretization of equation (3) changes on the basis of if a jump y_t occurs at time t. The jumps are Poisson distributed and thus have either 0 or 1 as their realization as the number of jumps is limited to one per day in this thesis. A likelihood function for the case that $y_t = 0$ or $y_t = 1$ is true is needed. The likelihood functions and jump probabilities from Christoffersen et. al. (2008) were used. The final likelihood is then a combination of the two likelihoods weighted by the probability of a jump occurring or not.

$$
\begin{aligned}
L_j(x|\theta, y_t = 0) &= \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi\hat{\sigma_t}^2}} \exp\left[\frac{\left(R_t - r + \frac{\hat{\sigma_t}^2}{2} + r_J\right)^2}{2\hat{\sigma_t}^2}\right] \\
L_j(x|\theta, y_t = 1) &= \prod_{t=1}^{T} \frac{1}{\sqrt{2\pi(\hat{\sigma_t}^2 + \sigma_J^2)}} \exp\left[\frac{\left(R_t - r + \frac{\hat{\sigma_t}^2}{2} + r_J - \mu_J\right)^2}{2(\hat{\sigma_t}^2 + \sigma_J^2)}\right] \\
Pr(y_t = 0) = 1 - \lambda_J \times dt; &\qquad Pr(y_t = 1) = \lambda_J \times dt \\
\text{Max}[ln(L_j(x|\theta))] = \text{Max} \sum_{i=1}^{T} ln\,(&Pr(y_t = 0) \times L_j(R_t|\theta, y_t = 0) + Pr(y_t = 1) \times L_j(R_t|\theta, y_t = 1))
\end{aligned}
\tag{6}
$$

The combination of the two Log-likelihoods yields the final Log-likelihood function which is dependent on the GARCH and jump parameters. The functions are maximized to obtain all parameters for each model combination. Comparisons across the different models are done with the Likelihood Ratio test as defined in the Appendix - section C.

That way, it is shown if each extension of a model combination explains the estimation data set significantly better. The parameters can be used in simulating stock price paths and ultimately value the options (further explanation of the estimation procedure in the Appendix - section B).

3.3 Option pricing evaluation

After estimating the model parameters and obtaining the model prices (further details in the Appendix - section E), a pricing error for each of the 9 model combinations j and option n with its properties (e.g. maturity) is obtained $E_{t,j}^{(n)} = C_{t,j}^{(n)}(\hat{\sigma}_t^2) - C_t^{(n)}$. To solidify supremacy claims across model specifications multiple tests are carried out. To test hypothesis **1**, a t-test is employed to see whether specification j has significantly lower squared pricing errors than a Monte Carlo simulation based on GBM and constant volatility (MC) and the Black Scholes model (BS). A Geometric Brownian Motion with constant volatility has the same assumption as the Black Scholes model. Thus, BS and MC should yield approximately same results and verify this thesis' approach. The test-statistic has to be higher than the 99.5% percentile of the standard normal distribution (about 2.57) in order to show a 1% significance.

$$t_{MC_i} = \frac{MSE_j - MSE_{MC}}{\sqrt{\frac{Variance(SE_{t,j}^{(i)})}{N} + \frac{Variance(SE_{t,MC}^{(i)})}{N}}} \tag{7}$$

$$t_{BS_i} = \frac{MSE_j - MSE_{BS}}{\sqrt{\frac{Variance(SE_{t,j}^{(i)})}{N} + \frac{Variance(SE_{t,BS}^{(i)})}{N}}} \tag{8}$$

Furthermore, for each option on each day squared pricing errors can be calculated $SE_{t,j}^{(i)} = (E_{t,j}^{(i)})^2$ and compared over the time frame of the option data sample $1, ..., T$ as well as aggregated to one number as a mean across all options, and times $MSE_j = \frac{1}{T}\frac{1}{N}\sum_{t=1}^{T}\sum_{i=1}^{N} SE_{t,j}^{(i)}$ to test the hypotheses **2 & 3** for the different model combinations. The same is done for relative pricing errors $e_{t,j}^{(i)} = \frac{C_{t,j}^{(i)}(\hat{\sigma}_t^2) - C_t^{(i)}}{C_t^{(i)}}$ that can be aggregated by means or medians as well. The MRPE is the mean while the mRPE is the median of the relative pricing error $e_{t,j}^{(i)}$ and is another way to aggregate pricing errors over the day. For investigating hypothesis **2** the aggregation over the error whole error calculation timeframe is used (as in most literature). For the examination of hypothesis **3** the daily aggregation of the errors over the timeframe is looked at to solidify supremacy findings.

3.4 Investigation of option pricing errors

After obtaining the pricing errors as described above they can be investigated to gain knowledge about the properties of the model combinations and financial (option) markets.

To test hypothesis **4** a linear regression of the errors on several market factors is done in order to see their explanatory power. Following literature such as Bakshi et. al (1997) and Jang et. al. (2014), the percentage errors are regressed upon several factors. The used linear regression is:

$$\epsilon_{j,t}^{(n)} = \beta_0 + \beta_1 \frac{S_{i,t}}{K^{(n)}} + \beta_2 SPREAD_t^{(n)} + \beta_3 r_{i,t} + \beta_5 \mathrm{VIX}_t + \beta_6 \tau_t^{(n)} + u_{j,t}^{(n)}. \tag{9}$$

The regression tests whether the pricing errors of model combination j for option n on stock index i depend on moneyness $\frac{S}{K}$, the Bid-Ask Spread, the index return, implied volatility (VIX) and the time left to maturity τ.

3.5 Option pricing during the COVID-19 crisis

The pricing errors during the crisis time are calculated and examined to see whether they have increased during recent market turbulence and test hypothesis **5**. Furthermore, the change in pricing precision through the crisis is examined to get an overview of how pricing errors have developed during this time. This may yield results on whether pricing advantages of certain models shown before are robust over a time of heightened market volatility. The pricing error behavior during the crisis gives indication on whether hypothesis **6** is true.

Chapter 4

Data & Description

This chapter describes which data I use in this thesis and how I collected it. Furthermore, there are descriptive statistics about the data to give the reader an overview of the statistical measures of the data sets and their implication for this thesis.

4.1 Risk-free rate and stock index returns

An important input in parameter estimation, as well as option pricing, is the risk-free rate. As a proxy, the 10-year yield of the US-Treasury is used. I obtained the data via the Yahoo! Finance API with Python. For parameter estimation purposes the daily yield on the corresponding day of the realized index return during the estimation window is used. For option pricing, the corresponding yield on which the given option is valued is used. Descriptive statistics of the full sample of 10 year Treasury yields can be found in the Appendix - section G.

The second piece in calibrating the model combinations as well as explaining option pricing errors are the returns of the option's underlying stock indices. In this thesis options on the S&P 500 and ESTOXX 50 are priced with the defined 9 model combinations. The stock log-returns are obtained from stock prices downloaded from the Yahoo! Finance API and defined as

$$R_t = \log\left(\frac{S_t}{S_{t-1}}\right)$$

which results in a time series of daily log-returns of the stock indices. Below, descriptive statistics for the time series of stock index Log-returns are shown.

	S&P 500	ESTOXX 50
N	3166	3166
Start date	02-04-2007	02-04-2007
Stop date	19-03-2020	19-03-2020
Min	-12.77%	-13.24%
Max	10.96%	10.44%
Daily average	0.02%	-0.02%
Daily median	0.07%	0.03%
Daily standard deviation	1.31%	1.48%
Daily skewness	-0.70	-0.40
Daily kurtosis	13.21	7.49

Table 4.1: Descriptive statistics of stock index Log-returns.
The characteristics and statistical measures are based on daily Log-returns within the whole data set and were calculated with pre-installed functions of Microsoft Excel.

The descriptive statistics show that the indices have a near-zero mean, above 1.3% daily standard deviation and slightly negative skewness. The most glaring measure of this table is that the kurtosis of daily Log-returns is much higher than the Kurtosis of a normally distributed random variable (which is 3) for both indices. These facts hint at a non-normal underlying distribution as the return realizations show a smaller skew, as well as fatter tails, than what is expected by a normally distributed random variable. The kurtosis of the S&P 500 is considerably bigger than the kurtosis of the ESTOXX 50 returns. Thus, extreme realizations are more common for the S&P 500 returns. In general the data description indicates that the two index return series do not follow the same underlying distribution. This is further verified when looking at cumulative return of the two indices as shown in Figure 7.2 in the Appendix - section G.

4.2 Option data

Empirical option prices are obtained from Thomson Reuters DataStream via their Eikon platform. From all available option chains Bid & Ask quotes as well as closing prices are downloaded via a Python script. Furthermore, the option properties such as Strike price, expiration date etc. are obtained as well. Combining those data sources with the underlying index prices and risk-free rate from Yahoo! Finance a data set of option prices, properties and market conditions for each underlying index is attained (a more detailed description can be found in the Appendix - section F).

To adjust for market imperfections and prevent biases in the results multiple filters are employed on each option data set. A lot of active options are rarely traded. Because of that the observed option prices don't accurately reflect a market's opinion on its worth. To have a meaningful data set those options need to be filtered out to prevent biases in the resulting pricing errors. Following Bakshi et. al. (1997) and Huang et. al. (2004) common filters are used and detailed below.

1. No Close and Bid/Ask spread are simultaneously available on Eikon.
2. The option identifier from Eikon is used for several options with different characteristics.
3. Strictly positive Bid/Ask price spread.
4. Strictly positive Option close price.
5. Option price P of less than $\frac{1}{2}$ USD/EUR.
6. Violation of the arbitrage condition. The option price is higher than its current payoff. For Call options $C_t^i(\tau) \geqslant \max(S_{i,t} - K^n, 0)$.

The filters result in the following changes in the option data sets. For both indices, a considerable amount of data points have been filtered out. The main criteria were that no Bid/Ask Quote and Close price data were present on the same day for the same option and that the arbitrage condition of the option was not met. This indicates that some parts of the active option chains are not actively traded. For ultimate validation, further examination with consideration of open interest and trading volume would be necessary, however.

Criterion	S&P 500	ESTOXX 50
Bid Ask Quotes	580,914	134,007
Close Quotes	117,402	97,622
No Bid/Ask & Close	116,205	93,473
Unambiguous key	106,900	93,473
Positive spread	104,971	89,196
Positive price	104,971	89,196
Price smaller 0.50 USD/EUR	104,163	87,340
Arbitrage Condition not met	102,266	67,705

Table 4.2: Available option data points after filtering.
The above-described filter were applied to all available data downloaded from Eikon via Python and Microsoft Excel. The data manipulation was done with Python's Pandas library.

