ES155 Homework 8

Contents

Problem 1

```
clear; clc;

% Given requirements
maxSSerr = 0.01;      % 1% at zero frequency
minPM = 30 * pi / 180;  % 30 degrees in radians
maxFreqError = 0.25;    % Bandwidth defined as max freq with less than 25% error

% Input the Plant transfer functions
P_1a = tf(1, [1, 10, 3, 10])
P_1b = tf([1.5, 0.75], [1, 0.7, 0.05])
```

```
P_1a =

             1
  -----------------------
  s^3 + 10 s^2 + 3 s + 10

Continuous-time transfer function.


P_1b =

     1.5 s + 0.75
  ------------------
  s^2 + 0.7 s + 0.05

Continuous-time transfer function.
```

1.a Disk Drive

For SSerr = 1/(1 + L(0)) <= 0.01 requires L(0) > 100 (+40dB) P_1a(0) = -20dB, so need to add +60dB. Adding a "safety margin", try +70dB (3162.3) This results in a phase margin of -53 degrees. Add a zero to improve this. Default frequency of 1 gives PM of 9.14 degrees. Moving frequency to 3.5 maximized the PM to 12 degrees. This required a second zero to achieve a 90 degree PM (zero frequency at 1) However, this no longer constitutes a PID controller, as there are two zeros and no poles. Introducing an integrator can allow the system to reach a steady state error of 0. Simply including an integrator and a zero at 1 rad/s results in 0 steady state error with a 94 degree phase margin. Thus,

```
C_1a = tf([1, 1], [1, 0])
L_1a = P_1a*C_1a
sys_1a = feedback(L_1a, 1)

% Check that design goals were met
SSerr = 1 - dcgain(sys_1a)
dbdrop = mag2db(maxFreqError);
BW = bandwidth(sys_1a, dbdrop)
[Gm, Pm] = margin(L_1a)                % Computed with Open Loop

if (SSerr < maxSSerr) & (Pm > minPM) & isstable(sys_1a)
    fprintf("The system satisfies the requirements\n")
else
    fprintf("The system failed to satisfy the requirements\n")
end
```

```
C_1a =

  s + 1
  -----
    s

Continuous-time transfer function.


L_1a =

             s + 1
  -------------------------
  s^4 + 10 s^3 + 3 s^2 + 10 s

Continuous-time transfer function.


sys_1a =

               s + 1
  -------------------------------
  s^4 + 10 s^3 + 3 s^2 + 11 s + 1

Continuous-time transfer function.


SSerr =

     0


BW =

    1.2651


Gm =
```

```
    2.1699


Pm =

   94.0404

The system satisfies the requirements
```

1.b Drug Administration

Again, want L(0) > 100 (+40dB). P_1b(0) = 15 (+23.5dB), so need to add about +20dB (10). The resulting phase margin is 90.8 degrees, so the controller is fine

```
C_1b = 10
L_1b = P_1b*C_1b
sys_1b = feedback(L_1b, 1)

% Check that design goals were met
SSerr = 1 - dcgain(sys_1b)
dbdrop = mag2db(maxFreqError);
BW = bandwidth(sys_1b, dbdrop)
[Gm, Pm] = margin(L_1b)                % Computed with Open Loop

if (SSerr < maxSSerr) & (Pm > minPM) & isstable(sys_1b)
    fprintf("The system satisfies the requirements\n")
else
    fprintf("The system failed to satisfy the requirements\n")
end
```

```
C_1b =

    10


L_1b =

      15 s + 7.5
  ------------------
  s^2 + 0.7 s + 0.05

Continuous-time transfer function.


sys_1b =

      15 s + 7.5
  -------------------
  s^2 + 15.7 s + 7.55

Continuous-time transfer function.


SSerr =

    0.0066
```

```
BW =

   58.4553


Gm =

   Inf


Pm =

   90.7636

The system satisfies the requirements
```

