CNY520501: Pattern Recognition

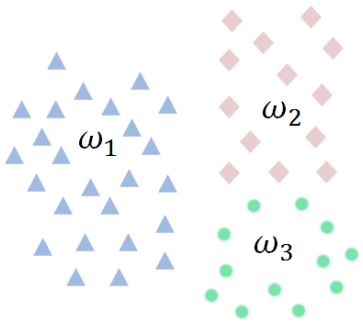
Homework #2

March 25, 2018

1. What is Minimum Distance Classifier? Compute the centroid samples of the following dataset.

$$\omega_1: X_1 = (-1,1)^T, X_2 = (1,-1)^T;\ \omega_2: X_3 = (1,1)^T, X_4 = (1,2)^T$$

2. Given data samples below:

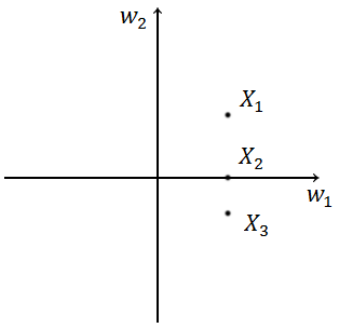


   a) Does each of the categories linear separable with the rest ones? You may demonstrate by illustration.

   b) Does any pair of two categories linear separable? You may demonstrate by illustration.

3. In what condition the linear classifier $G(X) = W^T X$ is able to classify the two classes below? Fill the corresponding area in the W space.

$$\omega_1: X_1 = (1,1)^T, X_2 = (1,0)^T;\ \omega_2: X_3 = \left(1, -\frac{1}{\sqrt{3}}\right)^T$$



4. Given data samples below, design a linear classifier using Perceptron Algorithm. (Initialize weights W with all 0, set ρ to 1)，Write down the procedure of each iteration.

$$\omega_1: X_1 = (1,0)^T;\ \omega_2: X_2 = (-1,0)^T, X_3 = (0,1)^T$$

5. Given data samples below, design linear classifiers using Perceptron Algorithm with no undetermined area in the data space. (Initialize weights *W* with all 0, set ρ to 1)，Write down the procedure of each iteration.

$$\omega_1: X_1 = (1,1)^T;\ \omega_2: X_2 = (-2,1)^T;\ \omega_3: X_3 = (2,-2)^T$$

6. Given a linear discriminant function $g(X) = 4x_1 - 3x_2 + 5$
   a) Write down the function $g(X)$ in the matrix form: $g(x) = W^T X + w_{n+1}$
   b) Convert the function $g(X)$ into *Y* space $f(Y) = W^T Y$ by replacing variables using $Y = (y_1, y_2, y_3)^T = (2x_1, x_2, 1)^T$.

7. The data samples below are linear inseparable

$$\omega_1: X_1 = (1,0)^T, X_2(-1,0)^T;\ \omega_2: X_3 = (0,1)^T, X_4 = (0,-1)^T$$

   a) To make the two classes linear separable, design several functions $y_i = f_i(x_1, x_2), i = 1,2, \ldots$ and convert the data samples into *Y* space.
   b) Write down a linear discriminant function $g(Y) = W^T Y$.

8. When applying the Gradient Descent Algorithm, given the loss function $J(W, X) = \frac{1}{2}(1 - W^T X)^2$ and parameters $W = (2,1)^T$, $X_1 = (2,3)^T$, $X_2 = (1,4)^T$. Determine the values of $\nabla J(W, X_1)$ and $\nabla J(W, X_2)$.

9. Use the Fisher's Linear Discriminant Analysis to classify the two classes below. Write down the projection vector.

$$\omega_1: X_1 = (0,0)^T, \mathrm{X}_2 = (1,2)^T; \omega_2: X_3 = (1,-1)^T, X_4 = (3,0)^T$$

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