The resulting data sets are further examined for their statistical moments in Table 4.3 for the S&P 500 and in Table 4.4 for the ESTOXX 50 index options. Following Lehar et. al. 2002 seven percentiles are depicted in the table to show the diversity of the data sets. There are options available in the data sets with relatively small and very high strikes (indicating far out and in the money options) as well as options with long and short maturities giving a whole picture of option market dynamics. For the S&P 500 options over 40,000 Call and over 60,000 Put options are available while over 20,000 Call and 40,000 Put options on the ESTOXX 50 are available.

Option price properties

	Minimum	10%	25%	50%	75%	90%	Maximum
S&P500	2,237.40	2,470.50	2,659.41	2,881.40	3,112.76	3,283.66	3,386.15
Strike Price	500	1850	2325	2760	3150	3425	4900
Option close price	0.51	7.2	27	92.3	201.39	334.94	2694
Bid/Ask Spread	0	0.6	1.2	2.8	7.8	15.9	654.5
Time to maturity in days	19	46	82	188	335	536	755

Table 4.3: Option price sample properties for S&P 500 options.

For the whole data set of 102,266 options the basic statistical percentiles are shown in the table as calculated with Microsoft Excel for 41,765 Call and 60,501 Put options.

Option price properties

	Minimum	10%	25%	50%	75%	90%	Maximum
ESTOXX 50	2385.82	2795.97	3165.2	3403.51	3569.45	3752.52	3865.18
Strike Price	500	2100	2650	3200	3550	3750	5000
Option close price	0.6	4.7	29	114.5	272.9	517.88	2356.2
Bid/Ask Spread	0	0.9	2.4	6.5	11.7	35.5	3702.2
Time to maturity in days	19	45	74.5	206	393	505	723

Table 4.4: Option price sample properties for options.

For the whole data set of 67,705 options the basic statistical percentiles are shown in the table as calculated with Microsoft Excel for 23,979 Call and 43,726 Put options.

Chapter 5

Results

5.1 Estimation results

Based on the aforementioned methodology the parameters for the model combinations between equation (1) to (3) and the three variations of equation (4) are estimated via maximum likelihood estimation (MLE). The basis of the parameter estimation are the 10 year yield r and stock return data R described in Chapter 3 and 4. The results of this estimation for the S&P 500 return data series are displayed in Table 5.1.

The estimation results show common properties of GARCH models with a small short- and bigger long-term persistence. Furthermore, the inherent stock price process parameters are estimated. For model combination (4)-(6) that are reliant on the stock price process from equation (2) the market risk measure is estimated. This measure captures a market risk premium in comparison to the risk-free rate adjusting for the fact that market participants are risk-averse. This parameter - consistent with theory and literature - is positive for all three model combinations. This implies that the returns compensate besides the risk free rate for variance in the returns. This effect can be quantified at roughly 0.02-0.74% per day. The last three columns show the estimated parameters of the Jump diffusion stock price process and indicate a Poisson intensity from 1.3 to 0.4. The negative jump mean suggests that abrupt movements in stock return data are expected to be negative which is in line with the fact that jumps are often caused by bad news.

The log-likelihood is constantly increasing along with each specification. Given this fact the Likelihood-Ratio test (defined in Appendix - section C) also suggests that the model combinations incorporating asymmetric News Information Curves provide additional information and improve the fit of the model for all three stock price processes. The steepening parameter γ in model combinations (3), (6) and (9) does not improve model fit significantly (further parameter explanation in Appendix - section H).

Parameter	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
ω	2.18E-06	2.48E-06	2.51E-06	2.18E-06	2.48E-06	2.49E-06	2.27E-06	2.45E-06	2.46E-06
β	8.62E-01	7.63E-01	7.66E-01	8.62E-01	7.63E-01	7.64E-01	8.57E-01	7.63E-01	7.64E-01
α	1.23E-01	5.83E-02	8.62E-02	1.23E-01	5.82E-02	8.32E-02	1.28E-01	5.86E-02	8.60E-02
θ		1.20E+00	8.82E-01		1.19E+00	9.37E-01		1.19E+00	9.40E-01
κ		2.92E-01	4.71E-01		2.94E-01	4.14E-01		2.90E-01	3.92E-01
γ			8.43E-01			8.67E-01			8.65E-01
λ				1.96E-04	7.35E-03	5.87E-03			
λ_J							1.54E+00	1.32E+00	3.66E-01
μ_J							-3.87E-04	-2.66E-05	-1.11E-04
σ_J							9.82E-04	1.02E-03	5.40E-05
Properties									
N	2625	2625	2625	2625	2625	2625	2625	2625	2625
Log-Likelihood	8457.93	8549.77	8550.36	8457.97	8549.84	8550.46	8465.50	8549.79	8550.43
LR p-value		0.00	0.30		0.00	0.30		0.00	0.30

Table 5.1: Parameter for the S&P 500 return estimation window

All parameters are estimated upon daily S&P 500 Log-returns R_t during the parameter estimation window. The given model combination uses the relevant log-likelihood given in equation (5) and (6) that are maximized to estimated each parameter. Following Christoffersen (2012) the starting value for volatility at $t = 0$ is the standard deviation of the returns from the estimation window. Afterward, each likelihood based on the GARCH forecast and implied standardized shocks z are calculated. The log-likelihoods are finally maximized to obtain the shown parameters. The parameters from (1)-(3) are used for the equations (1) & (4), from (4)-(6) for equations (2) & (4) and the parameters obtained in column (7)-(9) for model combinations from equations (3) & (4). See Appendix - section H for an overview.

Following the estimation, the difference in volatility models can be demonstrated. Table 5.1 shows that the additional GARCH parameters for the NGARCH and NPGARCH specifications are similar for the three stock price processes. Their influences in comparison to the standard GARCH model (columns (1), (4), (7)) are demonstrated in Figure 5.1 that displays the News Information Curve (influence of random number on the volatility forecast) for each of the different specifications.

One can observe that the influence of negative shocks are higher for all deviations from the standard GARCH model. This implies that a high past negative return (negative shock) has a higher influence on next period's conditional variance than a position return which is in line with stylized facts about return time series (see Cont (2001)). This asymmetry is more pronounced for NGARCH model specifications while the NPGARCH specifications are less responsive to negative and positive shocks for all three stock price processes. The true influence of z on conditional variance varies with α, however. In table 5.1, α for the NGARCH models is shown to be smallest, negating part of the higher influence of negative z's on $\hat{\sigma}_t{}^2$ induced by θ and κ.

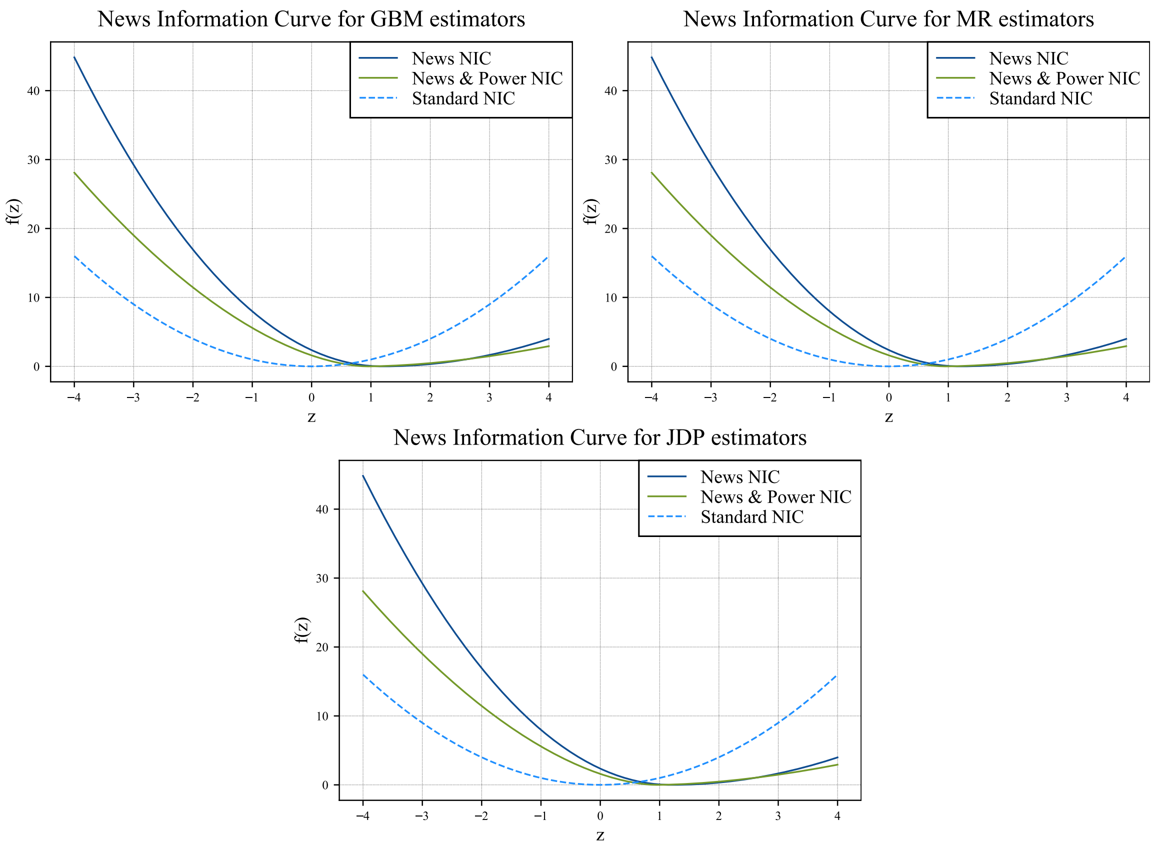

Figure 5.1: NIC for the estimated parameters of all combinations on the S&P 500 index.

The Figure shows varying News Information Curves and illustrates differences in the influence of shocks z on next period's conditional variance of the given GARCH model. Specifications (1)-(3) are on top, (4)-(6) in the middle and (7)-(9) on the bottom.

5.2 Pricing error examination

Based on the estimated parameters from Table 5.1, the methodology from Chapter 3.2 and the option data set each model's empirical option prices are calculated. Now, empirical errors for each model combination as well as a Monte Carlo simulation with constant volatility and the Black Scholes model are obtained. To test hypothesis **1** the errors are transformed into squared errors and t-tests (equation (7) and (8)) are employed. The test-statistic and p-value of the test for significantly different means in squared errors are presented in Table 5.2. They show that each model has a significantly lower MSE than the MSE of the MC (4,937) and BS (4,940) model. The insignificant difference between the MSE_{MC} and MSE_{BS} shows the approach's validity as the two models have the same underlying assumptions. Even though the pricing errors are probably not i.i.d. normally distributed the change in magnitude and the result of the t-test give enough reason to confirm hypothesis **1**.

