Due Wednesday, May 23, by 4:00pm, to Crowdmark.

All submitted work must be the student's own.

Question 1 (12 marks).

[Learning Goals: Find dead code.]

Suppose we have the following fragment of C code:

```c
int x;
int y;
//Read x and y as input from keyboard.
scanf("%d", &x);
scanf("%d", &y);
if(x < y){
  if((y < 2) && (x > 3)){
    //P1
  } else if ((y < 2) ||  ((x % 3) == (x % 2))){
    //P2
  } else if (((y % 2) == 0) && (y > 0) && (x > 7)){
    //P3
  }
}
else if ((((2*y) % 8) == 0) && ((x % 2 == 0) || (x % 2 == 1))){
   if(x >= y){
     //P4
   }
} else {
  if (y == 12){
    //P5
  } else if ((x*x + y*y) % 4 == 3) {
    //P6
  }
}
```

Determine which of *P1* to *P6* form dead code. If they are dead code, justify why. If they are not dead code, give an example of x and y values that reach this part of the code.

[2] (a) *P1*

[2] (b) *P2*

[2] (c) *P3*

[2] (d) *P4*

[2] (e) *P5*

[2] (f) *P6*

Question 2 (6 marks).

Learning goal: Design a circuit given a description of how it must function.
In the living room of your house, one light is controlled by three switches. Design a circuit to control the light through the switches such that the light changes state whenever exactly one of the switch is flipped. Assume that the light is off when all three switches are off.
In more detail, the light changes state (changes from on to off or vice versa) whenever exactly one switch is flipped (changes from on to off or vice versa).

[2] (a) Let a, b, and c represent the states of the three switches respectively (i.e. a is true/false when the first switch is on/off respectively). Let o represent the state of the light. Complete the given truth table, which shows the value of o for each combination of values of a, b, and c.

Remember that the light is off (false) when all three switches are off (false).

a	b	c	o
T	T	T	
T	T	F	
T	F	T	
T	F	F	
F	T	T	
F	T	F	
F	F	T	
F	F	F	

[2] (b) Write down a propositional formula, which is true if and only if the light is on (true). Briefly justify how you came up with the formula based on the truth table.

[2] (c) Draw the circuit diagram corresponding to the propositional formula. You ARE allowed to use XOR gates to solve this question.

c b a

output

Question 3 (8 marks).

[Learning Goal: Translate English sentences into compound propositions; Determine whether a semantic entailment holds or not.]
In each part of this question, you are given a set of propositions which are **premises** (call the set Σ), and a single proposition which is a **conclusion** (call it φ). In each part, you must

i) Translate each proposition from English into Propositional Logic. Explicitly state the definitions of all of your atomic propositions, as you did on A01.
ii) Determine with proof whether the resulting semantic entailment, $\Sigma \models \varphi$, holds or not.

[4] (a) Premises:
- If I study before my mid-term, and I get eight hours' sleep before my mid-term, then I will pass my mid-term.
- I did not pass my mid-term.

Conclusion:
- I did not study before my mid-term, or I did not get eight hours' sleep before my mid-term.

[4] (b) Premises:
- I will get enrolled in CS 245 only if I will course select CS 245.
- If my wi-fi is not working on course selection day, then I will not course select CS 245.

Conclusion:
- I will get enrolled in CS 245 if and only if my wi-fi is working on course selection day.

Question 4 (12 marks).

[Learning Goals: Determine whether a semantic entailment holds by using truth tables, valuation trees, and/or logical identities.]
Prove or disprove each of the following semantic entailment statements. If a semantic entailment statement does not hold, then give a valuation that demonstrates this. If a semantic entailment statement does hold, then either give a truth table marking the relevant rows, or otherwise explain why when every formula to the left of the $\vDash$ evaluates to `T` under some truth valuation then it follows that the formula to the right also evaluates to `T` under the same truth valuation.

[3] (a) $\{((q \vee r) \to s)\} \vDash (q \to s)$

[3] (b) $\{((\neg r) \to s),\ (q \vee (\neg r)),\ (s \to (\neg p)),\ (\neg(\neg p))\} \vDash q$

[3] (c) $\{((\neg r) \vee (\neg p)),\ ((\neg q) \to r),\ ((\neg s) \to p)\} \vDash (q \vee s)$

[3] (d) $\{((\neg r) \vee (\neg p)),\ ((\neg q) \to r),\ ((\neg s) \to p)\} \vDash ((\neg q) \vee s)$

Question 5 (8 marks).

Learning goal: Prove that a set of connectives is an adequate set for propositional logic by using truth tables and logical identities; Prove that a set of connectives is not an adequate set for propositional logic.

This problem is about **adequate sets of connective symbols**. This definition is in the slides.
Definition 1 *A set, S, of Propositional connectives is called **adequate for Propositional logic** if **every** Propositional connective (of any arity) can be implemented using the connectives from S.*

[4] (a) Let '$\downarrow$' be a connective symbol, with the value of $(p \downarrow q)$ being given in the following table.

p	q	$(p \downarrow q)$
T	T	F
T	F	F
F	T	F
F	F	T

Prove that $\{\downarrow\}$ is an adequate set of connectives for propositional logic.
Hint: You may use the fact that $\{\vee, \neg\}$ is an adequate set for Propositional Logic.

[4] (b) Prove that $\{\rightarrow, \vee\}$ is **not** an adequate set for propositional logic.
Hint: Prove the following Lemma, by Structural Induction on φ.
Lemma 2 *Let φ be any well-formed Propositional Formula, constructed using only the connective $\{\rightarrow, \vee\}$. Let t be a truth valuation that sets every Propositional variable in φ to* T. *Then $\varphi^t =$* T.

Question 6 (6 marks).

This exercise uses the following definition.

A **substitution** S is a function from propositional variables to formulas. We apply a substitution to a formula φ by simultaneously replacing each variable p in φ by the formula $S(p)$. Formally, we define $S(\varphi)$ by induction:

- If φ is a propositional variable p, then $S(\varphi)$ is the formula $S(p)$, or simply p if $S(p)$ is undefined.
- If φ is $(\neg\eta)$, then $S(\varphi)$ is $(\neg S(\eta))$.
- If φ is $(\eta \star \zeta)$ for a binary connective $\star$, then $S(\varphi)$ is the formula $(S(\eta) \star S(\zeta))$.

For example, if $S(p) = (q \wedge r)$ and $S(q) = (p \vee q)$ and $S(r)$ is undefined, then

$$S\big(\,(p \to (q \to r))\,\big) \text{ is } \big((q \wedge r) \to \big((p \vee q) \to r\big)\big) \ .$$

Fix an arbitrary substitution S, and an arbitrary valuation t. Prove that there is a valuation u such that for every formula φ, the value of φ under u is the same as the value of $S(\varphi)$ under t: in symbols, $\varphi^u = S(\varphi)^t$. Use structural induction on φ.

(Hint: your answer must specify how the valuation u depends on S and t. If you cannot see how to do that at first, try simply starting the proof without specifying u. Then ask yourself: what properties of u do you need in order to make the required arguments?)