University of Colorado - Boulder

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Guitar Project

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Abstract

This project will develop and evaluate mathematical models of plucked guitar strings. This will be done through the derivation of four separate models that consider various adjustments to the ideal wave equation. These will consider factors such as damping and string stiffness. Experimental data, collected by recording a guitar, will be applied to these models through the analysis of Fast Fourier Transform data. Resonance will be studied by looking at how the Fast Fourier Transform data changes as multiple strings are played. Finally, analysis with damping will be compared to basic wave equation and connections will be made between the mathematics and the art of music.

I. Introduction

Music is fundamentally grounded in the production and manipulation of waves. A standard guitar achieves the production of waves through the vibrating of a set of strings. The frequency, or pitch, produced by the strings varies due to multiple factors such as the length, thickness, stiffness, and tension of the strings. Stiffness and thickness are predetermined through the production of a guitar, but the length and tension of the strings can be freely modified by the user. Tuning pegs allow strings to be tightened to standardized notes and frets allow the length of the vibrating sections of the strings to be shortened while playing, to produce different notes and chords. The manipulation of guitar strings is something that can modeled using a modified form of the wave equation.

The main departure from the idealized wave is the presence of damping. When a guitar string is plucked, the note it produces fades enough to allow for the following note to be perceived clearly. The effect this has on the general form of the wave equation is the addition of a damping term. Another departure is string stiffness. This project will analyze multiple variations of the wave equation that attempt to model guitar strings. This will begin with the derivation of four equations, and will be followed by the inclusion and analysis of experimental data.

The collection of data will be achieved by recording strings of a guitar. These recordings will include the six individual strings and various sets of strings, including Fast Fourier Transform data and amplitude vs time data. These measurements will be used to analyze frequencies, resonance, and damping of the guitar strings compared to the mathematical models. The project will conclude with an analysis of mathematical principals and their relation to music production.

All time will be measured in seconds. All frequencies will be measured in Hz.

II. Damping PDE's, and the Relationship Between Frequency and Wave Number

A. The Wave Equation

The simplest version of The Wave Equation takes the form below (Haberman 2004).

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \tag{1}$$

$$u(x,t) : position, c : speed$$

This equation can be solved using separation of variables with the following boundary conditions:

$$BC : u(0,t) = 0 = u(L,t)$$

$$IC : u(x,0) = U(x), u_t(x,0) = V(x)$$

Separation of variables will be defined in the following way:

$$u(x,t) = F(x)G(t) \neq 0$$

Equation 1, along with the line above imply that:

$$F(x)G''(t) = c^2 F''(x)G(t)$$

$$\frac{1}{c^2}\frac{G''(t)}{G(t)} = \frac{F''(x)}{F(x)} = \lambda$$

The boundary conditions imply that $F(0) = F(L) = 0$. We now have a time domain problem and an eigenvalue problem. When $\lambda < 0$ in the eigenvalue problem, we can find that the general solution is $F(x) = C_1 e^{-mx} + C_2 e^{mx}$ where $\lambda = -m^2$. Using the boundary conditions, we find that $C_1 = C_2 = 0$. When $\lambda = 0$ in the eigenvalue problem, we let $F(x) = e^{rx}$ which yields the characteristic polynomial $r^2 = -\lambda$. Thus, the trivial solution is $F(x) = C_1 \cos\sqrt{\lambda}x + C_2 \sin\sqrt{\lambda}x$. The boundary conditions tell us that $C_1 = 0$ and $C_2 \sin\sqrt{\lambda}L = 0$. From this we get the following: $\lambda_n = (\frac{n\pi}{L})^2$ where $n = 1, 2, ...$ and $f_n(x) = C_2 \sin\frac{n\pi x}{L}$. In the time domain problem, if we let $G(t) = e^{rt}$ then, similar to the eigenvalue problem, $\lambda < 0$ and $\lambda = 0$ yield the trivial solution $G(t) = 0$. When $\lambda > 0$, $G(t) = d_1 \cos \sqrt{\lambda} ct + d_2 \sin \sqrt{\lambda} ct$ and $G_n(t) = d_1 \cos \sqrt{\lambda_n} ct + d_2 \sin \sqrt{\lambda_n} ct$. Now that we have $F_n(x)$ and $G_n(t)$ we can find $u_n(x,t)$ since $u_n(x,t) = F_n(x)G_n(t)$. Using the superposition principle, the solution below was found.

$$u(x,t) = \sum_{n=1}^{\infty} \alpha_n u_n(x,t) = \sum_{n=1}^{\infty} A_n \sin(\frac{n\pi x}{L}) \cos(\frac{n\pi ct}{L}) + B_n \sin(\frac{n\pi x}{L}) \sin(\frac{n\pi ct}{L})$$

$$A_n = \frac{2}{L} \int_0^L U(x) \sin(\frac{n\pi x}{L}) dx$$

$$B_n = \frac{2}{n\pi c} \int_0^L V(x) \sin(\frac{n\pi x}{L}) dx$$

B. Equation 2: Damping PDE

Below is an altered version of the wave equation which accounts for damping.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} - \beta \frac{\partial u}{\partial t} \tag{2}$$