Option pricing errors properties

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
MSE	2,001	2,849	3,120	2,147	2,622	2,796	2,111	2,351	3,358
MRPE	28.9%	193.0%	205.5%	29.8%	177.7%	185.6%	40.8%	160.0%	219.2%
MC	76.53*	46.36*	39.54*	72.08*	52.53*	47.94*	69.13*	60.64*	33.26*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
BS	76.51*	46.32*	39.49*	72.07*	52.50*	47.91*	69.11*	60.17*	33.22*
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)

Table 5.2: Mean Squared error and t-test for S&P 500 model combinations.

In row 1 the means of squared pricing errors of each model combination across the S&P 500 option data are given. In the rows 3 & 4 as well as 5 & 6 the t-statistic and p-value for different mean squared error of each model combination with the Monte Carlo pricing with constant volatility and the Black Scholes model are displayed.

* 1% significance-level

The mean squared error is lowest for the simple GBM & GARCH (1) combination and generally lower when using model combinations with less parameters. In contrast to Table 5.1, this hints at changing underlying parameters of the model specification between the estimation and error calculation window. There are no apparent pricing advantages of model combinations that include more empirically sound assumptions such as the leverage effect or jumps. Those observations also hold true when the mean of relative pricing errors is examined. On, average options are more accurately priced by model combinations involving the simple GARCH model as their future volatility assumption.

Moneyness	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	BS	MC
<0.9	88.76%	404.11%	430.21%	419.84%	374.32%	385.78%	114.01%	324.78%	458.83%	459.70%	459.27%
>0.90, <0.95	12.81%	64.90%	74.78%	107.88%	60.58%	66.61%	17.04%	55.15%	73.91%	95.47%	95.62%
>0.95, <1.00	4.49%	22.73%	24.68%	24.75%	21.36%	22.80%	5.85%	19.48%	26.21%	29.08%	29.06%
>1.00, <1.05	4.12%	18.72%	20.38%	1.91%	17.60%	18.67%	5.28%	16.65%	21.54%	23.73%	23.75%
>1.05, <1.1	9.05%	26.87%	29.49%	-16.82%	25.31%	27.37%	10.50%	24.41%	30.49%	36.28%	36.31%
>1.1	37.15%	309.91%	328.49%	-61.13%	284.32%	297.14%	56.14%	258.23%	351.77%	387.47%	387.05%
Maturity	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	BS	MC
<0.4	5.86%	272.75%	310.60%	12.66%	248.98%	285.95%	17.06%	242.16%	310.95%	397.44%	397.75%
>0.4, <0.6	54.02%	235.42%	263.89%	-3.51%	218.89%	244.67%	67.96%	202.62%	265.19%	345.50%	345.32%
>0.6, <0.8	29.26%	203.62%	226.20%	17.02%	186.49%	202.57%	41.42%	169.85%	230.63%	314.21%	313.90%
>0.8, <1.0	38.12%	181.14%	196.84%	28.73%	167.39%	177.71%	49.28%	150.73%	205.29%	218.48%	218.60%
>1.0	26.79%	175.28%	178.42%	41.20%	161.41%	159.67%	36.90%	141.42%	199.55%	185.33%	185.14%

Table 5.3: S&P 500 option pricing errors for all model combinations.

The pricing error models (1)-(3) are based on equations (1) & (4), columns (4)-(6) on equations (2) & (4) and the errors in columns (7)-(9) from equations (3) & (4). With 25,000 simulations a Monte Carlo simulation with each GARCH models' conditional volatility as a starting point was employed to value the options.. The relative pricing errors are defined as $E_{t,j}^{(n)} = \frac{C_{t,j}^{(n)}(\hat{\sigma}_t^2) - C_t^{(n)}}{C_t^{(n)}}$ *and averaged.*

Additionally, mean percentage pricing errors are shown in Table 5.3 to further solidify model differences across moneyness and maturity. Generally, one can observe that pricing errors are smallest for options with moneyness close to 1. A difference in pricing precision based on maturity don't allow for such direct interpretation. The overall better performing combinations (1), (4) and (7) (i.e. the simple GARCH models) tend to perform better for short-term options which is not the case for the worse performing model combinations. Moyaert et. al. (2011) postulate that steeper implied return Skewness measures drive short term option prices. Thus, the results suggest that the simple GARCH model captures this dynamic better than the other pricing models.

Overall, it remains that the GBM & GARCH (1) specification performs best which is in line with results from Christoffersen et. al. (2004) who showed that out of sample pricing favors GARCH models with less parameters for option pricing when estimated parameters are not continuously re-estimated. The pricing advantages from model (1) can be seen across moneyness and maturities indicating that it represents the best model for out-of sample options pricing. These results contradict hypothesis **2**.

Those pricing advantages are also apparent over time. In Figure 5.2 Figure 5.3 aggregations of absolute and relative pricing errors over time are shown to solidify supremacy findings and test hypothesis **3**. For some days, averages are very high or low across all combinations because of outliers in moneyness and maturity. Because of that the aggregated squared and relative errors are quite volatile and don't show a clear picture. Therefore, a 30 day moving average of the daily mean squared and mean relative pricing errors for each combination and the MC as well as BS model are shown.

From the two Figures, it becomes apparent that the pricing advantages of model (1) shown in the previous tables and tests are stable over time and thus confirm hypothesis **3**. The pricing advantages of the model combinations which assume non-constant future volatility in comparison to the MC and BS model are also stable over time. The GARCH & GBM combination shows small squared pricing errors throughout the testing period. The NGARCH and NPGARCH combinations' pricing errors are higher. The NGARCH model performs slightly better while both are advantageous in comparison to the models with constant volatility. Additionally, the middle graph shows that the inclusion of the risk aversion parameter changed the dynamic of the pricing errors of the GARCH pricing model. The risk aversion parameter that changes the mean of the underlying stock price process in combination with a symmetric NIC leads to a better pricing performance towards the end of the out of sample period. That change coincides in the drop of interest rates (risk-free rate - shown in the Appendix - section G) and may be apparent of the need of continuous model re-estimation for higher pricing precision (as done in Duan et. al. 2001).

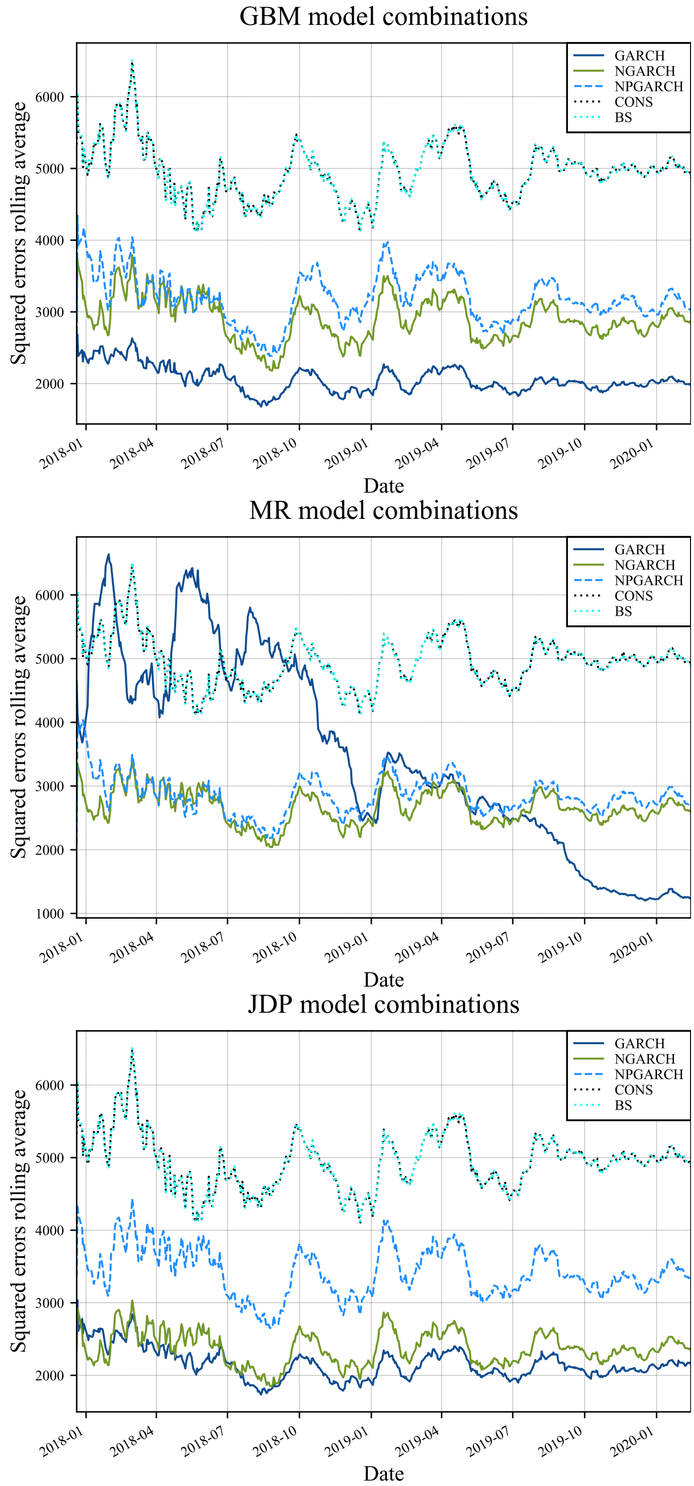

Figure 5.2: Aggregated squared pricing errors over time.

The Figures depict the rolling average of daily averages of squared pricing errors for each model combination. To aggregate the errors and reduce singular noise a 30 day moving average is used. Specifications (1)-(3) are on top, (4)-(6) in the middle and (7)-(9) on the bottom.