Problem 2

```
clear; clc;

% Given Constants
g = 9.8;
l = 0.05;
m = 1.5;
J = 0.0475;
c = 0.05;
r = 0.25;

% The given plant transfer function
P_2 = tf(r, [J, c, m*g*l])

% Design Goals
maxSSerr = 0.02        % SS error < 2%
BWerr = 0.1    % 10% tracking bandwidth from 0 to 1 rad/s
minBW = 1
minPM = 40             % Phase Margin > 40 degrees

% Use controlSystemDesigner(P2)

%Want SSerr = 1/(1 + L(0)) < 0.02, so L(0) > 50 (+34dB)
% The open loop P(0) gain is 0.3401 (-9.4dB), so need to add >44 dB of gain
% Try +50dB = 316.28

% Then to acheive BW tracking, need to have T = L(s)/(1 + L(s)) < 0.1 up to
% s = 1.  Assuming large L(s), this gives L(s) > 10 (+20dB).  This is
% satisfied.

% To improve the phase margin, need to add a zero at s = 1.  This gives a
% 90 degree phase margin.

C_2 = 316.28 * tf([1,1],1)
L_2 = P_2*C_2
sys_2 = feedback(L_2, 1)

% Check
SSerr = 1 - dcgain(sys_2)
dbdrop = mag2db(BWerr);
BW = bandwidth(sys_2, dbdrop)
[Gm, Pm] = margin(L_2)     % Computed with Open Loop
```

```
if (SSerr < maxSSerr) & (Pm > minPM) & (BW > minBW)
    fprintf("The system satisfies the requirements\n")
else
    fprintf("The system failed to satisfy the requirements\n")
end
```

```
P_2 =

             0.25
  ---------------------------
  0.0475 s^2 + 0.05 s + 0.735

Continuous-time transfer function.


maxSSerr =

    0.0200


BWerr =

    0.1000


minBW =

     1


minPM =

    40


C_2 =

  316.3 s + 316.3

Continuous-time transfer function.


L_2 =

        79.07 s + 79.07
  ---------------------------
  0.0475 s^2 + 0.05 s + 0.735

Continuous-time transfer function.


sys_2 =

        79.07 s + 79.07
  ---------------------------
  0.0475 s^2 + 79.12 s + 79.8

Continuous-time transfer function.
```

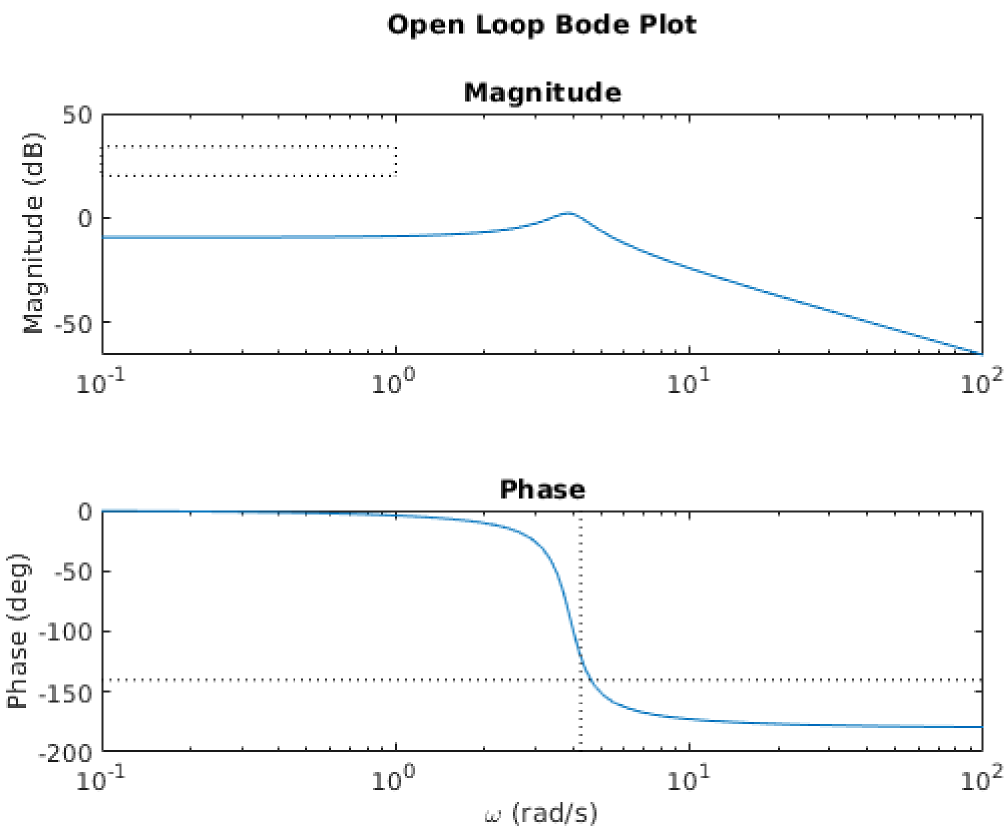
```
SSerr =

    0.0092


BW =

   1.6718e+04


Gm =

   Inf


Pm =

   90.0018

The system satisfies the requirements
```