Separation of variables is defined in the same way which yields a time problem and a space problem:

$$\frac{1}{c^2} \frac{G''(t)}{G(t)} + \frac{\beta}{c^2} \frac{G^1(t)}{G(t)} = \frac{F''(x)}{F(x)} = -\lambda$$

This implies that $\lambda = (\frac{n\pi}{L})^2$ and $F_n(x) = \sin \frac{n\pi x}{L}$ for the space problem, and $r = \frac{-\beta \pm \sqrt{\beta^2 - 4c^2(\frac{n\pi}{L})^2}}{2}$ for the time problem. From this, we can set $\alpha = \frac{\beta}{2}$ and $\omega_n = \sqrt{c^2} (\frac{n\pi}{L})^2 - \frac{\beta^2}{4}$. All of which yields the solution:

$$u(x,t) = e^{\frac{-\beta}{2} t} \sum_{n=1}^{\infty} (a_n \cos \omega_n t + b_n \sin \omega_n t) \sin \frac{n\pi x}{L}$$

C. Equation 3: Damping PDE

Below is another altered version of the wave equation which accounts for damping.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} + \gamma \frac{\partial}{\partial t} (\frac{\partial}{\partial x})^2 u \tag{3}$$

To solve this we are going to assume $u(x,t) = e^{ikx - i\omega t}$. The second partial derivatives of this are: $\frac{\partial^2 u}{\partial t^2} = (i\omega)^2 e^{ikx - i\omega t} = -\omega^2 u$, and $c^2 \frac{\partial^2 u}{\partial x^2} = c^2 (ik)^2 e^{ikx - i\omega t} = -c^2 k^2 u$. From here, we will find $\gamma \frac{\partial}{\partial t} (\frac{\partial}{\partial x})^2 u$. We know that $(\frac{\partial}{\partial x})^2 u = -k^2 u$, and $\gamma \frac{\partial}{\partial t} (-k^2 e^{ikx - i\omega t}) = \gamma i \omega k^2 e^{ikx - i\omega t}$. Using the results above, it can be found that $\omega(k) = \sqrt{c^2 - \gamma i \omega}$. This means that $u_{1,k}(x,t) = e^{ik(x - \sqrt{c^2 - \gamma i \omega t})}$ and $u_{2,k}(x,t) = e^{ik(x + \sqrt{c^2 - \gamma i t})}$. Using the superposition principle, the solution is as follows:

$$u(x,t) = \int_{\infty}^{-\infty} [A(k) e^{ik(x - \sqrt{c^2 - \gamma i \omega t})} + B(k) e^{ik(x + \sqrt{c^2 - \gamma i \omega t})}] dk$$

$$A(k) = \frac{1}{4\pi} \int_{\infty}^{-\infty} [U(x) - \frac{V(x)}{ik \sqrt{c^2 - \gamma i \omega}}] e^{ikx} dx$$

$$B(k) = \frac{1}{4\pi} \int_{\infty}^{-\infty} [U(x) + \frac{V(x)}{ik \sqrt{c^2 - \gamma i \omega}}] e^{ikx} dx$$

D. Equation 4: Damping PDE

Below is another altered version of the wave equation which accounts for damping.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} - \alpha \frac{\partial^4 u}{\partial x^4} \tag{4}$$

To solve this, we are going to assume $u(x,0) = U(x)$, $\frac{\partial}{\partial t}u(x,0) = V(x)$, and $u(x,t) = e^{ikx-i\omega t}$ where k is real. We then will use dispersive waves to find the dispersion relation $\omega(k)$ to solve the PDE. To start, we will find the second partial derivative of u with respect to t and with respect to x. This yields $\frac{\partial^2 u}{\partial t^2} = (i\omega)^2 e^{ikx-i\omega t} = -\omega^2 u$, and $c^2 \frac{\partial^2 u}{\partial x^2} = c^2(ik)^2 e^{ikx-i\omega t} = -c^2 k^2 u$. Next, we know that $\alpha \frac{\partial^4 u}{\partial x^4} =^4 u$, which yeilds $\omega(k) = \pm k\sqrt{c^2 + \alpha k^2}$ when solved for ω. This means that $u_{1,k}(x,t) = e^{ik(x-\sqrt{c^2+2}t)}$ and $u_{2,k}(x,t) = e^{ik(x+\sqrt{c^2+2}t)}$. Using superposition, the solution is as follows:

$$u(x,t) = \int_{\infty}^{-\infty} [A(k)e^{ik(x-\sqrt{c^2+\alpha k^2}t)} + B(k)e^{ik(x+\sqrt{c^2+\alpha k^2}t)}]dk$$

$$A(k) = \frac{1}{4\pi}\int_{\infty}^{-\infty} [U(x) - \frac{V(x)}{ik\sqrt{c^2+\alpha k^2}}]e^{ikx}dx$$

$$B(k) = \frac{1}{4\pi}\int_{\infty}^{-\infty} [U(x) + \frac{V(x)}{ik\sqrt{c^2+\alpha k^2}}]e^{ikx}dx$$

