ES 155: Systems and Control

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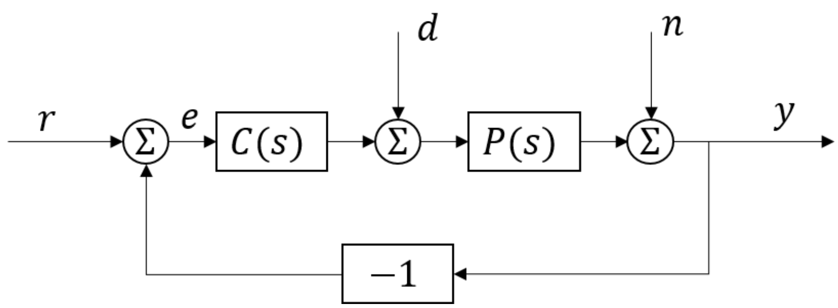
Homework 7, Due on November 20th 2018, 5pm, A box outside of Lina's office, MD 345

Note: In the upper left hand corner of the first page of your homework set, please put the number of hours that you spent on this homework set (including reading).

1. Consider the following block diagram



   In the block diagram, $C(s)$ is the controller transfer function, $P(s)$ is the plant transfer function. There are three inputs to the system, the reference signal r, the external disturbance d and the measurement noise n. We are interested in two outputs, e the control error, and y the measurement. Compute the following transfer functions: $G_{r\to e}$, $G_{d\to e}$, $G_{n\to e}$, $G_{r\to y}$, $G_{d\to y}$, $G_{n\to y}$. (Hint: for example, when computing $G_{d\to y}$, you can assume $r = n = 0$ and calculate $Y(s)/D(s)$.)

2. For each of the following systems, sketch the frequency response (magnitude and phase) by hand (you may use Matlab first with specific parameter values if you wish). Make sure that the gain at zero frequency is marked, the slope and key frequencies are indicated, but don't worry about the details. All parameters (a, b, ω, ζ) below are positive.

   (a) $G_1(s) = \dfrac{1}{(s+a)(s+10a)}$.

   (b) $G_2(s) = \dfrac{1+s/a}{s+10a}$.

   (c) $G_3(s) = \dfrac{s+a}{s+2a}$.

   (d) $G_4(s) = \dfrac{1}{s^2+2\zeta\omega s+\omega^2}$.

3. Plot the (open loop) Nyquist and Bode plots for the following systems and compute the gain and phase margin of each. You should annotate your plots to show the gain and phase margin computations. For the Nyquist plot, mark the branches corresponding to the following sections of the Nyquist "D" contour: negative imaginary axis, positive imaginary axis, semicircle at infinity (the curved part of the "D").

   (a) Disk drive read head positioning system, using a lead compensator(described in Chapter 11):

$$P(s) = \frac{1}{s^3+10s^2+3s+10}, \quad C(s) = 1000\frac{s+1}{s+10}$$

   (b) Second-order system with PD compensator:

$$P(s) = \frac{100}{(100s+1)(s+1)}, \quad C(s) = s+10$$

Note: you may find it easier to sketch the Nyquist plot from the Bode plot (taking some liberties with the scale) rather than relying on MATLAB. But you're welcome to use Matlab to help you answer this question.

4. In this problem we will design a PI controller for a cruise control system (the system is the same as the one in problem 2 of homework 6). Use the following transfer function to represent the vehicle and engine dynamics:

$$P(s) = \frac{Tba/m}{(s+a)(s+c/m)}$$

where $b = 25$ is the transmission gain, $T = 200$ is the conversion factor between the throttle input and steady state torque, $a = 0.2$ is the engine lag coefficient, $m = 1000kg$ is the mass of the car, and $c = 50$N s/m is the viscous damping coefficient. Throughout the problem, we assume there is no disturbance and measurement noise. We only consider the reference tracking of the closed-loop system.

(a) Consider a proportional+integral controller for the car,

$$C(s) = k_p + \frac{k_i}{s}$$

Fill in the following table (make sure to show your work), where g_m is the gain margin, φ_m the phase margin,[1] SSerr the steady state error for step response, T_r the rise time for step response, and M_p the overshoot for step response.[2] (Note that the steady state error is zero in each stable case, due to the integral term in the control law. The gain and phase margins are not defined if the system is unstable, nor are rise time or overshoot.)

k_p	k_i	Stable?	g_m	φ_m	SSerr	T_r	M_p
0.5	0.1						
0.05	1						
0.05	0.001						
0.005	0.001						

(b) For each entry in the table, plot the pole zero diagram (pzmap in Matlab) for the *closed loop* system and the step response.

(c) Use your results in (a) and (b), discuss how the choice of k_p and k_i affect the closed system stability, steady state error, rising time, overshoot, the gain and phase margins, the closed system poles, (Suggestion: look for relationships between the various quantities you are computing and plotting. This problem should give you some insight into the relationship between some of the quantities.)

[1]Hint 1: For gain margin and phase margin, you can use Matlab command "margin(sys)". But please note that "sys" should the openloop system since Matlab close the system by default for you. For more details, see `https://www.mathworks.com/help/control/ref/margin.html`.

[2]Hint 2: See Fig 6.9 on Section 6.3 of the textbook 2nd edition for the definition of steady state error, rise time, and overshoot. You do not need to be exact. Matlab command "stepinfo(sys)" will report many characteristics of step response. Moreover, you can change the specification of rising time and settling time in Matlab. Please note that "sys" should be the closedloop system. For more details, see `https://www.mathworks.com/help/control/ref/stepinfo.html`