



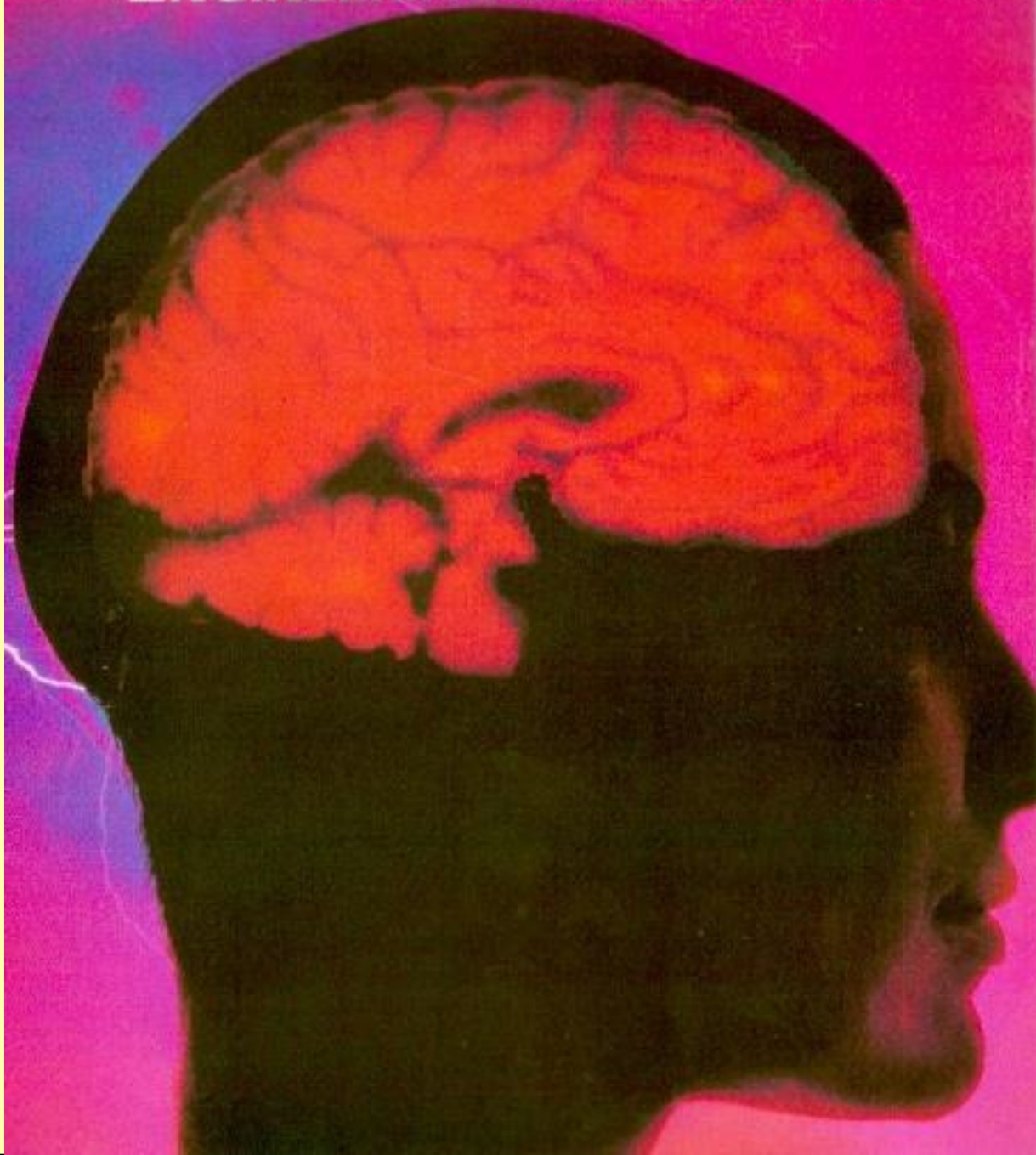
# **Artificial Neural Networks and Applications**

**Systems Modeling and Control Using  
Dynamic Neural Networks and Fuzzy-  
Neural Networks**

**Antonio Moran, Ph.D.**

**[amoran@ieee.org](mailto:amoran@ieee.org)**

# ENGINEERING INTELLIGENCE



# The Human Being is Intelligent

**It has the capacity for:**

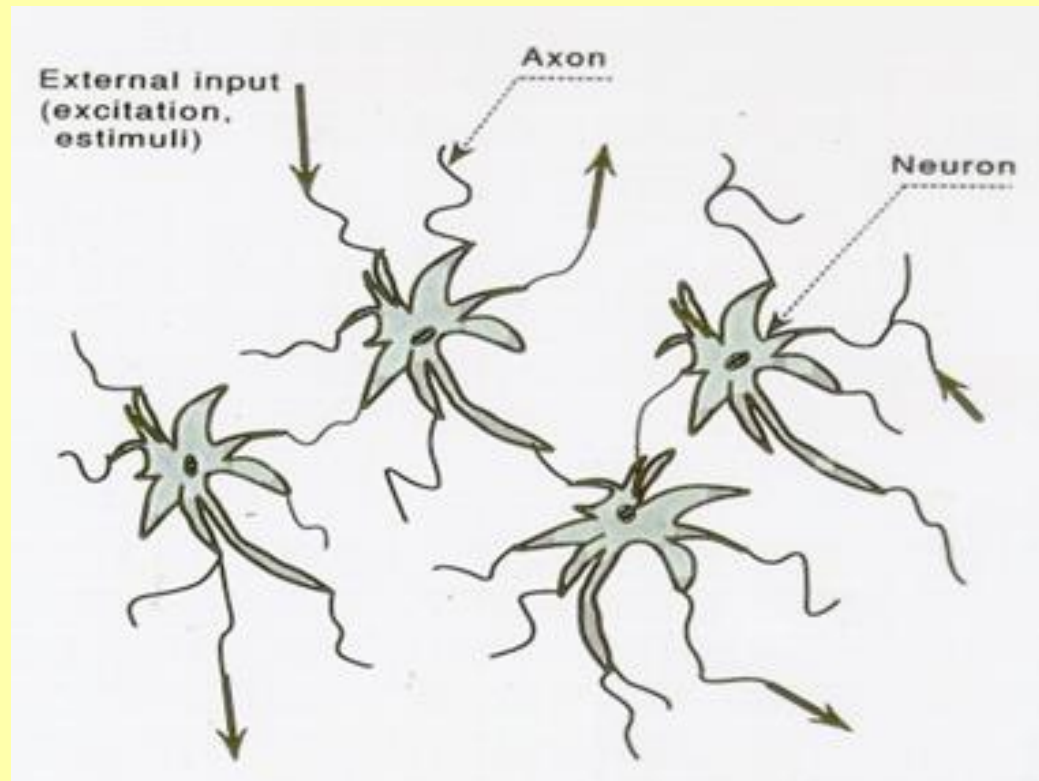
- **Thinking**
- **Reasoning**
- **Learning**
- **Improving**
- **Adapting**
- **.....**



**Able to Work in an  
Autonomous Way**

# The Brain

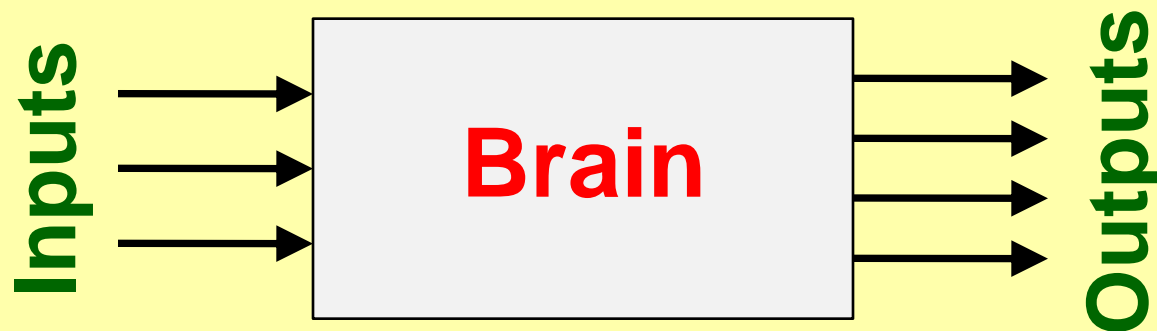
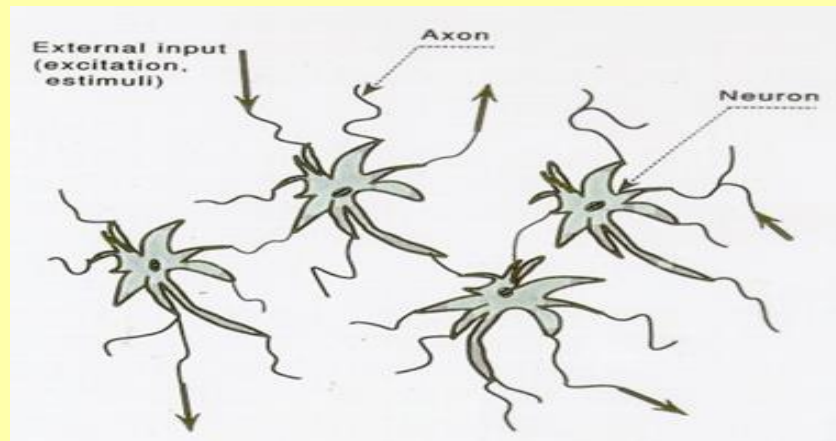
## A Natural Neural Network



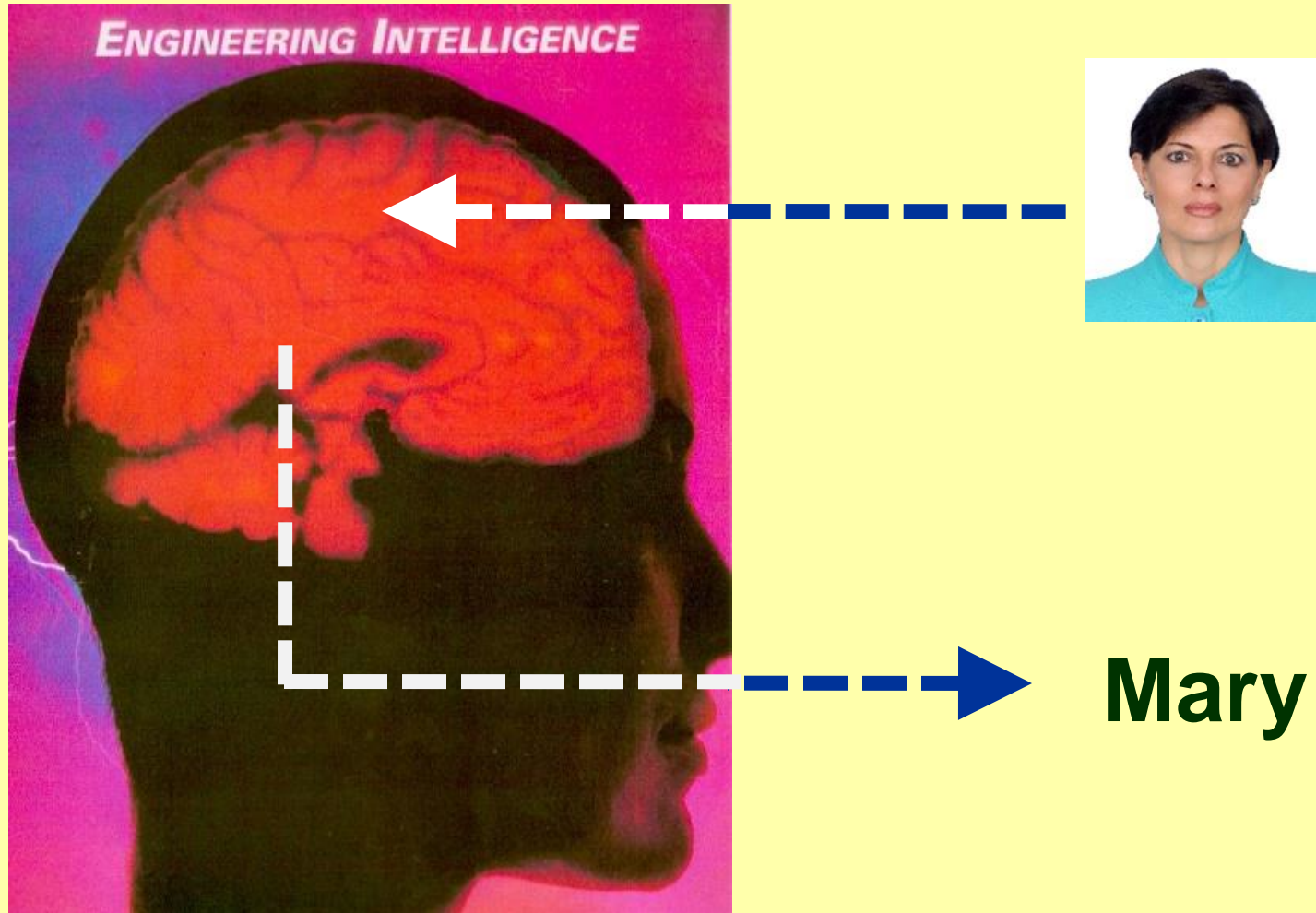
**Millions of highly interconnected neurons**

# The Brain

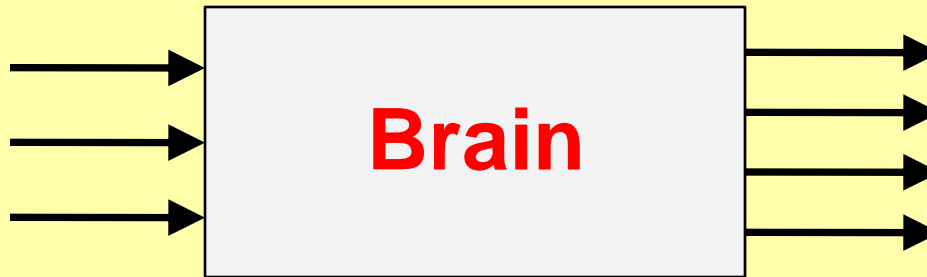
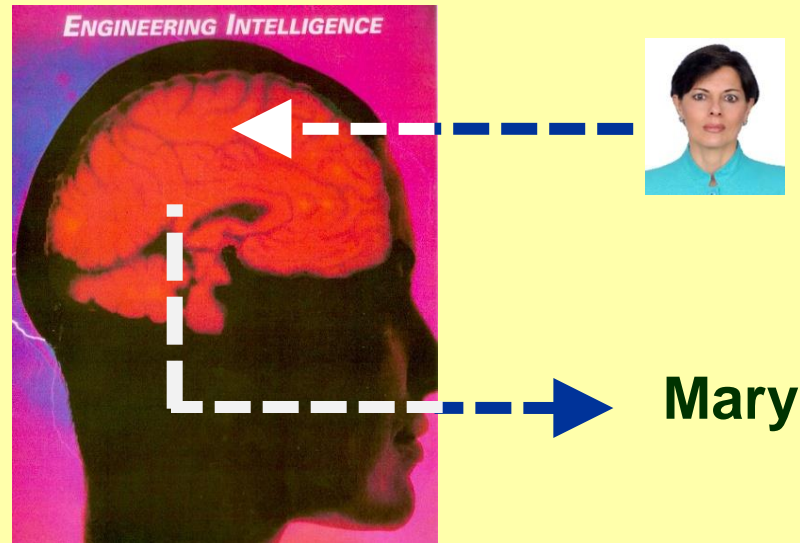
Behaves as a System with  
Inputs and Outputs



# Face Recognition



# Face Recognition



Mary



# Car Driving



**Present Position**

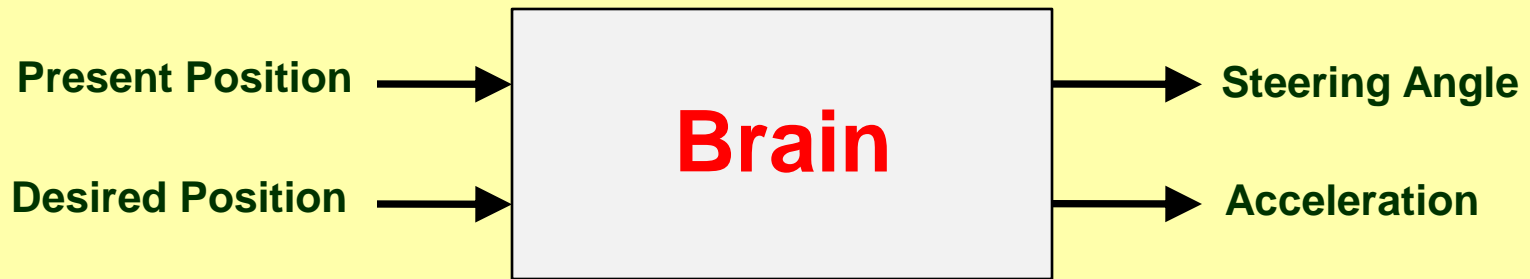
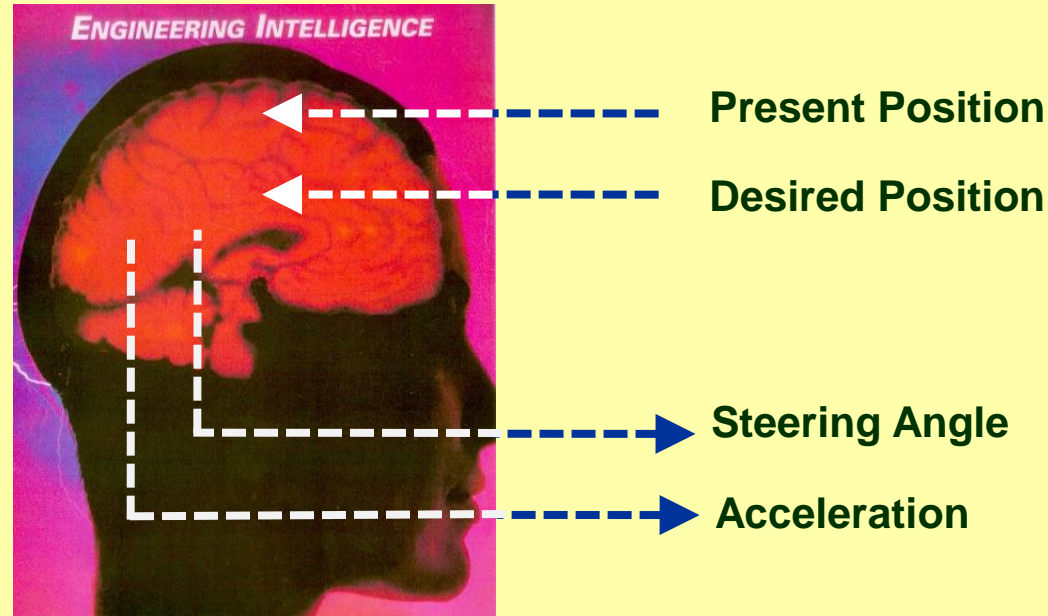
**Desired Position**

**Steering Angle**

**Acceleration**

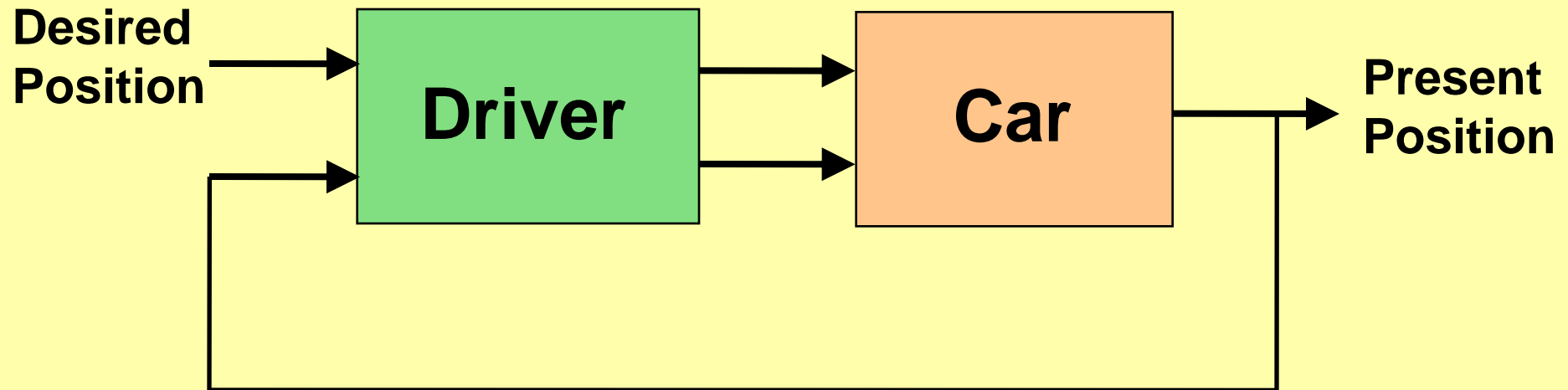


# Car Driving



# **Car Driving**

## **A Control Problem**



# Medical Treatment

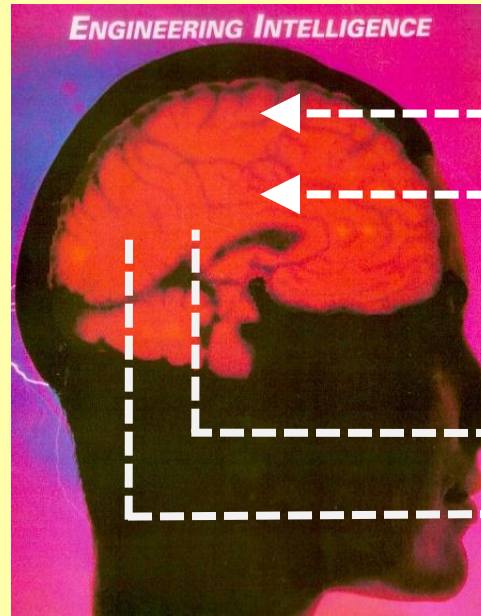


**Blood  
Pressure**

**Cardiac  
Pulse**

**Medicine  
Dose**

# Medical Treatment



Present Levels (Pressure, Pulse)

Desired Levels (Pressure, Pulse)