III. Discussion

A. Frequency Peaks

To proceed, experimental data is required. The procedure to acquire this data involves recording signals produced by a guitar. Individual notes were played, as well as chords and combinations of notes, and their signals were recorded. A Fast Fourier Transform Algorithm was then used to convert the signal format of the recordings to frequency representations. These representations allow analysis of wave frequency peaks to be conducted.

When analyzing the frequency peaks of the four equations, it is evident that the different theories of dampening and stiffness impact the values of these peaks. Damping causes a decrease in an oscillation's amplitude, as energy is being used. This directly translates to the fact that damping shifts the frequency of a wave downward. Stiffness, on the other hand, increases the frequency of a system, meaning this characteristic is shifted upward. We found our experimental data and mathematical model to be consistent with these results. For instance, compared to the simplest version of the wave equation identified as equation 1, the damping term in equation 3 that is added results in a frequency shifted down. Equation 4, which accounts for the stiffness of the string, illustrates a wave that is shifted up.

In the wave equation (1), c is the propagation speed of the wave on the guitar string. This means that c can be estimated using our solution to equation (1) and our data. In the solution to the wave equation, we see c in $\sin(\frac{n\pi ct}{L})$ and $\cos(\frac{n\pi ct}{L})$. From this, we know that the period is $\frac{2L}{nc}$ which means that the frequency is $\frac{nc}{2L}$ where n is the node, and L is the length of the string(s). To test this, we looked at the FFT plots for strings 1 and 2 and found the frequency of the first node. We then set these frequencies equal to $\frac{nc}{2L}$ where $n = 1$ and L is the length of that string. String 1 had a length of $62.5cm$ and string 2 had a length of $62.6cm$. This made our estimations for c on strings 1 and 2 equal to $245.8\frac{m}{s}$ and $307.7\frac{m}{s}$ respectively.

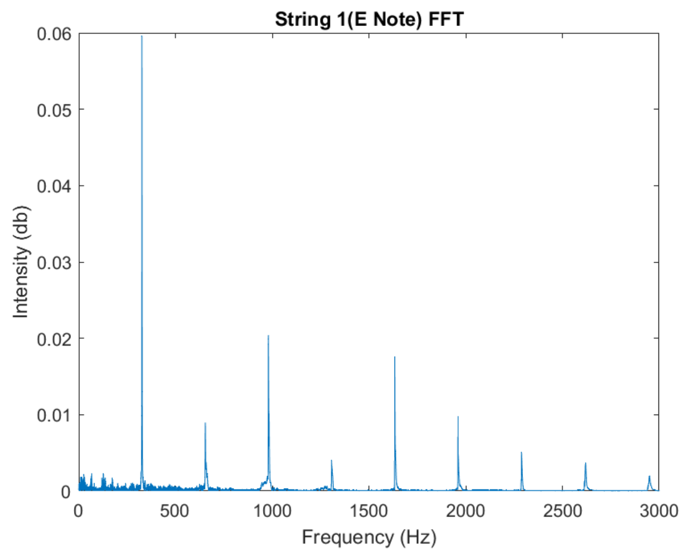

Figure 1: String 1 FFT

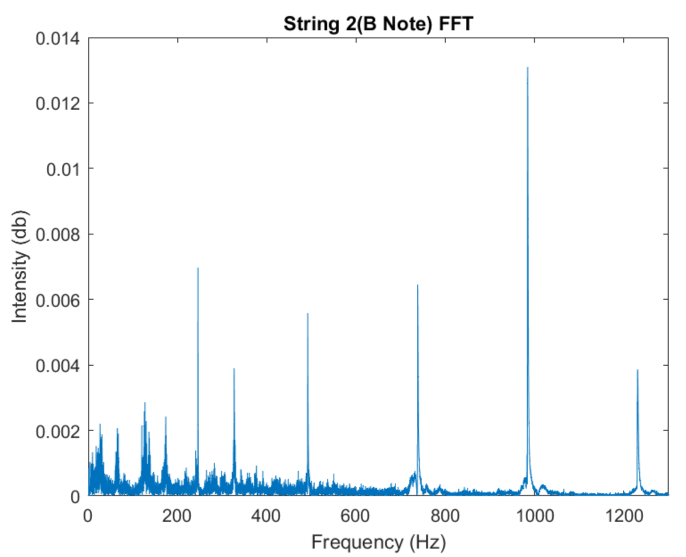

Figure 2: String 2 FFT

The boundary conditions we assumed in our calculations are consistent with those we have utilized throughout the semester:$u(0,t) = 0 = u(L,t)$ These conditions indicate fixed ends of a string, which is representative of the guitar string. Our solutions are sensitive to these boundary conditions; L is reciprocally related to frequency by the equation $frequency = nc/2L$. For instance, if we pluck string 1 without pressing down on the string the frequency is $327Hz$. When the first string is pressed down between the 2nd and 3rd fret, L is now $52cm$. This results in a frequency of $251.6Hz$ which shows how sensitive frequency is to the boundary conditions.

