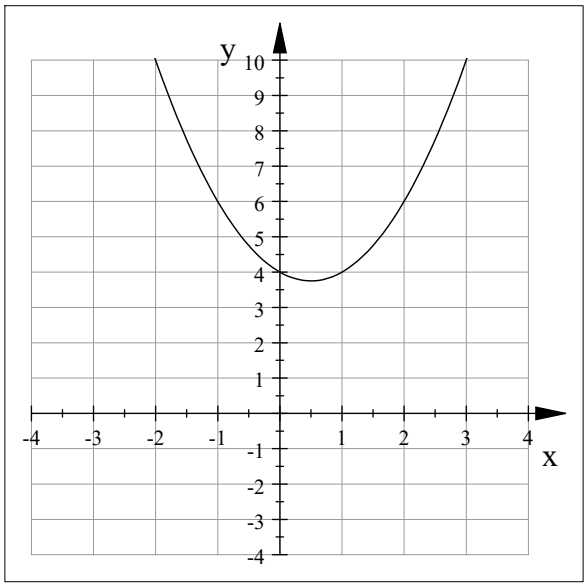
1.3 - The Limit of a Function

Example 1: Consider the graph of the function $f(x) = x^2 - x + 4$.



Describe the behavior of the function as x approaches 1 from the left and right side.

Thus, we say that $\lim_{x \to 1} (x^2 - x + 4) =$

Now consider the same function $f(x) = x^2 - x + 4$ and complete the table below:

x	.9	.99	.999	.9999	1	1.0001	1.001	1.01
f(x)								

Definition 1: The function f(x) has the limit L as x approaches a, written $\lim_{x->a} f(x) = L$, if the value of f(x) can be made as close to the number L as we please by taking x sufficiently close to a (but not equal to a). The limit is not affected by whether $f(a)$ is defined or not.

Note: You must approach a from both sides when finding a limit unless stated otherwise.

Diagram:

Notes:

1. X approaching a from the left: $\lim_{x->a^-} f(x)$

2. X approaching a from the right: $\lim_{x->a^+} f(x)$

3. For the limit to exist, the limit value from the left must be the same as the limit value from the right side. Otherwise, the limit does not exist.

$\lim_{x->a} f(x) = L$ if and only if $\lim_{x->a^-} f(x) = L$ and $\lim_{x->a^+} f(x) = L$

Example 2: Consider the function $f(x) = \frac{\sin x}{x}$.

a. State the domain of the function.

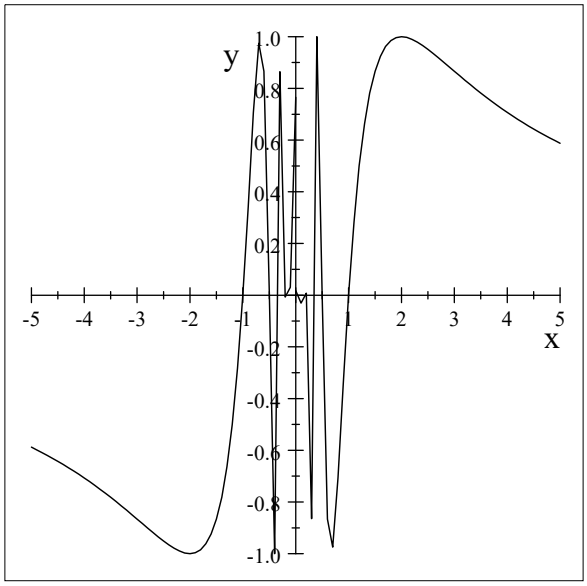
b. What do you think value of $\lim_{x->0} \frac{\sin x}{x}$ is?

c. Use a graphing calculator to graph the function near x = 0.

Example 3: Let's now investigate $\lim_{x->0} \sin(\frac{\pi}{x})$. Don't forget to also consider the values smaller but very close to zero.

$f(1) =$ $f(1/2) =$ $f(1/3) =$ $f(1/4) =$

$f(.1) =$ $f(.01) =$



Example 4: Consider the function $G = \begin{cases} 5, x=1 \\ \frac{x^2-1}{x-1}, x \neq 1 \end{cases}$.

Graph the function and find the limit, if it exists.

a. $\lim_{x \to -2} G$

b. $\lim_{x \to 3^-} G$

c. $\lim_{x \to 1} G$

Example 5: Consider the function $H(t) = \begin{cases} 0, t<0 \\ 1, t\geq 0 \end{cases}$.
Graph the function and find the limit, if it exists.

a. $\lim_{t\to -3} H$

b. $\lim_{t\to 0^-} H$

c. $\lim_{t\to 0^+} H$

d. $\lim_{t\to 0} H$

e. $\lim_{t\to 5} H$

Example 6: Find $\lim_{x\to \pi} 5\sin x$, if it exists.