Medicine 1

Medicine 2

Present Levels →

Desired Levels →

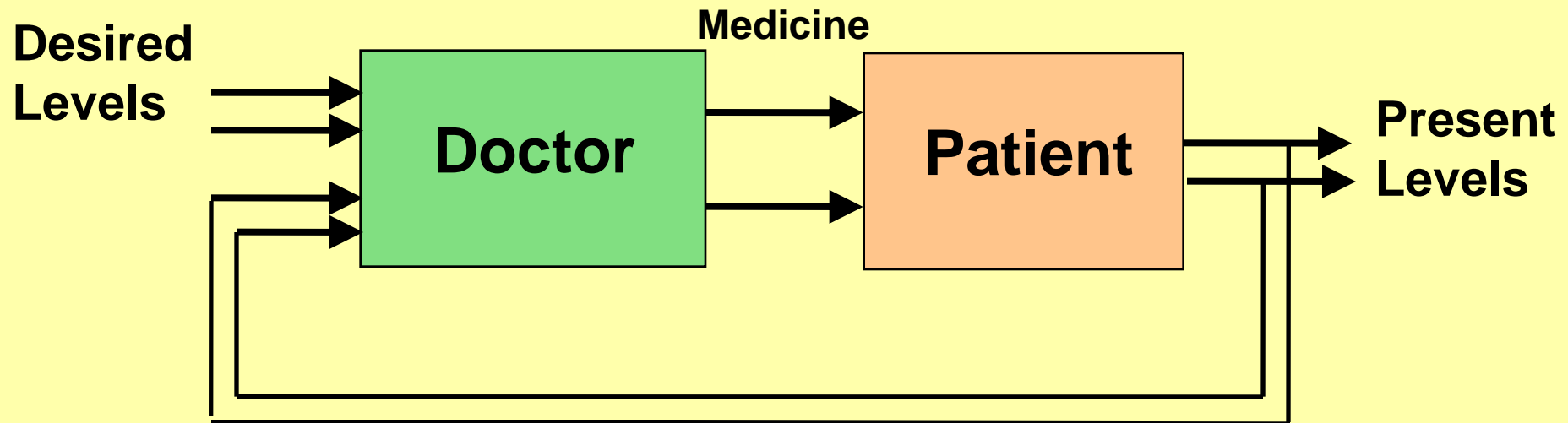
**Brain**

→ Medicine 1

→ Medicine 2

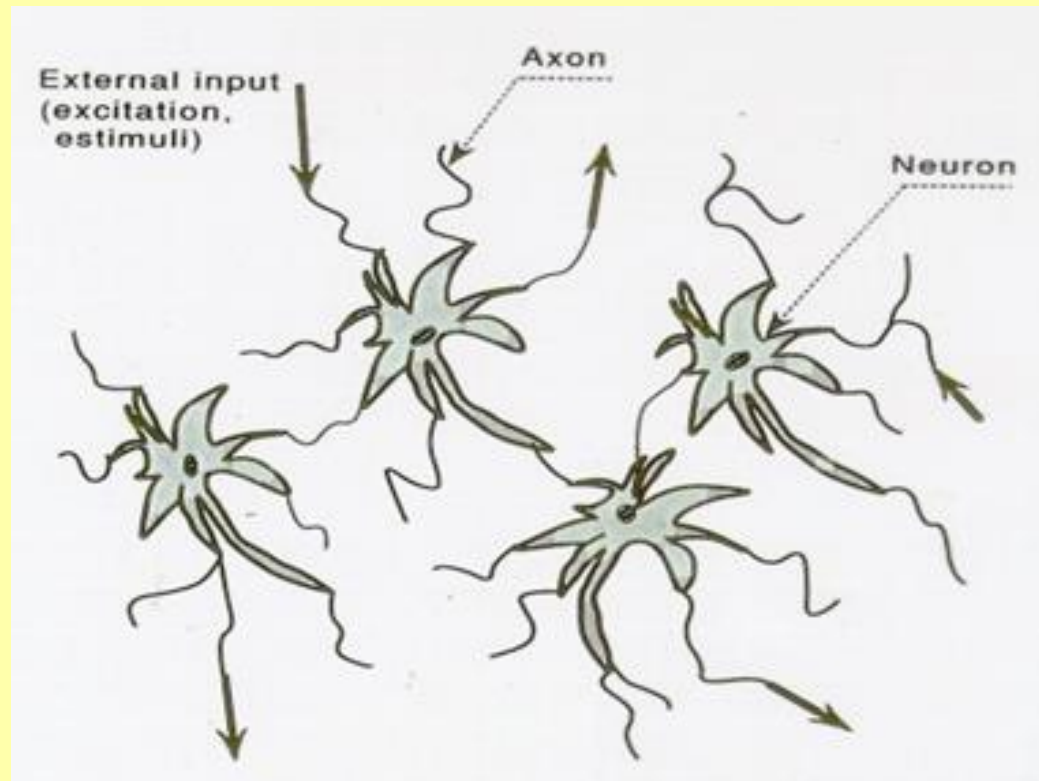
# Medical Treatment

## A Control Problem



# The Brain

## A Natural Neural Network

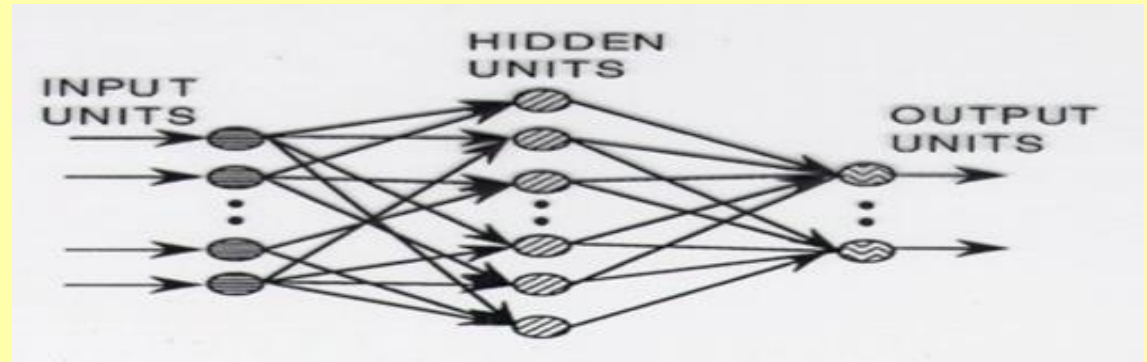


**Millions of highly interconnected neurons**



# Artificial Neural Network Models

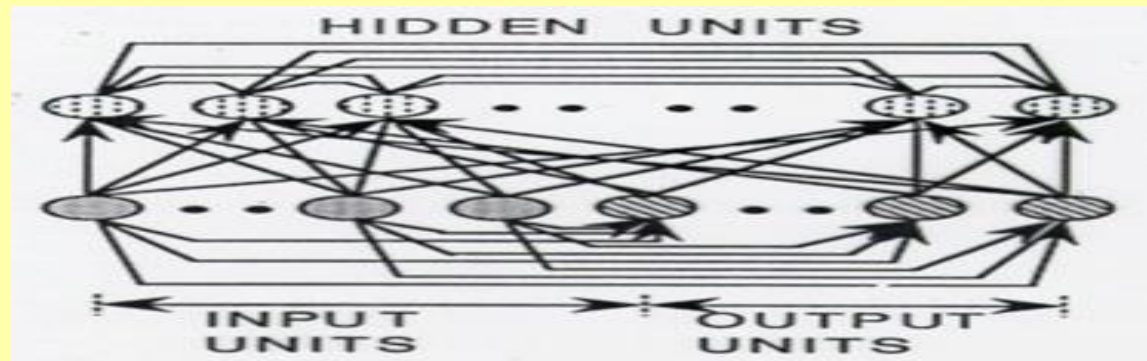
**Multilayer  
Neural Network**



**Self-Organizing  
Map**

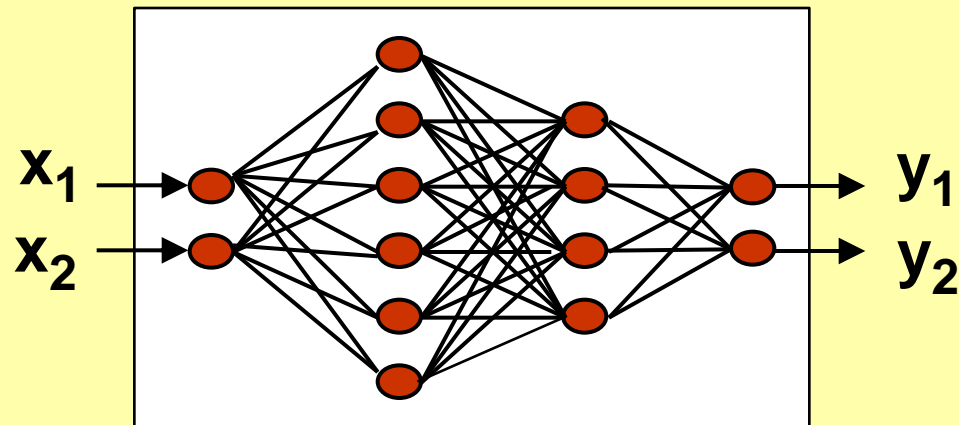


**Boltzmann  
Completion Network**

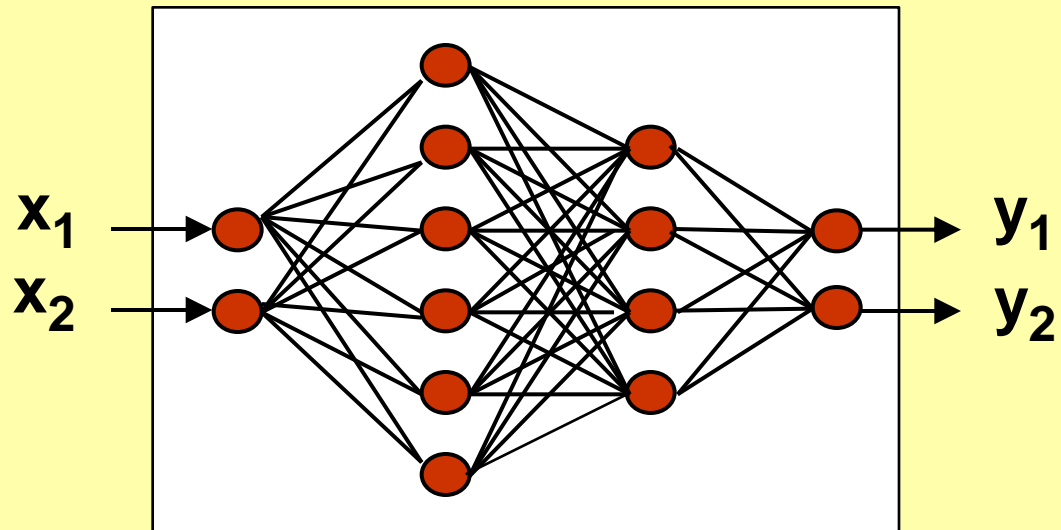


# Neural Networks

**Systems capable of estimating functions of several inputs and outputs using input-output data**

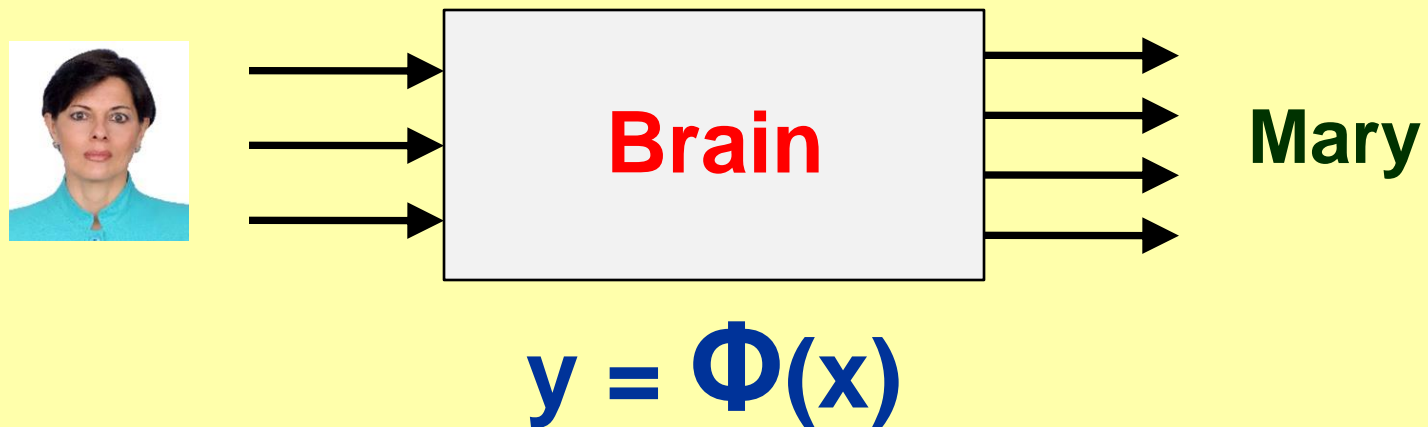
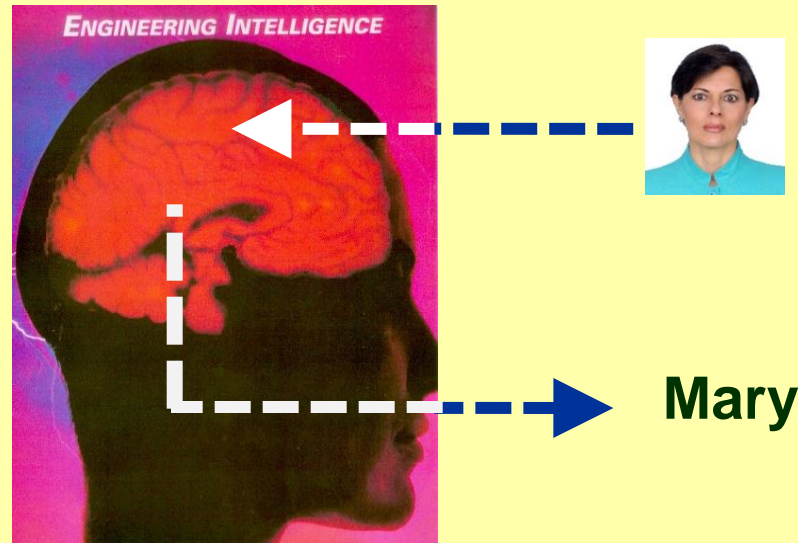


# Neural Networks

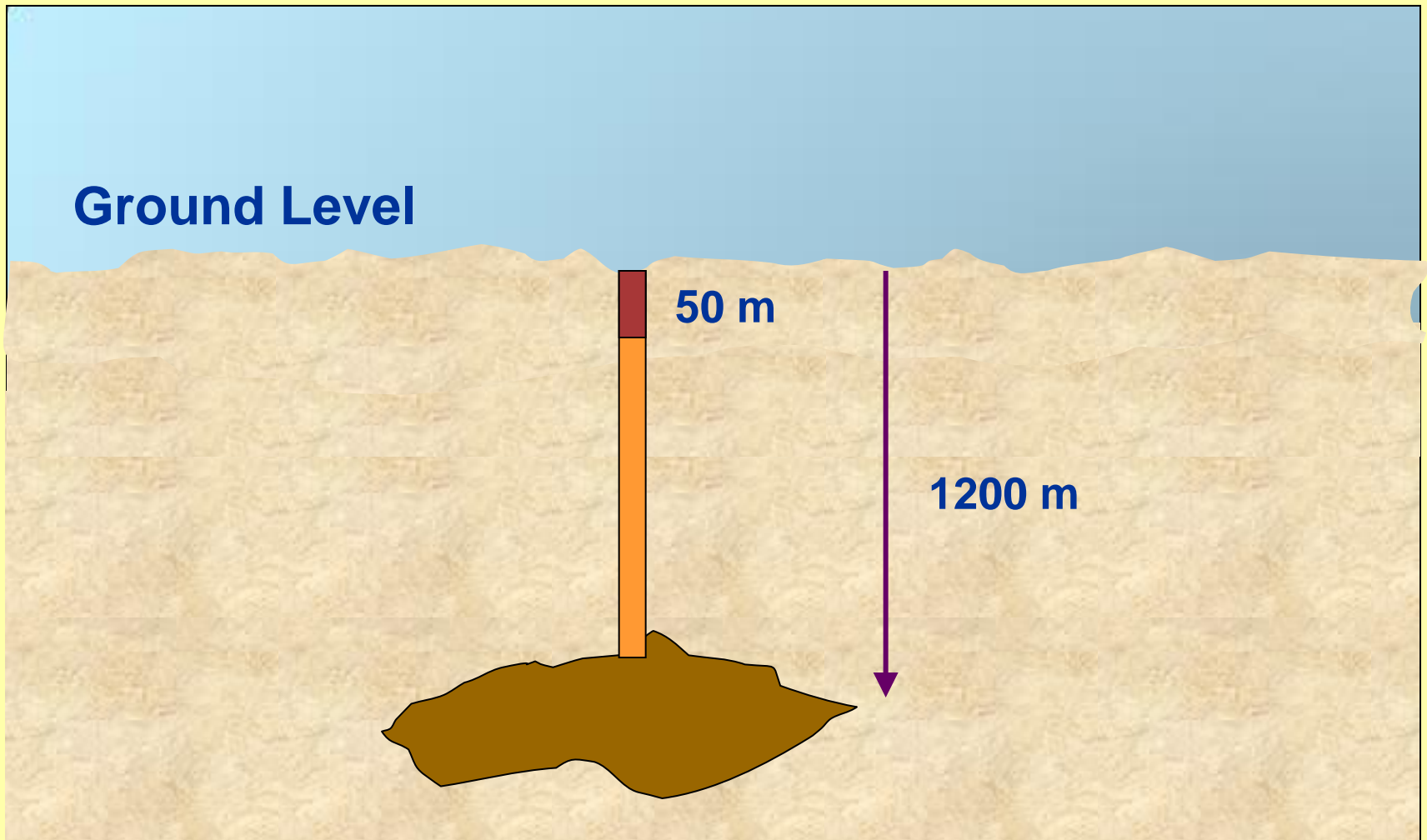


$$y = \Phi(x)$$

# Face Recognition

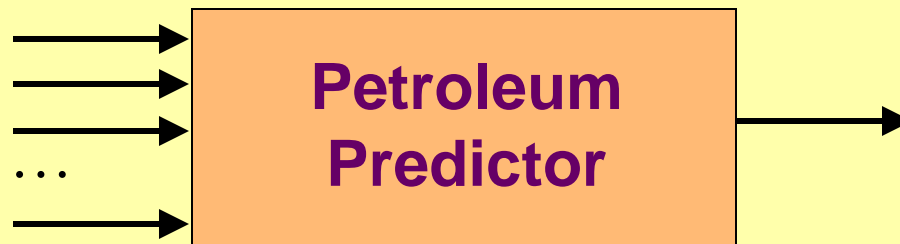


# Petroleum Prediction



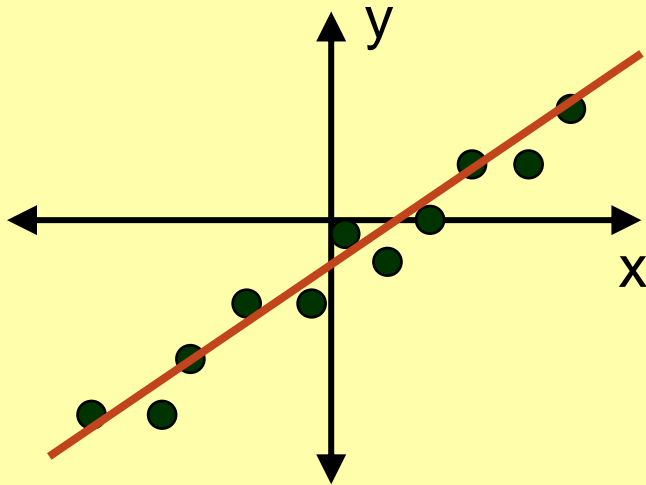
# Petroleum Prediction

	10m				.....	50m				
	Tem	Hum	Ca	Su		Tem	Hum	Ca	Su	Petroleum
Well 1	42	55	14	2		56	42	12	1	1
Well 2	39	62	20	4		54	40	18	1	0
Well 3	33	31	36	1		51	40	31	2	0
..		..		..			..			..
..		..		..			..			..
Well 50	45	51	19	5		60	48	21	3	1

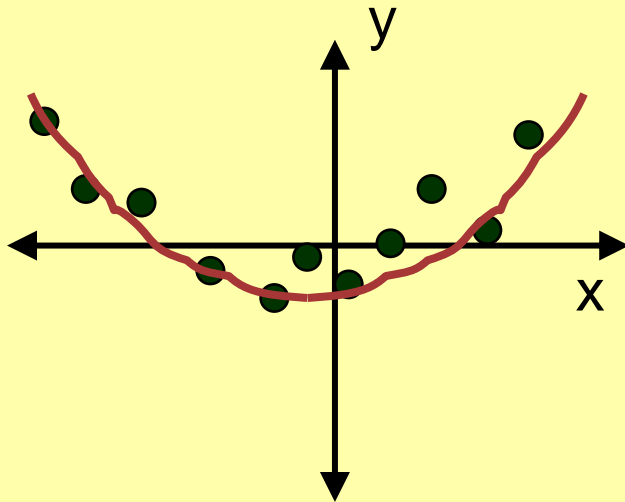




# Function Estimation



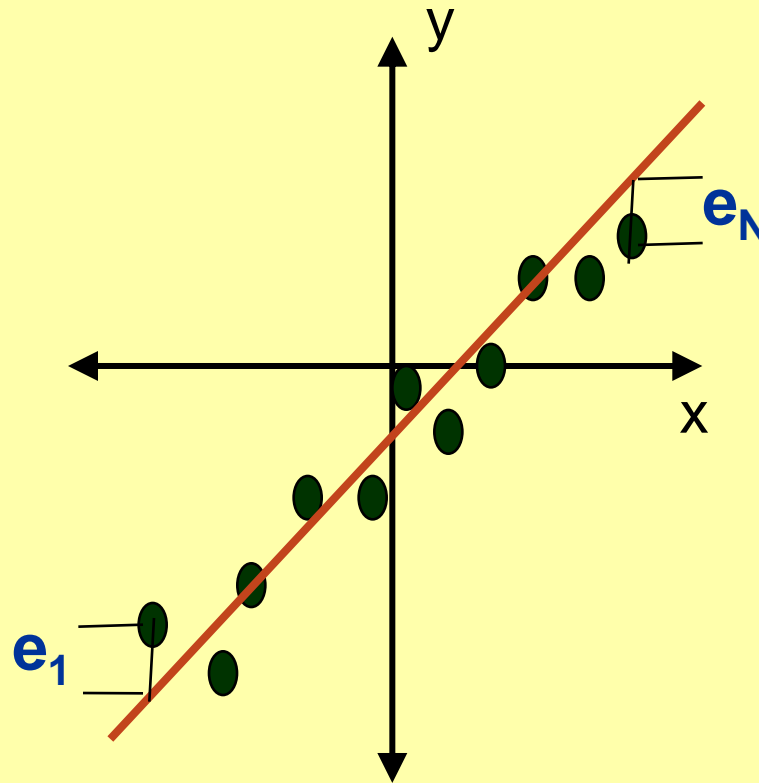
$$y = ax + b$$



$$y = ax^2 + bx + c$$

# Function Estimation

Data	
x	$\bar{y}$
$x_1$	$\bar{y}_1$
$x_2$	$\bar{y}_2$
$x_3$	$\bar{y}_3$
$\vdots$	$\vdots$
$x_N$	$\bar{y}_N$



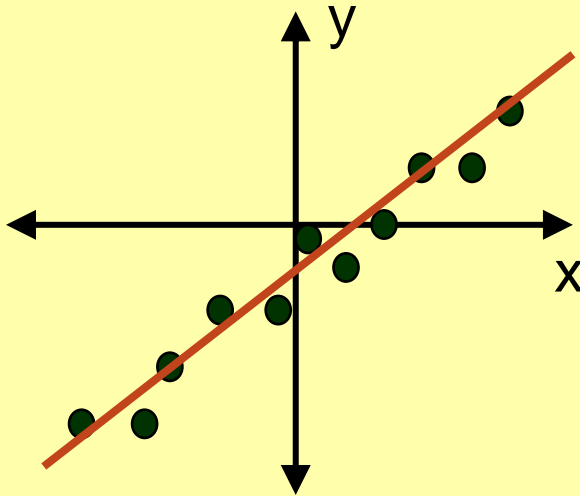
$$y = ax + b$$

## Sum of Errors Squares

$$J = 0.5 e_1^2 + 0.5 e_2^2 + \cdots + 0.5 e_N^2$$

$$J = 0.5 (y_1 - \bar{y}_1)^2 + 0.5 (y_2 - \bar{y}_2)^2 + \cdots + 0.5 (y_N - \bar{y}_N)^2$$

# Function Estimation



$$y = ax + b$$

$$J = 0.5 (y_1 - \bar{y}_1)^2 + 0.5 (y_2 - \bar{y}_2)^2 + \dots + 0.5 (y_N - \bar{y}_N)^2$$

**Problem: Find a and b that minimize J**

# Function Estimation

$$y = ax + b$$

$$J = 0.5 (y_1 - \bar{y}_1)^2 + 0.5 (y_2 - \bar{y}_2)^2 + \cdots + 0.5 (y_N - \bar{y}_N)^2$$

**Problem:** Find  $a$  and  $b$  that minimize  $J$

---

## **Solution**

**Exact Method:**  $\frac{\partial J}{\partial a} = 0 \quad \frac{\partial J}{\partial b} = 0$

**Iterative Method:**

$$a = a - \eta \frac{\partial J}{\partial a} \quad b = b - \eta \frac{\partial J}{\partial b}$$

# Function Estimation

## Iterative Method


$$a = a - \eta \frac{\partial J}{\partial a}$$

$$b = b - \eta \frac{\partial J}{\partial b}$$

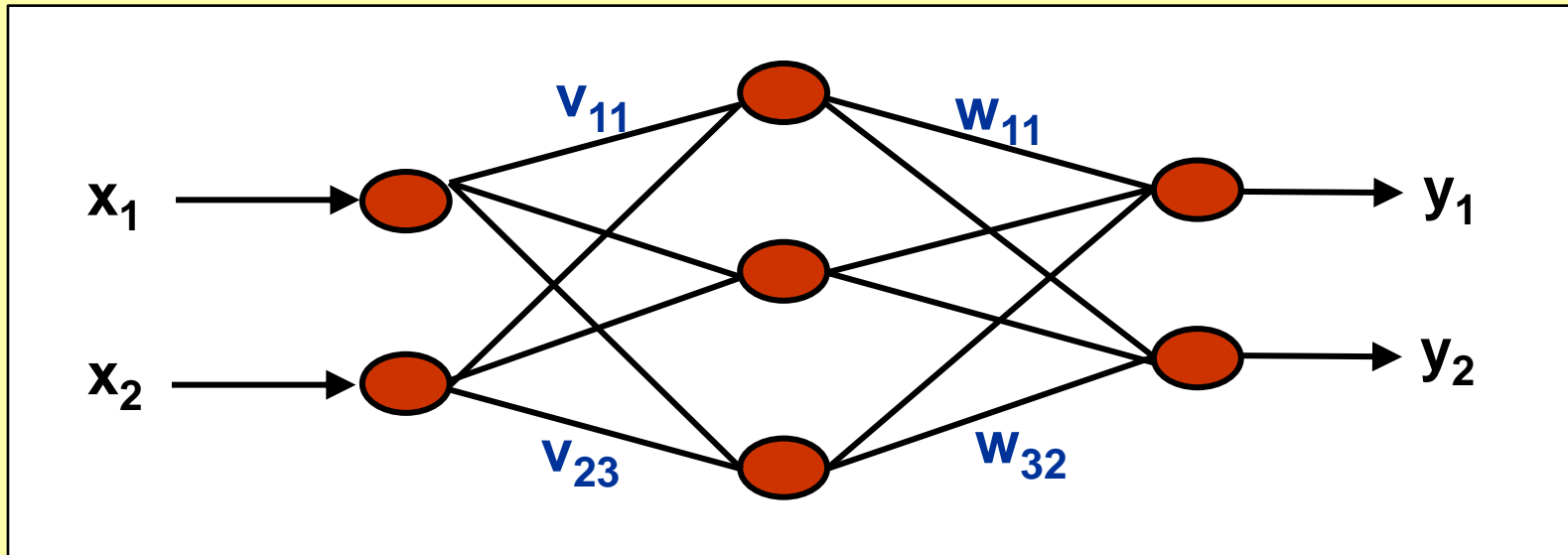
$\eta$  : Learning rate

Fix value of  $\eta$

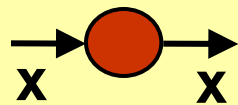
Initial values of  $a$  and  $b$

- 
- Compute derivatives  $\partial J / \partial a$  and  $\partial J / \partial b$
  - Update  $a$  and  $b$
  - Verify convergence condition

# Neural Network

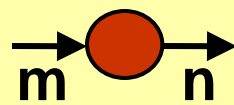


Input  
Layer



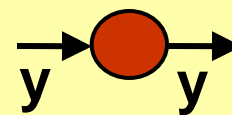
Linear

Hidden  
Layer



Non-Linear

Output  
Layer



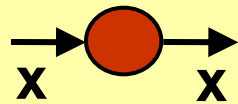
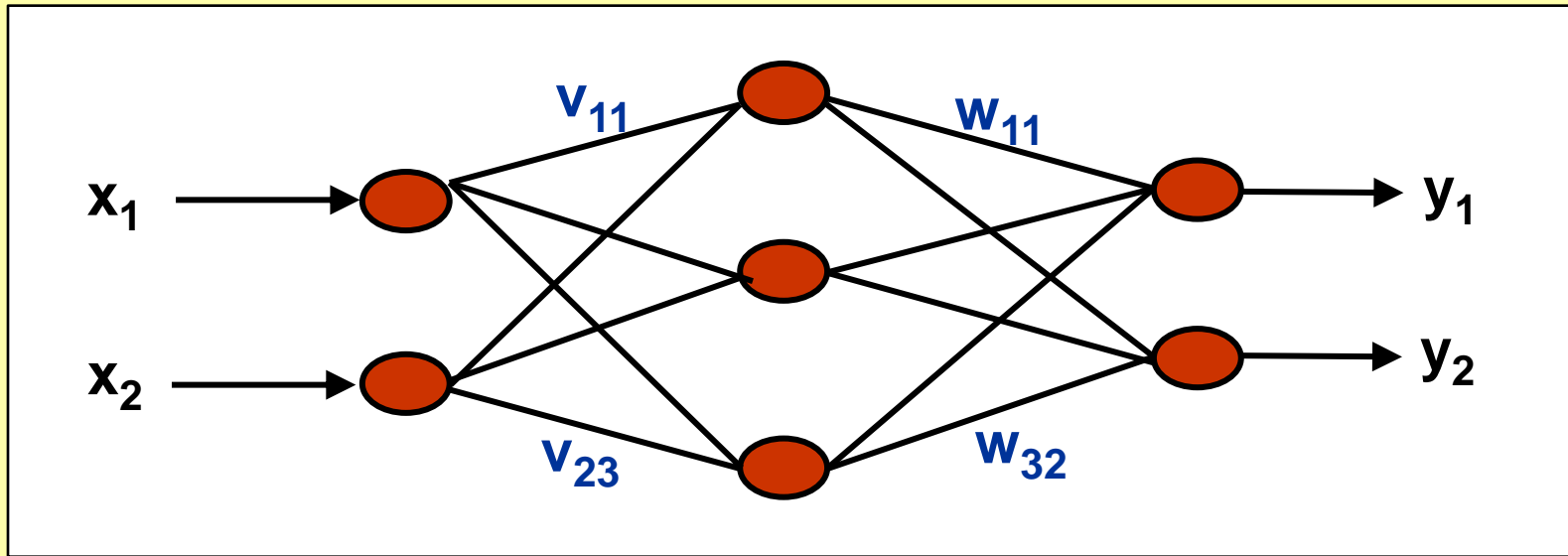
Linear