B. Chords

A chord is a combination of notes that when played together create a satisfying musical sound. There are 6 primary categories of recognized chords, determined by the number of half steps between each note in the chord (J. Wild 2002). These categories are: Major, Minor, Diminished, 7th, Minor 7th, and Major 7th.

This is how chords are formed from a musical perspective. This next section will analyze chords from a mathematical perspective.

To proceed, graphs of the Fast Fourier Transform data from combinations of strings will be analyzed. Figure 3 contains data from the guitar's six strings being strummed at once.

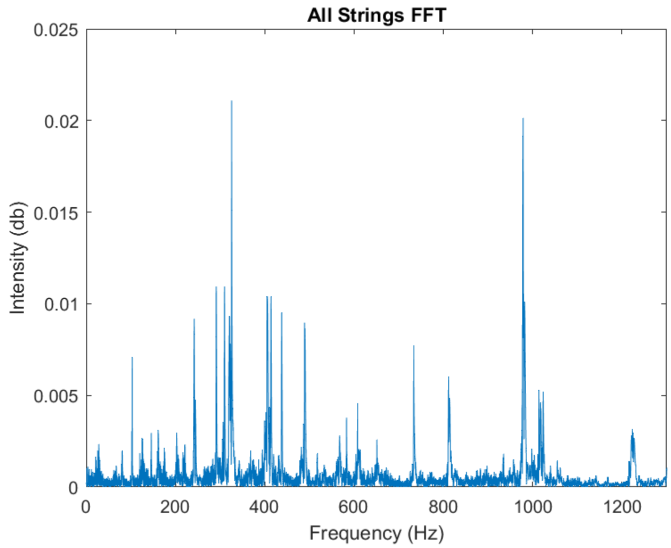

Figure 3: All Strings FFT

There is a clear departure from the Fast Fourier Transform data from individual strings. Single notes produce frequency peaks that trend downward with an even interval between peaks. The data from all six strings shows more variation to this pattern. If the two largest spikes are removed from the data set, there still remains an overall downward trend, but the presence of the peaks reveals something interesting about the composition of the frequencies produced by these strings. The first peak occurs around 330 Hz, which is the frequency of an E. Because strings one and six are both E strings, an octave apart, it makes sense that the frequencies of these two strings resonate and result in a increased amplitude at 330 Hz. From this we can conclude that playing multiple strings at once produces a series of frequencies rather than an averaged frequency which is a common intuition.

Next, the A Chord and E Minor Chords will be analyzed.

