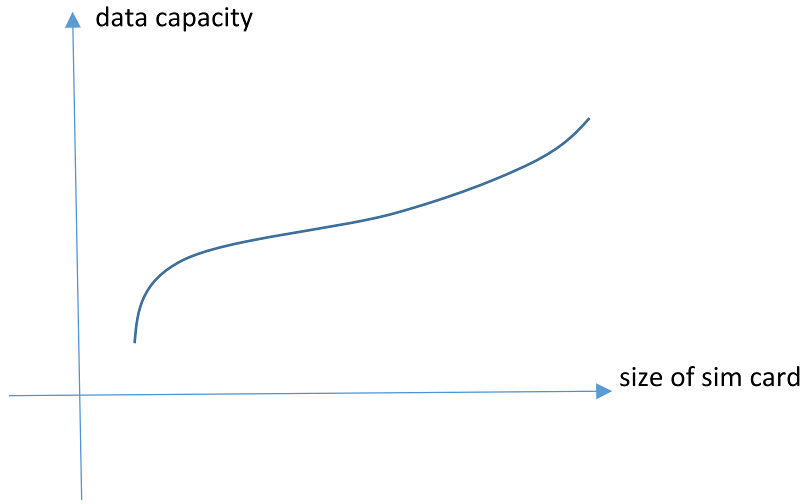
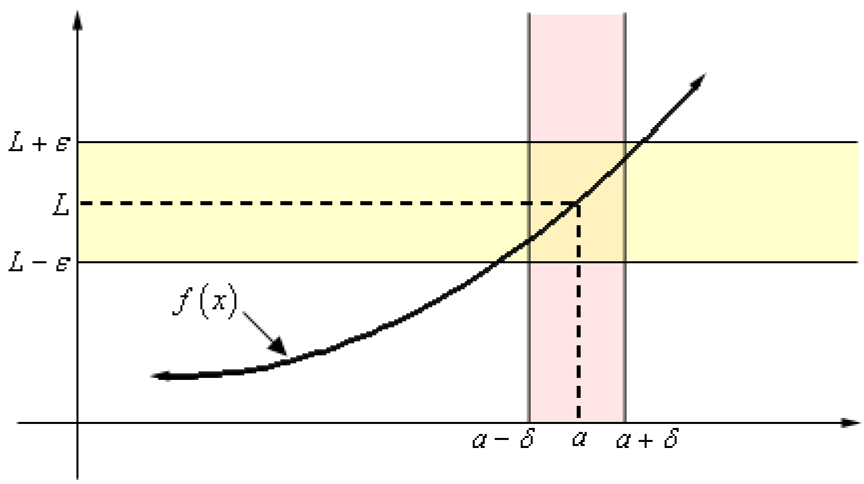
1.3 continued… (The Precise Definition of a Limit)

Goal: Understand the precise definition of a limit using δ (delta) and ε (epsilon).

Example 1: In today's world most people own a cellphone. Suppose that the phone's data capacity depends on the size of the sim card, that sim cards of a particular size are made with some small degree of variation, and that the data capacity for the (approximately)same size sim card holds a certain amount of data.



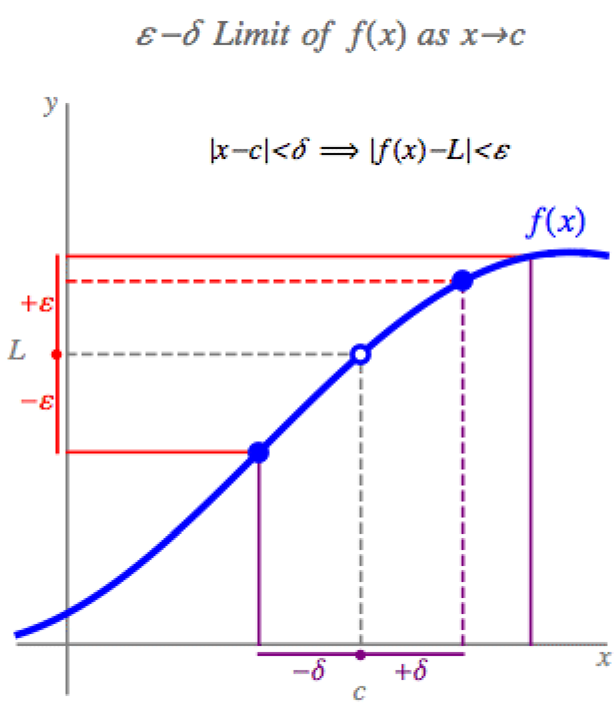
Definition: Let $f(x)$ be defined on some open interval that contains the number a , except possibly at a itself. Then $\lim_{x \to a} f(x) = L$ if for every $\varepsilon > 0$, there exists $\delta > 0$ such that if $0 < |x-a| < \delta$, then $|f(x) - L| < \varepsilon$.



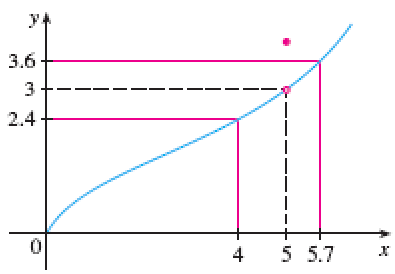
Example 2: Consider the function f(x) whose graph is given by below. Notice that this function is not defined at x = c since there is an open circle at (c,L).

Given any positive value of ε, our mission is to always find the appropriate positive value of δ so that any point in the open interval on the x-axis containing δ will produce a y-value in the desired interval on the y-axis.

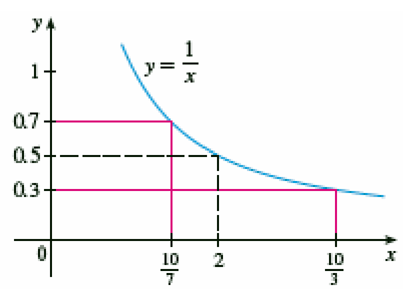
What happens if you choose a value outside the interval located on the x-axis?



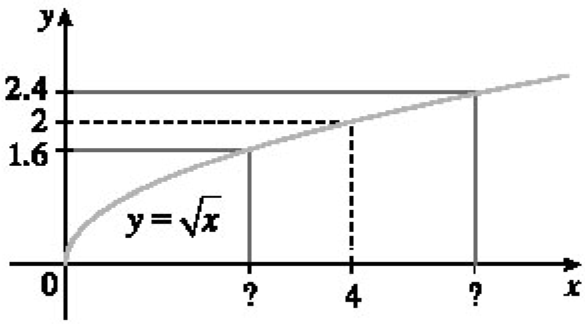
Example 3: Use the graph of f(x) to find a number δ such that if $0 < |x - 5| < \delta$, then $|f(x) - 3| < 0.6$.



Example 4: Use the graph of f(x) to find a number δ such that if $0 < |x - 2| < \delta$, then $|f(x) - 0.5| < 0.2$.



Example 5: Use the graph of $f(x) = \sqrt{x}$ to find the number δ such that if $|x-4| < \delta$, then $\left|\sqrt{x} - 2\right| < 0.4$.



Example 6: Use the ε, δ definition of a limit to prove the following limits.

a. $\lim\limits_{x \to 2}(x^2 - 4x + 5) = 1$

b. $\lim\limits_{x \to 1} \dfrac{2+4x}{3} = 2$