2.a Open Loop Bode Plot

```
% get the bode plot values
[mag, phase, wout] = bode(P_2);
mag = reshape(mag, [length(mag), 1]);
phase = reshape(phase, [length(phase), 1]);



% plot
figure(1); clf;
subplot(2,1,1)
semilogx(wout, mag2db(mag))
title({'Open Loop Bode Plot', '', 'Magnitude'})
ylabel('Magnitude (dB)')

subplot(2,1,2)
semilogx(wout, phase)
title("Phase")
xlabel('\omega (rad/s)')
ylabel('Phase (deg)')

% given the desired tracing error, compute the necessary gain in the BW
% T = L(s)/(1 + L(s)) <= BWerr, so L(s) > 1/BWerr in the BW
BWgain = 1/BWerr

% given the desired steady state error, compute necessary gain at zero
% frequency.  S = 1/(1 + L(s)) <= SSerr, so L(s) > 1/SSerr
SSgain = 1/maxSSerr

subplot(2,1,1)
Xlim = xlim;
x = Xlim(1);
y = mag2db(BWgain);
w = minBW - x;
h = mag2db(SSgain) - y;
rectangle('Position', [x,y,w,h], 'LineStyle', ':')
```

```
% Show phase margin line
[Gm,Pm,Wcg,Wcp] = margin(P_2);
phaseAtMargin = -180 + minPM;
subplot(2,1,2)
Ylim = ylim;
hold on;
plot([Xlim(1), Xlim(2)], [phaseAtMargin, phaseAtMargin], ':k')
plot([Wcp, Wcp], [Ylim(1), Ylim(2)], ':k')
hold off;
```

```
BWgain =

    10


SSgain =

    50
```