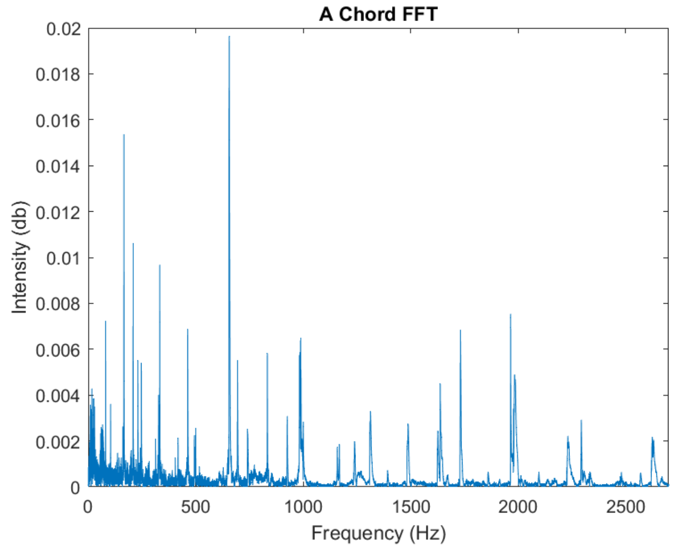

Figure 4: A Chord FFT

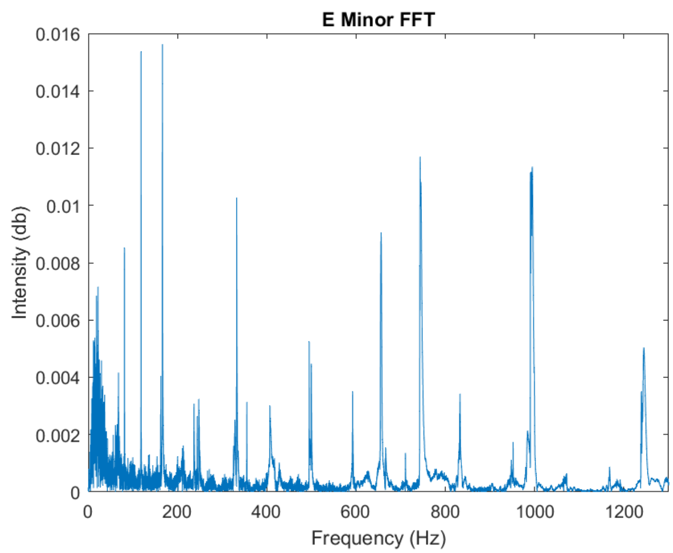

Figure 5: E Minor Chord FFT

These two graphs show similar results to the the graph of all six base strings, however there is more resonance occurring. This is because chords contain a lower quantity of unique notes with more repeats across the six strings. Therefore, we can conclude that the harmonics of guitar chords use resonance to proportionally increase the amplitudes of notes within chords to produce clear and pleasant sounds. Another indicator of resonance is the slight jitter that occurs at some of the peaks. This can be seen clearly at 1000 Hz in Figure 5. The cause of this phenomenon is likely tuning inconsistencies. If the strings are tuned slightly off, there will be imperfections in the resonance of notes.

C. Effects of Damping

Damping is the main component that differentiates a guitar string from an ideal string. Through experimental data we can see proof of damping, however proving the existence of dampening can be easily observed by listening to the immediate amplitude decrease of the sound produced. Below are a selection of graphs that show the damping effect for three of the guitar's strings.

