ES 155: Systems and Control

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Homework 1, Due on September 19th 2018, In class.

Note: In the upper left hand corner of the first page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. [Astrom and Murray, Exercise 1.3] (Balance systems) Balance yourself on one foot with your eyes closed for 15s. Using Figure 1.3 as a guide, describe the control system responsible for keeping you from falling down.

2. [Astrom and Murray, Exercise 1.2] Identify three feedback systems that you encounter in your everyday environment. For each system, identify the sensing mechanism, actuation mechanism and control law. Describe the uncertainty with respect to which the feedback system provides robustness and/or the dynamics that are changed through the use of feedback.

3. [Build a mathematical model] Pick one system that you describe in the previous question and build a mathematical model using ODE. Please provide some rationale behind your model. (Note: This is an open-ended question. As long as the model makes sense, your answer is good.)

4. [PI cruise-control] Consider the following cruise-control example with dynamics

$$m\dot{v} = -av + u + \omega$$

where u is the control input (force applied by engine) and ω the disturbance input (force applied by hill, etc.), which will be ignored below, i.e., $\omega = 0$. An open-loop control strategy to achieve a given reference speed v_{ref} would be to choose

$$u = \hat{a}v_{\text{ref}}$$

where $\hat{a}$ is your estimate of a, which may not be accurate. Assume m,a, and $\hat{a}$ are all positive.

   (a) Compute the equilibrium points (which is also called as steady-state [1]) for both the open-loop strategy above, and for the feedback law

$$u = -k_p(v - v_{\text{ref}})$$

   and compare the steady-states as a function of $\beta = a/\hat{a}$ when $k_p = 10\hat{a}$. (You should solve the problem analytically, and then plot the response v_{ss}/v_{ref} as a function of β for both the open-loop and proportional-gain feedback law. Here V_{ss} stands for the steady states.)

   (b) Now consider a proportional-integral (PI) control law

$$u = -k_p(v - v_{\text{ref}}) - k_i \int_0^t (v - v_{\text{ref}})dt$$

   Compute the steady state solution. Then compare the response with the proportional gain case from above. (Note that if you define $q = \int_0^t (v - v_{\text{ref}})dt$ then $\dot{q} = v - v_{\text{ref}}$.)

[1] Given a dynamical system described by ODE $\dot{x} = f(x)$, a steady state or equilibrium point is the state x^* such that $f(x^*) = 0$. Note that if $x(t) = x^*$, we have $\dot{x} = 0$, therefore $x(t) = x^*$ all the time. This is why x^* is called as a steady state/equilibrium point.

(c) Next, simulate and plot the response of the system (using ode45 in Matlab[2]) with the PI control law above with $m = 1, a = 0.1, w = 0$, and reference speed $v_{\text{ref}} = \sin(\omega_0 t)$, for $\omega_0 = 0.01$, 0.1, 1, and 10 rad/sec. In each case, you should simulate at least 10 cycles; after some initial transient, the response should be periodic. Compute the peak-to-peak amplitude of the final period for the error $v - v_{\text{ref}}$, and plot this as a function of frequency on a log-log scale, for the following control gains:

i. $k_p = 1, k_i = 0$
ii. $k_p = 1, k_i = 1$
iii. $k_p = 1, k_i = 10$

[2]Please check the lecture notes and the sample codes on the course website for lecture 2. You need to modify parameter values for this homework.