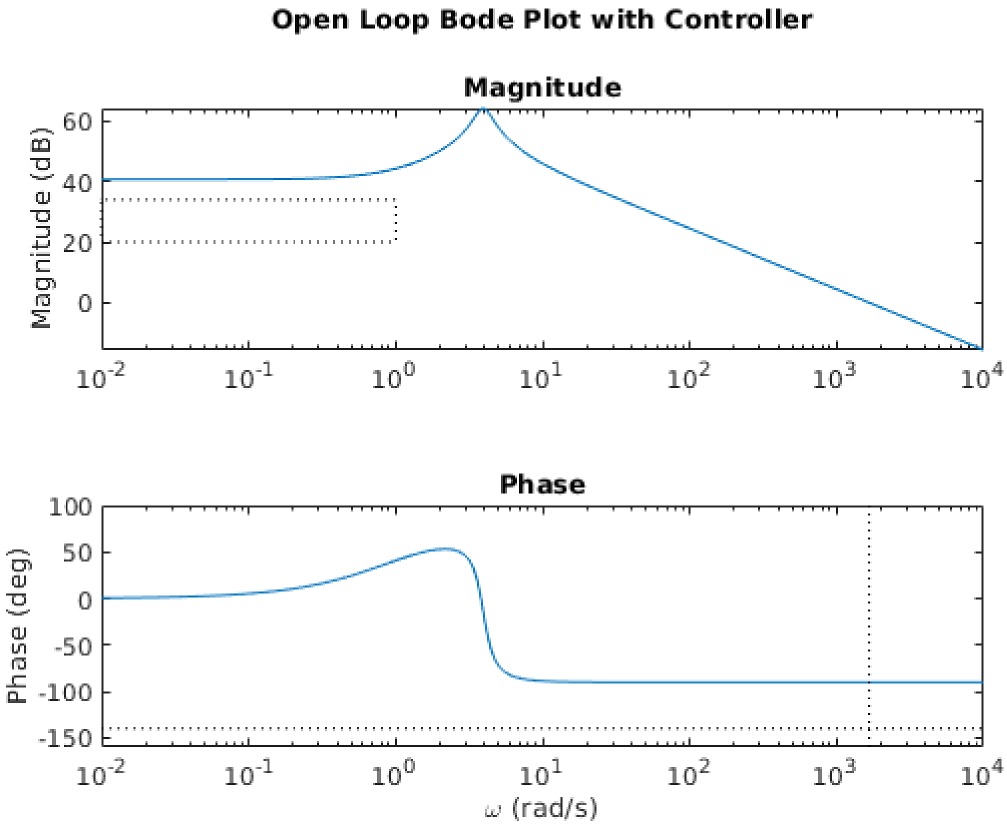


2.b Show that assumptions are met

```
% get the bode plot values
[mag, phase, wout] = bode(L_2, {10^-2, 10^4});
mag = reshape(mag, [length(mag), 1]);
phase = reshape(phase, [length(phase), 1]);


% plot
```

```
figure(2); clf;
subplot(2,1,1)
semilogx(wout, mag2db(mag))
title({'Open Loop Bode Plot with Controller', '', 'Magnitude'})
ylabel('Magnitude (dB)')

subplot(2,1,2)
semilogx(wout, phase)
title("Phase")
xlabel('\omega (rad/s)')
ylabel('Phase (deg)')

% given the desired tracing error, compute the necessary gain in the BW
% T = L(s)/(1 + L(s)) <= BWerr, so L(s) > 1/BWerr in the BW
BWgain = 1/BWerr

% given the desired steady state error, compute necessary gain at zero
% frequency.  S = 1/(1 + L(s)) <= SSerr, so L(s) > 1/SSerr
SSgain = 1/maxSSerr

subplot(2,1,1)
Xlim = xlim;
x = Xlim(1);
y = mag2db(BWgain);
w = minBW - x;
h = mag2db(SSgain) - y;
rectangle('Position', [x,y,w,h], 'LineStyle', ':')

% Show phase margin line
[Gm,Pm,Wcg,Wcp] = margin(L_2);
phaseAtMargin = -180 + minPM;
subplot(2,1,2)
hold on;
plot([Xlim(1), Xlim(2)], [phaseAtMargin, phaseAtMargin], ':k')
Ylim = ylim;
Ylim = [Ylim(1) - 10, Ylim(2)];
ylim(Ylim);
plot([Wcp, Wcp], [Ylim(1), Ylim(2)], ':k')
hold off;
```

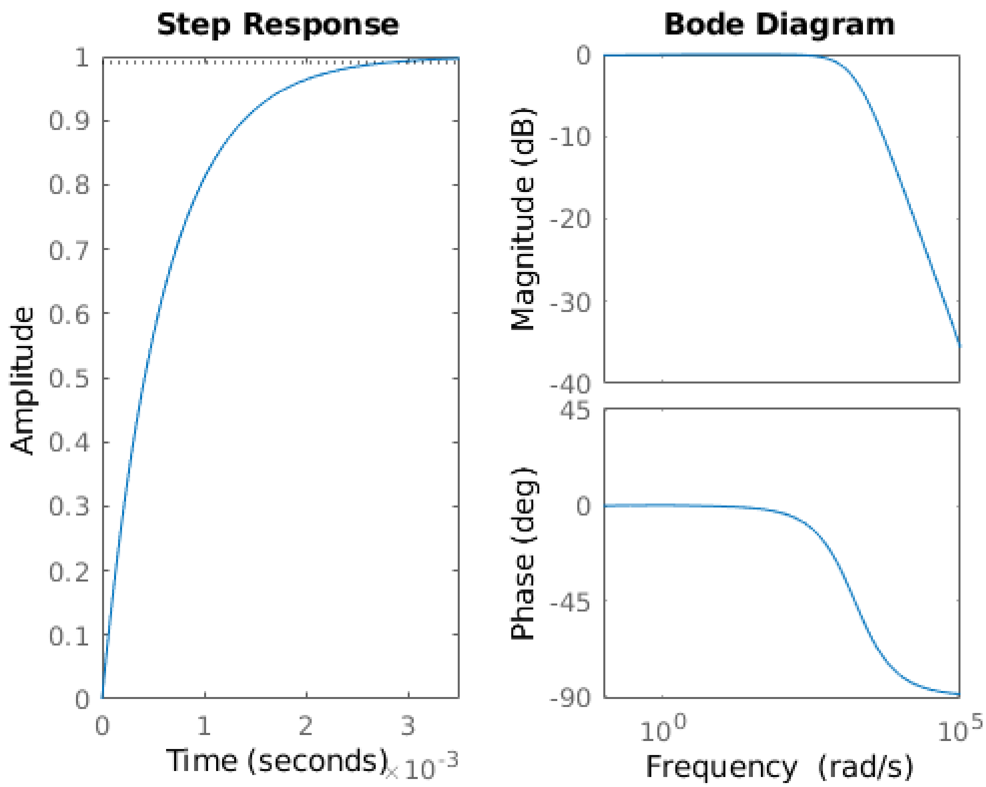
```
BWgain =

    10


SSgain =

    50
```