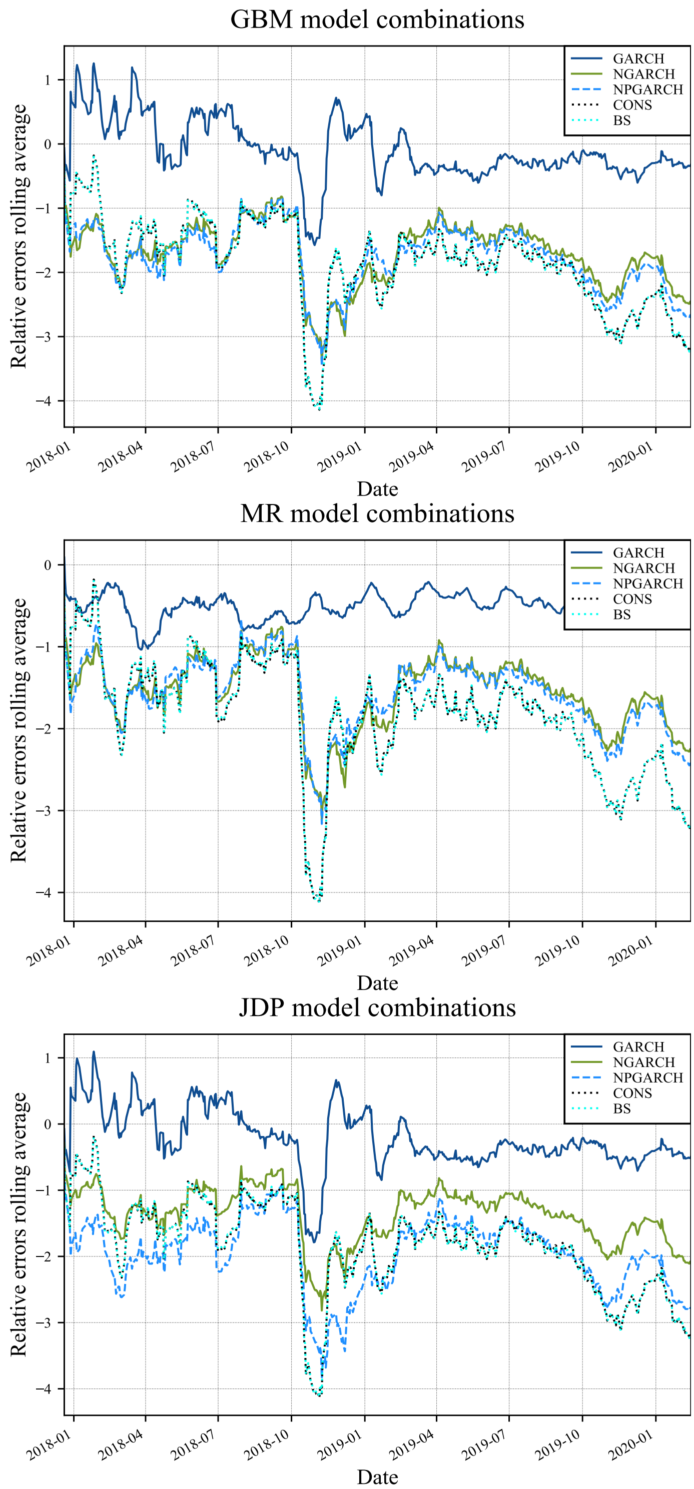

Figure 5.3: Aggregated relative pricing errors over time.

The Figures depict the rolling average of daily averages of relative pricing errors for each model combination. To aggregate the errors and reduce singular noise a 30 day moving average is used. Specifications (1)-(3) are on top, (4)-(6) in the middle and (7)-(9) on the bottom.

After the classification of the error magnitude of the employed model combinations, the relative errors are regressed upon several market variables as shown in equation (7). The results can be seen in Table 5.4. The pricing errors can be partly explained by option characteristics (moneyness and maturity), market conditions such as return and volatility measures (VIX - explanation in Table 7.1 in the Appendix) and market imperfections like the Bid/Ask spread. The explanatory power of these models vastly differs for the combinations only employing the simple GARCH model. The regressions show that apart from the market imperfections no other factors significantly explain pricing errors of combination (1) and (7). This result further hints at parsimonious model combinations performing better than the more estimation driven option pricing models without constant re-estimation.

The regression is limited in its explanatory power. It is almost certain that there is an omitted variable bias (OVB) as pricing errors are probably dependent on a whole range of variables that are not in the regression (e.g. daily liquidity of the option or intraday based statistical measures - see Ferreira et. al. 2005). Furthermore, the regression is likely to suffer from multicollinearity as returns and implied volatility (VIX) are correlated. Resulting errors in parameter estimations are not of importance for this thesis, however. The regression is not used to test causality between the market factors and pricing errors. As stated in Ferreira et. al. (2005) the regression is a means to understand the structure and behavior of pricing errors. Thus, the regression quantifies dependencies of pricing errors that allow conclusions about model combinations and market behavior. Additionally, the changes in the regression results give us clues about differences in pricing accuracy of the different model combinations and their implication of how markets perceive future returns.

The results are in line with literature as Bakshi et. al. (1997) and Jang et. al. (2014) who find that models with relatively high pricing errors are dependent on market conditions. They have higher R^2 and significant regression parameters. The only parameter that is significant along all model combinations is the $SPREAD$ parameter β_2. As this measure shows market imperfectness (as a result of definite liquidity) the correlation of pricing errors with this factor cannot be erased by picking the best-performing model. In conclusion, the results show that pricing models with non-constant volatility are able to reduce institutional sources of pricing errors of other models that stem from unrealistic model assumptions (i.e. constant volatility) or out-of-sample errors stemming from changing underlying distributions.

$$\epsilon_{j,t}^{(n)} = \beta_0 + \beta_1 \frac{S_{i,t}}{K^{(n)}} + \beta_2 SPREAD_t^{(n)} + \beta_3 r_{i,t} + \beta_5 \text{VIX}_t + \beta_6 \tau_t^{(n)} + u_{j,t}^{(n)}$$

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	BS	MC
cons	-0.2913	7.1569***	6.8810***	-1.6974***	6.4807***	5.9612***	0.1464	5.6057***	8.2376***	7.1501***	7.1215***
	(0.66)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.82)	(0.00)	(0.00)	(0.00)	(0.00)
S/K	0.1657	-7.5709***	-7.7170***	1.9250***	-6.8809***	-6.7231***	-0.2927	-6.1059***	-8.6924***	-8.5501***	-8.5318***
	(0.74)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.56)	(0.00)	(0.00)	(0.00)	(0.00)
Spread	0.0403***	0.1166***	0.1159***	0.1149***	0.1085***	0.1077***	0.0472***	0.0924***	0.1319***	0.1191***	0.1193***
	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
return	3.9856	7.6301*	8.0965*	0.0290	7.5451*	6.5075	4.4278	6.9598*	8.6665*	11.8124*	12.0334*
	(0.42)	(0.10)	(0.08)	(0.98)	(0.09)	(0.14)	(0.37)	(0.10)	(0.08)	(0.09)	(0.08)
VIX	-0.0165	-0.0531***	-0.0451***	-0.0307***	-0.0487***	-0.0446***	-0.0207	-0.0404***	-0.0596***	-0.0542**	-0.0535**
	(0.31)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.20)	(0.00)	(0.00)	(0.02)	0.02
Maturity	-0.0707	0.1913***	0.3523***	-0.6444***	0.1717***	0.3274***	-0.0517	0.2257***	0.2116***	0.7585***	0.7566***
	(0.25)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.41)	(0.00)	(0.00)	(0.00)	(0.00)
R^2	0.04%	16.18%	16.29%	16.43%	14.84%	13.91%	0.10%	13.13%	18.24%	10.46%	10.44%

Table 5.4: Linear regression of S&P 500 option pricing errors.

The pricing error models (1)-(3) are based on equations (1) & (4), columns (4)-(6) on equations (2) & (4) and the errors in columns (7)-(9) from equations (3) & (4). The relative pricing errors are defined as $e_{t,j}^{(n)} = \frac{C_{t,j}^{(n)}(\hat{\sigma}_t^2) - C_t^{(n)}}{C_t^{(n)}}$*. For the regression heteroscedasticity-consistent standard errors are employed. The p-values of each estimator's t-test is shown in parentheses.*

*** 1% significance-level, ** 5% significance-level, * 10% significance-level

5.3 Option pricing during the COVID-19 financial turmoil

For further assessment of GARCH-model option pricing, pricing errors for recent empirical option prices are examined. In Q1 2020 a sharp drop in asset prices occurred. As it was the first time since the financial crisis of 2008 that implied volatilities (explained in Table 7.1 in the Appendix) rose that quickly an examination of the model combination's behavior during that time is warranted. The problem with the construction of a classical event study in option pricing - especially during the COVID downturn - is that there is no singular date on which the news broke and markets corrected. Indicated by the VIX for example option prices steadily rose during the pandemic induced downturn (see Figure 5.4). Thus, I focus on the overall performance of the models during this time of high and rising implied volatility. It can be shown that in the time up until late February a combination of an underlying Geometric Brownian Motion with a GARCH model was most precise in empirical option pricing. For further investigation, several pricing error measures during the downturn are displayed in Table 5.5 & Table 5.6 and show that the model specification (1), (4) and (5) display the smallest empirical pricing errors in relative terms. Model (5) shows smallest squared overall and relative pricing errors when compared across moneyness. When looking at the numerical results of Table 5.5 & Table 5.6, model combinations, that include the leverage effect seem to price options more precisely now. In comparison to the pricing errors from Chapter 5.2 the pricing seems to be less precise (support hypothesis **5**). This is supported by bigger mean squared pricing errors.

Option pricing errors properties

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	MC	BS
MSE	6,161	3,918	21,515	6,076	2,265	3,676	6,547	3,734	4,265	6,908	6,907
mRPE	9.2%	33.2%	52.6%	8.3%	17.9%	33.5%	7.3%	31.1%	36.2%	41.5%	41.4%

Table 5.5: Mean Squared errors and median relative pricing errors of the event window.

In row 1 the mean of squared pricing errors of each model combination across all the S&P 500 option data during the event window is given. Rows 2 shows the median of relative pricing errors for all options during the crisis.

For further evaluation of the model performances, the median relative pricing error for on the money options is examined over time in Figure 5.4. It is appended by the depiction of the VIX index indicating the rise of option prices during this time. The Figure shows that pricing errors for the constant volatility (MC) and Black Scholes model (BS) rise with the VIX from overpricing to underpricing the options. This is logical as implied volatility rises (see VIX) but the used volatility is assumed to be constant (see Black Scholes pricing explanation in Appendix - section E). This development is not apparent in the other option pricing models. For each stock price process, median relative pricing error remains fairly constant over time. This indicates that non-constant volatility models perform better in option pricing during times without major financial turmoil. There is no clear performance difference between the GARCH models, however, with the numerical relative results favoring model combination (5). This result suggests that hypothesis **6** is correct in that pricing advantages change during financial crises. An explanation is that option markets assume different future distribution of returns due to new information from the crisis being priced into the market.