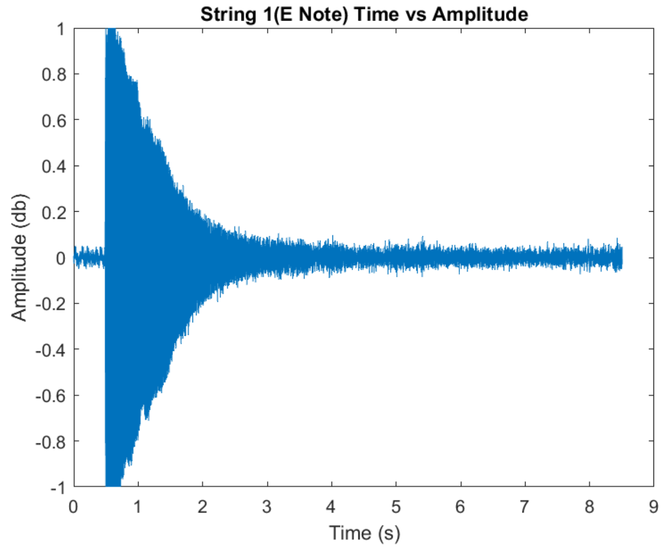

Figure 6: String 1 Damping

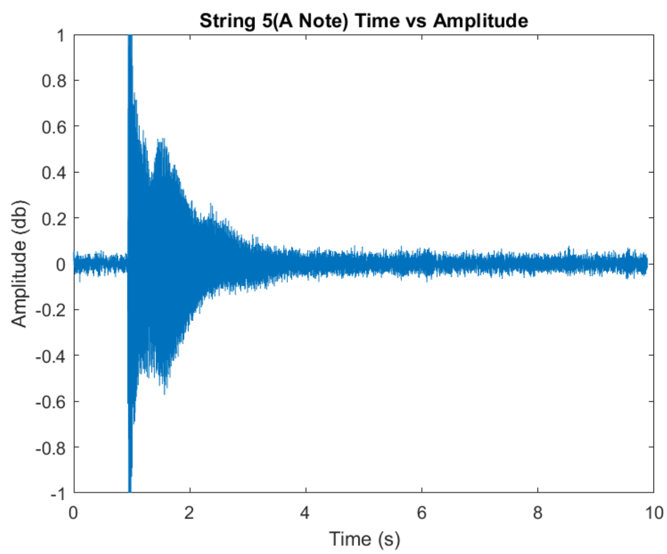

Figure 7: String 3 Damping

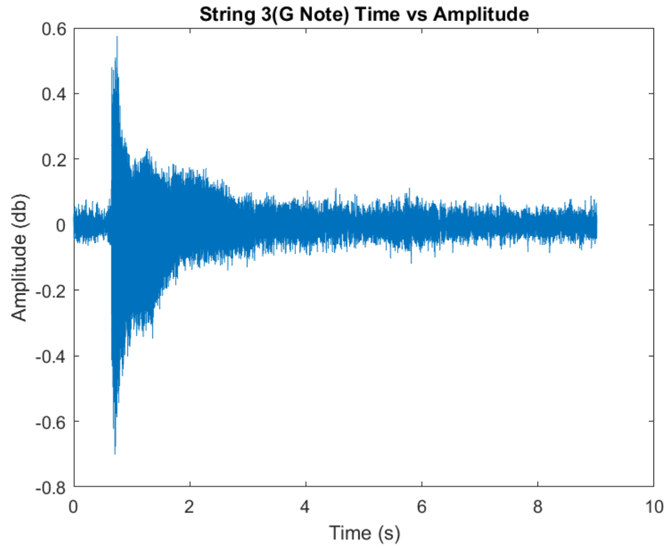

Figure 8: String 5 Damping

The damping trends are very similar between the three graphs. Through the data collection phase, it was observed that the thinner strings were more stable in the sounds they produced, so the relative smoothness of these three graphs is aligned with our observations. The guitar strings operate on a timescale of approximately three seconds. This means that approximately three seconds after a string is plucked, its motion closely resembles the string's steady state. This data does not show a significant difference in the timescale between strings.

The stiffness model described by equation 4 differs from the model described by the undamped, nonstiff wave equation for numerous reasons. The process of solving these equations revealed one of the apparent differences: equation 4 can be solved by using dispersive waves to find the dispersion relation $\omega(k)$. Essentially, our solution to equation 1 was made up of sines and cosines while that of equation 4 was comprised of exponential functions.

The relationship between ω and k described by equation (4) is $\omega(k) = \pm k\sqrt{c^2 + \alpha k^2}$. Since ω can't be negative, as it is the temporal frequency, the relationship between ω and k is an upward parabola with its vertex at the origin. This means that when $k = 0$, so does ω. When $k < 0$ and decreasing, ω increases, and when $k > 0$ and increasing, ω also increases.

D. Applications to Music

The analysis thus far has significant connections to the way music is produced. Most modern music has a simple four chord structure, meaning that four chords are played after each other and repeated throughout the course of a song. The amount of time between the chords depends on the BPM of the song, however a chord is usually played as the previous chord fades. This project has shown that for the specific guitar we used, the strings have an approximate timescale of three seconds, which would imply that an appropriate amount of time between chords would be about three seconds. If a song has a BPM of 130, which is common among pop music, and the chords played at the start of each eight count, the space between chords would be 3.7 seconds. From this we can observe that the pace of music is partially influenced by the damping that occurs when musical instruments are played.

The discussion of chords can be further continued. Resonance has been previously discussed, but dissonance is also a tool used to create unique sounds. Resonance occurs when notes that are the same are stacked, done through the mechanism of chords, however, dissonance refers to the adverse effect where different notes are stacked. In music this mostly occurs during the overlap between chords. Resonance and dissonance together modify the quality of music as well as the Fast Fourier Transform Data. Through this analysis, there is a clear back and forth between the mathematics of music and art of music.

IV. Conclusion

This project has developed a series of mathematical models of the strings on a guitar. This was done through the derivation of four equations, which modeled an undamped string, two separate interpretations of a damped string, and a stiff string. These equations were then applied to Fast Fourier Transform data recorded from a guitar. The results following our analysis showed that playing multiple strings showed both resonance and dissonance, where common guitar chords tended to be resonant, while random combinations tended towards dissonance. Damping proved, through data and observation, to be the main fundamental difference between a guitar string and an ideal string modeled through the basic wave equation (equation 1). Finally, the analysis developed in this project revealed connections between the production of music and the mathematics behind it.

V. References

Haberman, R. Applied Partial Differential Equations with Fourier Series and Boundary Value Problems. 4th edition. Pearson Prentice Hall; 2004.

J. Wild, ”The computation behind consonance and dissonance,” Interdisciplinary Science Reviews, Vol 27, No. 4, p 299 (2002).

