Center of mass of an n-point system

For a system consisting of *n* masses where a mass m_i is located at a point (x_i, y_i) we may obtain the center of mass as a point $(\overline{x}, \overline{y})$ where

- $m = \sum_{i=1}^{n} m_i,\ M_x = \sum_{i=1}^{n} m_i y_i,\ M_y = \sum_{i=1}^{n} m_i x_i,\ \overline{x} = \frac{1}{m} M_y$ and $\overline{y} = \frac{1}{m} M_x$

Center of mass of a lamina of constant density

For a lamina whose region is an ordinate region, $\mathcal{R}$, bounded above by $y = f(x)$ and below by the *x*-axis, on the left by $x = a$ and on the right by $x = b$, we may obtain the center of mass as the point $C(\overline{x}, \overline{y})$ where

- $A = Area(\mathcal{R}) = \int_a^b f(x)\,dx$ and $m = \rho A$ and $M_x = \rho \int_a^b \frac{1}{2}\left[f(x)\right]^2 dx$ and $M_y = \rho \int_a^b x f(x) dx$,
- $\overline{x} = \frac{M_y}{M} = \frac{1}{A}\int_a^b x\, f(x)\, dx$ and $\overline{y} = \frac{M_x}{M} = \frac{1}{A}\int_a^b \frac{1}{2}\left[f(x)\right]^2 dx$

If the region of the lamina is bounded above by $y = f(x)$ and below by $y = g(x)$, then the calculation of the center of mass must be modified slightly, as follows:

- $A = Area(R) = \int_a^b f(x) - g(x)\,dx$ and $m = \rho A$
- $M_x = \rho \int_a^b \frac{1}{2}\left[\left[f(x)\right]^2 - \left[g(x)\right]^2\right] dx$ and $M_y = \rho \int_a^b x\left[f(x) - g(x)\right] dx$
- $\overline{x} = \frac{M_y}{m} = \frac{1}{A}\int_a^b x\left[f(x) - g(x)\right] dx$ and $\overline{y} = \frac{M_x}{m} = \frac{1}{A}\int_a^b \frac{1}{2}\left[\left[f(x)\right]^2 - \left[g(x)\right]^2\right] dx$

Symmetry principle of centroids and centers of mass

Often times, one or both of the coordinates of the centroid may be determined via the principle of symmetry. If a lamina has constant density then its centroid must lie along every line of symmetry of the corresponding region. Consequently, if the region has two distinct lines of symmetry then the centroid lies at the intersection of these two lines.

For example, every diameter of a circle is a line of symmetry of the circle hence the centroid must lie at the center of the circle where the diameters intersect. As another example, suppose the *x*-axis is a line of symmetry, then we must have $\overline{y} = 0$. Similarly, if the *y*-axis is a line of symmetry, then $\overline{x} = 0$.