$v_{11}$  .....  $v_{23}$   
 $w_{11}$  .....  $w_{32}$

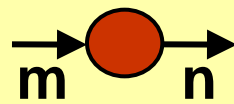
Weights, Connection Coefficients



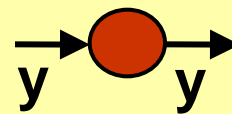
# Neural Network



Linear



Non-Linear



Linear

Sigmoid

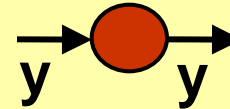
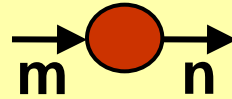
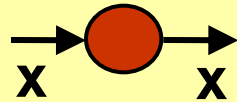
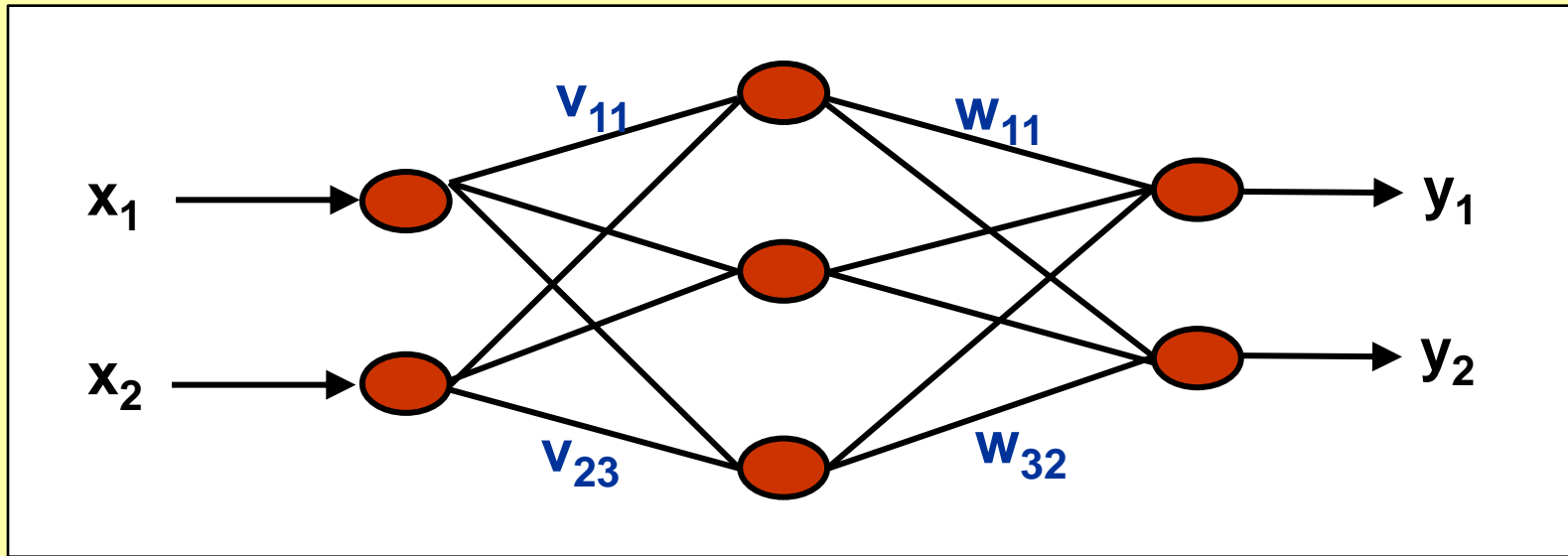
$$n = \frac{1}{1 + e^{-m}}$$

$$n = f(m)$$

Gaussian

$$n = e^{-m^2}$$

# Neural Network



$$m_1 = v_{11} x_1 + v_{21} x_2$$

$$m_2 = v_{12} x_1 + v_{22} x_2$$

$$m_3 = v_{13} x_1 + v_{23} x_2$$

$$n_1 = f(m_1)$$

$$n_2 = f(m_2)$$

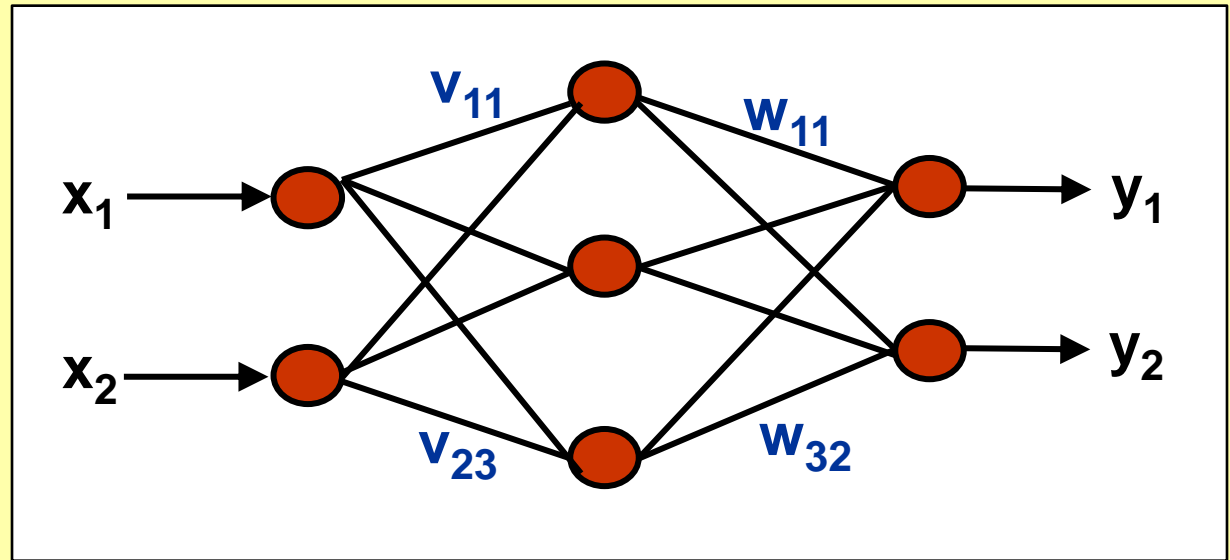
$$n_3 = f(m_3)$$

$$y_1 = w_{11} n_1 + w_{21} n_2 + w_{31} n_3$$

$$y_2 = w_{12} n_1 + w_{22} n_2 + w_{32} n_3$$

# Training of Neural Network

Data			
$x_1$	$x_2$	$\bar{y}_1$	$\bar{y}_2$
*	*	*	*
*	*	*	*
*	*	*	*
*	*	*	*

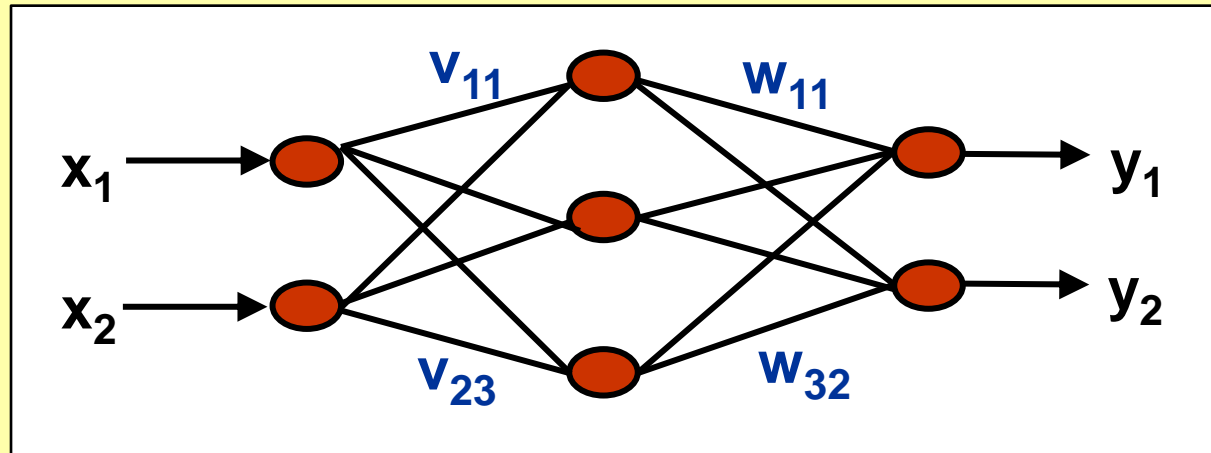


**Cost function to be minimized:**

$$J = 0.5 (y_{(1)} - \bar{y}_{(1)})^T (y_{(1)} - \bar{y}_{(1)}) + \dots + 0.5 (y_{(N)} - \bar{y}_{(N)})^T (y_{(N)} - \bar{y}_{(N)})$$

$$y_{(k)} = [y_{1(k)} \quad y_{2(k)}]^T$$

# Training of Neural Network



$$J = 0.5 (y_{(1)} - \bar{y}_{(1)})^T (y_{(1)} - \bar{y}_{(1)}) + \dots + 0.5 (y_{(N)} - \bar{y}_{(N)})^T (y_{(N)} - \bar{y}_{(N)})$$

## Problem

Find  $V_{11} \dots V_{23}$  that minimize  $J$   
 $W_{11} \dots W_{32}$

# Training of Neural Network

$$J = 0.5 (y_{(1)} - \bar{y}_{(1)})^T (y_{(1)} - \bar{y}_{(1)}) + \dots + 0.5 (y_{(N)} - \bar{y}_{(N)})^T (y_{(N)} - \bar{y}_{(N)})$$

## Problem

Find  $\begin{matrix} v_{11} & \dots & v_{23} \\ w_{11} & \dots & w_{32} \end{matrix}$  that minimize J

## Iterative Method

$$v_{ij} = v_{ij} - \eta \frac{\partial J}{\partial v_{ij}}$$

$$i = 1, 2$$

$$j = 1, 2, 3$$

$$w_{jk} = w_{jk} - \eta \frac{\partial J}{\partial w_{jk}}$$

$$k = 1, 2$$

# Training of Neural Network

$$J = 0.5 (y_{(1)} - \bar{y}_{(1)})^T (y_{(1)} - \bar{y}_{(1)}) + \dots + 0.5 (y_{(N)} - \bar{y}_{(N)})^T (y_{(N)} - \bar{y}_{(N)})$$

## Iterative Method

$$v_{ij} = v_{ij} - \eta \frac{\partial J}{\partial v_{ij}}$$

$$w_{jk} = w_{jk} - \eta \frac{\partial J}{\partial w_{jk}}$$

**Fix value of  $\eta$**

**Initial values of  $v_{ij}$  and  $w_{jk}$**

**Compute derivatives  $\partial J / \partial v_{ij}$  and  $\partial J / \partial w_{jk}$**

**Update  $v_{ij}$  and  $w_{jk}$**

**Verify convergence condition**



# Training of Neural Network

How to compute the derivatives

$$\frac{\partial J}{\partial v_{ij}}$$

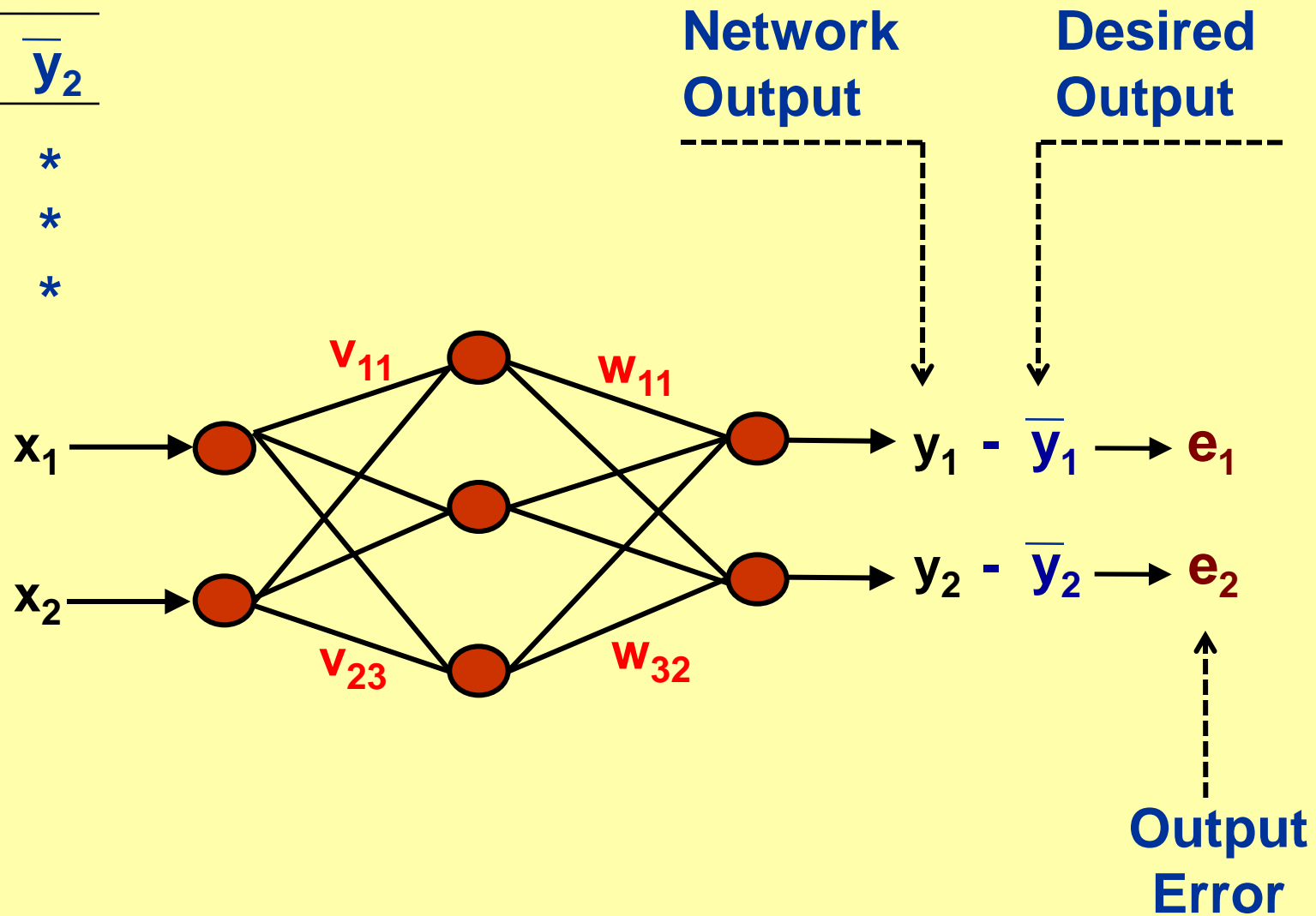
$$\frac{\partial J}{\partial w_{jk}}$$



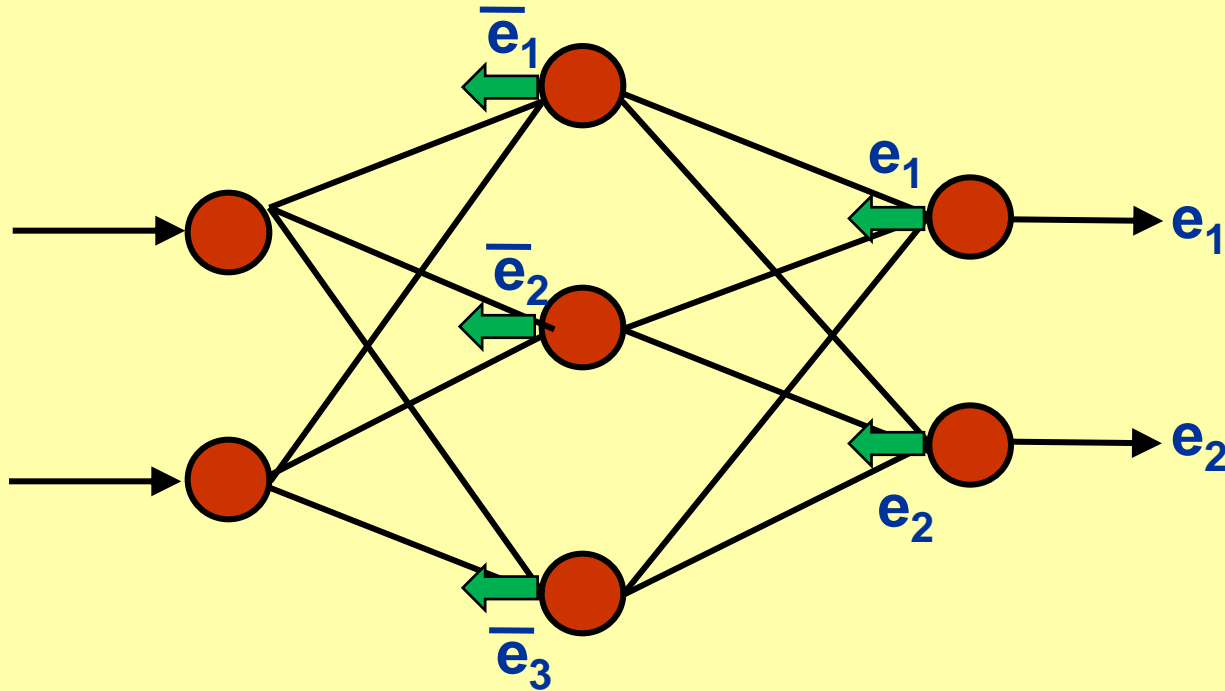
**Error Back Propagation Algorithm**  
**Delta Rule**

# Error Back Propagation

Data			
$x_1$	$x_2$	$\bar{y}_1$	$\bar{y}_2$
*	*	*	*
*	*	*	*
*	*	*	*



# Error Back Propagation



$$e_1 = (y_1 - \bar{y}_1)$$

$$e_2 = (y_2 - \bar{y}_2)$$

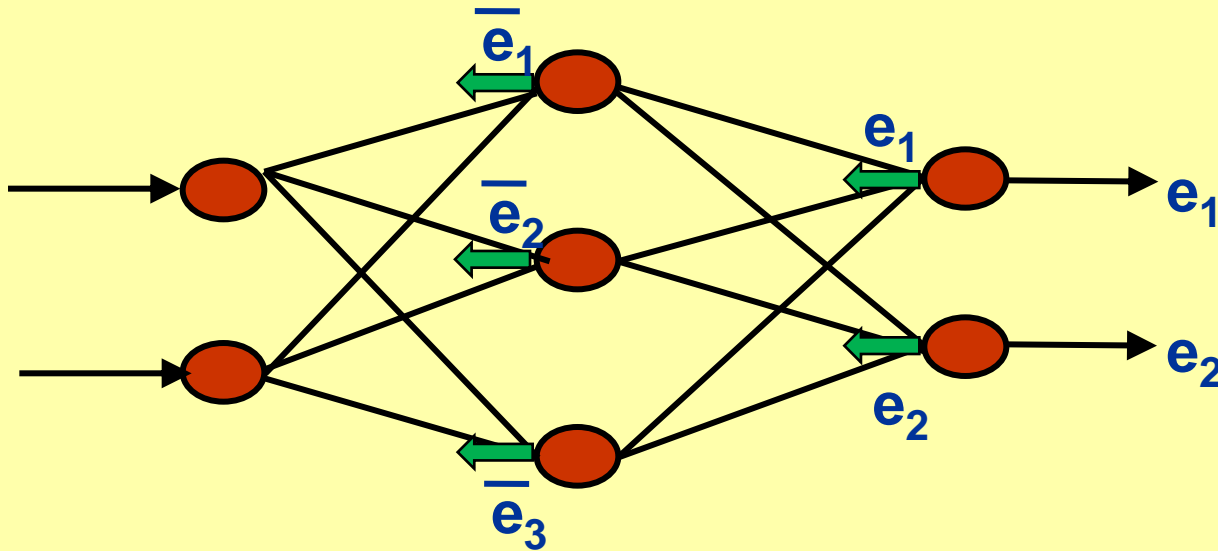


$$\bar{e}_1 = (w_{11}e_1 + w_{12}e_2) f'(m_1)$$

$$\bar{e}_2 = (w_{21}e_1 + w_{22}e_2) f'(m_2)$$

$$\bar{e}_3 = (w_{31}e_1 + w_{32}e_2) f'(m_3)$$

# Error Back Propagation



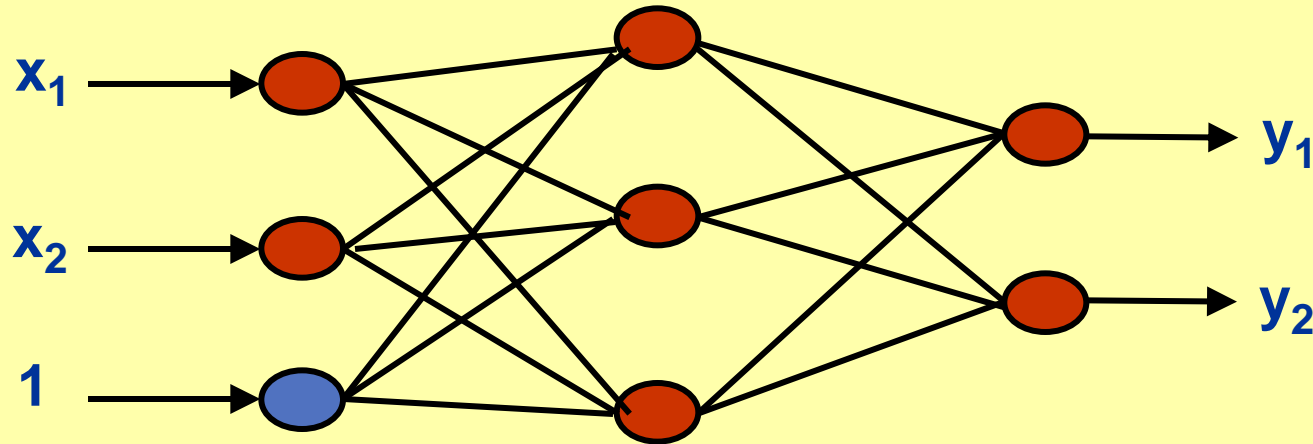
**Computing Derivatives:**

(back propagated error) (output of previous neuron)

$$\frac{\partial J}{\partial v_{ij}} = e_j x_i$$

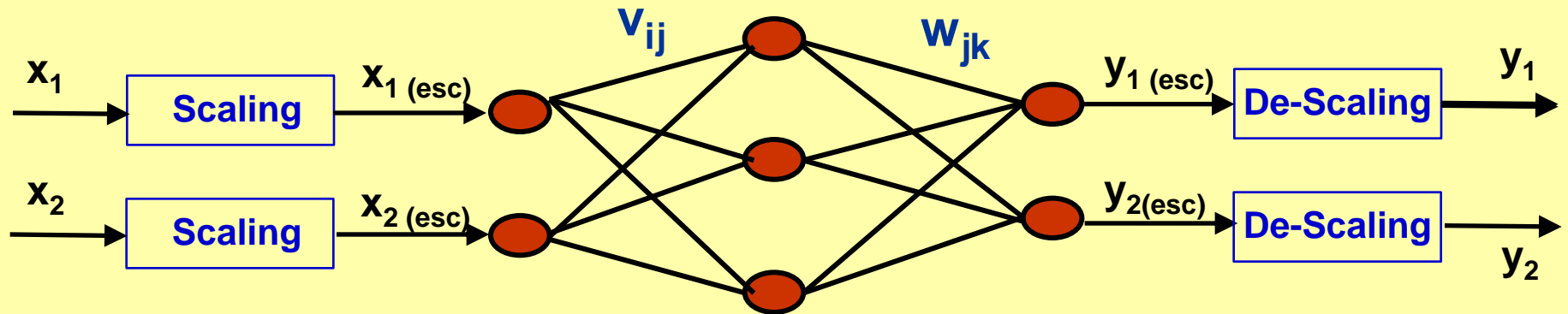
$$\frac{\partial J}{\partial w_{jk}} = e_k n_j$$

# Bias Neuron



**In some cases, learning significantly improves with an additional input neuron having a constant input. This is the bias neuron.**

# Input – Output Scaling



Given the saturation characteristics of neuron activation functions (sigmoid, gaussian) is desirable that the inputs to the neuron do not be of large value. To achieve that:

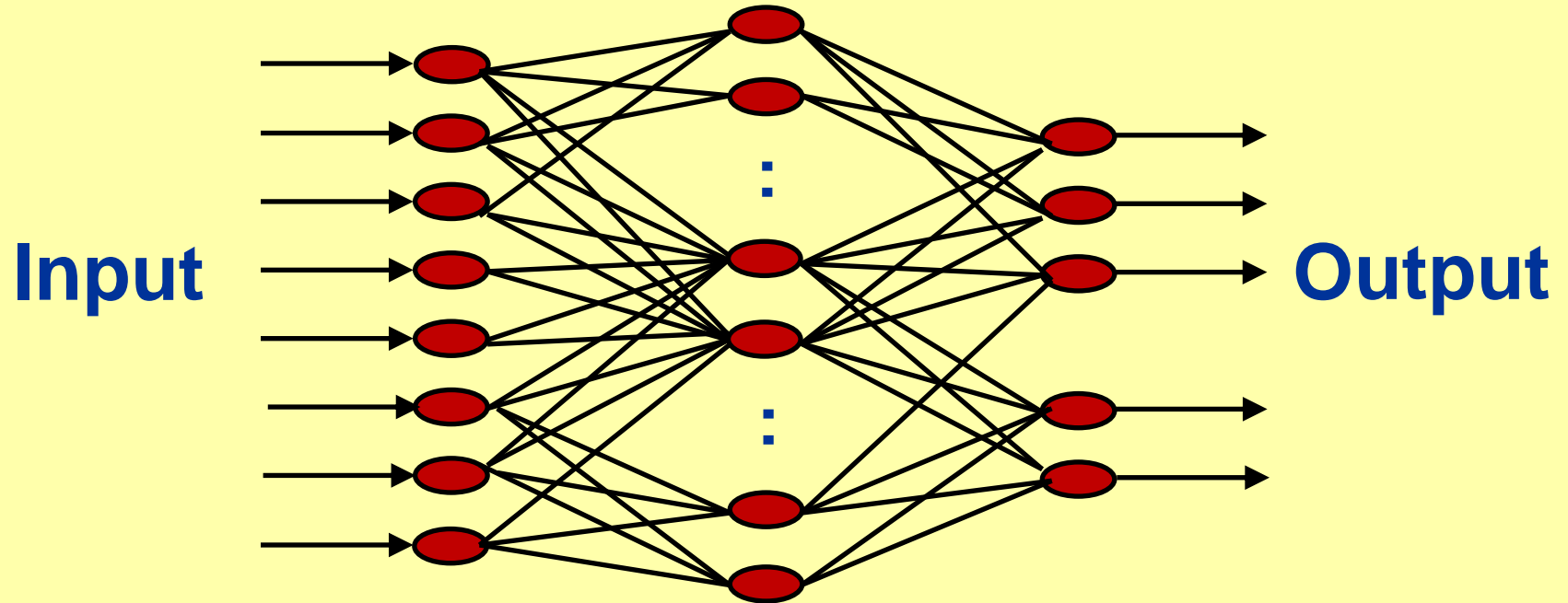
- Inputs (and corresponding outputs) should be scaled to the range  $[-2 \ 2]$  (for instance). Linear scaling.
- Weights  $v_{ij}$  and  $w_{jk}$  should be of small value.

