# Pupil Centre Coordinates Detection Using the Circular Hough Transform Technique

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***Abstract:*** *This paper presents a robust technique for pupil center coordinates detection that uses the circular Hough transform technique. The algorithm of the proposed application is based on detecting the circle which represents the pupil edge from a set of eye images captured using an infrared camera which employs the dark-pupil technique. The accuracy of the pupil detection algorithm is evaluated on the basis of the pupil center positioning error for a set of 52 eye images of the same subject, with gaze oriented in all directions.*

## 1. Introduction

In the last years, various techniques of eye detection have been proposed for use in many applications. The most common application is people identification, a technique increasingly used in the network society [1–2]. This application is based on face recognition and uses biometric signal processing [3]. Obviously, face detection is a most important step in the face recognition process [3]. One of the techniques used in face recognition that has enjoyed great interest lately is based on iris recognition [1–12]. This has the advantage that the iris of every person is unique and never changes during a person's lifetime [1, 4].

Almost all methods used for iris localization are based on the incorrect assumptions that the center of the iris is the same as the pupil and that the iris has perfectly circular shape. In fact, the pupil shape and its position is affected by many error sources, such as: corneal reflection, involuntary blinking, physiological tremor of the eye, and others.

In this paper is presented a method for pupil center detection based on a circular Hough transform technique used for eye tracking application. A method for testing the accuracy of the proposed algorithm, used for pupil center detection is also presented.

Experimental tests with different subjects, realized in various conditions have shown a high accuracy of the pupil center detection, confirming the possibility of using this method for eye tracking applications.

## 2. Circular Hough Transform

The Hough transform (HT) is a robust algorithm presented by Paul Hough in 1962 for the detection of features of a particular shape like lines or circles in digitalized images [1, 9]. The circular Hough transform can be used to deduce the radius and the centre coordinates of the pupil, and consequently in gaze direction detection [1, 8].

The main advantage of the Hough transform technique is that it is tolerant of gaps in feature boundary descriptions and remains relatively unaffected by image noise, unlike edge detectors [1, 10].

The general principle of the Hough transform is represented by the projections of the N-dimensional image space to a parameter space with a dimension M, which are related by a specified mathematical model [1].

Since a circle is represented by the equation,

$$(x-a)^2+(y-b)^2-r^2=0 \tag{1}$$

it is obvious that in this case the parameter space has only three parameters: two parameters for the center of the circle and one parameter for the radius of the circle. For more complex geometrical shapes (e.g. ellipses, which approximate better the eye pupil), the dimension of the Hough space increases with the number of the variables, and the computation time of the algorithm increases considerably.

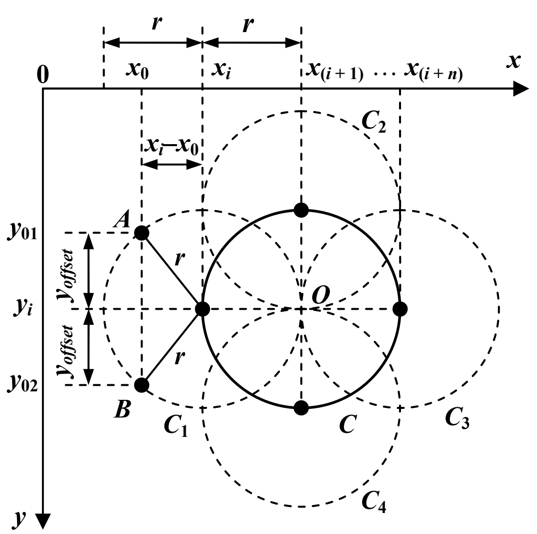



**Fig. 1.** The Hough transform principle.

According to the Hough Transform for circles, each pixel in the image space corresponds to a circle in the Hough space and vice versa [1]. All points of the circle $C$ in the image are transformed in several circles ($C_1$, $C_2$, $C_3$, $C_4$, …) with the same radius $r$, where their intersection, $O$, is the center of the detected circle (Fig. 1). The coordinates on the $y$ axis of the two symmetrical points A and B (with the same coordinates $x_0$ on the $x$ axis,) on the Hough circle $C_1$ in Fig. 1, are given by the following equations:

$$\begin{cases} y_{01} = y_i - y_{offset} \\ y_{02} = y_i + y_{offset} \end{cases} \tag{2}$$

where the offset on the $y$ axis is given by:

$$y_{offset} = \sqrt{r^2 - \left(x_i - x_0\right)^2} \tag{3}$$

The result of the Hough transform is stored in a table cell of the image size named the Hough accumulator. The accumulator value is updated for each circle generated by using the Hough transform. Thus, the accumulator terms having the address provided by the points on the Hough circles ($C_1$, $C_2$, $C_3$, $C_4$, …) are increased by one unit. The maximum value of the accumulator represents the center $O$ of the detected circle, $C$.