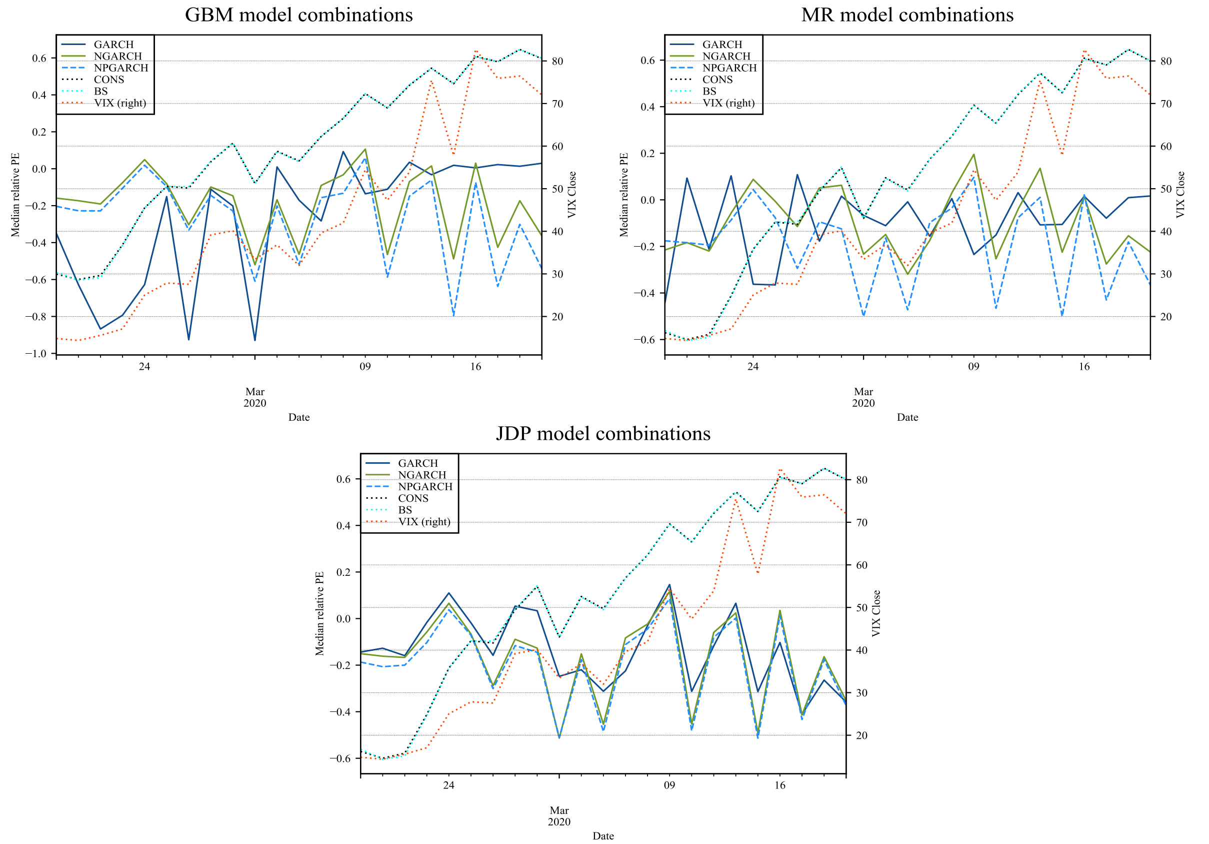

Figure 5.4: Median relative pricing errors over time.

The median relative pricing errors for each day of the crisis are shown in these Figures. The median is used since averages are more distorted for relative numbers. Specifications (1)-(3) are on top, (4)-(6) in the middle and (7)-(9) on the bottom.

Table 5.6 shows that mean relative pricing errors for GARCH specifications during the crisis tend to be smaller for models that include the leverage effect while being overall higher than during the error calculation period (see Figure 3.1). The pricing errors for those parametric models is in contrast to results described in Lento et. al. (2014) for non-parametric models. He showed that pricing precision for non-parametric models decreased below the parametric BS model's precision. In Moyaert et. al. (2011) option pricing precision during the financial crisis of '08 remained better for GARCH option models. Thus, this thesis further solidifies this finding and contributes new, more in-depth evidence for the supremacy of GARCH options pricing during a financial crisis. The results of Duan et. al. (2001) also indicate that GARCH model option pricing performs better during times of financial crisis. Rising errors during the time of financial turmoil which are then relatively stable confirm their results. This thesis' indication that option models with incorporated leverage effect price option markets better during a crisis is in line with findings of Brownlees et. al. (2012). A possible explanation is that during a crisis financial markets assume the future distribution to include more negative returns which is reflected in today's implied volatilities.

Relative errors dependent on moneyness

Moneyness	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
<0.9	381.4%	368.4%	1127.1%	517.1%	225.1%	379.6%	1040.4%	358.7%	394.2%
>0.90, <0.95	99.2%	73.2%	97.1%	74.8%	47.6%	73.2%	100.2%	71.0%	76.2%
>0.95, <1.00	34.8%	27.2%	40.5%	25.5%	16.8%	27.5%	26.4%	26.1%	28.9%
>1.00, <1.05	30.6%	21.0%	33.0%	21.0%	10.2%	21.1%	10.2%	20.0%	22.5%
>1.05, <1.1	43.0%	17.9%	30.4%	30.3%	6.3%	17.9%	-4.2%	16.5%	19.7%
>1.1	376.3%	84.6%	107.35%	304.1%	48.9%	84.5%	-35.3%	80.1%	91.7%

Table 5.6: Moneyness dependent mean relative pricing errors during the event window.

The table depicts the average relative pricing errors of each model combination sorted by moneyness.

Chapter 6

Robustness Checks

In this section, the estimation fit of the GARCH models for the S&P 500 log-returns is assessed. Additionally, parts of the results of Chapter 5 are replicated for ESTOXX 50 index options.

6.1 GARCH model fit

The GARCH model estimation from Chapter 5.1 results in estimated conditional variances that are used as the starting input in the option pricing Monte Carlo simulations (see Figure 3.2 and Appendix - section E). The conditional volatility of each GARCH model for the S&P 500 returns is shown below in Figure 6.1. There are no clear deviations in the models, each showing the characteristic spikes and returns to long term volatility of GARCH models. The Figure indicates that the GARCH estimation done in this thesis was successful in producing time-varying conditional volatilities with spikes in times of high abnormal returns (e.g. 2008). A big difference in them could lead to vastly different option pricing by the algorithm. This could mean that pricing errors calculated depend on the assumed starting volatility. The similar conditional variances, however, imply that differences in option pricing between the combinations come from the assumed behavior of future volatility and not the starting volatility.

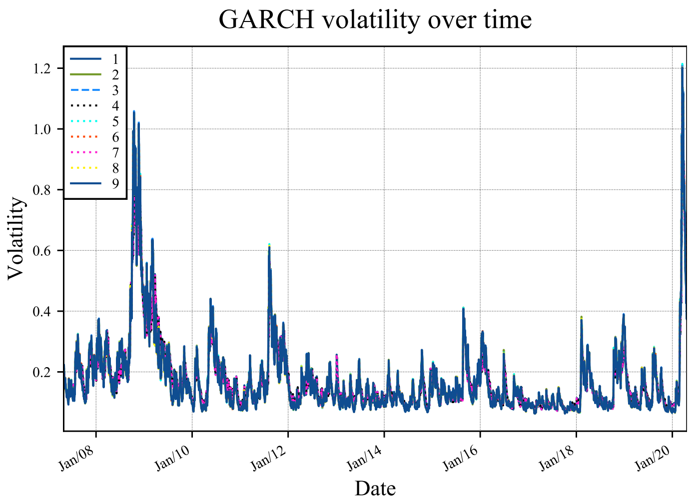

Figure 6.1: GARCH volatilities of the S&P 500 over time.

The Figure presents the conditional volatility of model combinations (1) to (9).

It is also important to verify that the GARCH estimations are resulting in models that exhibit proximity to assumed properties. Namely, it is first verified that the standardized GARCH residuals are reasonably similar to each model's assumption. The standardized residuals of each GARCH model are obtained by subtracting each stock price process' mean of the stock's return at time t from the empirical log-return R_t and dividing it by the conditional volatility stemming from the GARCH model at time t.

$$\hat{z}_t = \frac{R_t - \mu}{\hat{\sigma}_t}$$

GARCH standardized residuals statistics

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
Mean	0.02	0.01	0.01	0.02	0.01	0.01	-0.05	0.01	0.01
Variance	0.99	1.00	0.99	0.99	1.00	1.00	0.98	0.98	1.00

Table 6.1: Mean and variance of standardized GARCH residuals.

For assessing the properties of the standardized GARCH shocks their mean and variance are written down in this table for the S&P 500 models.

Theses standardized residuals are simulated for forecasts according to the combination of equation (1)-(3) and equation (4) (see Figure 3.1). They are simulated from a standard normal distribution. Thus, the in-sample standardized residuals ought to have a mean of roughly 0 and a variance of 1. One can observe in Table 6.1 that this is the case for each of the 9 employed GARCH models. Furthermore, a QQ-plot is depicted in Figure 6.2 to compare the standard residuals' percentiles with the theoretical percentiles of the standard normal distribution. It can be observed that there are deviations for the empirical percentiles on the left tail. This is an indication that z is not standard normally distributed as it is the case for all model combinations which is in line with established literature. This fact is reduced but still remains the case for models with an asymmetric NIC (see Figure 6.2 - on the right). To investigate how changes in the distribution assumptions of z impact pricing performance is beyond the scope of this thesis.

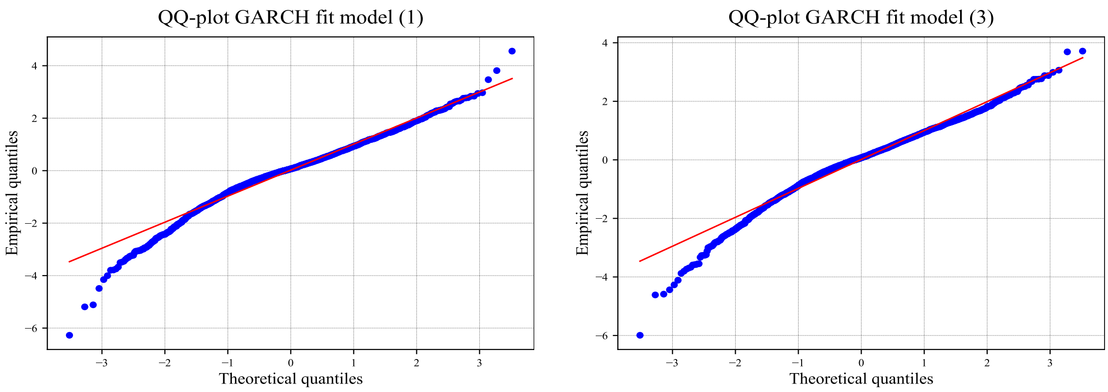

Figure 6.2: QQ-plot of standardized residuals .

