Appendix II

Integral Table

The integrals that routinely arise in propagation problems are frequently of a non-elementary nature and involve a variety of special functions (see Appendix I). Many of these integrals have previously been evaluated and tabulated in various integral tables. In practice, therefore, it is useful for the theoretician to have access to a fairly extensive reference source of integrals. However, for most of the integrals encountered in this text, the following short table of integrals should prove adequate.

Table of Integrals

1. $$\int_0^\infty e^{-st} t^{x-1} dt = \frac{\Gamma(x)}{s^x}, \quad x > 0, \ s > 0$$

2. $$\int_0^\infty e^{-a^2x^2} dx = \frac{\sqrt{\pi}}{2a}, \quad a > 0$$

3. $$\int_0^1 t^{x-1}(1-t)^{y-1} dt = \int_0^\infty \frac{t^{x-1}}{(1+t)^{x+y}} dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}, \quad x > 0, \ y > 0$$

4. $$\int_0^x \frac{t^{\mu-1}}{(1+\beta t)^\nu} dt = \frac{x^\mu}{\mu} \ {}_2F_1(\nu,\mu;1+\mu;-\beta x), \quad \mu > 0$$

5. $$\int_0^\infty x^{\mu-1} e^{-ax} \sin bx \, dx = \frac{\Gamma(\mu)}{(a^2+b^2)^{\mu/2}} \sin\left(\mu \ \arctan \frac{b}{a}\right), \quad \mu > -1, \ a > 0$$

6. $$\int_0^\infty x^{\mu-1} e^{-ax} \cos bx \, dx = \frac{\Gamma(\mu)}{(a^2+b^2)^{\mu/2}} \cos\left(\mu \ \arctan \frac{b}{a}\right), \quad \mu > 0, \ a > 0$$

7. $$\int_0^\infty x^{\mu-1} e^{-a^2x^2} \sin bx \, dx = \frac{b}{2a^{\mu+1}} \Gamma\left(\frac{1+\mu}{2}\right) e^{-b^2/4a^2} \, {}_1F_1\left(1-\frac{\mu}{2}; \frac{3}{2}; \frac{b^2}{4a^2}\right),$$
$$\mu > -1, \ a > 0, \ b > 0$$

8. $$\int_0^\infty x^{\mu-1}e^{-a^2x^2}\cos bx\,dx = \frac{\Gamma(\mu/2)}{2a^u}\,{}_1F_1\left(\frac{\mu}{2};\frac{1}{2};-\frac{b^2}{4a^2}\right),$$

$$\mu > 0,\ a > 0,\ b > 0$$

9. $$\int_0^{2\pi}\exp(\pm ix\cos\theta)\,d\theta = 2\pi J_0(x)$$

10. $$\int_0^\infty xe^{ia^2x^2}J_0(bx)\,dx = \frac{i}{2a^2}\exp\left(-\frac{ib^2}{4a^2}\right),\quad a > 0,\ b > 0$$

11. $$\int_0^\infty xe^{-a^2x^2}J_0(bx)\,dx = \frac{1}{2a^2}\exp\left(-\frac{b^2}{4a^2}\right),\quad a > 0,\ b > 0$$

12. $$\int_0^\infty e^{-ax}J_0(bx)\,dx = \frac{1}{\sqrt{a^2+b^2}},\quad a > 0,\ b > 0$$

13. $$\int_0^\infty e^{-t}t^{n+p/2}J_p(2\sqrt{xt})\,dt = n!e^{-x}x^{p/2}L_n^{(p)}(x),\quad p \ge 0,\ n = 0, 1, 2, \ldots$$

14. $$\int_0^\infty x^\mu e^{-a^2x^2}J_p(bx)\,dx = \frac{b^p\Gamma\left(\frac{p+\mu+1}{2}\right)}{2^{p+1}a^{p+\mu+1}\Gamma(p+1)}\ {}_1F_1\left(\frac{p+\mu+1}{2};p+1;-\frac{b^2}{4a^2}\right),$$

$$\mathrm{Re}(\mu+p) > -1,\ a > 0,\ b > 0$$

15. $$\int_0^\infty \frac{J_p(bx)x^{p+1}}{(x^2+a^2)^{\mu+1}}\,dx = \frac{a^{p-\mu}b^\mu}{2^\mu\Gamma(\mu+1)}K_{p-\mu}(ab), -1 < \mathrm{Re}(p) < \mathrm{Re}(2\mu+3/2),$$

$$a > 0,\ b > 0$$

16. $$\int_0^\infty x^\mu K_p(ax)\,dx = \frac{2^{\mu-1}}{a^{\mu+1}}\Gamma\left(\frac{1+\mu+p}{2}\right)\Gamma\left(\frac{1+\mu-p}{2}\right),$$

$$p \ge 0,\ \mu > p-1,\ a > 0$$

17. $$\int_0^\infty \kappa^{2\mu}\frac{\exp\left(-\kappa^2/\kappa_m^2\right)}{\left(\kappa_0^2+\kappa^2\right)^{11/6}}\,d\kappa = \frac{1}{2}\kappa_0^{2\mu-8/3}\Gamma\left(\mu+\frac{1}{2}\right)U\left(\mu+\frac{1}{2};\,\mu-\frac{1}{3};\frac{\kappa_0^2}{\kappa_m^2}\right),$$

$$\mu > -\frac{1}{2}$$

18. $$\int_0^\infty x^{p-1}\exp\left(-\frac{a}{x}-bx\right)dx = 2\left(\frac{a}{b}\right)^{p/2}K_p\left(2\sqrt{ab}\right),\quad \mathrm{Re}(a) > 0, \mathrm{Re}(b) > 0.$$