ES 155: Systems and Control

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Homework 5, Due on October 31th 2018, In class.

Note: In the upper left hand corner of the first page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. (*Observer for nutrient model*) Continue with Exercise 1 in Homework 4. We will study the observability of the problem and build an observer.

   (a) Compute the observability matrix for the system and determine whether the system is observable.

   (b) Consider an observer for the system in the form $\frac{d\hat{x}}{dt} = A\hat{x} + Bu + L(y - C\hat{x})$. Here $L = [\ell_1 \quad \ell_2]^T$ is called as an estimator gain matrix. Define $e(t) := x(t) - \hat{x}(t)$. Please write down the dynamics of $e(t)$ in the form of $\frac{de}{dt} = A_e e$. In other words, you need to write down A_e using A, B, C, L (Note: you might not need all of them.) Then compute L such that the eigenvalues of A_e are the two roots of equation $\lambda^2 + 2\zeta_e\omega_e\lambda + \omega_e^2 = 0$, where ζ_e, ω_e are considered given.

   (c) Now we combine the controller you obtained in Homework 4 and the estimator in part (b). Let $u = -K\hat{x} + k_r r$, where $\hat{x}$ is the estimated states in part (e), K and k_r are what you obtained in part (c), and r is a reference signal. Note here $\hat{x}$ follows the dynamics given in part (e), with L being whatever you calculated in part (e). Let $e = x - \hat{x}$ be the estimation error. Stack x and e into one 4-dimensional vector $\tilde{x} = [x_1, x_2, e_1, e_2]^T$. Let the state be $\tilde{x}$, the input be $\tilde{u} = r$, and the output be $\tilde{y} = x_1$, derive a state space model for the closed loop system that takes into account the controller and the estimator. That is to say, find matrices $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ in the following state space model

   $$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}\tilde{u}$$

   $$\tilde{y} = \tilde{C}\tilde{x}$$

   (Note: you can represent $\tilde{A}$, $\tilde{B}$ and $\tilde{C}$ just using k_1, k_2, k_r, l_1, l_2)
   Find the eigenvalues of $\tilde{A}$. Assume $\tilde{u} = 0$ and $\omega_0 > 0, \omega_e > 0$, $0 < \zeta_0$, $0 < \zeta_e$, determine the stability of the above system.

2. (*Observer for Whipple bicycle model*) Consider the Whipple bicycle model given in Exercise 2 of Homework 4.

   (a) Let $y = \phi$. Is the system observable from this measurement output? Why?

   (b) If the system is observable, design an observer for the system with eigenvalues at -4, -20, and at $-2 \pm 2i$.

   (c) Design an output feedback for the system using your observer from part (b) above and the first set of state feedback gains designed in Exercise 2 of Homework 4 (case (i) in Exercise 2 of Homework 4), but with $u = -K\hat{x} + k_r r$ rather than $u = -Kx + K_r r$. Again simulate the response to a step change in the reference value r for the steering angle of 0.002 rad and plot both the steering angle δ and the torque input T(assuming $x(0) = 0$). Compare with the results obtained with full state feedback in Exercise 2 of Pset 4. For the initial estimate value, plot the results with a perfect initial estimate $\hat{x}(0) = x(0)$, and also with a non-zero error in the estimated value of δ at time zero, e.g., $\tilde{\delta}_0 = \hat{\delta}_0 - \delta(0) = 0.0002$.