6.2 ESTOXX 50

This chapter continues by showing the extension of the methodology in Chapter 3 for another set of options on a different stock index. The chosen options are on the ESTOXX 50 stock index. This subsection will focus on the pricing errors of the ESTOXX options of the three model combinations (1) - (3) that are based on Geometric Brownian Motion (equation (1)) and the GARCH specifications of equation (4). Thus, this section aims to solidify findings of the Results chapter. Namely, that the introduction of non-constant volatility measures increases pricing precision and that throughout the crisis pricing errors went up for the constant volatility models while remaining fairly constant for the model combinations.

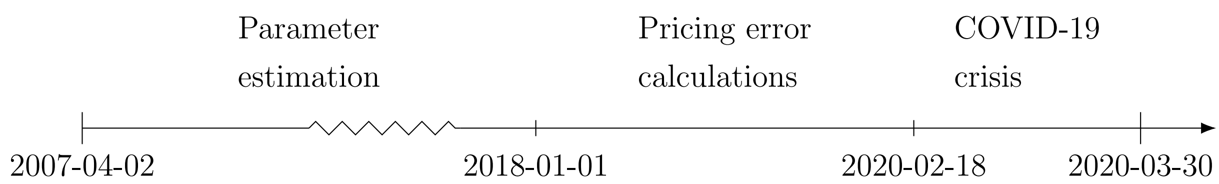

Figure 6.3: Methodology timeline for ESTOXX 50 returns and options.

The time frame for the ESTOXX 50 index options consideration is detailed in Figure 6.3 as pricing errors are calculated between the 1st of January 2018 and the 18th of February 2020 and is followed by the COVID-19 calculations in February/March 2020. The estimated parameters based on returns between 2007 and 2018 are shown in Table 6.2.

The estimation show similar results as in Table 5.1 in the Results chapter which increases robustness of the maximum likelihood approach. Furthermore, Figure 6.4 demonstrates that the resulting News Information curves are very similar to the ones shown in Figure 5.1.

Parameter estimation results

	(1)	(2)	(3)
ω	4.37e-06	3.79e-06	3.66e-06
β	8.82e-01	7.97e-01	8.05e-01
α	1.00e-01	1.88e-02	5.13e-01
κ		1.32e+00	9.33e-01
θ		8.83e-01	8.68e-01
γ			7.81e-01

Table 6.2: Parameter estimation for the ESTOXX 50 Log-returns.

All parameters are estimated upon daily ESTOXX 50 Log-returns R_t of the parameter estimation window. The given model combination uses the relevant log-likelihoods given in equation (5) that are maximized to estimate each parameter. The parameters from (1)-(3) are used for the equations (1) & (4).

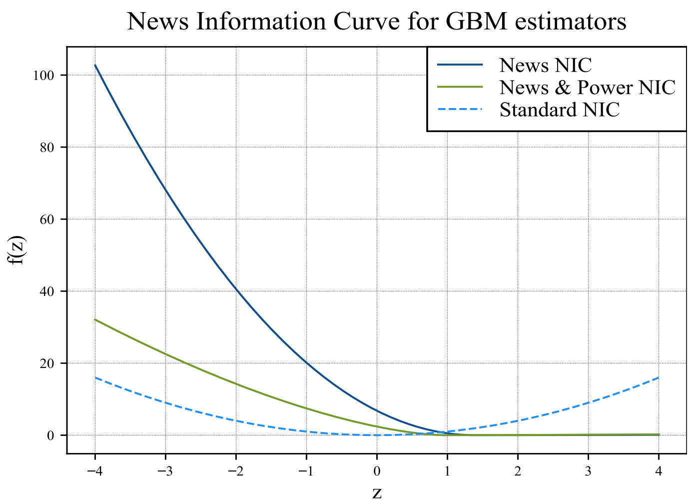

Figure 6.4: NIC for the estimated parameters of the ESTOXX 50 index.

To demonstrate that absolute as well as relative pricing errors are lower for the non-constant volatility model specifications (1) - (3) Figure 6.5 depicts the rolling mean of squared errors and rolling pricing errors in % over time. The Figure indicates that constant volatility models perform worse as in Chapter 5 referring to results from Figure 5.2. A clear distinctions in pricing supremacy is not significant. To further investigate pricing differences for this index a deeper dive into the pricing errors would be needed.

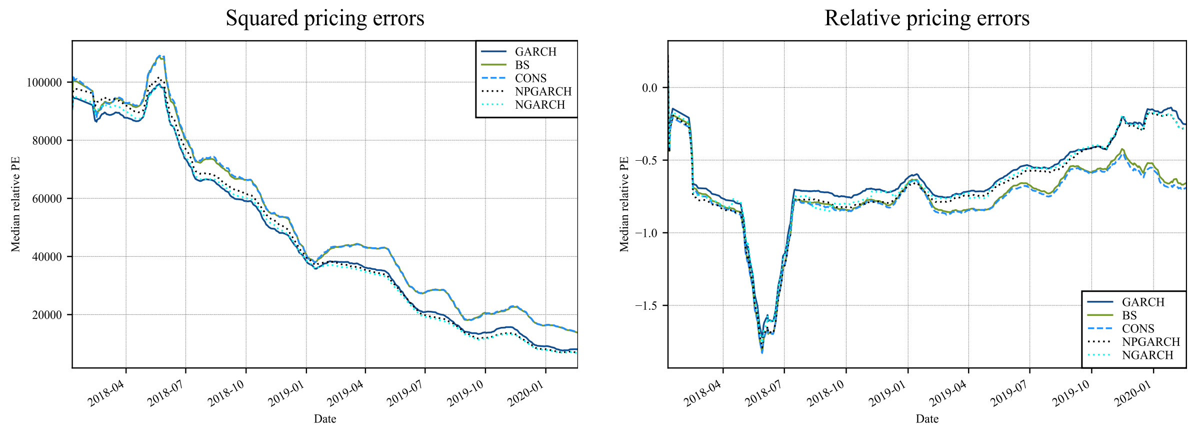

Figure 6.5: Mean squared and median relative pricing errors over time for ESTOXX 50 options.

The left figure shows the rolling 30 day average of squared pricing errors before the crisis. The 30 day rolling average of median relative pricing errors for each day before the crisis window are depicted on the right.

Analogous to Figure 5.3, the relative pricing errors and VIX during the COVID-19 induced crisis are shown in Figure 6.6 and depict similar general developments. The valuation of options rises during the event window with pricing errors of the constant volatility models increasing as well. The same is not true for the three GARCH model combinations. Like in Figure 5.3 they remain rather constant, further providing weak evidence that pricing models based on GARCH volatility processes are a superior means of option valuation during crises.

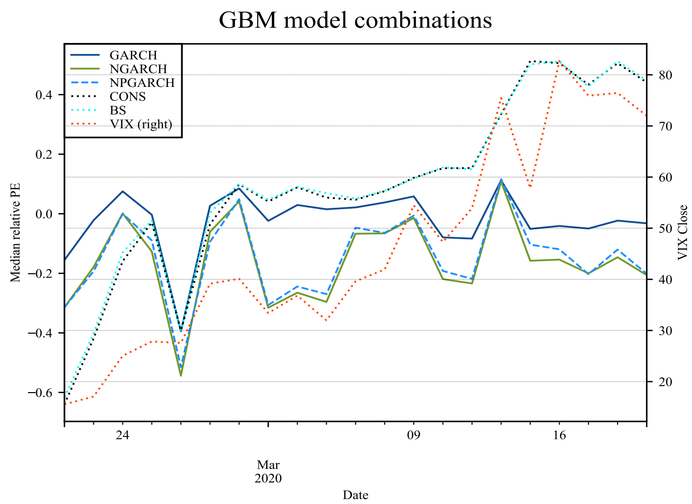

Figure 6.6: Relative pricing errors throughout the crisis for ESTOXX 50 options.

Chapter 7

Conclusion

This thesis's main objective is to outline a holistic option pricing framework and show its validity in pricing exchange traded options while ranking them and extracting information about market behavior from this process. I showed certain pricing supremacies and behavior during the current financial turmoil.

Conclusive remarks

In general, this thesis successfully introduced an extension of GARCH option pricing as used in Christoffersen et. al. (2004) by using different stock price processes. The empirical pricing of exchange traded options further showed that options can be priced more precisely by the use of GARCH models with an indication that more parsimonious models outperform estimation-heavy model combinations. This may be due to the fact that for out-of-sample option pricing these models have to be continuously re-estimated to accurately describe current market behavior. This would mean that option markets imply constant changes in underlying return distributions.

The option pricing errors are both absolutely (MSE) as well as relatively (MRPE) aggregated smaller for the non-constant GARCH pricing models which prevailed over time. The pricing errors for all models were further investigated to deduce market dynamics from them. It is shown that especially well-performing combinations produce pricing errors that are not explainable by market conditions. This indicates that institutional biases arise from models that are not as well suited as others for option pricing. Another explanation is the aforementioned bigger estimation bias. The previously estimated parameters are probably not reflective of the expected value of the parameters anymore. Market dynamics change which results in biased pricing errors of models that are reliant on parameters that are estimated on past returns. The resulting forecast errors from these biases can be reduced by picking and estimating the right model. The Bid & Ask spread for option prices is the only significant factor in the regression. Thus, it still remains the case that better-performing models are subject to market imperfections.

Afterward, it is verified that the introduced option pricing models were less precise in the sharp financial downturn of Q1 2020 as absolute as well as relative pricing errors rose for models with constant and non-constant volatility. Furthermore, the thesis has shown that during the COVID-19 downturn of financial markets the pricing errors of the constant volatility model continuously increased from over to underpricing with the rise of option prices. This is not the case for the GARCH models as they tend to perform worse in this crisis than they did before but don't accelerate. Thus, the adjustment of the volatility and the assumption of its future behavior of each model may reflect on how markets price the behavior of volatility in the future and thus option prices today.

