Task 6

Solve the string equation:

$$\frac{\partial^2 y(x,t)}{\partial t^2} = c^2 \frac{\partial^2 y(x,t)}{\partial x^2}$$

where $y(x,t)$ determines the shape of the string at the moment t

For the needs of the solution, we assume c = 1 (c wave propagation speed)

Right side:

$$\frac{\partial^2 y(x,t)}{\partial x^2}$$

Approximated:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{y(x_{i+1},t) - 2 * y(x_i,t) + y(x_{i\text{-}1},t)}{\Delta x^2} = a(x_i,t)$$

L – string length (π)

N – numer of points(10)

$$\Delta x = \frac{L}{N}$$

For each point of the string we solve the equation of motion by the MidPoint method:

$$\begin{cases} \frac{dy}{dt} = v \\ \frac{dv}{dt} = a \end{cases}$$

The string is attached at the ends.

Determine the potential, kinetic and total energy of the string:

Ek,Ep the sum of the energy of individual points

$$E_k = \sum_{i=0}^{N} \frac{dx * V^2(x_i)}{2}$$

$$E_p = \sum_{i=0}^{N} \frac{\left(y(x_{i+1}) - y(x_i)\right)^2}{2\Delta x}$$

Present Ep, Ek, Et on the graph