ES 155: Systems and Control

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Homework 5, Due on November 9th 2018, 5pm, A box outside of Lina's office, MD 345

Note: In the upper left hand corner of the first page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. Consider the block diagram for the following system

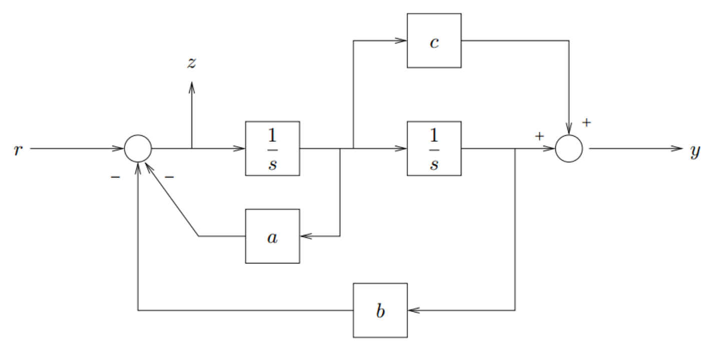

Figure 1: Block Diagram

   (a) Compute the transfer function H_{yr} between the input r and the output y.

   (b) Show that the following state space system has the same transfer function, with the appropriate choice of parameters:

$$\frac{d}{dt}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -a_2 & -a_1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} r$$

$$y = \begin{pmatrix} b_2 & b_1 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + d \cdot r$$

   Give the values of a_i, b_i, and d that correspond to the transfer function you computed in (a).

   (c) Compute the transfer function H_{zr} between the input r and the output z.

2. Consider the following simplified equations of motion for a cruise control system:

$$m\frac{dv}{dt} = -cv + b\tau + F_{\text{hill}}, \tag{1}$$

where $m = 1000kg$ is the mass of the vehicle, $c = 50Ns/m$ is the viscous damping coefficient, $b = 25$ is the conversion factor between engine torque and the force applied to the vehicle, v is the vehicle speed, τ is the engine torque input, and F_{hill} is the external disturbance due to the hill. We model the engine using a simple first-order equation

$$\frac{d\tau}{dt} = a(-\tau + Tu),$$

where $a = 0.2$ is the lag coefficient and $T = 200$ is the conversion factor between the throttle input and the steady state torque, and u is the throttle input. The simplest controller for this system is a proportional control, $u = k_p e$, where $e = (r - v)$ (r is the reference speed).

(a) Draw a block diagram for the system, with the controller dynamics, the engine dynamics and the vehicle dynamics in separate blocks and they are represented by transfer functions. In the block diagram, label the reference input to the closed loop system as r, the disturbance due to the hill (the F_{hill} in (1)) as d, and the speed v as output y.

Note: For the rest of this problem, assume the disturbance due to the hill, d, is 0.

(b) (MATLAB) Use MATLAB to compute the transfer function from r to y and plot the step response (use function step) and frequency response (use function bode),[1] for the following two choices of k_p: $k_p = 0.01$ and $k_p = 0.1$. What is the difference between the two cases?

(c) Consider a more sophisticated control law of the form

$$\frac{dx_c}{dt} = r - v, \quad u = k_p e + k_i x_c.$$

The control law contains an integral term, which uses the controller state x_c to integrate over the error. Compute the transfer function for this control law and redraw your block diagram from part (a) with the proportional controller replaced by this one.

(d) (MATLAB) Let $k_p = 0.5$ and $k_i = 0.1$. Use MATLAB to compute the transfer function from r to y and plot the step response as well as the frequency response for this system. What's the difference between the new controller and the proportional controller?

[1] To plot frequency response, you need to plot the magnitude of $G(j\omega)$ and the phase of $G(j\omega)$ along with frequency ω. Matlab code bode(sys) will automatically generate the two plots for you. You can wait until we finish Wednesday class Nov 7th to do the frequency plots. We will explain bode plots in details.