Also, pricing error supremacy across the non-constant volatility models changed. During the crisis window, models with asymmetric NICs performed relatively better than the standard GARCH model that is performing better in the window before the crisis. This may be the result of better reflection of past negative returns and their incorporation in today's option prices. Option markets may follow the assumption that future volatilities and thus today's implied volatilities will be more sensitive to future negative returns. This may lead to a point where before postulated biases stemming from model estimations from past returns are compensated and reduced as the estimators "are true again". Due to these findings, no clear model ranking across the introduced model combinations can be done for different market regimes. Furthermore, the introduction of stochastic processes that aim to more accurately explain market behavior does not seem to play a significant role in increasing pricing precision.

Approach limitations

The results of this thesis are somewhat limited in several aspects. A higher number of simulations would reduce pricing error variances. In this thesis, the error variance is minimized by the use of a big sample size of options during the short time span of the COVID-19 crisis. Also, the data set used in this thesis was manually collected via Thomson Reuters Eikon. There, only active option chains and their current properties and historical prices can be gathered which introduces a bias. As only options with long maturities at this time are available the further you go back in history the data set only approximates the complete option market at each point in time. Ideally, a data set consisting of the same number of option data with the same properties for each day in a continuous time series is used.

The used risk-free rate in this thesis is the 10Y Treasury yield which is assumed to stay constant at each valuation point. This violates observed market behavior. Typically, a rising term structure that is subject to changes over time can be observed. Thus, it would be suitable to include a model that introduces changes to the risk free rates into the Monte Carlo pricing framework. Such approaches have been pioneered by Amin et. al. (1993).

As this thesis among other things aimed at solidifying results from Christoffersen et. al. (2004) a standard normal distribution of residuals of the GARCH model is assumed. After the parameter fit, the empirical standardized shocks are shown to be fat-tailed on the left side. Thus, it would be an extension for this thesis' approach to use a leptokurtic distribution for estimating the parameters and modeling the shocks. Examples are (asymmetric) t-distributions. Their use in the generation of the random number z may also improve pricing precision of the framework. Furthermore, it makes sense to include loss-functions into the estimation of the parameters. It would be feasible to estimate parameters by minimizing the mean squared errors for an in-sample of option prices to get more empirically sound parameter estimators.

Further research

To further research in GARCH model option pricing the imperfections used in this thesis (e.g. data set) and in the literature (e.g. Christoffersen et. al. 2004 - constant risk-free rate of 5%) should be resolved. Such research may solidify findings of this thesis and show apart from the constant volatility assumption how changes in the deviation of the constant risk-free rate assumption influences pricing errors. This would give additional insights into option market behavior, how option prices depend on interest rate assumptions and how to hedge this exposure most efficiently. Also, constant parameter re-estimation may yield clearer answers to which model combinations show improved option pricing capabilities. The indication supported by this thesis is that the introduction of more complicated stock price processes and asymmetric News Information Curves doesn't necessarily increase option pricing precision.

The uniqueness of this thesis is that it used well-established option pricing models with extensions on a newly acquired sample of empirical option prices with a focus on the recent COVID-19 induced downturn in stock index markets. The approach of this thesis is not methodologically sophisticated, however, as only nominal statistics and graphs were used to assess changes in option pricing during the recent crisis. An extension of the used approach could be further investigation of the option prices as well as model mispricings during the crisis with the use of price implied density functions. Since option prices are based on the assumed distribution of future underlying returns the option prices can be used to deduce implied probability density functions and statistical measures like the implied volatility and kurtosis of the underlying. Those measures may yield results in showing how markets operate during the crisis and if the crisis may have been partly priced into the market beforehand.

An application to exchange-traded options that are path-dependent such as American options (e.g. with Exchange Traded Funds as the underlying) may give further robustness of this thesis' results. Finally, another extension of this thesis is to test besides pricing capabilities of the GARCH models, their ability to accurately hedge stock or volatility exposure (Delta, Gamma & Vega) and how these capabilities have changed during the financial downturn of Q1 2020. Orientation could be Moyaert et. al. (2011) who have been doing similar research for option pricing models during the financial crisis of 2007/08.

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Appendix

Term	Meaning
Stock index	Weighted group of stocks to measure the group's performance.
Yield	Earnings generated by a fixed income product. A measure of how risky the fixed payments of the product are. Generally, the higher the yield the riskier the product.
Derivative	Financial product that derives its value from the (expected) value of another financial product.
Stock option	Derivative that gives you the right to buy or sell a stock at a later time for a pre-determined price.
Option underlying	The stock the option gives you the right to buy/sell.
Option maturity	The date on which you can buy/sell the underlying stock of the option.
Strike price	The price for which the option buyer can buy/sell the underlying security.
Moneyness	Relation of the current underlying's price to the strike price.
In-, At- and Out-of-money	When the underlying's price is considerably bigger, approximately the same and considerably smaller than the option strike (for a Call).
Implied volatility	Annual Volatility of the underlying in the Black Scholes model that would result in a BS price equal to the market price. Measure of market price implied risk of the underlying.
S&P 500 Index	Stock index of the biggest 500 US-American companies.
ESTOXX 50 Index	Stock index of the biggest 50 European companies.
VIX Index	Index showing the expected annual volatility of the S&P 500 as implied by its options' implied volatilities.
Mean	Arithmetic average of a sample or time series.
Volatility	Degree of variation of a sample of time series.
Skewness	Measure of asymmetry of a statistical distribution around the mean.
Kurtosis	Measure of how frequent extreme realizations of a random number given its statistical distribution are.
News Information Curve (NIC)	Measure of influence of the random number on the conditional variance of a GARCH model.

Table 7.1: Glossary of financial & statistics terms.

A: Model combinations

In this thesis three stock price processes are introduced in equation (1) to (3). Also, three variations of the GARCH model in equation (4) are detailed. I use combinations of each stock price process and GARCH model variation to price options via a Monte Carlo framework. The Table below details all model combinations to give an overview which model is referred to.

Model combination	Stock price process	GARCH model NIC
(1)	Geometric Brownian Motion: equation (1)	$f(z_t) = z_t^2$
(2)	Geometric Brownian Motion: equation (1)	$f(z_t) = (\|z_t - \theta\| - \kappa(z_t - \theta))^2$
(3)	Geometric Brownian Motion: equation (1)	$f(z_t) = (\|z_t - \theta\| - \kappa(z_t - \theta))^{2\gamma}$
(4)	Market Risk model: equation (2)	$f(z_t) = z_t^2$
(5)	Market Risk model: equation (2)	$f(z_t) = (\|z_t - \theta\| - \kappa\|z_t - \theta\|)^2$
(6)	Market Risk model: equation (2)	$f(z_t) = (\|z_t - \theta\| - \kappa(z_t - \theta))^{2\gamma}$
(7)	Jump diffusion process: equation (3)	$f(z_t) = z_t^2$
(8)	Jump diffusion process: equation (3)	$f(z_t) = (\|z_t - \theta\| - \kappa(z_t - \theta))^2$
(9)	Jump diffusion process: equation (3)	$f(z_t) = (\|z_t - \theta\| - \kappa(z_t - \theta))^{2\gamma}$

Table 7.2: Overview of model combinations.

B: Parameter estimation

The estimation process for the 9 model combination is detailed in the methodology chapter 3.1. I use the equations (5) and (6) for parameter estimation. In the parameter estimation window from roughly 2007 to 2018 both equations are maximized for all model combinations to gain the necessary parameters shown in Table 5.1.

A starting value for the conditional variance is needed to start the likelihood estimation. For that, I use the standard deviation over the whole return time series (see Christoffersen 2012). The conditional variance $\hat{\sigma}_t$ is calculated by obtaining the standardized shock z (see Chapter 6) of $t-1$ to get $f(z_{t-1})$ and used it with $\hat{\sigma}_{t-1}$ to get this day's likelihood. With this procedure the Likelihood function for a given parameter vector θ is defined and maximized afterward. For maximization the Python library *SciPy* is used. The library has different minimization algorithms that each require a starting value for θ. For that reason I changed the sign of the likelihood and minimized it. I use the Nelder-Mead optimization algorithm. An approximation of the local maximum is achieved by continously comparing three different θ results in such a way that only the biggest remains after each iteration. For the next iteration the two worse θs are replaced. Eventually, θ that maximizes the likelihood is obtained that way. For starting values of model (1) $\omega = 0.001$, $\alpha = 0.09$ and $\beta = 0.89$ were chosen. A full list of starting values as well as the estimation script is available upon request.

C: Likelihood-Ratio test

The Likelihood-Ration test is employed to test whether a specification of a used model provides further information in comparison to its simpler version. This is done by comparing if there are significant differences in Log-likelihoods between the general model 0 and its specification 1. The test-statistic:

$$LR = 2(L_1 - L_0)$$

is χ^2_m distributed with m being the number of extra parameters of model 1. The NPGARCH model for example has 1 more parameter than the NGARCH model. The test-statistic LR is compared to the corresponding percentile of the χ^2_m distribution which are detailed below. If the test-statistic is bigger than the critical value the model provides additional information with an error likelihood of the corresponding 1 - percentile of the critical value.

m	50%	75%	90%	95%	97.5%	99%
1	0.454936	1.32330	2.70554	3.841459	5.023886	6.63489
2	1.386294	2.77258	4.60517	5.991465	7.377759	9.21034

Table 7.3: Critical values of the χ^2_m distribution.

D: Empirical Martingale Simulation

In their paper, Duan and Simonato (1998) explain a procedure to correct simulated stock prices in such a way that they fulfill the martingale condition of option prices. The aim is to get non-biased stock price paths and thus, have more precise asset prices obtained via Monte Carlo simulation. The martingale property was already defined as:

$$E^Q[e^{-rt}S^*_t|S_0] = S_0.$$

The correction algorithm is started by defining the first corrected stock price as the starting stock price $S^*_j(t_0) = S_j(t_0) = S_0$. Then the algorithm employed is for path j and time t defined as follow[1]:

$$Z_j(t) = S^*_j(t-1)\frac{S_j(t)}{S_j(t-1)}, \text{for } j = 1, ..., m$$

$$Z_0(t) = \frac{1}{m}e^{-rt}\sum_{j=1}^{m} Z_j(t)$$

[1] see Chan et. al. (2013), p. 87-89.

$$S_j^*(t) = S_0 \frac{Z_j(t)}{Z_0(t)}.$$

E: Option pricing explanation

A stock option is explained in the Glossary of Table 7.1. The right to buy buy an option at a pre-determined price results in the payoff at maturity: $\Pi(S_T) = max(S_T - K, 0)$ for a Call and $\Pi(S_T) = max(K - S_T, 0)$ for a Put option. The stock price at maturity is S_T (S_{t+h} in Figure 3.2) while the strike price of the option is denoted by K. With the $m = 25,000$ different Stock prices at maturity 25,000 possible payoffs are generated by the Monte Carlo simulation procedure. The option prices for Calls and Puts are defined as:

$$C_0 = e^{-rT} E_0^Q(\Pi(S_T)) \qquad P_0 = e^{-rT} E_0^Q(\Pi(S_T)).$$

According to these formulas the risk-neutral expected value of the final simulated payoffs $\Pi(S_T)$ need to be discounted to the present to obtain an estimate of today's option value. The estimator of the discounted risk-neutral expected value is given by:

$$C_0 = e^{-rT} \sum_{i=1}^{T} (\Pi(S_T^i)) \qquad P_0 = e^{-rT} \sum_{i=1}^{T} (\Pi(S_T^i)).$$

Thus, the options are valued by (1) calculating conditional variances, (2) simulating future conditional volatility and stock price paths resulting in option payoffs and (3) estimating the risk-neutral expected value of the payoffs that is discounted to the option's valuation date.