# Face Recognition



**Neural network for  
recognizing 10 faces**

# Neural Network for Face Recognition



**Input: Face**

**Output: Code for each face**



# Face Recognition

## Assigning a code to each face

Considering 10 faces, the code will be of 10 digits of 1's and 0's in an orthogonal scheme

Face 1: 1 0 0 0 0 0 0 0 0 0 0

Face 2: 0 1 0 0 0 0 0 0 0 0 0

Face 3: 0 0 1 0 0 0 0 0 0 0 0

Face 4: 0 0 0 1 0 0 0 0 0 0 0

:

:

:

Face 9: 0 0 0 0 0 0 0 0 0 1 0

Face 10: 0 0 0 0 0 0 0 0 0 0 1

# Face Recognition

## Neural Network Inputs

Given that an image contains great amount of information, it should be reduced to be processed by the neural network.

There are several ways to accomplish this reduction:

- Principal Components Analysis PCA.
- Discrete Cosine Transformation Coefficients.
- Pixeling (used in this report)



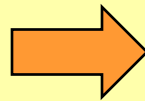
**Full Color**  
**2808 x 2425**

# Face Recognition

## Reducing the size of images - Pixeling

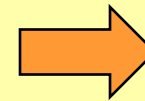


**Full Color**  
**2808 x 2425**



**Gray Scale**  
**1826 x 1529**

The face occupies the  
most of the image



**Monocromatic**  
**40 x 30**  
**1200 pixels**

# Face Recognition

# Network Input



# Matrix 40x30

[illegible]

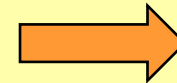
**The matrix should be transformed into vector**

# Face Recognition

**Network Input: Converting 40x30 matrix  
into 1200x1 vector**

0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	1	1	1	1	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1	1	1	1	1	1	1	0	0	0	0	0	0
:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:	:
0	0	0	0	1	0	1	1	1	1	1	1	1	0	0	1	0	0	0	0
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

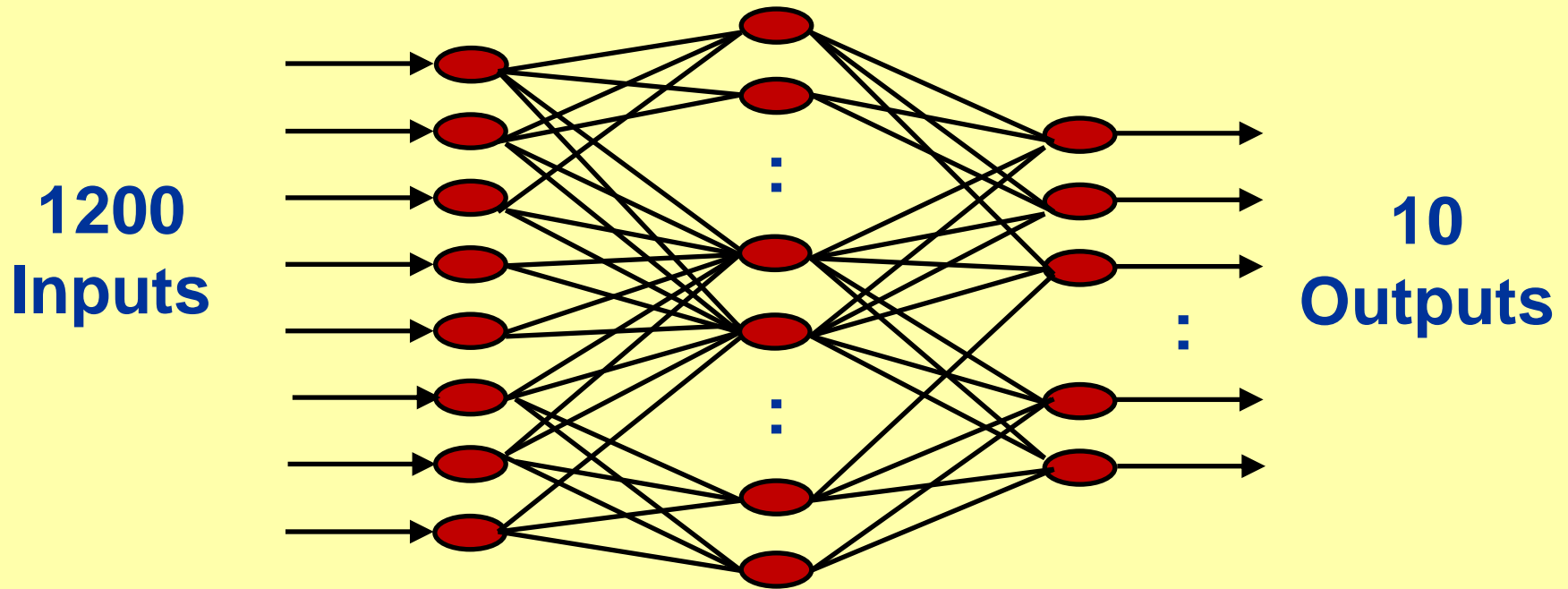
**40x30**



0
0
0
0
:
1
0
1
0
:
1
0
0
0
0

**1200x1**

# Neural Network for Face Recognition



To generate input-output training data, several faces of a person could be considered but all of them with the same output code

# Neural Network for Face Recognition

## Image Preprocessing - Pixeling



1213x1013



40x30



2644x2106



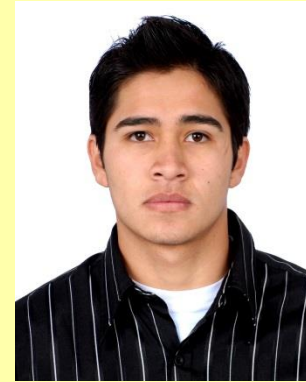
40x30



2854x2370



40x30



2446x2016



40x30

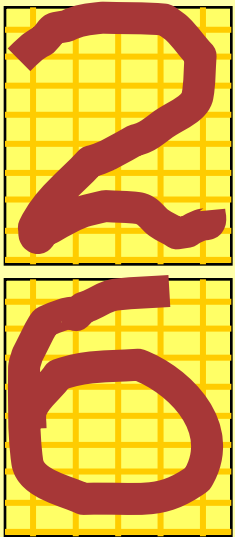


2507x2190

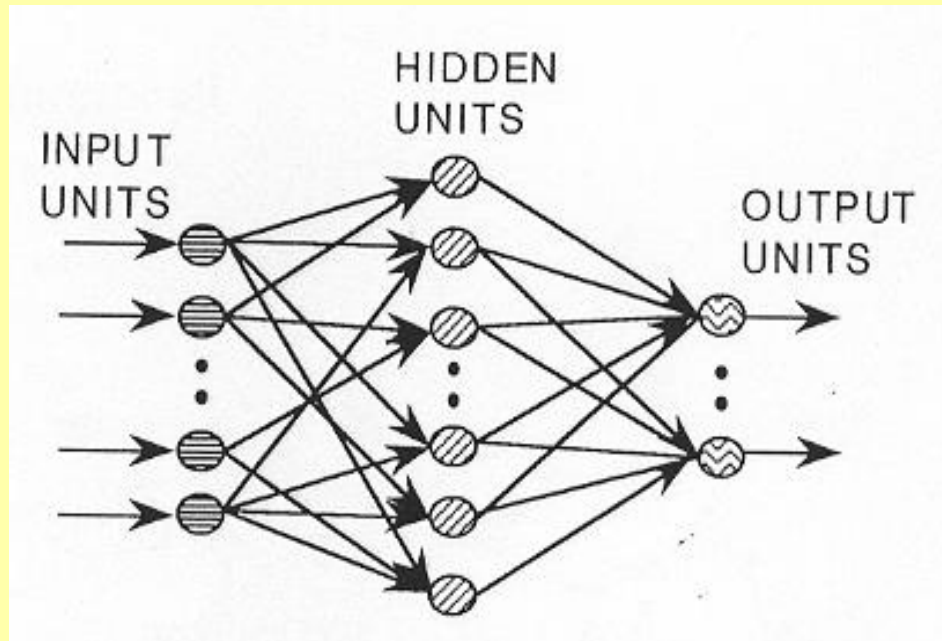


40x30

# Number Recognition



**9 x 6 = 54**  
**Inputs**

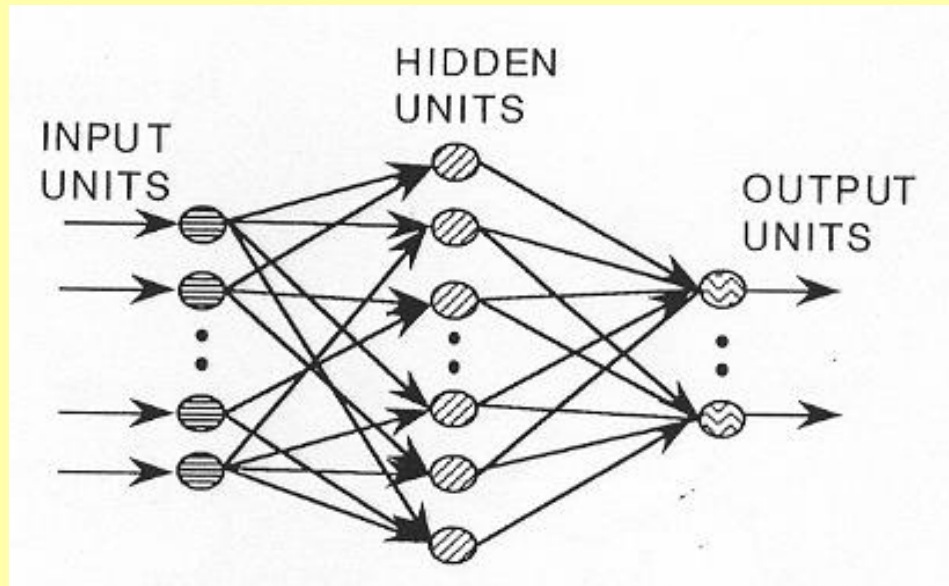


1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	0
0	0	0	0
0	0	0	...
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	1

**10 Outputs**



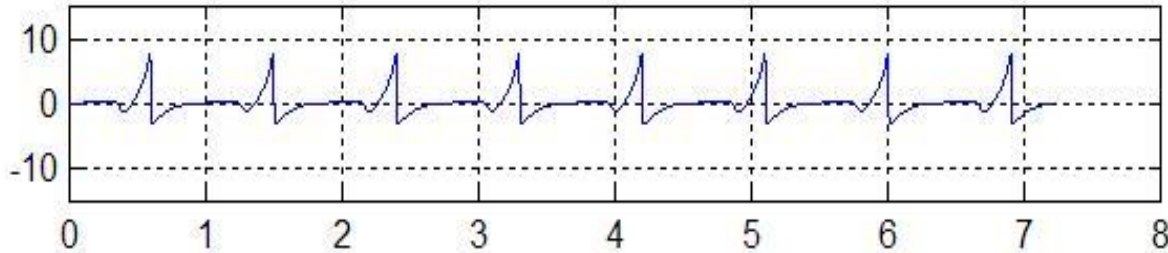
# Number Recognition



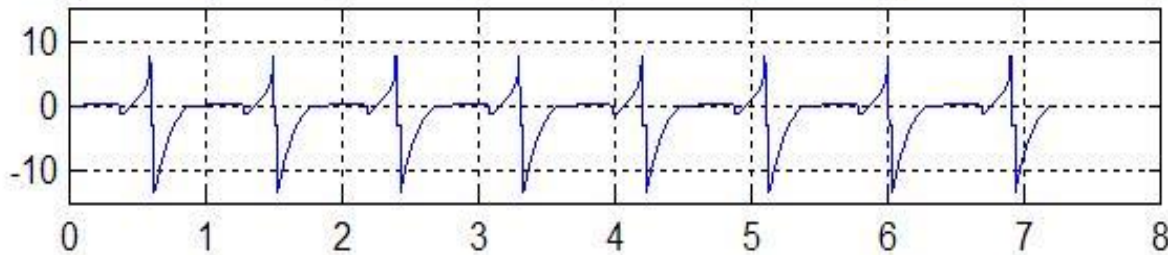
0  
1  
0  
0  
0  
0  
0  
0  
0  
0

**Recognition of 100% for training data**  
**Recognition of 92% for validation data**

# Detection of Cardiac Anomalies



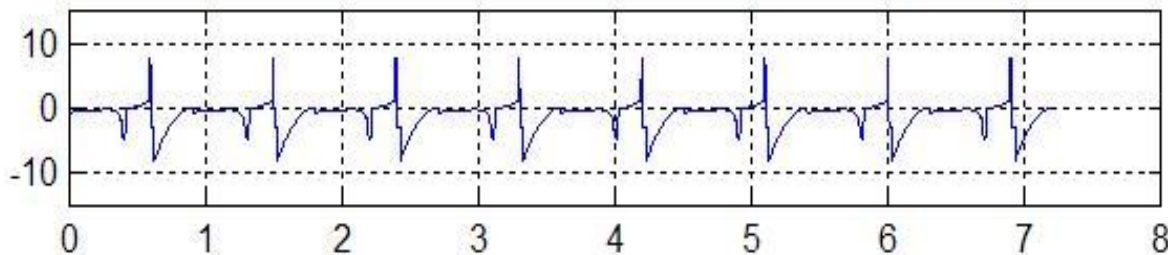
**Normal**



**Anomaly 1**

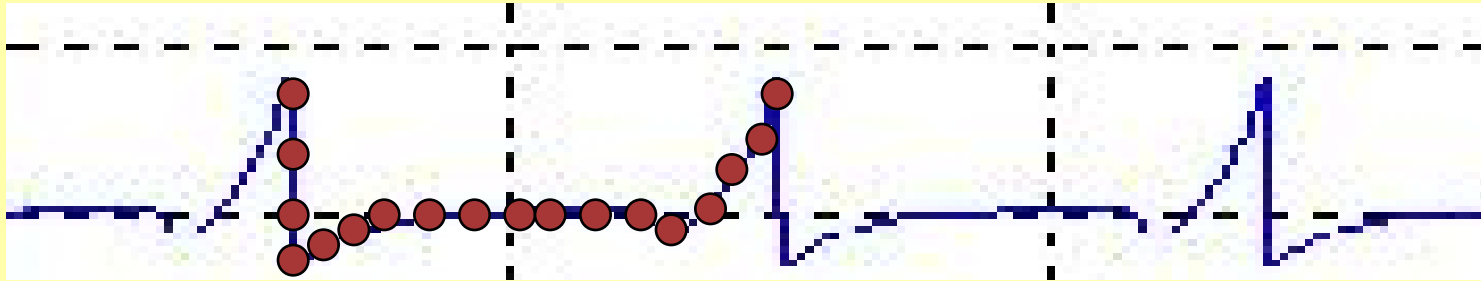


**Anomaly 2**



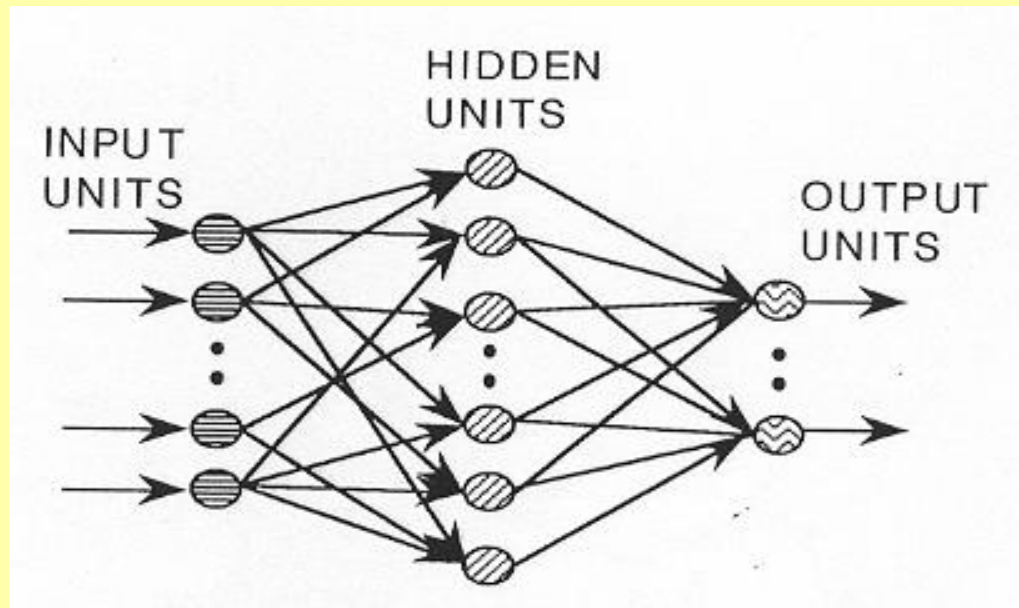
**Anomaly 3**

# Training of Neural Network



**600 samples in a period**

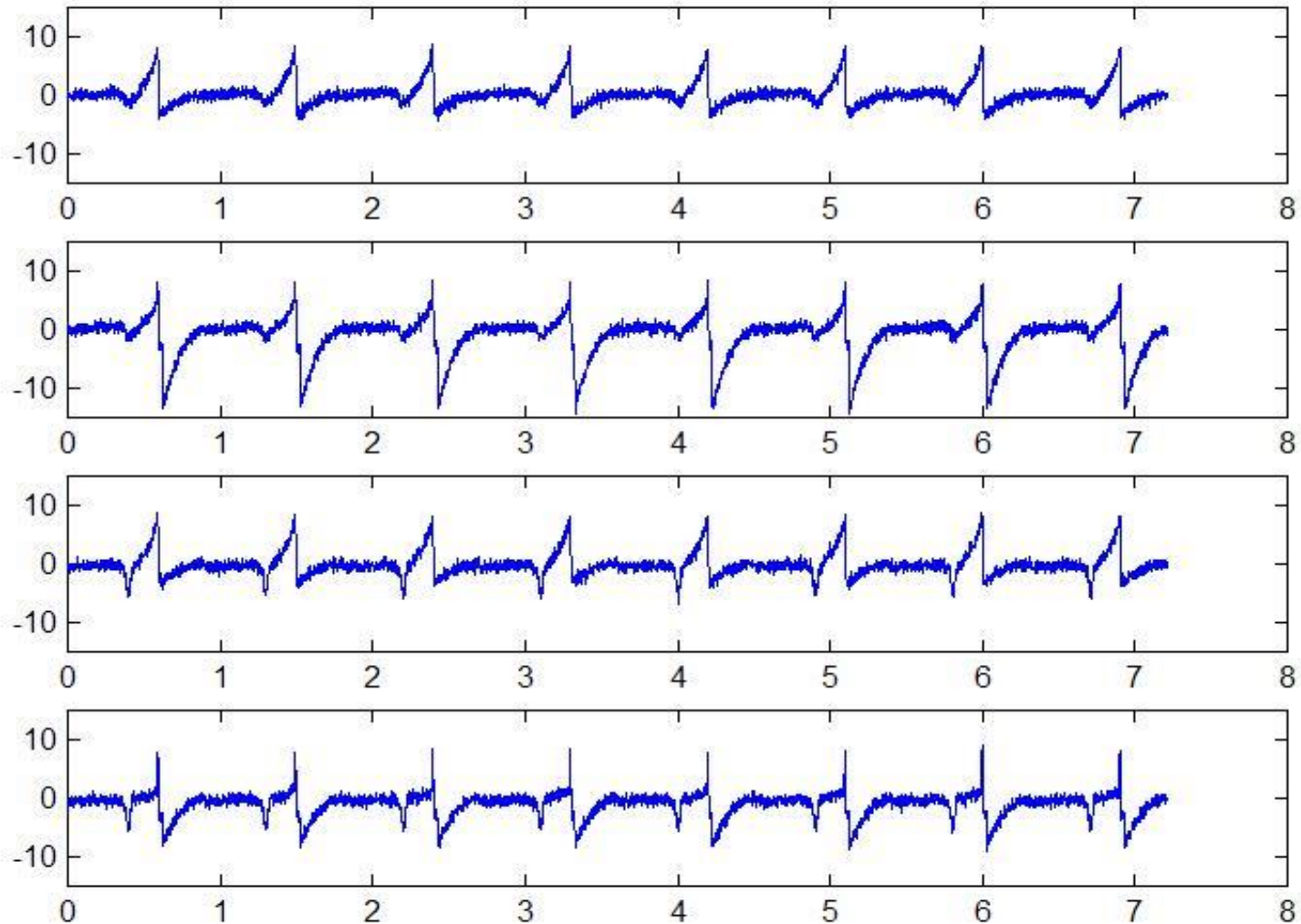
**600  
Inputs**



1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

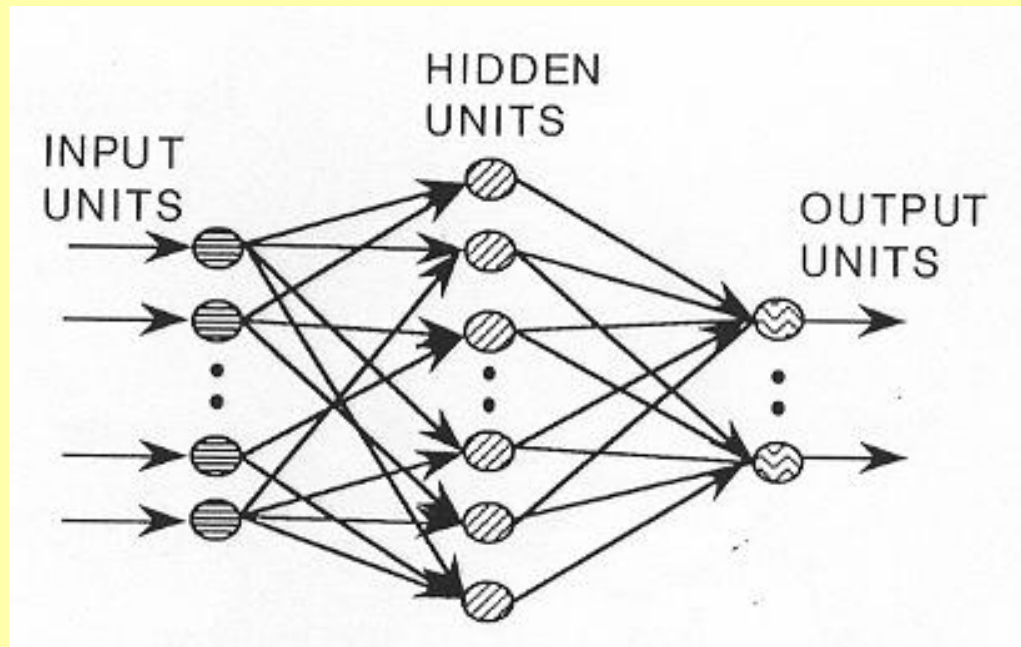
**4 Outputs**

# Validation with Noisy Signals



# Detection of Cardiac Anomalies

**600  
Inputs**



**4 Outputs**

**Recognition of 100% for low and medium level noise**

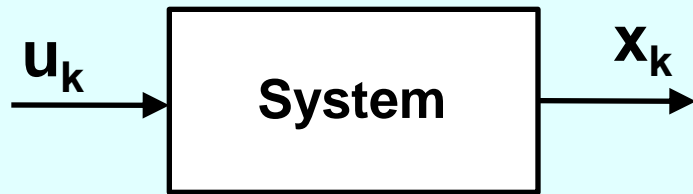
**Recognition of 90% for high level noise**

# **Dynamic Neural Networks**

## **Modeling of Dynamical Systems**

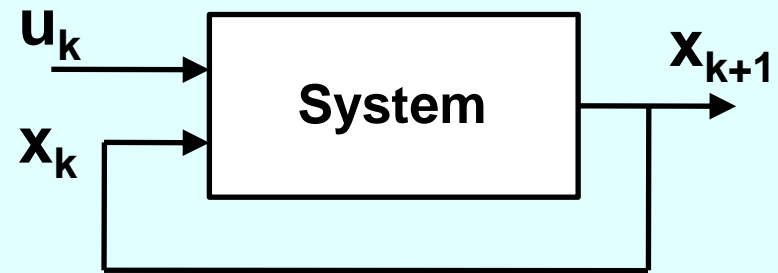
# Modeling of Dynamical Systems

## Static System



$$x_k = \Phi(u_k)$$

## Dynamic System

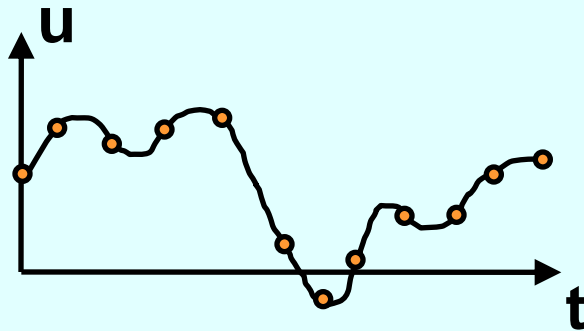


$$x_{k+1} = \Phi(x_k, u_k)$$

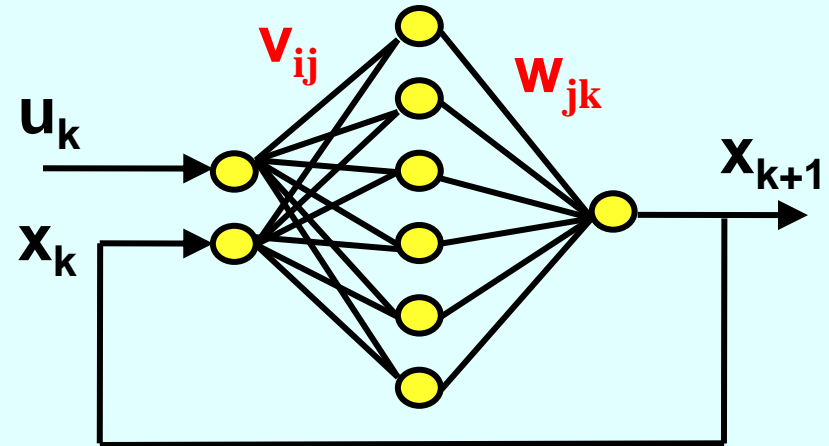
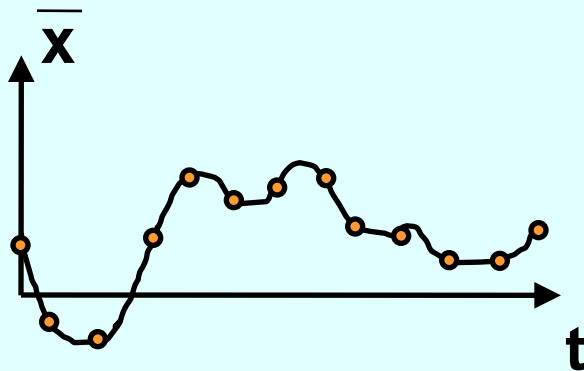
Output becomes input in the next step