VI. Appendix

```
%Kayla Johnson, Mandy Widner, Nick Larson
%Guitar Project

%% Wav to FFT
%All Strings
[y_AS,Fs_AS] = audioread('AllStrings.wav');
Length_AS = length(y_AS);
Mono_AS = (y_AS(:,1)+y_AS(:,2))/2;
y_AS = Mono_AS;
time_AS=(1:length(y_AS))/Fs_AS;
F_AS = 1./time_AS;
NFFT_AS = 2^nextpow2(Length_AS);
f_AS = Fs_AS/2*linspace(0,1,NFFT_AS/2+1);
Y_AS = fft(y_AS,NFFT_AS)/Length_AS;
figure(1)
plot(f_AS,2*abs(Y_AS(1:NFFT_AS/2+1)))
xlim([0,1300])
xlabel('Frequency (Hz)')
ylabel('Intensity (db)')
title('All Strings FFT')
dt_AS = 1/Fs_AS;
t_AS = 0:dt_AS:(length(y_AS)*dt_AS)-dt_AS;
figure(11)
plot(t_AS,y_AS);
xlabel('Time')
ylabel('Amplitude')
title('All Strings Time vs Amplitude')

%A Chord
[y_AC,Fs_AC] = audioread('AChord.wav');
Length_AC = length(y_AC);
Mono_AC = (y_AC(:,1)+y_AC(:,2))/2;
y_AC = Mono_AC;
time_AC=(1:length(y_AC))/Fs_AC;
F_AC = 1./time_AC;
NFFT_AC = 2^nextpow2(Length_AC);
f_AC = Fs_AC/2*linspace(0,1,NFFT_AC/2+1);
Y_AC = fft(y_AC,NFFT_AC)/Length_AC;
figure(2)
plot(f_AC,2*abs(Y_AC(1:NFFT_AC/2+1)))
xlim([0,2700])
xlabel('Frequency (Hz)')
ylabel('Intensity (db)')
title('A Chord FFT')
dt_AC = 1/Fs_AC;
t_AC = 0:dt_AC:(length(y_AC)*dt_AC)-dt_AC;
figure(12)
plot(t_AC,y_AC);
xlabel('Time')
ylabel('Amplitude')
title('A Chord Time Time vs Amplitude')

%Background
[y_BG,Fs_BG] = audioread('Background.wav');
Length_BG = length(y_BG);
Mono_BG = (y_BG(:,1)+y_BG(:,2))/2;
y_BG = Mono_BG;
time_BG=(1:length(y_BG))/Fs_BG;
F_BG = 1./time_BG;
NFFT_BG = 2^nextpow2(Length_BG);
f_BG = Fs_BG/2*linspace(0,1,NFFT_BG/2+1);
Y_BG = fft(y_BG,NFFT_BG)/Length_BG;
figure(3)
plot(f_BG,2*abs(Y_BG(1:NFFT_BG/2+1)))
xlim([0,3000])
xlabel('Frequency')
ylabel('Amplitude')
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```
title('Background FFT')
dt_BG = 1/Fs_BG;
t_BG = 0:dt_BG:(length(y_BG)*dt_BG)-dt_BG;
figure(13)
plot(t_BG,y_BG);
xlabel('Time')
ylabel('Amplitude')
title('Background Time vs Amplitude')

%E Minor
[y_EM,Fs_EM] = audioread('Eminor.wav');
Length_EM = length(y_EM);
Mono_EM = (y_EM(:,1)+y_EM(:,2))/2;
y_EM = Mono_EM;
time_EM=(1:length(y_EM))/Fs_EM;
F_EM = 1./time_EM;
NFFT_EM = 2^nextpow2(Length_EM);
f_EM = Fs_EM/2*linspace(0,1,NFFT_EM/2+1);
Y_EM = fft(y_EM,NFFT_EM)/Length_EM;
figure(4)
plot(f_EM,2*abs(Y_EM(1:NFFT_EM/2+1)))
xlim([0,1300])
xlabel('Frequency (Hz)')
ylabel('Intensity (db)')
title('E Minor FFT')
dt_EM = 1/Fs_EM;
t_EM = 0:dt_EM:(length(y_EM)*dt_EM)-dt_EM;
figure(14)
plot(t_EM,y_EM);
xlabel('Time')
ylabel('Amplitude')
title('E Minor Time vs Amplitude')

%String 1
[y_S1,Fs_S1] = audioread('String1.wav');
Length_S1 = length(y_S1);
Mono_S1 = (y_S1(:,1)+y_S1(:,2))/2;
y_S1 = Mono_S1;
time_S1=(1:length(y_S1))/Fs_S1;
F_S1 = 1./time_S1;
NFFT_S1 = 2^nextpow2(Length_S1);
f_S1 = Fs_S1/2*linspace(0,1,NFFT_S1/2+1);
Y_S1 = fft(y_S1,NFFT_S1)/Length_S1;
figure(5)
plot(f_S1,2*abs(Y_S1(1:NFFT_S1/2+1)))
xlim([0,3000])
xlabel('Frequency (Hz)')
ylabel('Intensity (db)')
title('String 1(E Note) FFT')
dt_S1 = 1/Fs_S1;
t_S1 = 0:dt_S1:(length(y_S1)*dt_S1)-dt_S1;
figure(15)
plot(t_S1,y_S1);
xlabel('Time (s)')
ylabel('Amplitude (db)')