2.3 Step and Frequency Response of Closed Loop

```
figure(3);clf;
subplot(1,2,1);
step(sys_2)
subplot(1,2,2);
bodeplot(sys_2)
stepinfo(sys_2)
SSerr
```

```
ans =

  struct with fields:

        RiseTime: 0.0013
    SettlingTime: 0.0021
     SettlingMin: 0.8953
     SettlingMax: 0.9989
       Overshoot: 0.8180
      Undershoot: 0
            Peak: 0.9989
        PeakTime: 0.0041


SSerr =

    0.0092
```



Problem 3

```
clear; clc;

% Given constants
k = 4000;
r = 25;

P_3 = tf(k, [1, 0, -r^2])

% The system has two poles, so to stabalize it, add a zero.  Adding a zero
% at frequency = 1 stabilizes it.

C_3 = tf([1, 1], 1)
L_3 = P_3*C_3
sys_3 = feedback(L_3, 1)

% Check
if isstable(sys_3)
    fprintf("The system was stabilized\n")
else
    fprintf("The system was not stabilized\n")
end
```

```
P_3 =

    4000
  ---------
  s^2 - 625

Continuous-time transfer function.
```

```
C_3 =

  s + 1

Continuous-time transfer function.


L_3 =

  4000 s + 4000
  -------------
    s^2 - 625

Continuous-time transfer function.


sys_3 =

     4000 s + 4000
  -------------------
  s^2 + 4000 s + 3375

Continuous-time transfer function.

The system was stabilized
```

3.a Compute Poles and Zeros

```
% Open Loop
fprintf("Open Loop zeros, poles, and gain\n")
[L_3_n, L_3_d] = tfdata(L_3, 'v');
[z, p, k] = tf2zpk(L_3_n, L_3_d)

% Closed Loop
fprintf("Closed Loop zeros, poles, and gain\n")
[sys_3_n, sys_3_d] = tfdata(sys_3, 'v');
[z, p, k] = tf2zpk(sys_3_n, sys_3_d)
```

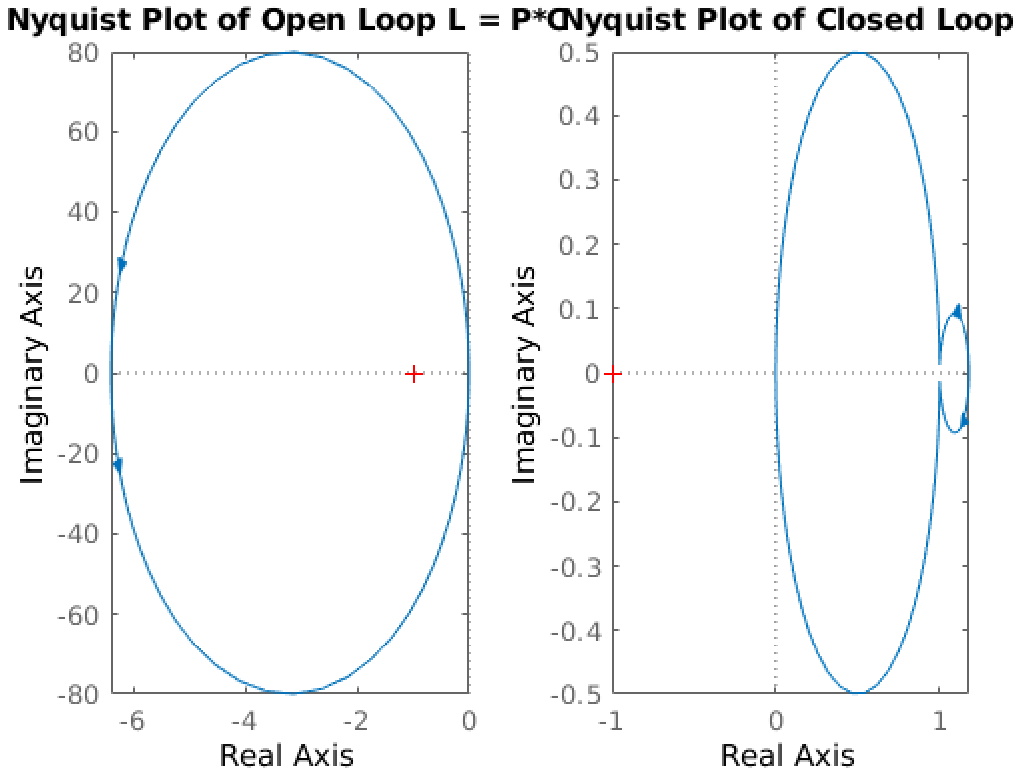
```
Open Loop zeros, poles, and gain

z =

    -1


p =

   25.0000
  -25.0000


k =

        4000

Closed Loop zeros, poles, and gain
```

```
z =

    -1


p =

   1.0e+03 *

  -3.9992
  -0.0008


k =

        4000
```