# Modeling of Dynamical Systems

Input  $u$



Desired Output  $\bar{x}$

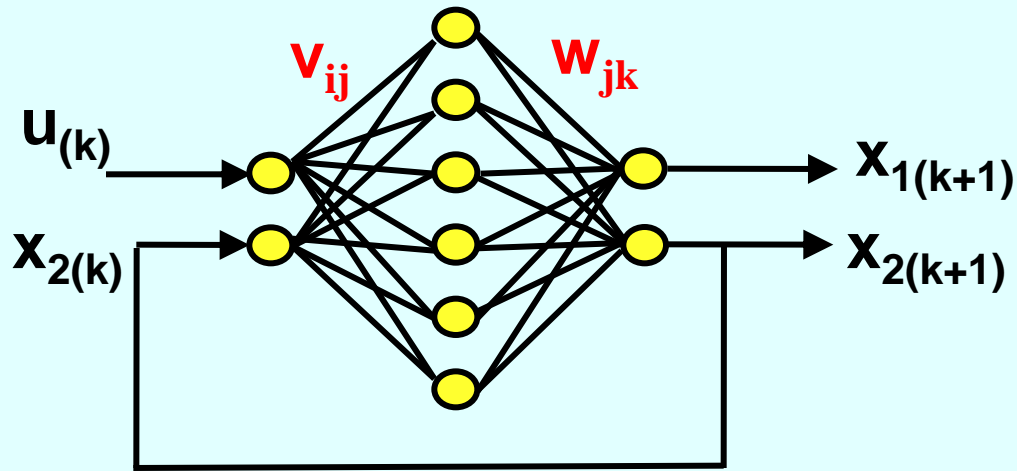


$$x_{k+1} = \Phi(x_k, u_k)$$

$$\begin{array}{lll} x_0, u_0 & \longrightarrow & x_1 \\ x_1, u_1 & \longrightarrow & x_2 \\ x_2, u_2 & \longrightarrow & x_3 \\ \vdots & & \vdots \\ x_N, u_N & \longrightarrow & x_N \end{array}$$



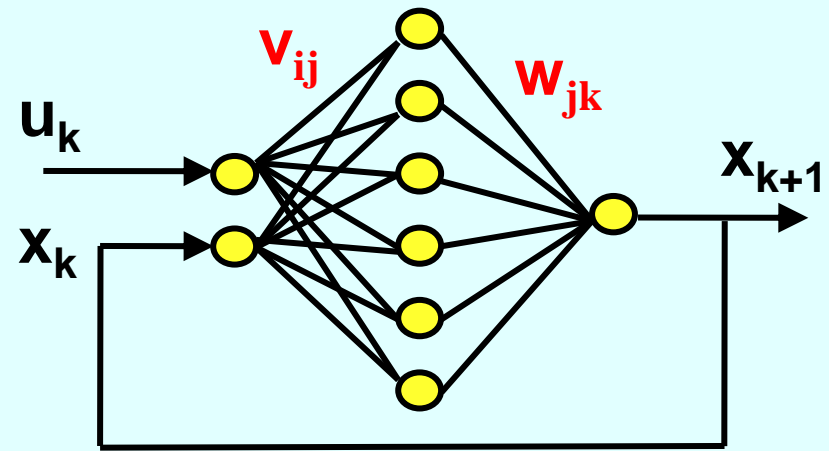
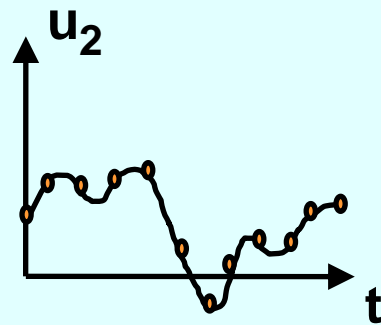
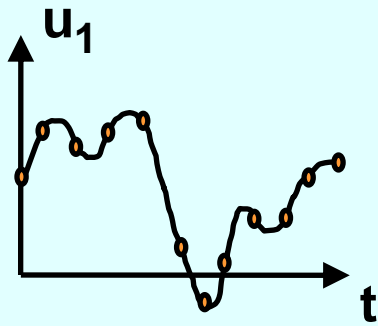
# Modeling of Dynamical Systems



$$x_{k+1} = \Phi(x_k, u_k)$$

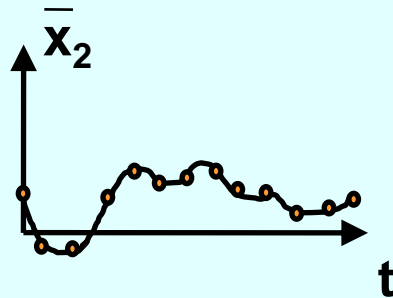
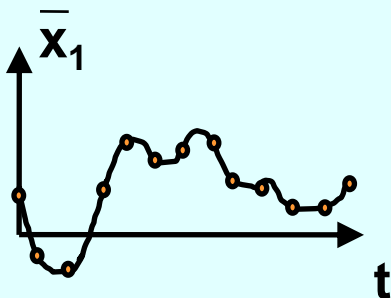
# Modeling of Dynamical Systems

Input  $u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$

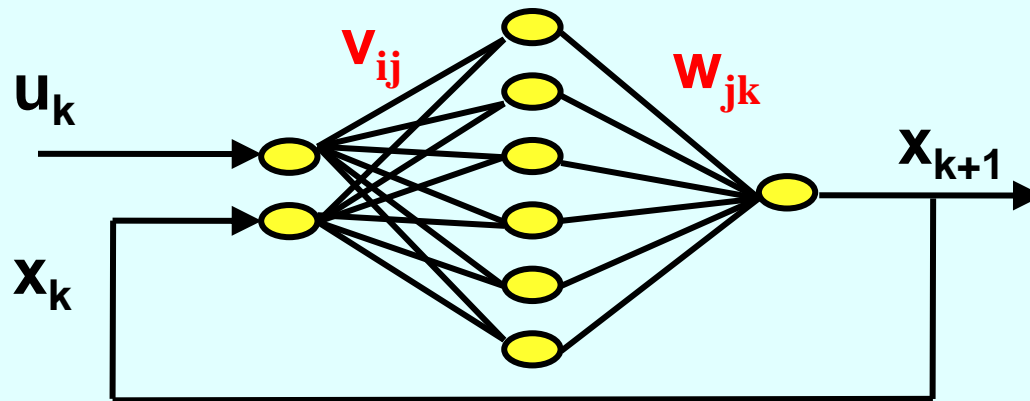


$$x_{k+1} = \Phi(x_k, u_k)$$

Desired Output  $\bar{x} = \begin{bmatrix} \bar{x}_1 \\ \bar{x}_2 \end{bmatrix}$



# Training of Dynamical Neural Networks



**Cost Function to be Minimized**

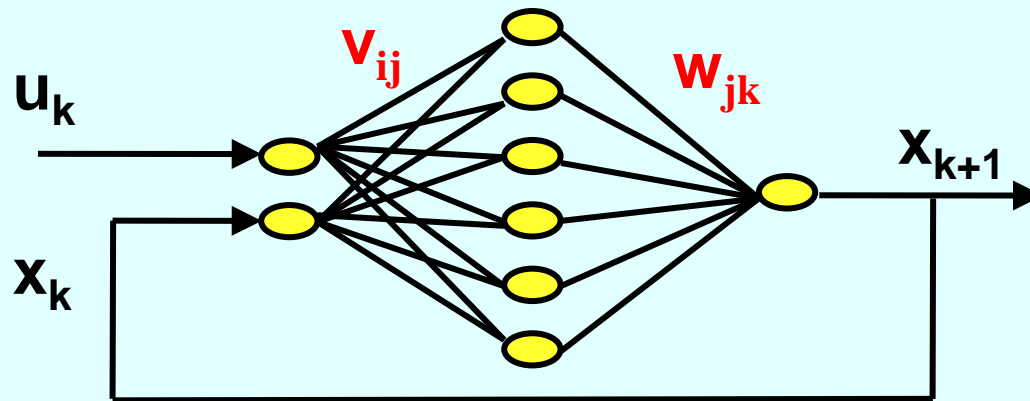
$$J = 0.5 (x_1 - \bar{x}_1)^2 + 0.5 (x_2 - \bar{x}_2)^2 + \dots + 0.5 (x_N - \bar{x}_N)^2$$

$$J = 0.5 \sum_{k=1}^{k=N} (x_k - \bar{x}_k)^2$$

$x_k \rightarrow$  Estado (Salida) de la red

$\bar{x}_k \rightarrow$  Salida deseada (data)

# Training of Dynamical Neural Networks

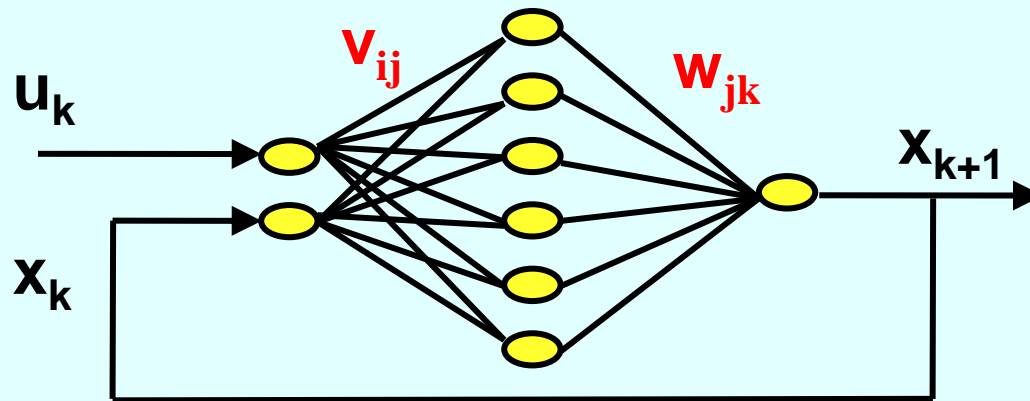


If  $x$  is a vector  $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Cost Function to be Minimized

$$J = 0.5 \sum_{k=1}^{k=N} (x_k - \bar{x}_k)^T (x_k - \bar{x}_k)$$

# Training of Dynamical Neural Networks



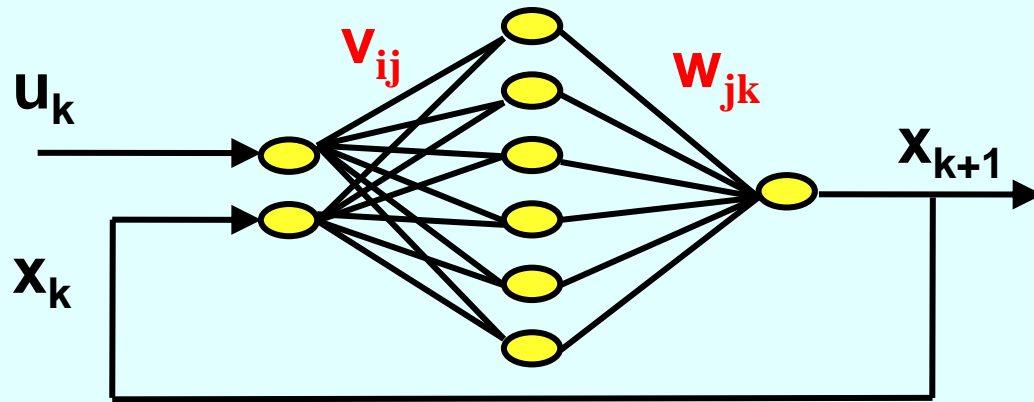
Cost Function to be Minimized

$$J = 0.5 (x_1 - \bar{x}_1)^2 + 0.5 (x_2 - \bar{x}_2)^2 + \dots + 0.5 (x_N - \bar{x}_N)^2$$

$$v_{ij} = v_{ij} - \eta \frac{\partial \bar{J}}{\partial v_{ij}}$$
$$w_{jk} = w_{jk} - \eta \frac{\partial \bar{J}}{\partial w_{jk}}$$

Total partial derivatives

# Training of Dynamical Neural Networks



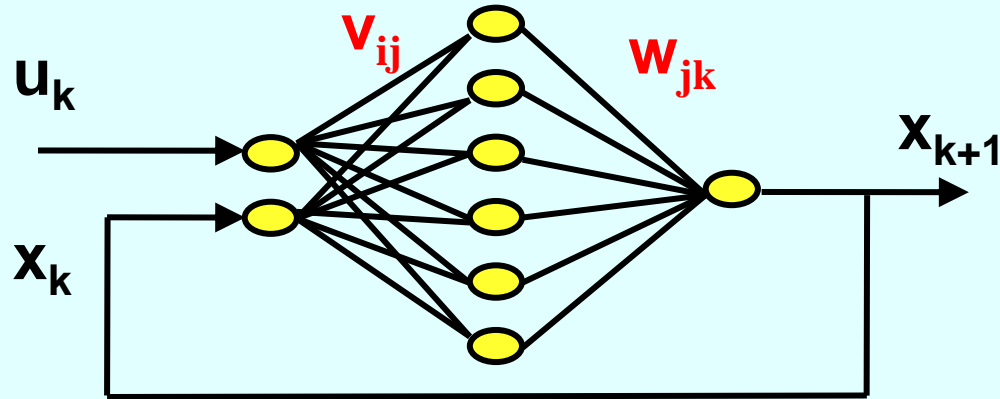
Cost Function to be Minimized  $J = 0.5 \sum_{k=1}^{k=N} (\mathbf{x}_k - \bar{\mathbf{x}}_k)^T (\mathbf{x}_k - \mathbf{x}_k)$

$$\frac{\partial J}{\partial \mathbf{v}} = \sum_{k=1}^{k=N} (\mathbf{x}_k - \bar{\mathbf{x}}_k)^T \frac{\partial \mathbf{x}_k}{\partial \mathbf{v}}$$

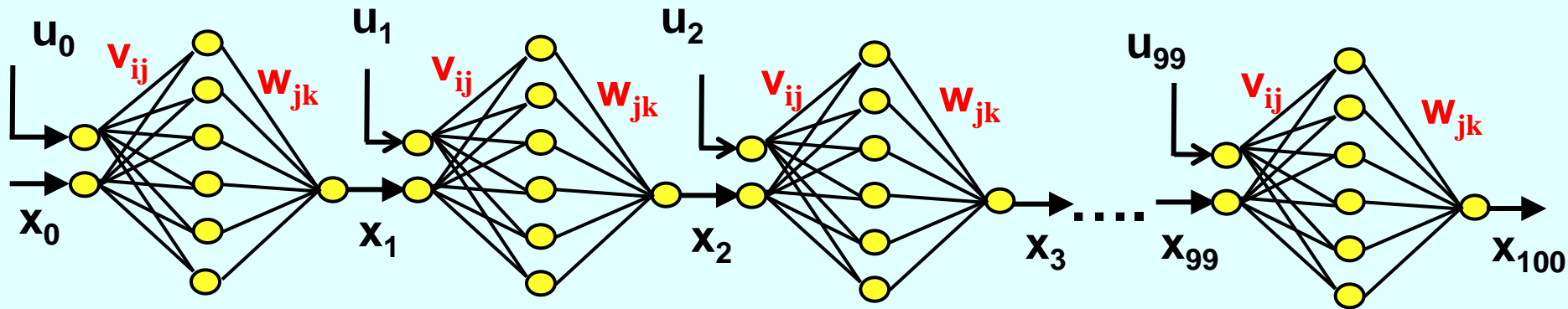
$$\frac{\partial J}{\partial \mathbf{w}} = \sum_{k=1}^{k=N} (\mathbf{x}_k - \bar{\mathbf{x}}_k)^T \frac{\partial \mathbf{x}_k}{\partial \mathbf{w}}$$

Total partial  
derivative of  $\mathbf{x}_k$

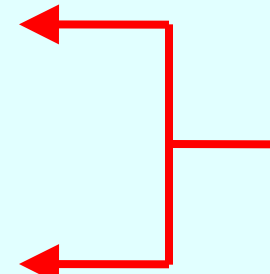
# Training of Dynamic Neural Networks



## Unfolding the Network Along Time



# Training of Dynamical Neural Networks

$$\begin{aligned} \mathbf{v}_{ij} &= \mathbf{v}_{ij} - \eta \frac{\partial \bar{J}}{\partial \mathbf{v}_{ij}} \\ \mathbf{w}_{jk} &= \mathbf{w}_{jk} - \eta \frac{\partial \bar{J}}{\partial \mathbf{w}_{jk}} \end{aligned}$$


Total partial derivatives

## Simple Derivative

$$z = 3y + 2x$$

$$y = 4x + 5r$$

$$r = 2x + 6s$$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial x} = 2$$

## Total Derivative

$$\frac{\partial \bar{z}}{\partial x} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial x} + \frac{\partial z}{\partial y} \frac{\partial y}{\partial r} \frac{\partial r}{\partial x}$$



# Training of Dynamical Neural Networks

## Computation of Total Partial Derivatives

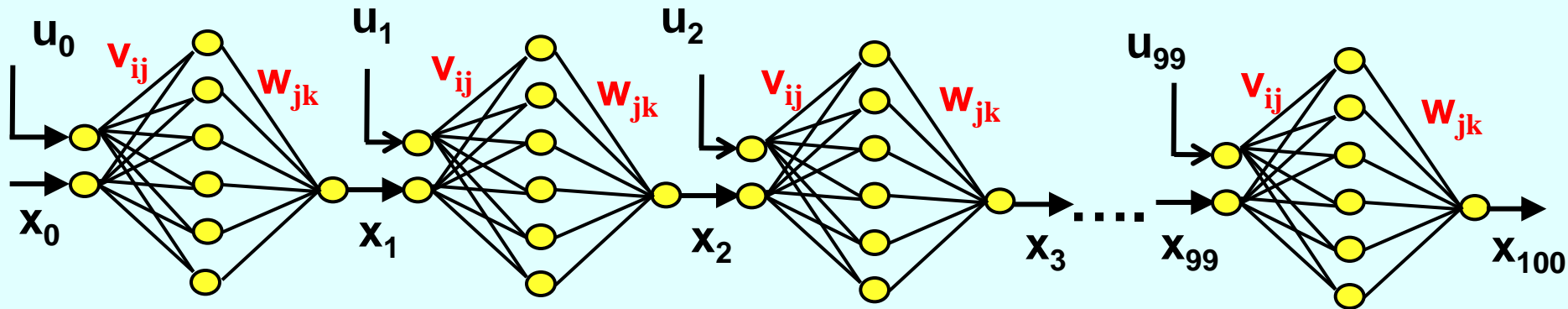
- **Back Propagation Through Time BPTT**

**Paul Werbos, 1972**

- **Dynamic Back Propagation DBP**

**Kumpati Narendra, 1989**

# Dynamic Back Propagation

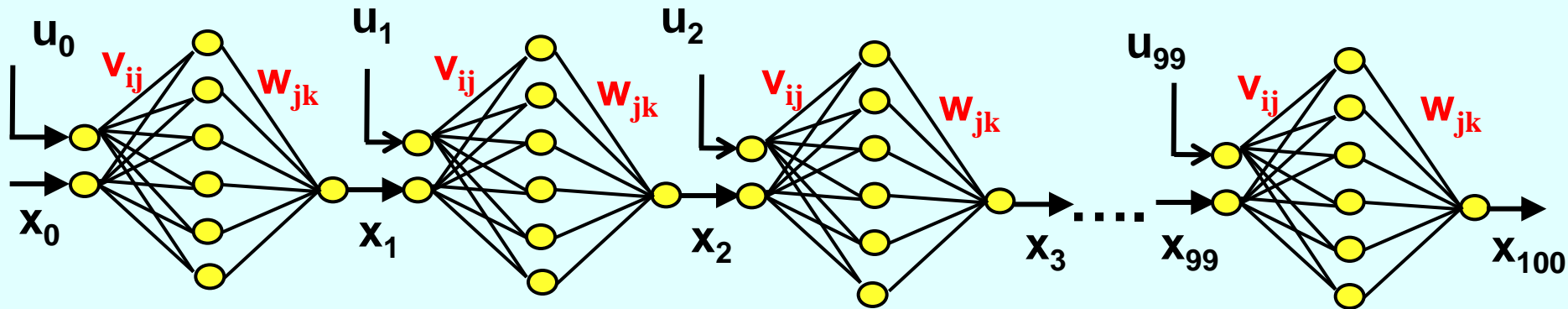


$$\frac{\partial \bar{x}_1}{\partial \bar{v}} = \frac{\partial x_1}{\partial v}$$

$$\frac{\partial \bar{x}_2}{\partial \bar{v}} = \frac{\partial x_2}{\partial v} + \frac{\partial x_2}{\partial x_1} \frac{\partial \bar{x}_1}{\partial \bar{v}}$$

$$\frac{\partial \bar{x}_3}{\partial \bar{v}} = \frac{\partial x_3}{\partial v} + \frac{\partial x_3}{\partial x_2} \frac{\partial \bar{x}_2}{\partial \bar{v}}$$

# Dynamic Back Propagation



$$\frac{\partial \bar{x}_{k+1}}{\partial v} = \frac{\partial x_{k+1}}{\partial v} + \frac{\partial x_{k+1}}{\partial x_k} \frac{\partial \bar{x}_k}{\partial v}$$

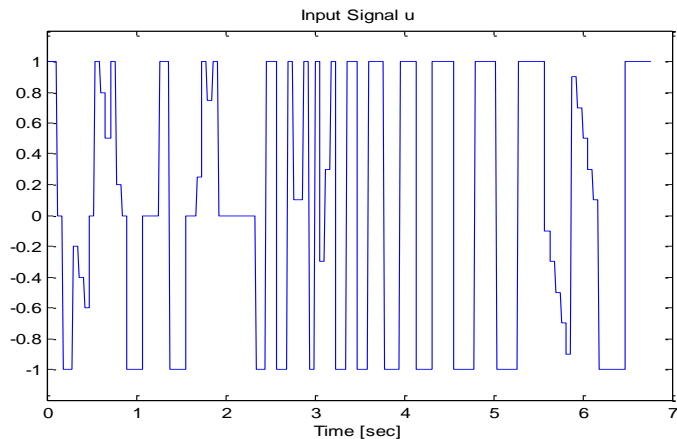
**Recursive expression for computation  
of total partial derivatives**

# Modeling of Nonlinear Dynamic System

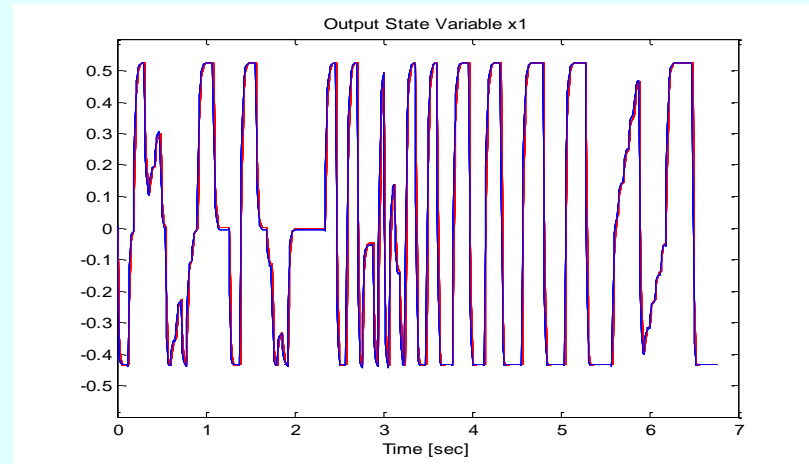
## One Input and Two Outputs

## Network Training

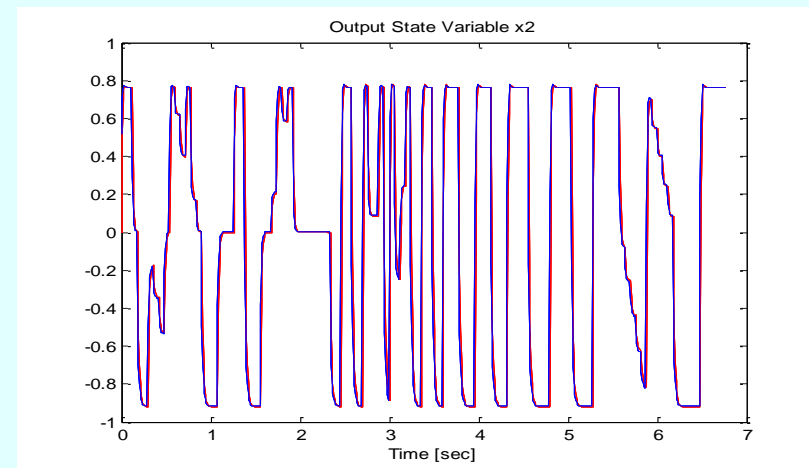
Input Signal  $u$



— Training Signal  
— Model Output



Output Signal  $x_1$

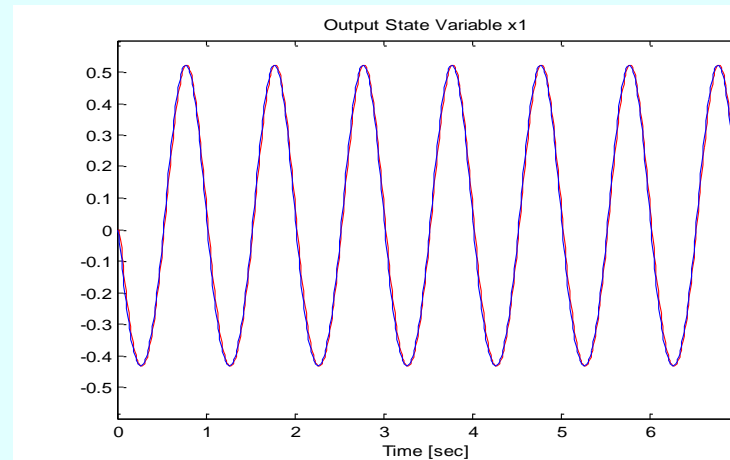
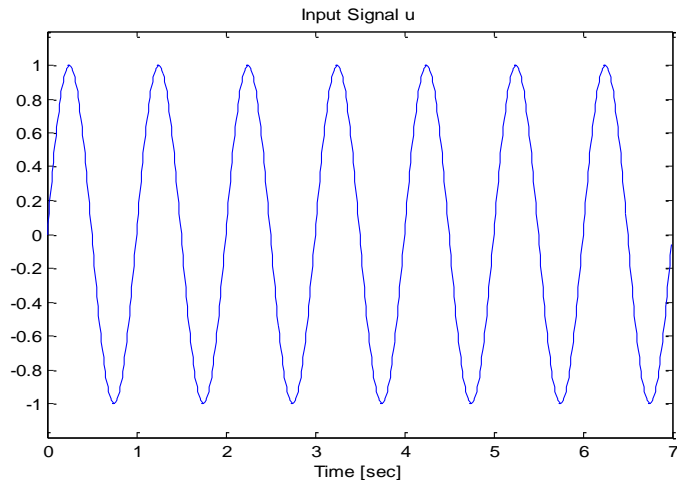