The Hough transform for circles can be used in the applications in two cases [1]:

- *The case of known radius* – in this case the search parameters are a pair of coordinates ($x_i$, $y_i$), representing the center of the Hough circles and thus the Hough space is bidimensional.

- *The case of unknown radius* – in this case the search parameters are a triplet ($x_i$, $y_i$, $r_i$), representing the coordinates of the Hough circles and their radii; the Hough space is tridimensional.

## 3. Pupil Center Detection Algorithm

The pupil center coordinates are detected by using model-based approaches [13]. Model-based approaches do not explicitly detect features, but rather find the best fitting model that is consistent with the image [13]. In our case, the HT is used to find the best-fitting circle for the eye pupil contour.

The hardware component of this pupil detection technique uses a head-mounted eye-tracking system, which consists of an infrared webcam mounted on frame glasses right underneath the eye, and a PC for image acquisition and processing. In order to improve eye pupil detection, the dark-pupil technique has been implemented. The result of the infrared illumination is that the pupil is clearly demarcated as a bright region due to the photo reflective nature of the back of the eye [13].

In this study, the eye images have been captured from a subject who was asked to place his head in a chin rest and look at the user screen placed approximately 60cm away.

The circular Hough transform for the case of unknown radius is used in order to detect the pupil edge and thus its center coordinates in the image. The main problem of this detection method is represented by the fact that in gaze recognition applications the pupil is not a perfect circle and its shape and radius depend on different factors, such as, pupil position, illumination, corneal reflection, blinking, physiological tremor of the eye, etc.

In order to find the best fitting circle for the pupil contour, the HT algorithm was computed for different values of the radius from a fixed range between 25 to 35 pixels detected experimentally. The proposed algorithm increases significantly the accuracy of the pupil center coordinates determination, but with the price of increased running time. This represents the main disadvantage of the algorithm, and is due to the fact that the radius of the detected circles is unknown in eye tracking applications.

## 4. Experimental Results

In order to appreciate the accuracy of the proposed pupil detection technique, a set of 52 eye images with the pupil placed in different positions and having different shapes and radii have been considered. For each eye image the relative error of the pupil center coordinates position has been calculated using the following equation:

$$\varepsilon = \left[d/(2R)\right]\cdot 100 \ \ (\%) \tag{4}$$

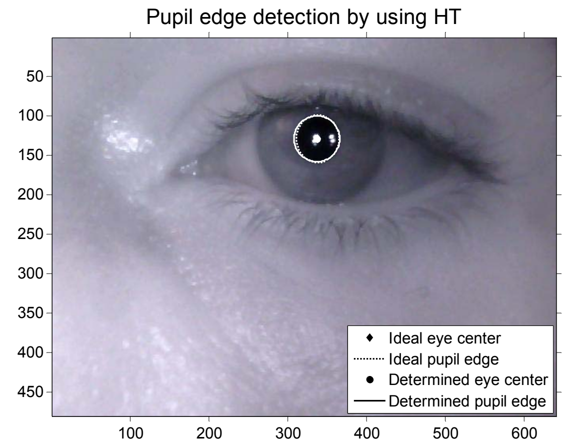
where $d$ is the Euclidian distance between the detected and the ideal center of the pupil, and $R$ the maximum radius of the pupil for the case when the pupil is placed in the center of sclera. For each image, the pupil edge and center have been precisely determined.

In Figs. 2.a) and b) the ideal pupil edge and the pupil edge detected by using the algorithm on the basis of circular HT are overlapped on the same eye image. The Euclidian distance between the two centers (the ideal one and that given by the Hough algorithm) is calculated for each image. According to Fig. 2.a), when the pupil is placed in the center of the sclera, the relative error of the pupil center positioning is minimal ($\varepsilon$ = 1.75 %). On the other hand, when the pupil is placed on the edge of the sclera (Fig. 2.b), this error increases ($\varepsilon$ = 7.23 %) due to the pupil change of shape from circle to ellipse and the minimizing of its radius.