3.b Nyquist Plot

The open loop system actually does work, as the encirclement is CCW, so N still equals 0. The closed loop shows that the system is in fact stable.

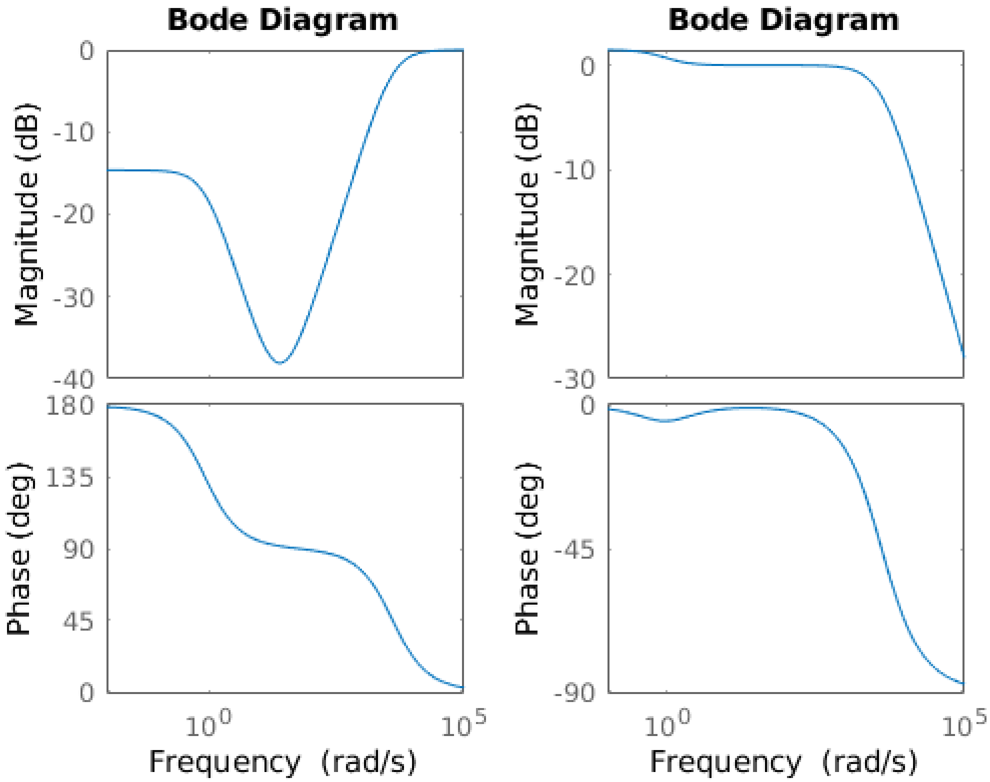
```
figure(4); clf;
subplot(1,2,1)
nyquist(L_3)
title("Nyquist Plot of Open Loop L = P*C")

subplot(1,2,2)
nyquist(sys_3)
title("Nyquist Plot of Closed Loop")
```



3.c Bode Plot

```
S = 1/(1 + L_3)
T = L_3/(1 + L_3)

figure(5); clf;
subplot(1,2,1)
bode(S)

subplot(1,2,2)
bode(T)
```

```
S =

       s^2 - 625
  -------------------
  s^2 + 4000 s + 3375

Continuous-time transfer function.


T =

      4000 s^3 + 4000 s^2 - 2.5e06 s - 2.5e06
  -----------------------------------------------
  s^4 + 4000 s^3 + 2750 s^2 - 2.5e06 s - 2.109e06

Continuous-time transfer function.
```



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