Output Signal  $x_2$

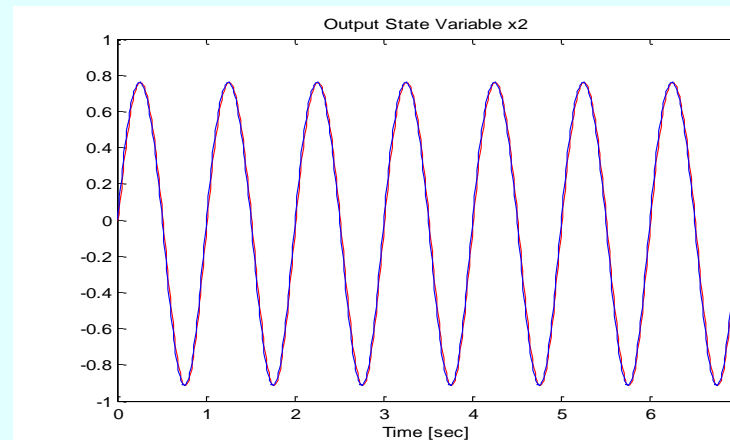
# Modeling of Nonlinear Dynamic System

## Validation: Input-Output Signals

Input Signal  $u$



Output Signal  $x_1$



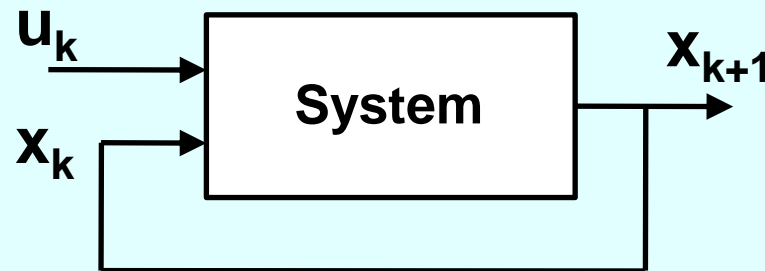
Output Signal  $x_2$

— Training Signal  
— Model Output

# Modeling of Nonlinear Dynamic System

## Matlab Simulation

Dynamical system with 1 input and 3 outputs



$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Nonlinear system

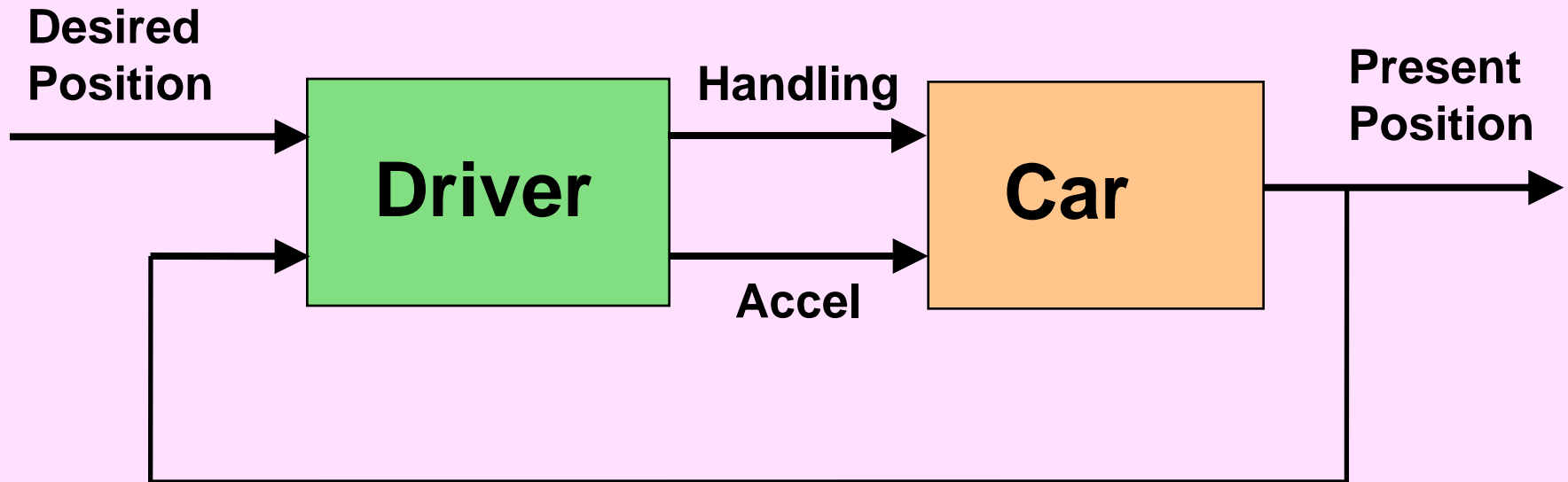
$$\mathbf{x}_k = \mathbf{A}\mathbf{x}_k + \mathbf{B}u_k + \mathbf{G}\mathbf{x}_k u_k$$

# **Dynamic Neural Networks**

**Control of Dynamical Systems**

# Car Driving

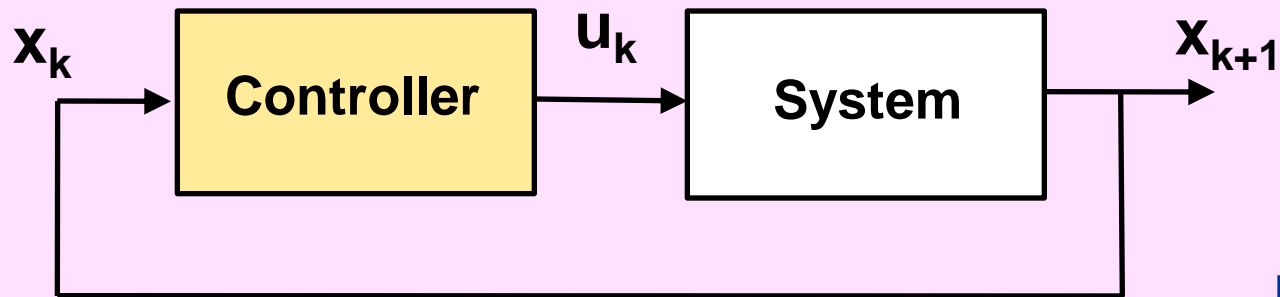
## A Control Problem





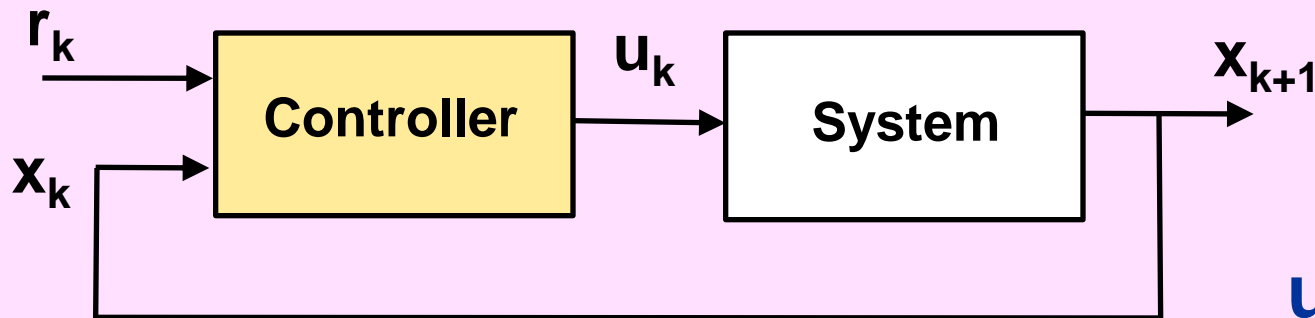
# Control of Dynamical Systems

## Stabilization



$$u_k = \Omega(x_k)$$

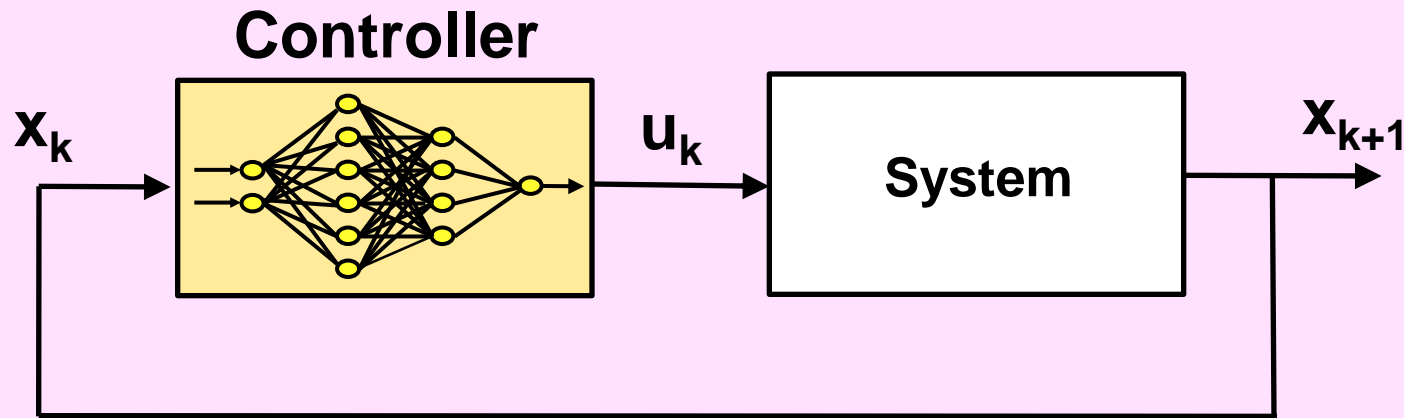
## Tracking



$$u_k = \Omega(x_k, r_k)$$

# Control of Dynamical Systems

## Stabilization



**Controller**

$$u_k = \Omega(x_k)$$

**System**

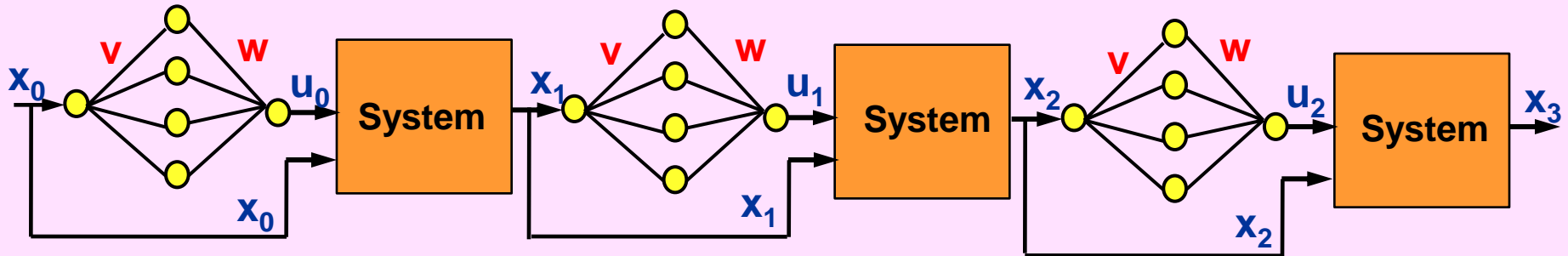
$$x_{k+1} = \Phi(x_k, u_k)$$

Represented by:

→ Neural Network

→ State Equation

# Training of Neuro-Controller



**Cost Function to be Minimized**

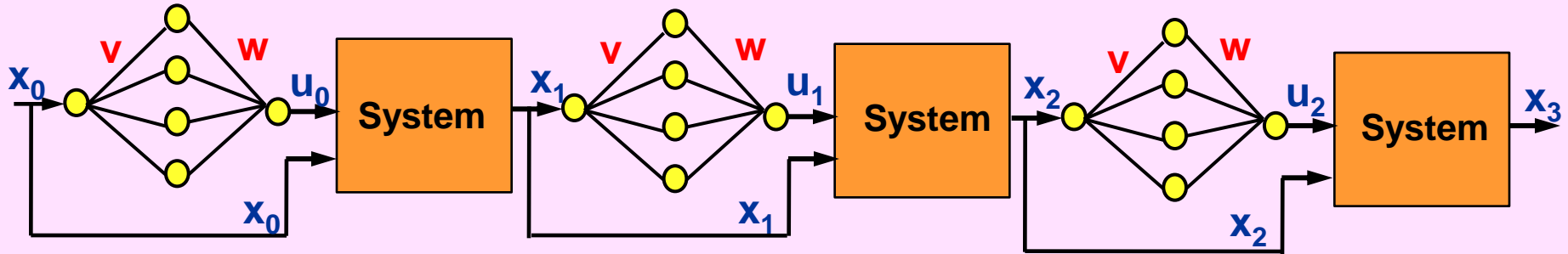
$$J = 0.5 (x_1 - \bar{x}_1)^2 + 0.5 (x_2 - \bar{x}_2)^2 + \dots + 0.5 (x_N - \bar{x}_N)^2$$

$$J = 0.5 \sum_{k=1}^{k=N} (x_k - \bar{x}_k)^2$$

$x_k \rightarrow$  Estado (Salida)

$\bar{x}_k \rightarrow$  Salida deseada

# Training of Neuro-Controller



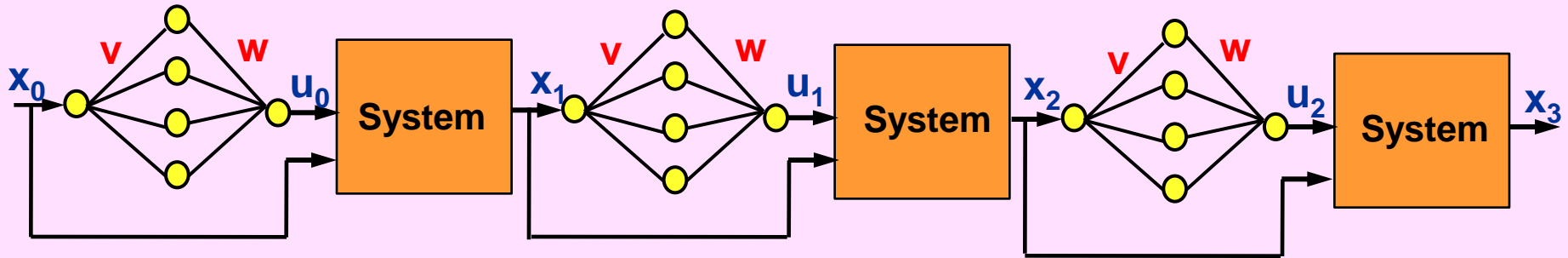
If  $x$  is a vector  $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

Cost Function to be Minimized

$$J = 0.5 \sum_{k=1}^{k=N} (x_k - \bar{x}_k)^T (x_k - \bar{x}_k)$$

$\bar{x}_k =$  Desired output

# Training of Neuro-Controller



**Cost Function to be Minimized**

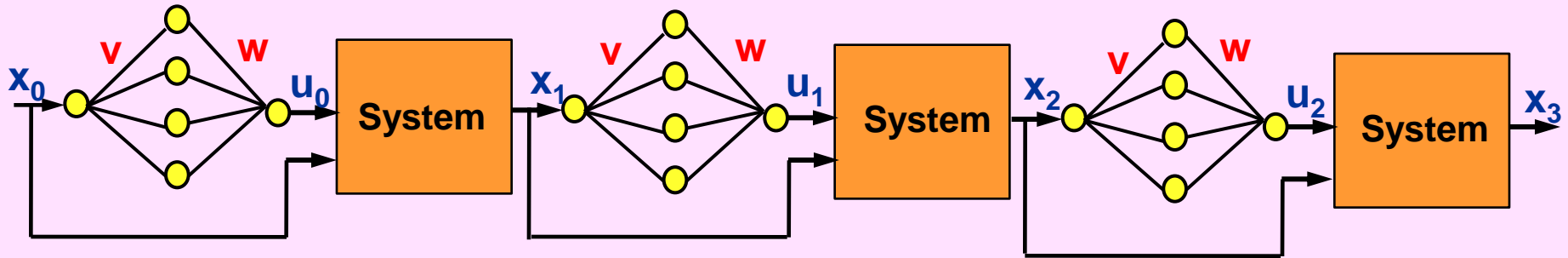
$$J = 0.5 (x_1 - \bar{x}_1)^2 + 0.5 (x_2 - \bar{x}_2)^2 + \dots + 0.5 (x_N - \bar{x}_N)^2$$

$$v_{ij} = v_{ij} - \eta \frac{\partial J}{\partial v_{ij}}$$

$$w_{jk} = w_{jk} - \eta \frac{\partial J}{\partial w_{jk}}$$

Total partial derivatives

# Training of Neuro-Controller



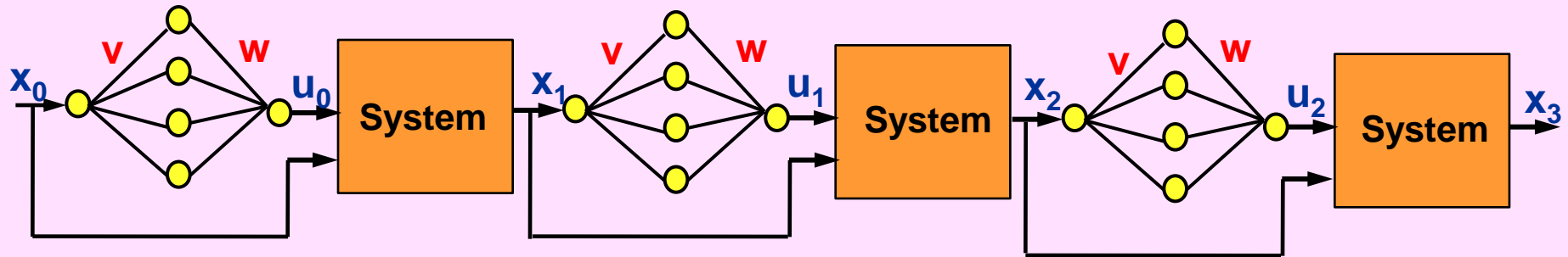
Cost Function to be Minimized  $J = 0.5 \sum_{k=1}^{k=N} (\mathbf{x}_k - \bar{\mathbf{x}}_k)^T (\mathbf{x}_k - \mathbf{x}_k)$

$$\frac{\partial \bar{J}}{\partial \mathbf{v}} = \sum_{k=1}^{k=N} (\mathbf{x}_k - \bar{\mathbf{x}}_k)^T \frac{\partial \bar{\mathbf{x}}_k}{\partial \mathbf{v}}$$

$$\frac{\partial \bar{J}}{\partial \mathbf{w}} = \sum_{k=1}^{k=N} (\mathbf{x}_k - \bar{\mathbf{x}}_k)^T \frac{\partial \bar{\mathbf{x}}_k}{\partial \mathbf{w}}$$

Total partial  
derivative of  $\mathbf{x}_k$

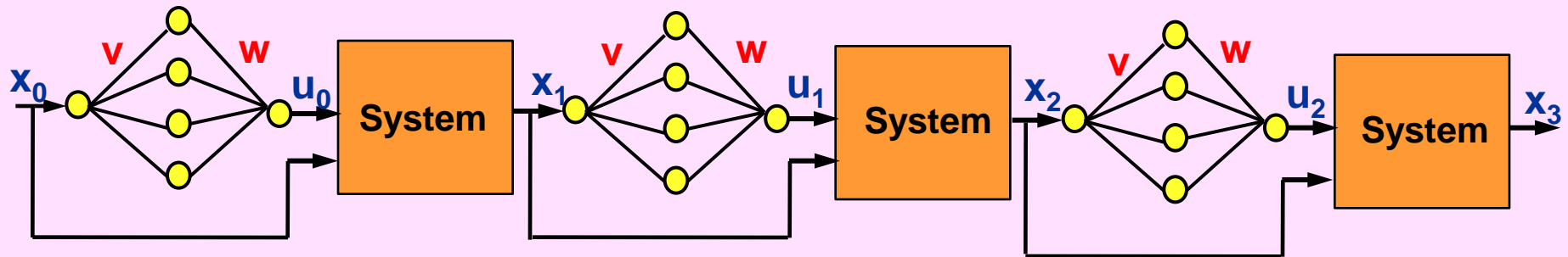
# Dynamic Back Propagation



$$\frac{\partial \bar{x}_{k+1}}{\partial v} = \frac{\partial x_{k+1}}{\partial u_k} \frac{\partial u_k}{\partial v} + \left( \frac{\partial x_{k+1}}{\partial x_k} + \frac{\partial x_{k+1}}{\partial u_k} \frac{\partial u_k}{\partial x_k} \right) \frac{\partial \bar{x}_k}{\partial v}$$

**Recursive expression for computation  
of total partial derivatives**

# Dynamic Back Propagation



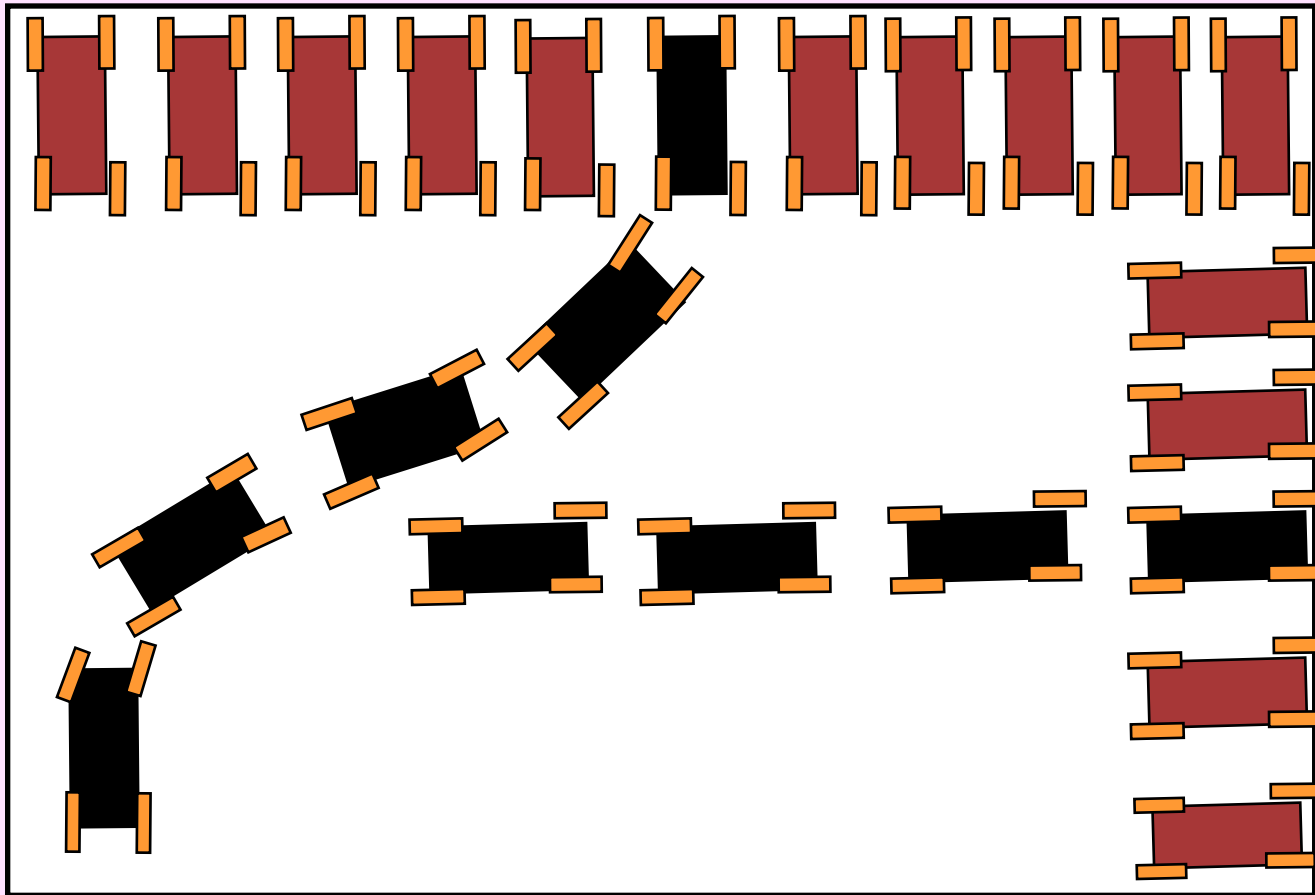
$$\frac{\partial \bar{x}_{k+1}}{\partial v} = \frac{\partial x_{k+1}}{\partial u_k} \frac{\partial u_k}{\partial v} + \left( \frac{\partial x_{k+1}}{\partial x_k} + \frac{\partial x_{k+1}}{\partial u_k} \frac{\partial u_k}{\partial x_k} \right) \frac{\partial \bar{x}_k}{\partial v}$$

Computed with the  
system model

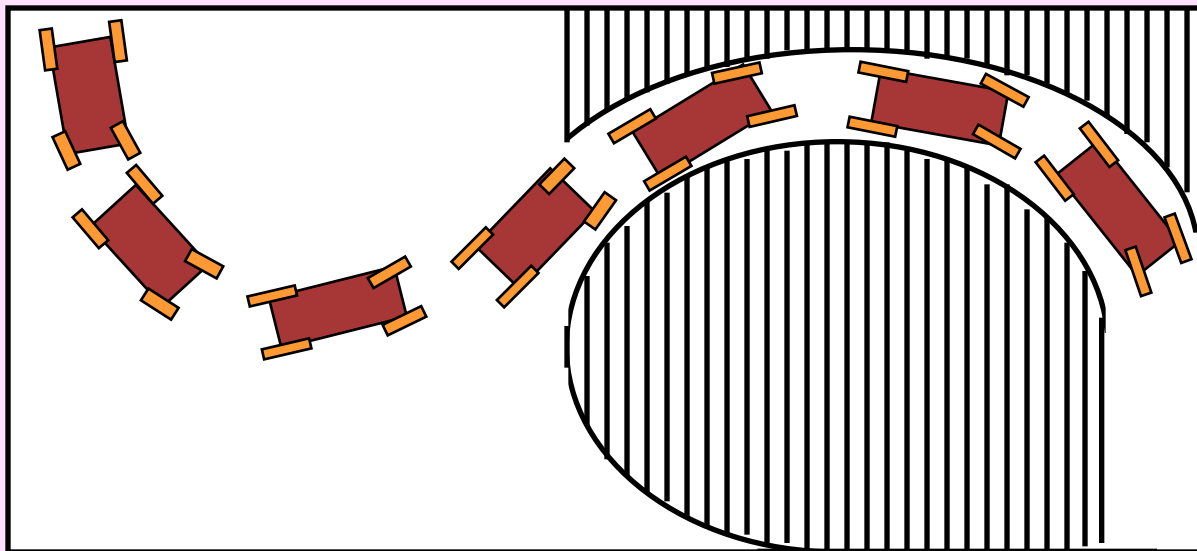
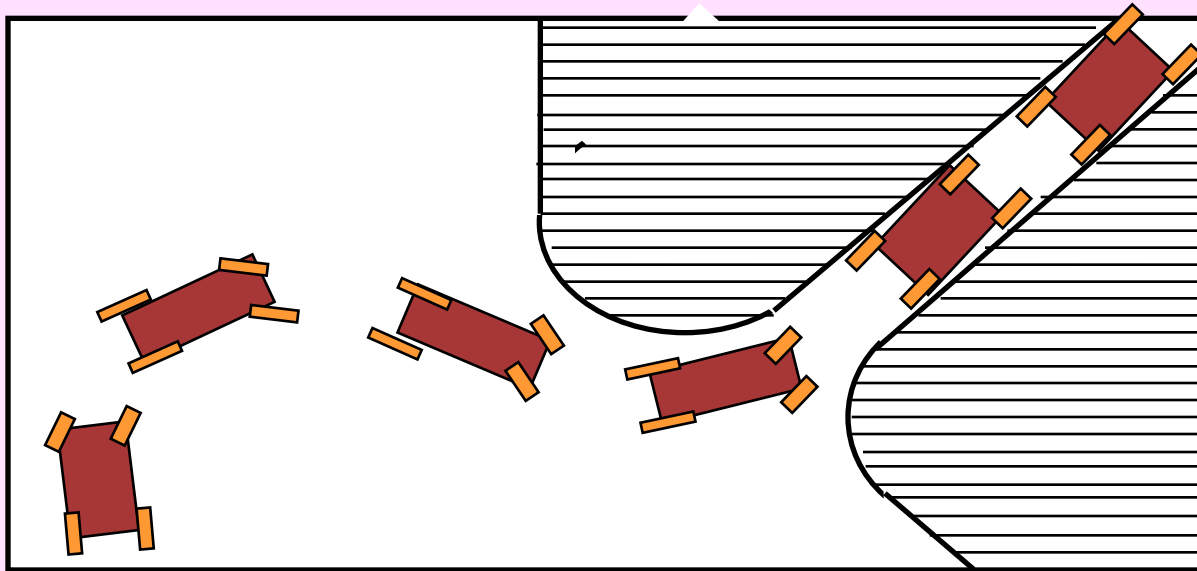
Computed with the  
neural controller