title('String 1(E Note) Time vs Amplitude')

%String 2
[y_S2,Fs_S2] = audioread('String2.wav');
Length_S2 = length(y_S2);
Mono_S2 = (y_S2(:,1)+y_S2(:,2))/2;
y_S2 = Mono_S2;
time_S2=(1:length(y_S2))/Fs_S2;
F_S2 = 1./time_S2;
NFFT_S2 = 2^nextpow2(Length_S2);
f_S2 = Fs_S2/2*linspace(0,1,NFFT_S2/2+1);
Y_S2 = fft(y_S2,NFFT_S2)/Length_S2;
figure(6)
plot(f_S2,2*abs(Y_S2(1:NFFT_S2/2+1)))
xlim([0,1300])
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xlabel('Frequency (Hz)')
ylabel('Intensity (db)')
title('String 2(B Note) FFT')
dt_S2 = 1/Fs_S2;
t_S2 = 0:dt_S2:(length(y_S2)*dt_S2)-dt_S2;
figure(16)
plot(t_S2,y_S2);
xlabel('Time')
ylabel('Amplitude')
title('String 2(B Note) Time vs Amplitude')

%String 3
[y_S3,Fs_S3] = audioread('String3.wav');
Length_S3 = length(y_S3);
Mono_S3 = (y_S3(:,1)+y_S3(:,2))/2;
y_S3 = Mono_S3;
time_S3=(1:length(y_S3))/Fs_S3;
F_S3 = 1./time_S3;
NFFT_S3 = 2^nextpow2(Length_S3);
f_S3 = Fs_S3/2*linspace(0,1,NFFT_S3/2+1);
Y_S3 = fft(y_S3,NFFT_S3)/Length_S3;
figure(7)
plot(f_S3,2*abs(Y_S3(1:NFFT_S3/2+1)))
xlim([0,1300])
xlabel('Frequency')
ylabel('Amplitude')
title('String 3(G Note) FFT')
dt_S3 = 1/Fs_S3;
t_S3 = 0:dt_S3:(length(y_S3)*dt_S3)-dt_S3;
figure(17)
plot(t_S3,y_S3);
xlabel('Time (s)')
ylabel('Amplitude (db)')
title('String 3(G Note) Time vs Amplitude')

%String 4
[y_S4,Fs_S4] = audioread('String4.wav');
Length_S4 = length(y_S4);
Mono_S4 = (y_S4(:,1)+y_S4(:,2))/2;
y_S4 = Mono_S4;
time_S4=(1:length(y_S4))/Fs_S4;
F_S4 = 1./time_S4;
NFFT_S4 = 2^nextpow2(Length_S4);
f_S4 = Fs_S4/2*linspace(0,1,NFFT_S4/2+1);
Y_S4 = fft(y_S4,NFFT_S4)/Length_S4;
figure(8)
plot(f_S4,2*abs(Y_S4(1:NFFT_S4/2+1)))
xlim([0,1300])
xlabel('Frequency')
ylabel('Amplitude')
title('String 4(D Note) FFT')
dt_S4 = 1/Fs_S4;
t_S4 = 0:dt_S4:(length(y_S4)*dt_S4)-dt_S4;
figure(18)
plot(t_S4,y_S4);
xlabel('Time')
ylabel('Amplitude')
title('String 4(D Note) Time vs Amplitude')

%String 5
[y_S5,Fs_S5] = audioread('String5.wav');
Length_S5 = length(y_S5);
Mono_S5 = (y_S5(:,1)+y_S5(:,2))/2;
y_S5 = Mono_S5;
time_S5=(1:length(y_S5))/Fs_S5;
F_S5 = 1./time_S5;
NFFT_S5 = 2^nextpow2(Length_S5);
f_S5 = Fs_S5/2*linspace(0,1,NFFT_S5/2+1);
Y_S5 = fft(y_S5,NFFT_S5)/Length_S5;
figure(9)
```

```
plot(f_S5,2*abs(Y_S5(1:NFFT_S5/2+1)))
xlim([0,1300])
xlabel('Frequency')
ylabel('Amplitude')
title('String 5(A Note) FFT')
dt_S5 = 1/Fs_S5;
t_S5 = 0:dt_S5:(length(y_S5)*dt_S5)-dt_S5;
figure(19)
plot(t_S5,y_S5);
xlabel('Time (s)')
ylabel('Amplitude (db)')
title('String 5(A Note) Time vs Amplitude')

%String 6
[y_S6,Fs_S6] = audioread('String6.wav');
Length_S6 = length(y_S6);
Mono_S6 = (y_S6(:,1)+y_S6(:,2))/2;
y_S6 = Mono_S6;
time_S6=(1:length(y_S6))/Fs_S6;
F_S6 = 1./time_S6;
NFFT_S6 = 2^nextpow2(Length_S6);
f_S6 = Fs_S6/2*linspace(0,1,NFFT_S6/2+1);
Y_S6 = fft(y_S6,NFFT_S6)/Length_S6;
figure(10)
plot(f_S6,2*abs(Y_S6(1:NFFT_S6/2+1)))
xlim([0,1300])
xlabel('Frequency')
ylabel('Amplitude')
title('String 6(E Note) FFT')
dt_S6 = 1/Fs_S6;
t_S6 = 0:dt_S6:(length(y_S6)*dt_S6)-dt_S6;
figure(20)
plot(t_S6,y_S6);
xlabel('Time')
ylabel('Amplitude')
title('String 6(E Note) Time vs Amplitude')
```