Property	Date	Identifier	Identity	K	S_0	r	T	$\hat{\sigma}_0^{(a)}$
Value	03-18-2020	/STXE35750R0.EX	PUT	3575	2385.82	1.27%	64	73.96%

Table 7.4: Example option properties.

This table details the inputs of the illustrative example of the procedure of Figure 3.2. They are: the option valuation date, the option identifier, the option identity, the strike price K, the starting underlying Stock price S_0, the risk-free rate r, the time to maturity in days and the conditional variance of this day according to the used GARCH model.

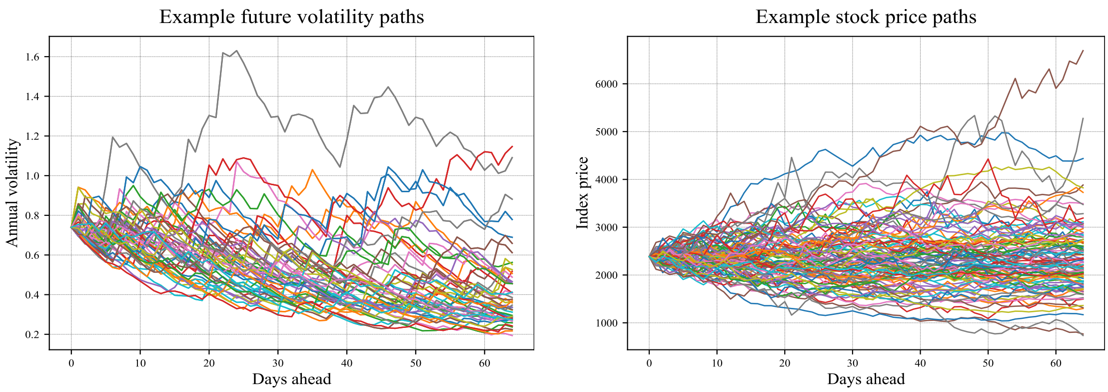

Figure 7.1: Example stock price and volatility progression.

This approach is demonstrated in Table 7.6 and Figure 7.1 with an illustrative example. The option properties and market conditions of this Put option with a strike of 3575 on the 18th of March 2020 are detailed in Table 7.6. The inputs are used to generate a future volatility progression that is used in the stock price process to generate a number m simulated stock paths (here 100). The illustration uses model combination (1). Thus it has an underlying Geometric Brownian Motion as the stock price process with a simple GARCH volatility model according to equations (1) and (4). The GARCH estimators are detailed in Table 6.2.

For each day until maturity (64 days ahead) 100 random numbers z are generated that are used to forecast the volatility progression. The random numbers z and the GARCH volatility forecasts $\hat{\sigma}$ are then used to calculate the stock price forecasts. The annual GARCH volatility starts at roughly 73%. This value is high since the observation date is in the midst of the COVID-19 downturn. One can observe the volatility forecast $\hat{\sigma}$ on the left side of Figure 7.2. The forecast shows a GARCH model's property of returning to a long-run volatility with shock induced spikes around this trend. The obtained stock price forecasts are then corrected by the Empirical Martingale Simulation algorithm and displayed on the right side of Figure 7.2. The resulting stock prices at day 64 can be used as S_T to calculate the payoffs $\Pi(S_T)$ and estimate the model's Put option price on the 18th of March 2020.

The model combinations' pricing errors are compared to a Monte Carlo option pricing model assuming constant future volatility and the Black Scholes model. The constant future volatility is the standard deviation of the log-returns of the underlying over the whole log-return samples (see Figure 3.1) which is also used in the Black Scholes model. The constant volatility is denoted by σ. The Black Scholes model is defined as:

$$d_1 = \frac{ln(S_0/K) + (r - \sigma/2)T}{\sigma\sqrt{T}} \quad d_2 = d_1 - \sigma\sqrt{T}$$

$$C_0(S_0) = S_0 \times N(d_1) - e^{-rT} K \times N(d_2)$$

for Call options and as

$$P_0(S_0) = e^{-rT} K \times N(-d_2) - S_0 \times N(-d_1)$$

for Put options. The variable S_0 is the underlying's price, K the strike price and T the maturity in years as $N(x)$ denotes the cumulative distribution function of a standard normal distribution.

F: Option data collection

I gathered the option data from Thomson Reuters' Eikon platform. There is no way to get all (historical) option data for a given index as only historical data of options that are still active (no matured options) are available. Data for a given option needs to be downloaded individually. I exported all active options and their identifier (e.g. /SPXq152017000.U). Afterward, I used them individually as an input to download all their historical Bid-, Ask- and Close-prices. Furthermore, the characteristics of each option (based on its instrument identifier) such as maturity, strike price and identity (Call or Put) are downloaded. The resulting option data sets are merged on date and Instrument. Furthermore, they are merged on date with the data of stock price, risk free rate (10Y Treasury Yield), VIX-value and conditional volatilities from the GARCH models (depicted in Figure 6.1) of that date to get a complete data set with:

- Option properties: Date, instrument, identity, maturity date, strike price
- Option quotes: Close-, Bid- and Ask-price.
- Market data: Underlying price, risk-free rate and VIX
- Conditional volatilities of the 9 GARCH variations.

This data set is then filter and ultimately used as inputs for option pricing (properties, market data and conditional volatilities) and error calculation (comparison with the close price). Additionally the properties, market data and the the resulting spread from bid- and ask-prices are used as input in the regression detailed in equation (9) of Chapter 3.4. The data scripts are available upon request.

G: Stock indices & risk-free rate

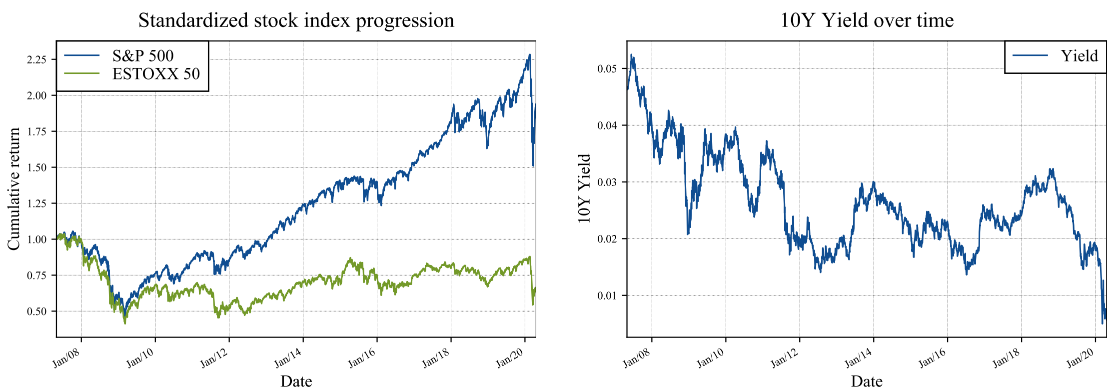

Figure 7.2: Progression of the stock indices and Yield over time.

	10y Trasury yield
N	3215
Start date	30-04-2007
Stop date	19-03-2020
Min	1.37%
Max	5.25%
Average yield	2.69%
Median yield	2.52%

Table 7.5: Descriptive statistics of 10Y Treasury Yield.
The characteristics and statistical measures are based on daily yields of the 10 year Treasury bond Yield obtained from Yahoo! Finance within the time frame of Figure 3.1 and were calculated with pre-installed functions of Microsoft Excel.

H: Estimated parameters

All of the model combinations depend on estimated parameters. This table gives an overview of the estimated parameters, in which combination they are used and their influence & meaning.

Parameter - name	Use	Meaning & interpretation
ω - Omega	All models (1)-(9)	Constant of the GARCH model from equation (4).
α - Alpha	All models (1)-(9)	Short-term persistence. Reaction of the shock from $f(z_t)$ on future conditional variance.
β - Beta	All models (1)-(9)	Long-term persistence. Influence of past conditional variance.
θ - Theta	(2), (3), (5), (6), (8), (9)	Shifts the NIC $f(z_t)$ (to the right if positive, to the left if negative).
κ - Kappa	(2), (3), (5), (6), (8), (9)	Tilts the NIC. The higher Kappa the more tilted the NIC $f(z_t)$ and the higher the influence of negative shocks relatively is.
γ - Gamma	(3), (6), (9)	Steepens/flattens the NIC. If above 1 the NIC $f(z_t)$ is steeper and for γ smaller 1 it is flatter.
λ - Risk parameter	(4)-(6)	The risk parameter shows how much compensation investors desire from a stock for its uncertainty.
λ_J - Jump intensity	(7)-(9)	Input of the Poisson process of the JDP. Drives the frequency of jumps. The higher λ_J the more jumps are likely to occur.
μ_J - Jump mean	(7)-(9)	The average magnitude of each jump.
σ_J - Jump volatility	(7)-(9)	The volatility of the magnitude of each jump.

Table 7.6: Overview of use and meaning of needed parameters.

Used computational tools

For computations and image creations I primarily used Python. The implementation of the Python codes is mostly done in Jupyter Notebooks that are available upon request. The notebooks have the *.ipynb* data form and are used for data download and error calculation & examination. Excel is used to get option properties, the calculation of statistical measures and data manipulation. Furthermore, data manipulation is done with *Pandas* and the linear regression with the Statsmodels library. All images were created with the *Matplotlib* Python library.

Environment	Library
Spyder 3.3.3	Numpy 1.19
Jupyter Notebook 6.0.3	Pandas 1.05
	Matplotlib 3.2.2
	Statsmodels 0.9.0
	Scipy 1.5.1
Microstoft Excel 2016	

Table 7.7: Used computational tools, environments and libraries.