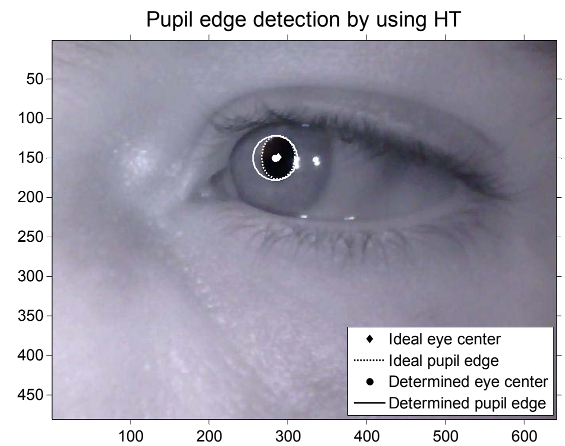
The mesh in Fig. 3 illustrates the 3D representation of the accumulator matrix values for the best pupil edge detection presented in Fig. 1.a). The address of the maximum value stored in the accumulator matrix represents the coordinates of the detected pupil center.

Fig. 4 represents the relative error of the pupil center coordinates position and its histogram for the entire set of 52 eye images calculated using equation (4).

The results obtained show a performing detection of the pupil center and the possibility of using an improved version of the Hough algorithm for eye tracking applications.




a)




b)

**Fig. 2.** Ideal pupil edge and the pupil edge detected by using the Hough algorithm for different positionings of the pupil; a) $\varepsilon$ = 1.75 %; b) $\varepsilon$ = 7.23 %.

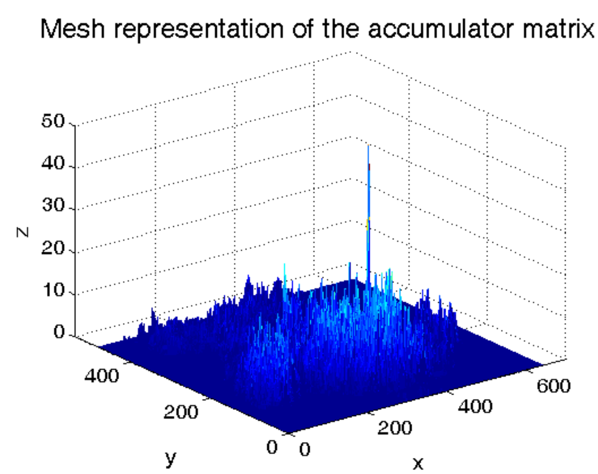



**Fig. 3.** 3D representation of the accumulator matrix values for the best pupil edge detection.

$\varepsilon_{average} = \mathbf{3.25\%}$

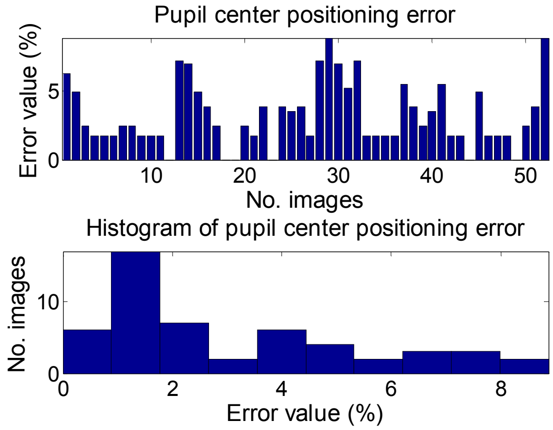



**Fig. 4.** Relative error of the pupil center position and its histogram for a set of eye images.

The algorithm speed can be increased by decreasing the number of radii considered by the algorithm for each frame of the eye image processing, if the relative error of the pupil center positioning does not exceed certain limits.

The best trade-off between accuracy and running time of the algorithm depends on the processing power of the PC calculation unit.

## 5. Conclusion

In this paper a new technique used for pupil center coordinates detection by using the HT algorithm has been presented.

The hardware component of the pupil detection algorithm uses a head mounted system which captures infrared eye images for processing.

The accuracy of the pupil detection algorithm is given by the relative errors of the pupil center positioning for a set of eye images captured from a subject with the pupil placed in different positions.

For higher accuracy, the algorithm running time increases due to a large number of radii considered in order to find the best fitting circle for the eye pupil contour.

## Acknowledgement


The work has been carried out within the program *Joint Applied Research Projects*, funded by the *Romanian National Authority for Scientific Research* (MEN – UEFISCDI), contract PN-II-PT-PCCA-2013-4-0761, no. 21/2014 (SIACT).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