ES 155: Feedback Control System

Lecturer: Na Li (nali@seas.harvard.edu)

Homework 8, Due on December 5 2018, in class.

Note: In the upper left hand corner of the first page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. For the systems below, design a controller (either P, PI, PD, PID, Lead compensator, OR Lag compensator control law) that stabilizes the system, gives less than 1% error at zero frequency (i.e., steady state error), and gives at least 30° phase margin. You may use any design method and you only need to design one type of controller (as long as it meets the specification) but be sure to explain why you choose your controller and include appropriate plots or calculations showing that all specifications are met. For the closed loop system, determine the steady-state error in response to a step input and the maximum frequency for which the closed loop system can track with less than 25% error.

   (a) Disk drive read head positioning system:

   $$P(s) = \frac{1}{S^3 + 10s^2 + 3s + 10}$$

   (b) Drug administration/compartment model (AM08, Section 3.6):

   $$P(s) = \frac{1.5s + 0.75}{s^2 + 0.7s + 0.05}$$

2. In this problem we will design a PID compensator for a vectored thrust aircraft (see Example 3.12 in the text 2nd edition for a description). Use the following transfer function to represent the dynamics from the lateral input to the roll angle of the aircraft:

   $$P(s) = \frac{r}{Js^2 + cs + mgl} \qquad \begin{array}{lll} g = 9.8\text{m/s}^2 & m = 1.5\text{kg/s} & c = 0.05\text{kg} \\ l = 0.05\text{m} & J = 0.0475\text{kg m}^2 & r = 0.25\text{m} \end{array}$$

   Design a feedback controller that tracks a given reference input with the following specifications:

   - Steady-state error of less than 2%
   - Tracing error of less than 10% from 0 to 1 rad/s.
   - Phase margin of at least 40°.

   (a) Plot the open loop Bode plot for the system and mark on the plot the various frequency domain constraints in the above specification, as we did in class.

(b) Design a compensator for the system that satisfies the specification. you should include appropriate plots or calculations showing that all specifications are met.

(c) Plot the step and frequency response of the resulting closed loop control. For the step response, compute the steady-state error, rise time, overshoot and settling time of your controller.

(Hint: You may not need all of the terms in a PID Controller.)

3. Consider the dynamics of the magnetic levitation system from Lecture 20. The transfer function from the electromagnet input voltage to the IR sensor output voltage is given by

$$P(s) = \frac{k}{s^2 - r^2}$$

with $k = 4000$ and $r = 25$. (Remark: These parameters are slightly different than those used in the lecture note.)

(a) Design a controller that stabilizes this process. Compute the poles and zeros for the open loop transfer function and for the closed loop transfer function between the reference input and measured output.

(b) Plot the Nyquist plot corresponding to your open loop transfer function L (i.e., compensator transfer function * P(s)), and verify that the Nyquist criterion is satisfied.

(c) Plot Bode plots for the Closed-loop sensitivity functions (i.e., $S := \frac{1}{1+L}$) and the closed-loop step response from a commanded input to the output (i.e., $T := \frac{L}{1+L}$).