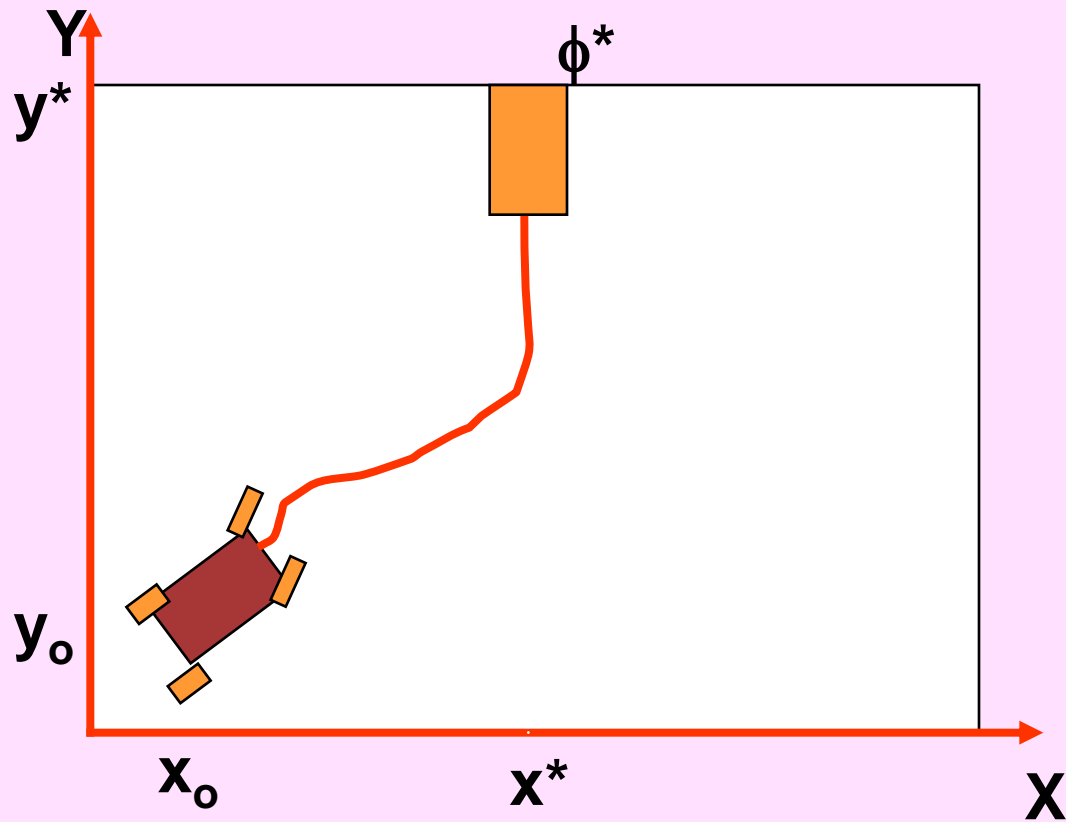
# Positioning of Mobile Robots



# Mobile Robot Following a Road



# Control Problem



How to compute  
steer angle  $\delta$

Initial  
Position

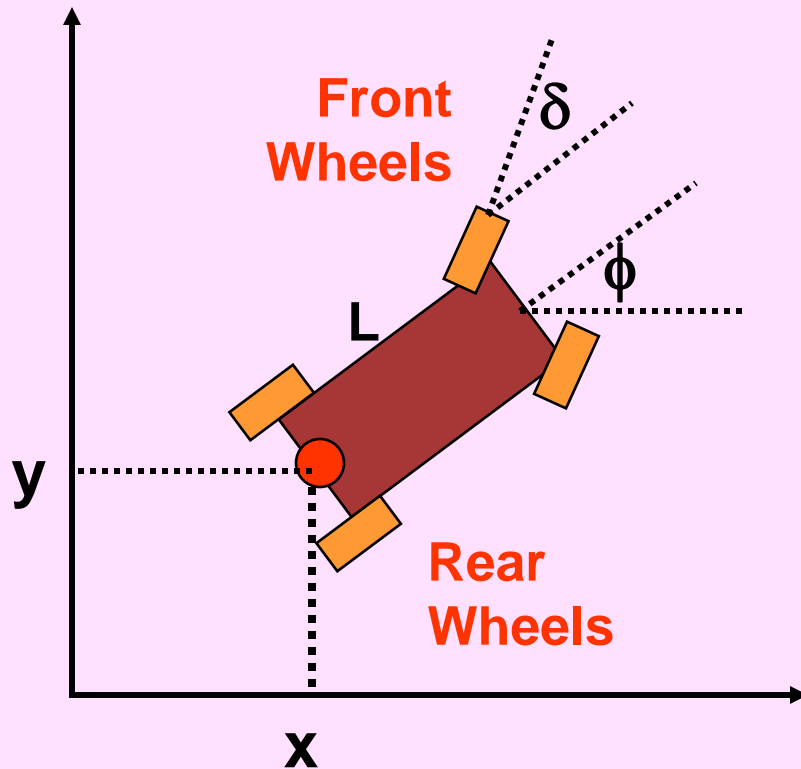
$x_o$   
 $y_o$   
 $\phi_o$

$\delta$

$x^* = 50$   
 $y^* = 100$   
 $\phi^* = \pi/2$

Desired  
Position

# Robot Model



$$x(k+1) = x(k) + v\Delta t \cos(\phi(k))$$

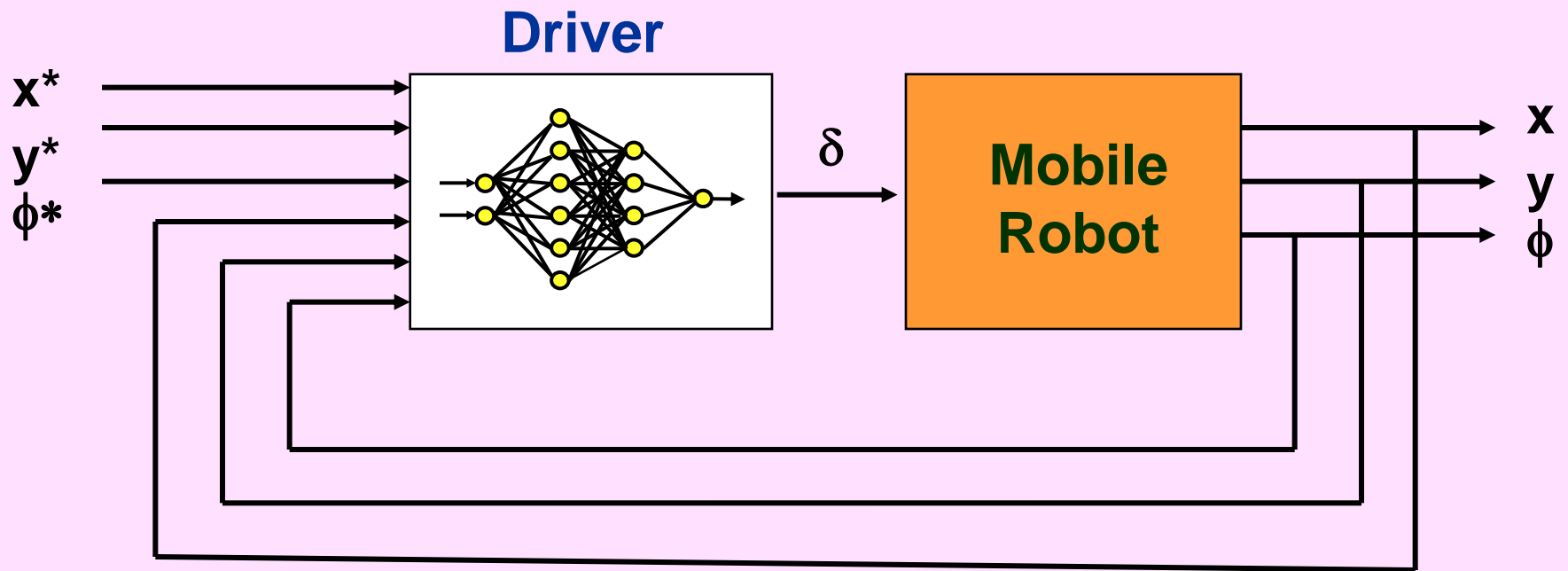
$$y(k+1) = y(k) + v\Delta t \sin(\phi(k))$$

$$\phi(k+1) = \phi(k) - v\Delta t / L \tan(\delta(k))$$

- Backward motion
- Constant speed
- No slipping – No skidding

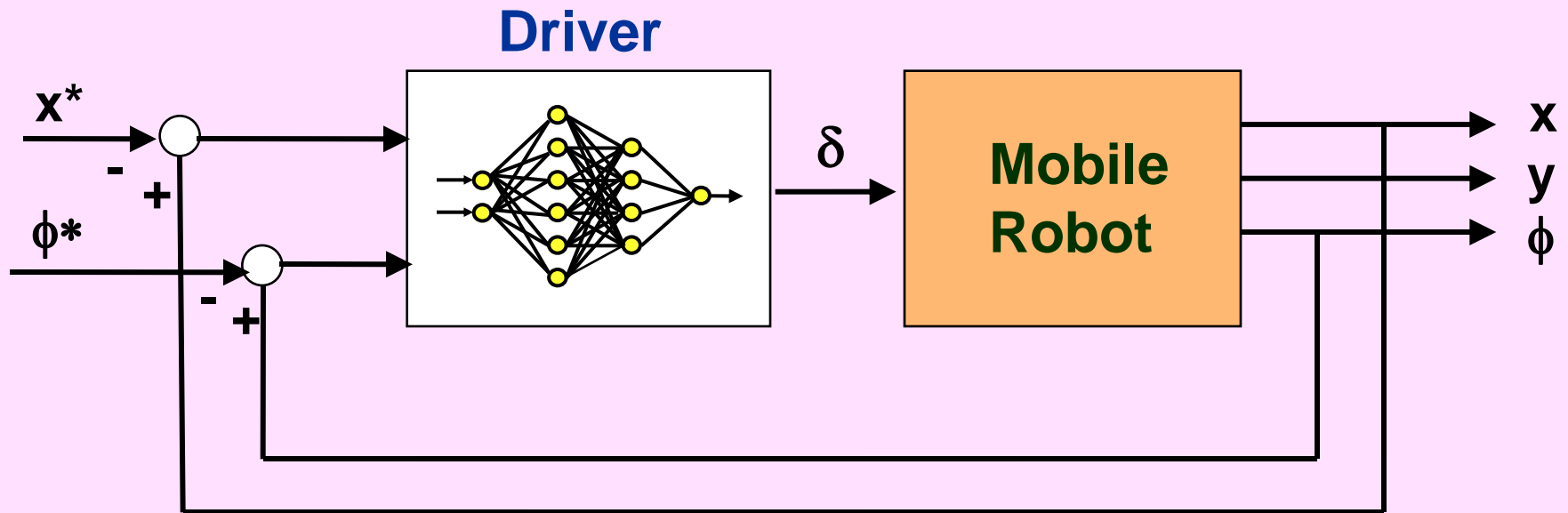
# Positioning of Mobile Robot

## Control Structure



# Positioning of Mobile Robot

## Control Structure



**Given problem characteristics,  
coordinate  $y$  is not used for control**

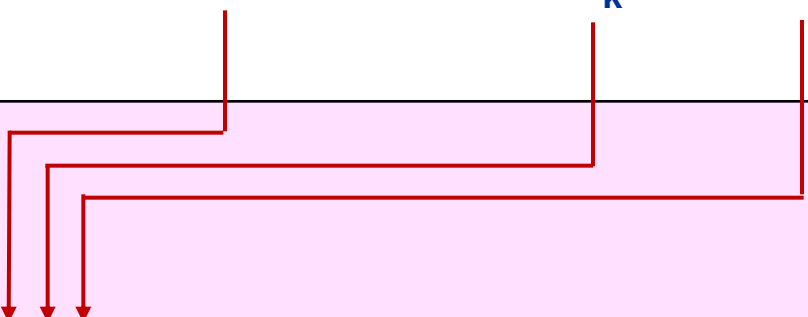
# Dynamic Back Propagation

## Robot Model

$$\mathbf{x}(\mathbf{k}+1) = \mathbf{x}(\mathbf{k}) + v\Delta t \cos(\phi(\mathbf{k}))$$

$$\phi(\mathbf{k}+1) = \phi(\mathbf{k}) - v\Delta t / L \tan(\delta(\mathbf{k}))$$

$$\mathbf{x}_k = \begin{bmatrix} x(k) \\ \phi(k) \end{bmatrix} \quad \mathbf{u}_k = \tan(\delta(k))$$

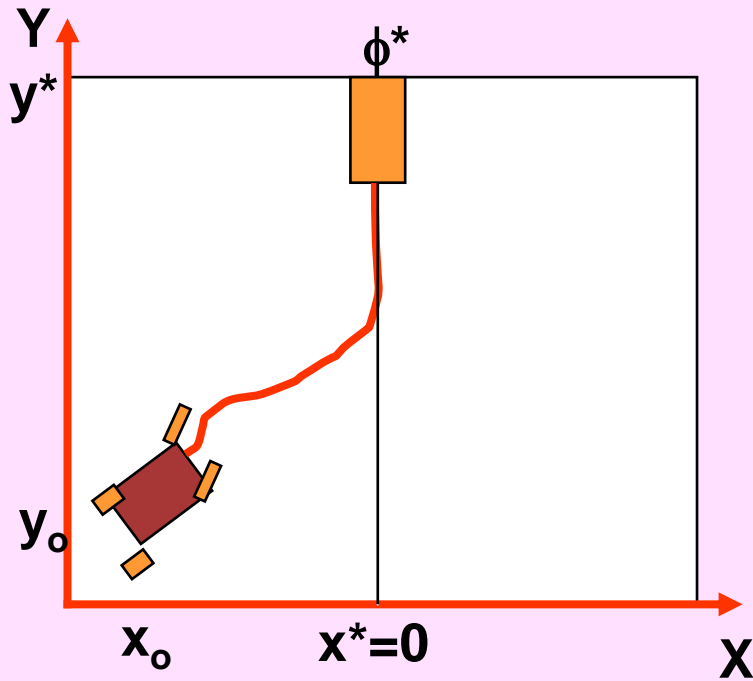
$$\frac{\partial \bar{\mathbf{x}}_{k+1}}{\partial \bar{\mathbf{v}}} = \frac{\partial \mathbf{x}_{k+1}}{\partial \mathbf{u}_k} \frac{\partial \mathbf{u}_k}{\partial v} + \left( \frac{\partial \mathbf{x}_{k+1}}{\partial \mathbf{x}_k} + \frac{\partial \mathbf{x}_{k+1}}{\partial \mathbf{u}_k} \frac{\partial \mathbf{u}_k}{\partial \mathbf{x}_k} \right) \frac{\partial \bar{\mathbf{x}}_k}{\partial \bar{\mathbf{v}}}$$


Computed with the  
system model

$$\frac{\partial \mathbf{x}_{k+1}}{\partial \mathbf{x}_k} = \begin{bmatrix} 1 & -v\Delta t \sin(\phi(k)) \\ 0 & 1 \end{bmatrix}$$

$$\frac{\partial \mathbf{x}_{k+1}}{\partial \mathbf{u}_k} = \begin{bmatrix} 0 \\ -v\Delta t / L \end{bmatrix}$$

# Incremental Learning



**Train the neural network for positions close to  $x^*=0$  (four positions)**

$x =$	-2	-2	2	2
$\phi =$	$-\pi/2$	$\pi/2$	$-\pi/2$	$\pi/2$

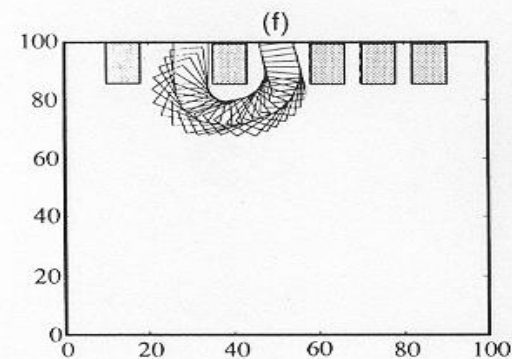
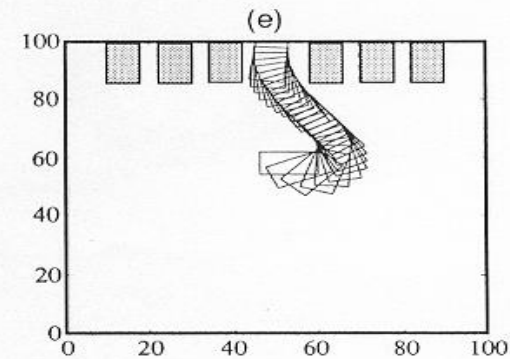
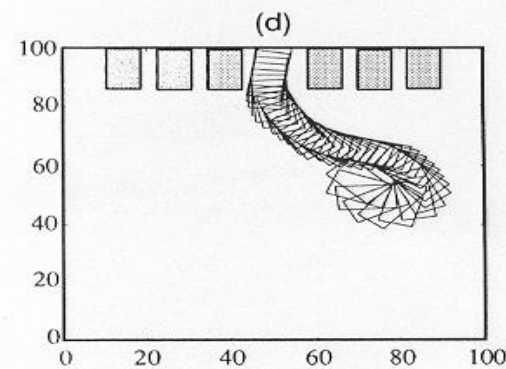
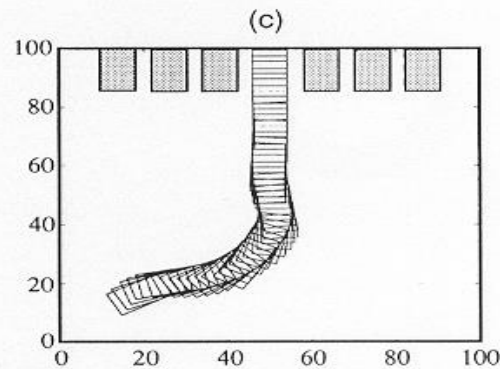
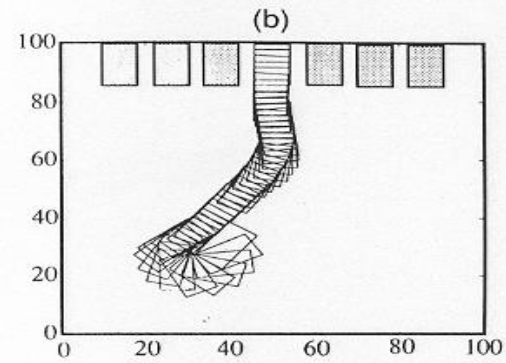
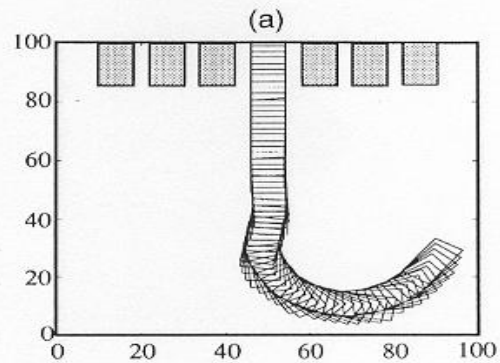
**Train the neural network for far away positions**

$x =$	-4	-4	4	4
$\phi =$	$-\pi/2$	$\pi/2$	$-\pi/2$	$\pi/2$

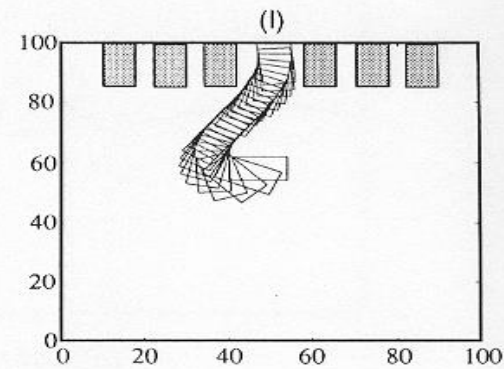
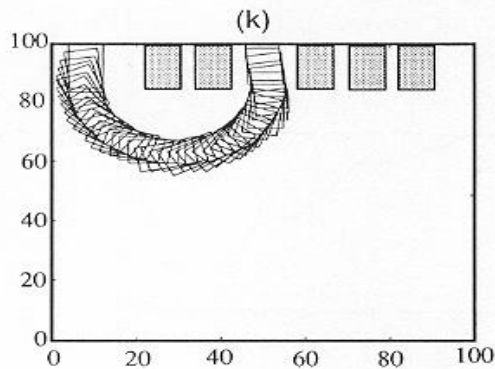
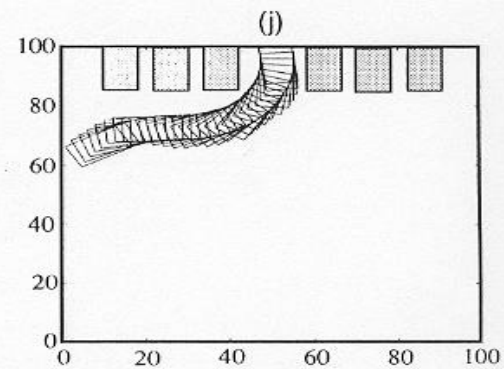
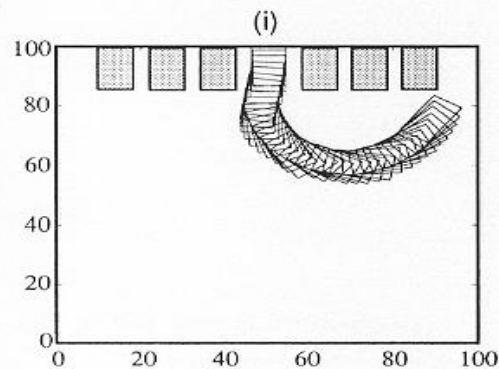
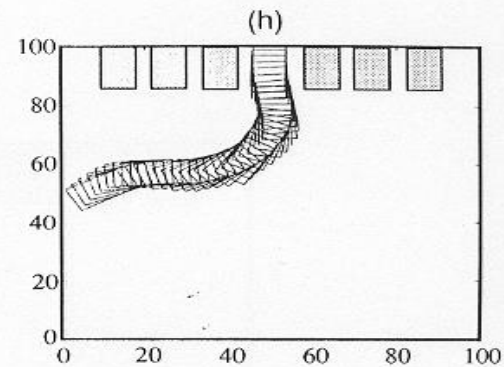
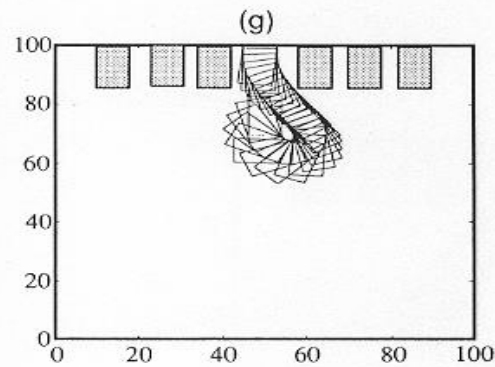
$x =$	-6	-6	6	6
$\phi =$	$-\pi/2$	$\pi/2$	$-\pi/2$	$\pi/2$



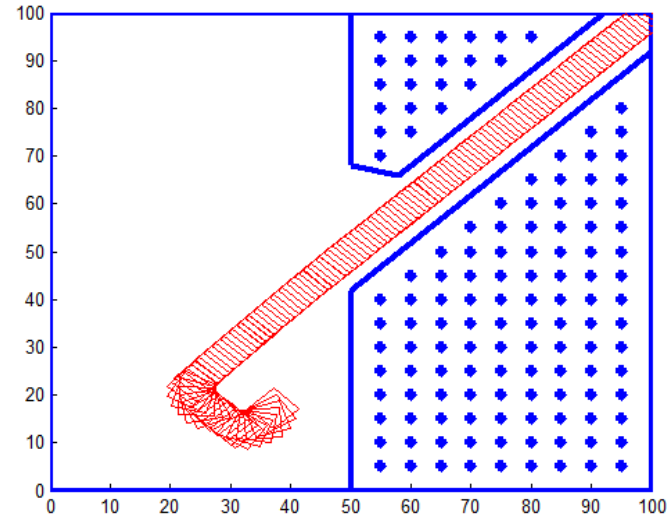
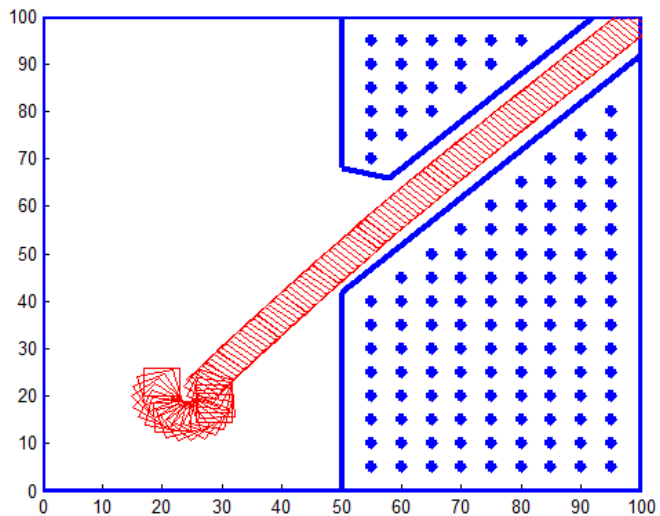
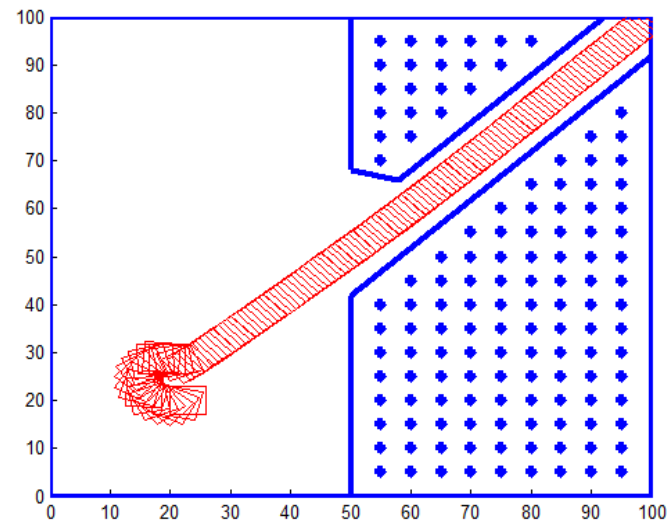
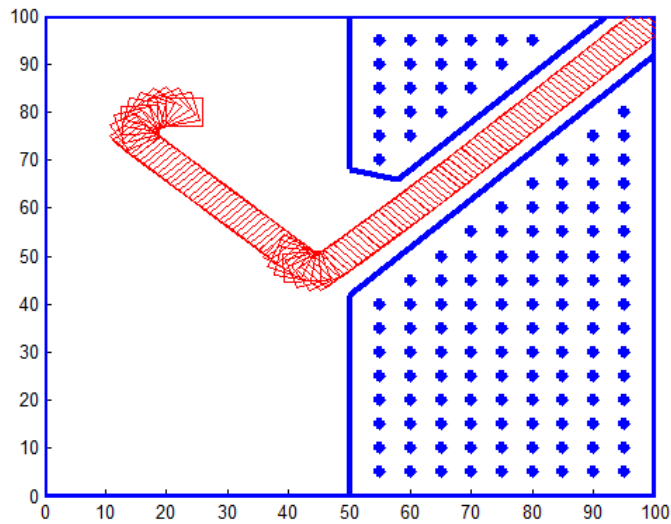
# Trajectories of Mobile Robot to Achieve a Final Desired Position



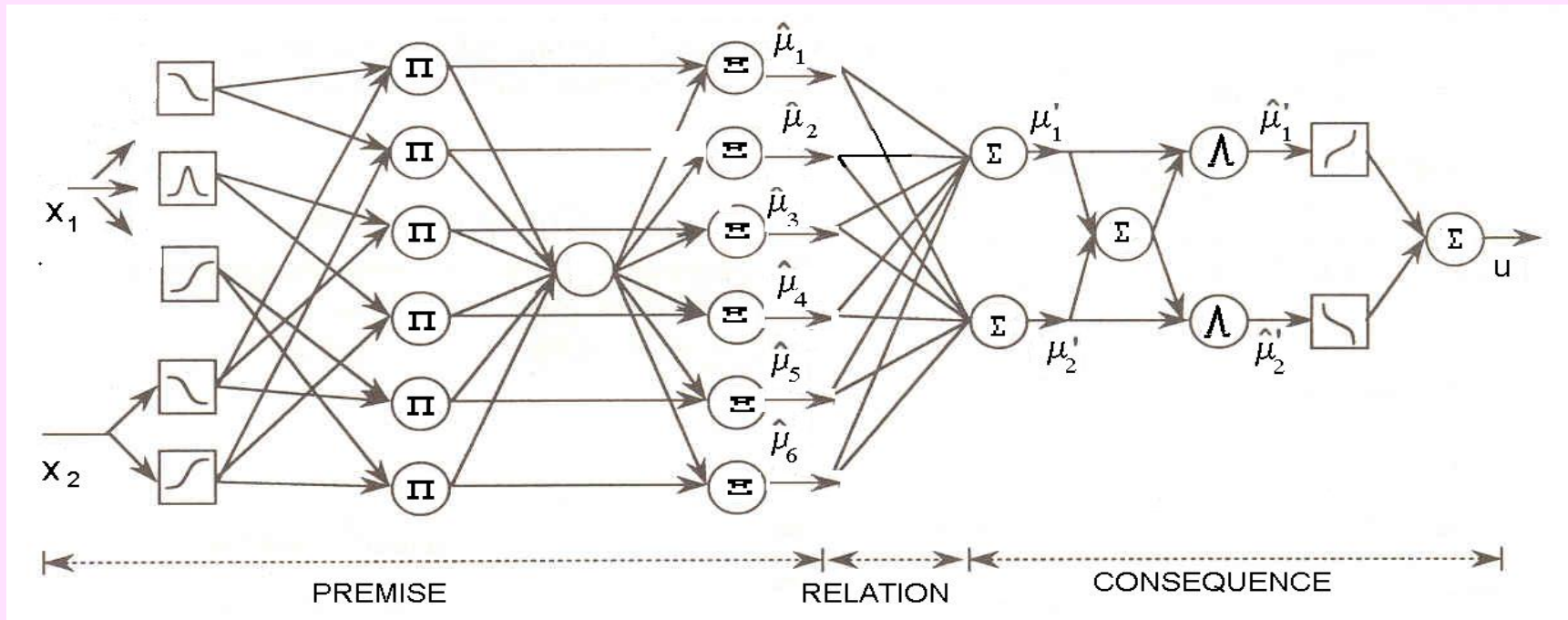
# Trajectories of Mobile Robot to Achieve a Final Desired Position



# Trajectories of Mobile Robot to Follow a Road



# Fuzzy Neural Network



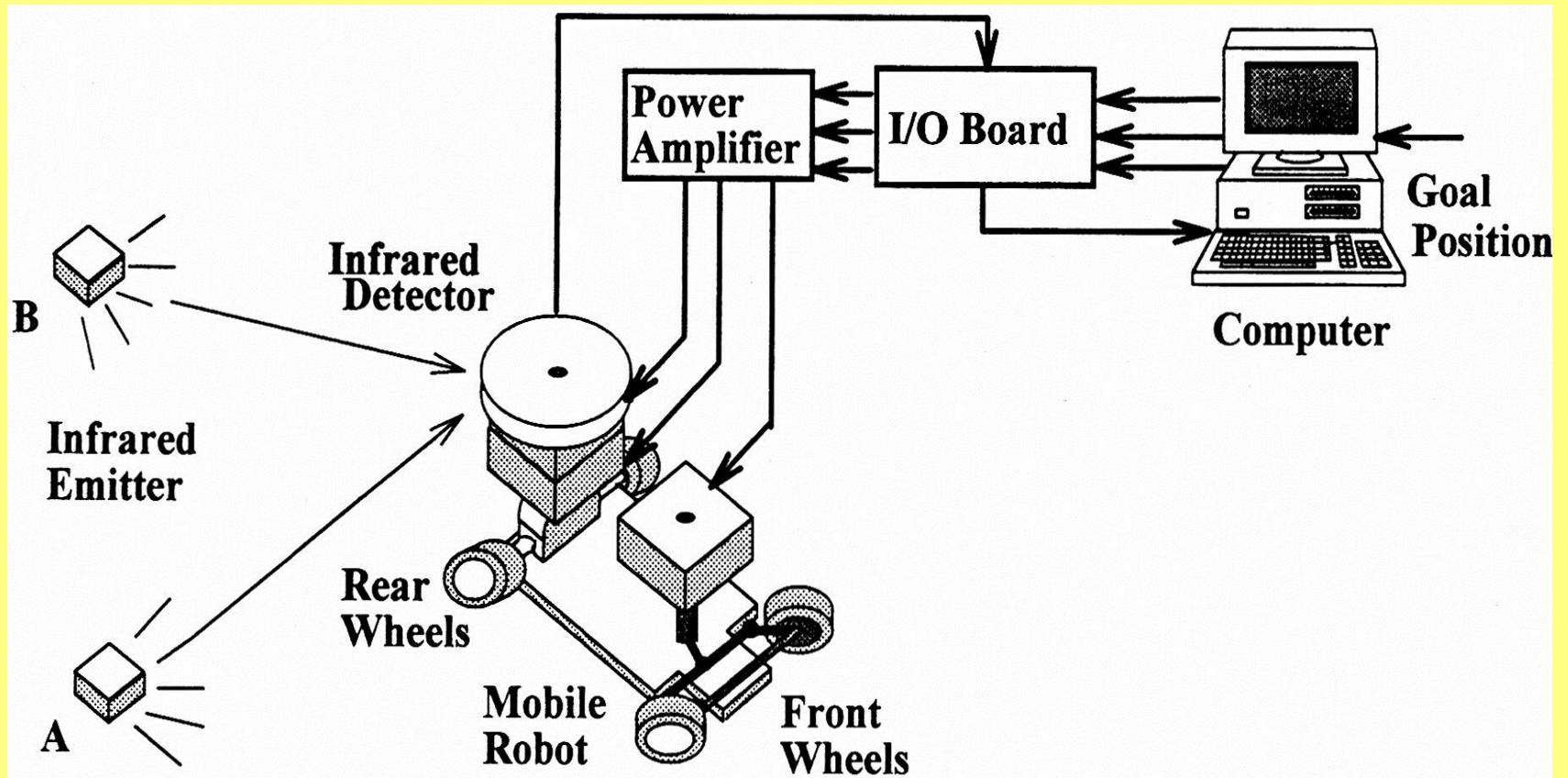
**Integrates:**

**Knowledge**  $\rightarrow$  **IF -THEN Rules (Fuzzy)**

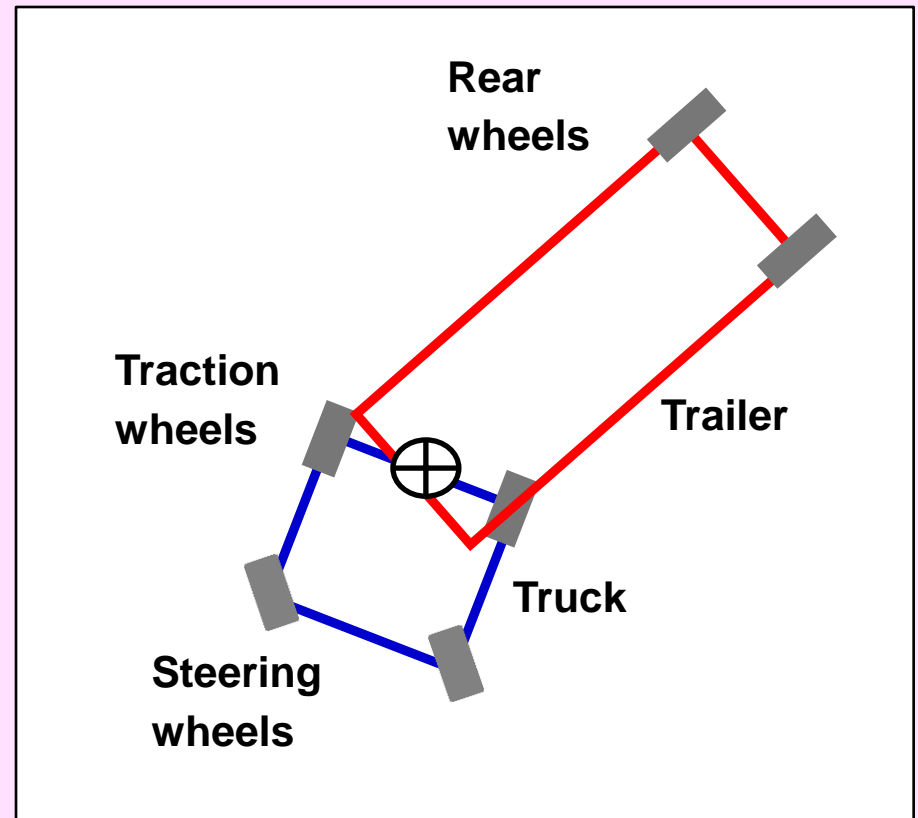
**Data**  $\longrightarrow$  **Training (Neural Network)**



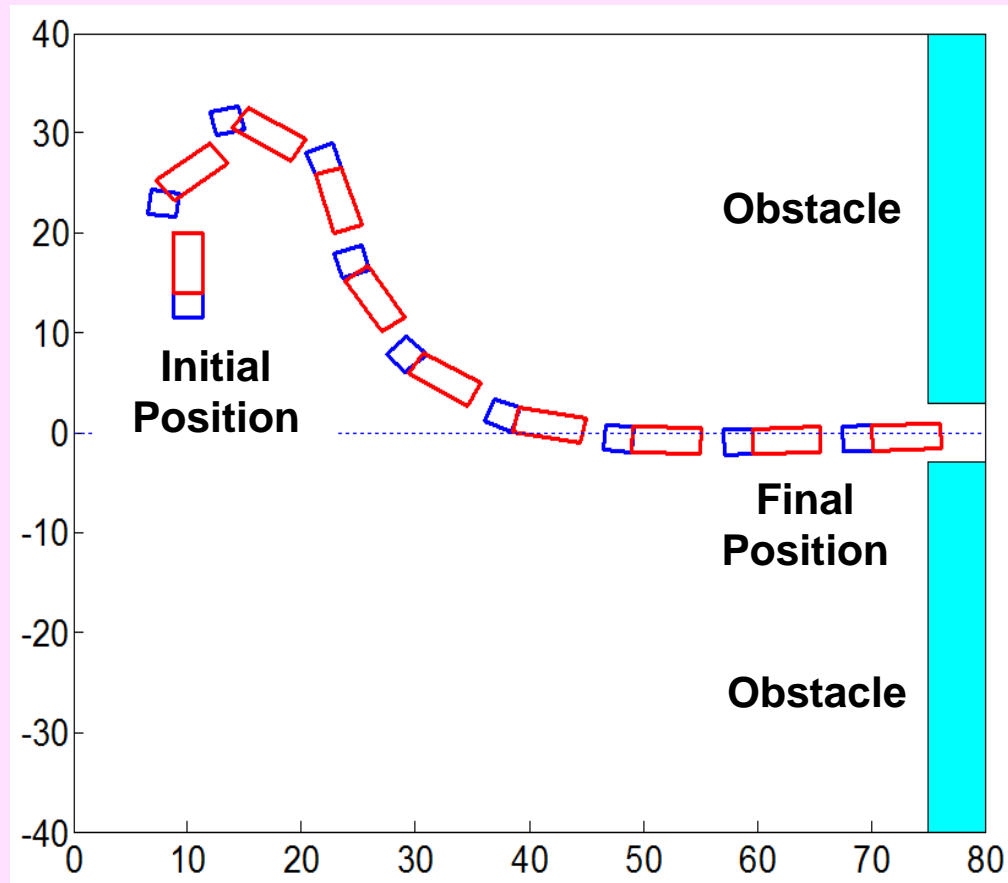
# Experimental Mobile Robot



# Control of a Truck-Trailer Mobile Robot

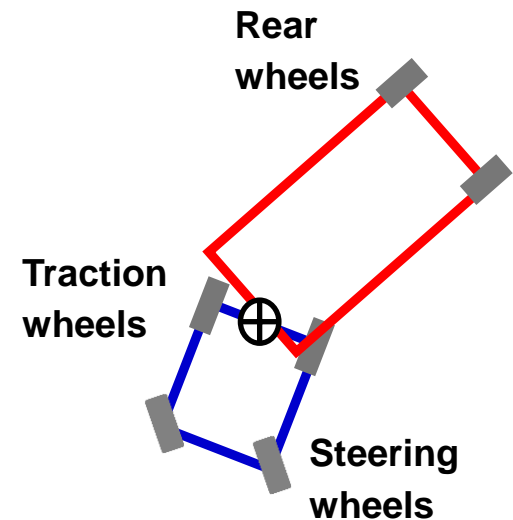
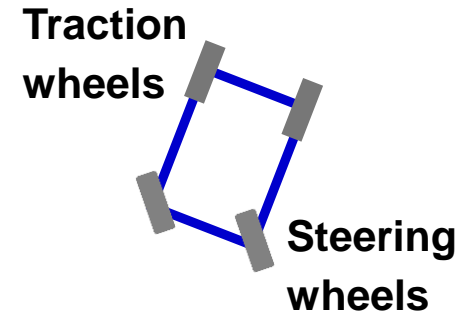


# Control of a Truck-Trailer Mobile Robot



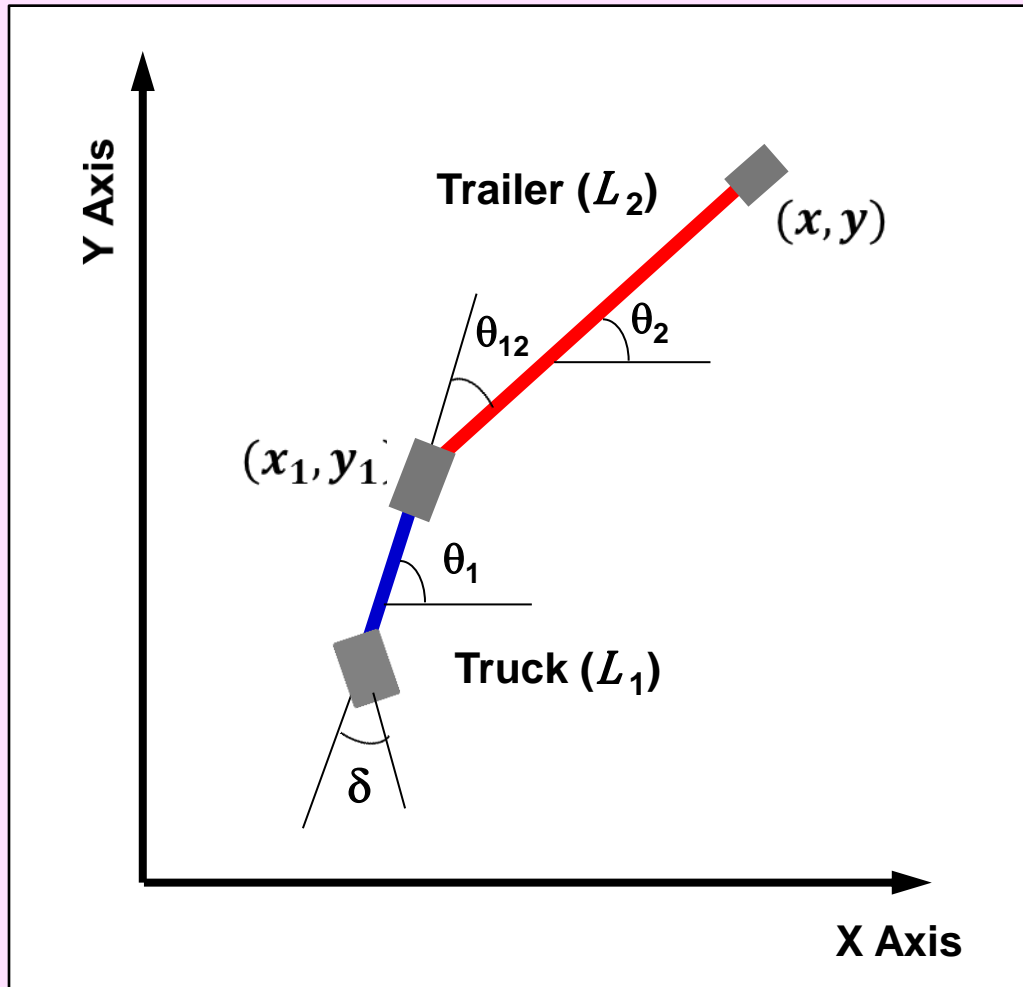
# Incremental Learning

- Train the neural network for controlling a car  $\theta_{12} = 0$ 
  - Close to the desired position
  - Away from the desired position
- Train the neural network for controlling a truck-trailer  $\theta_{12} \neq 0$ 
  - Small values of  $\theta_{12}$
  - Higher values of  $\theta_{12} < \pi/2$





# Control of a Truck-Trailer Mobile Robot



$$\dot{x} = v \cos \theta_{12} \cos \theta_2$$

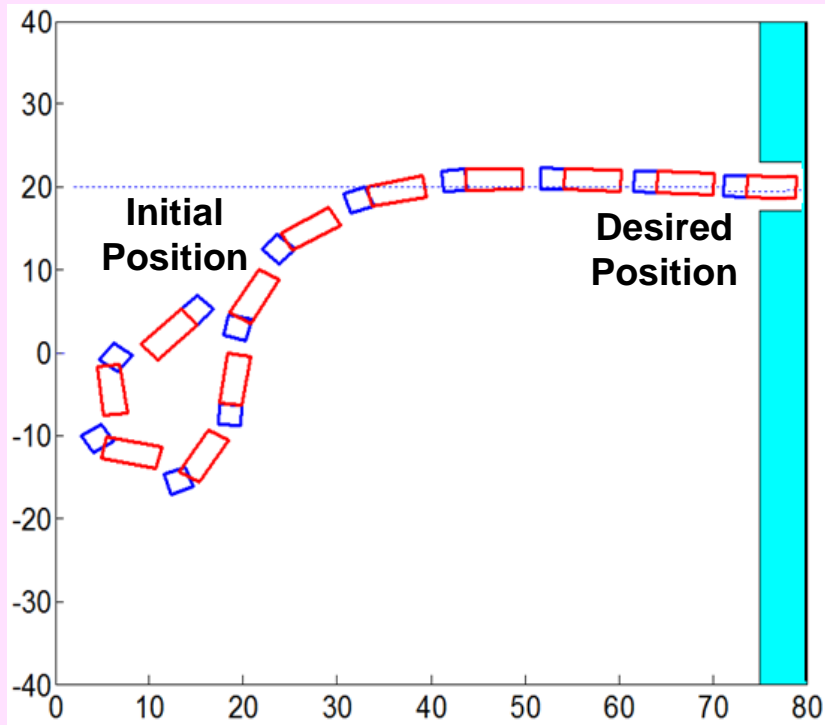
$$\dot{y} = v \cos \theta_{12} \sin \theta_2$$

$$\dot{\theta}_1 = -\frac{v}{L_1} \tan \delta$$

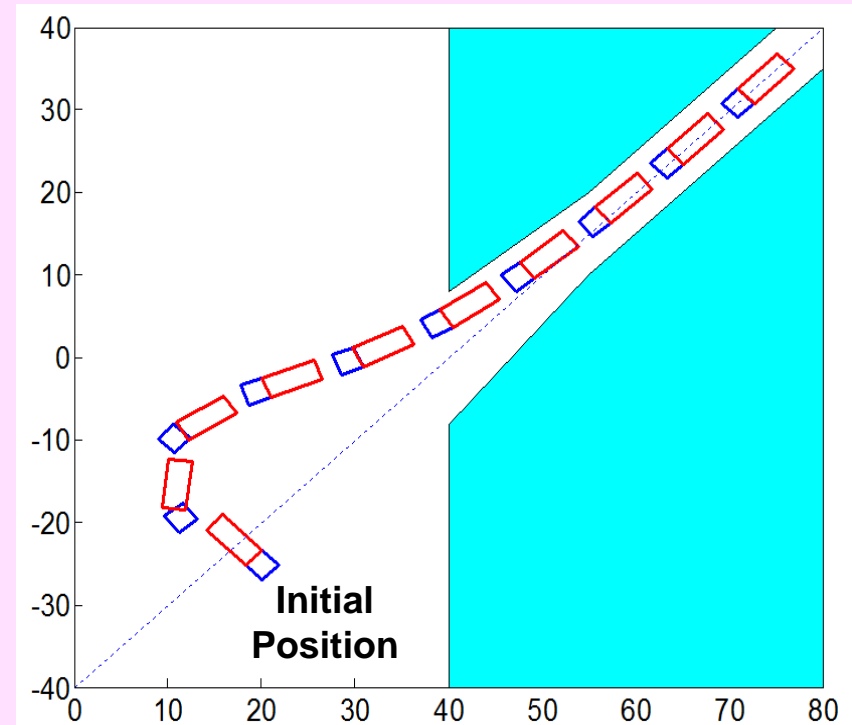
$$\dot{\theta}_2 = -\frac{v}{L_2} \sin \theta_{12}$$

# Control of a Truck-Trailer Mobile Robot

## Achieving a Goal Position

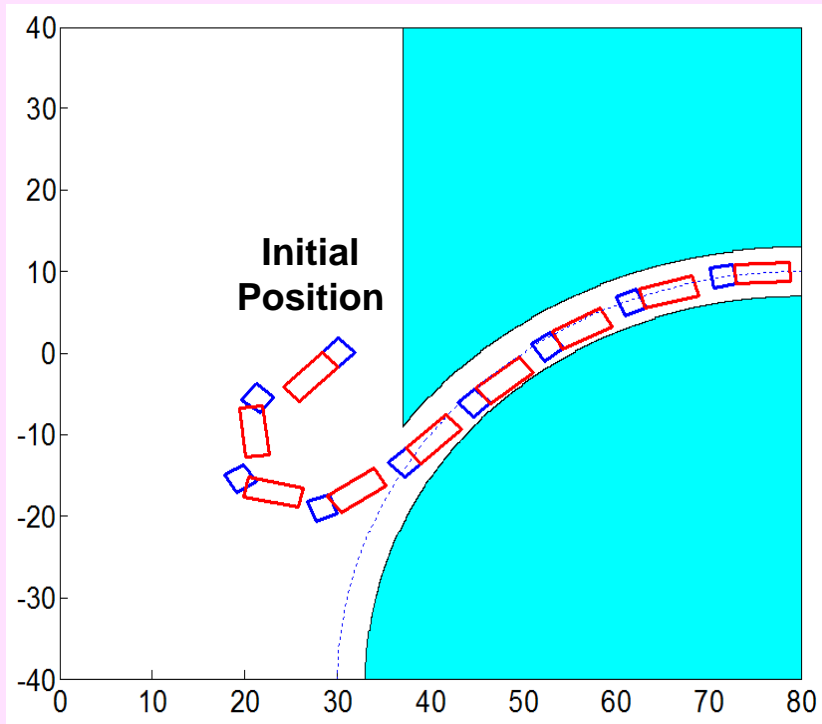


## Following a Straight Line

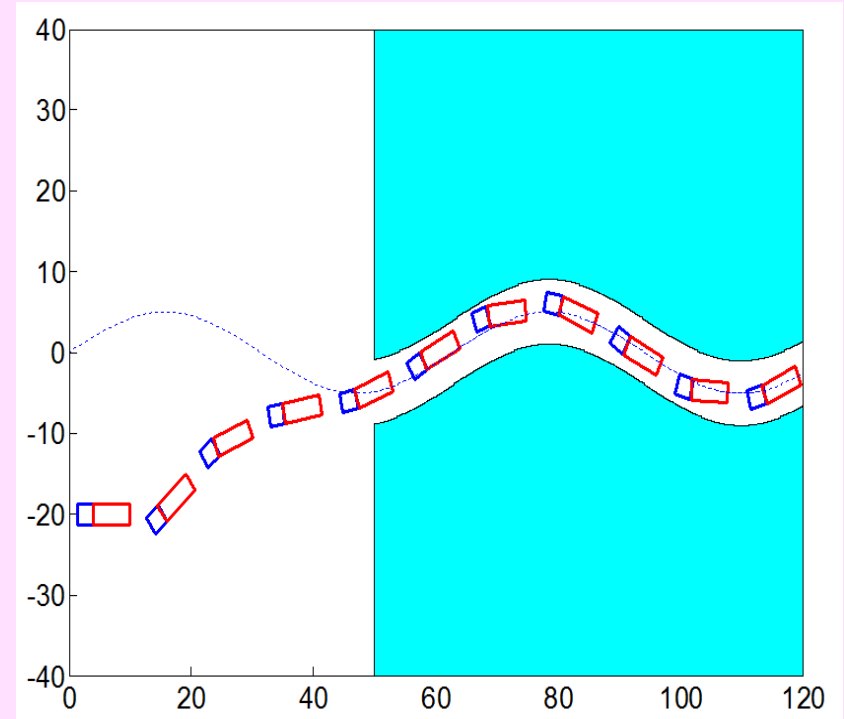


# Control of a Truck-Trailer Mobile Robot

## Following a Curved Path



## Following a Sinusoidal Path



**Thank you for your  
attention!**

**Antonio Moran, Ph.D.**

**[amoran@ieee.org](mailto:amoran@ieee.org)**