OLIN COLLEGE OF ENGINEERING
LINEARITY 2, 2016

STUDIO 1 PROBLEMS
Due Thursday September 8

(1) Practice with partial derivatives
Define

$$\mathbf{F}(x, y, z) := x(3 - x - y - z)\hat{\mathbf{i}} + y(6 - x - 2y - 3z)\hat{\mathbf{j}} + z(8 - 2x - 3y - 3z)\hat{\mathbf{k}}$$

(a) Use the approximation

$$\mathbf{F}(x+\Delta x, y+\Delta y, z+\Delta z) \approx \mathbf{F}(x, y, z) + \frac{\partial \mathbf{F}(x, y, z)}{\partial x}\Delta x + \frac{\partial \mathbf{F}(x, y, z)}{\partial y}\Delta y + \frac{\partial \mathbf{F}(x, y, z)}{\partial z}\Delta z$$

to find approximate values for

i. $\mathbf{F}(1.01, 0.99, 0.98)$
ii. $\mathbf{F}(.99, 1.01, .98)$
iii. $\mathbf{F}(2.02, 2.99, 1.98)$

(b) Find the matrix $A(x, y, z)$ so that the approximation above rewrites in matrix-vector form as

$$\mathbf{F}(x + \Delta x, y + \Delta y, z + \Delta z) \approx \mathbf{F}(x, y, z) + A(x, y, z) \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \end{bmatrix}$$

The matrix $A(x, y, z)$ is called the **derivative** of $\mathbf{F}$ and written $D\mathbf{F}(x, y, z)$.

I-3 p9 (modified)

(a) Write a formula for a vector function in two dimensions which is in the positive radial direction and whose magnitude is one.

(b) Write a formula for a vector function in two dimensions whose direction makes an angle of 45° with the x-axis and whose magnitude at any point (x, y) is $(x + y)^2$.

(c) Write a formula for a vector field in two dimensions whose direction at the point (x, y) is tangential to the circle centered at the origin through (x, y) and whose magnitude is equal to its distance from the origin.

(d) Write a function for a vector function in three dimensions which is in the positive radial direction and whose magnitude is one.

(e) Write a formula for a vector field in two dimensions whose direction at the point (x, y) is tangential to the ellipse centered at the origin and going through the points $((x^2 + 4y^2)^{\frac{1}{2}}, 0)$ and $(0, \frac{1}{2}(x^2 + 4y^2)^{\frac{1}{2}})$.

(f)* This problem is an optional exploration this week. It will be required on next week's homework. Think about how you might solve it. If you are ambitious, solve it.
Write a formula for a vector field in three dimensions whose direction at the point (x, y) is normal to the ellipsoid centered at the origin and going through the points $((x^2 + 4y^2 + 9z^2)^{\frac{1}{2}}, 0, 0)$, $(0, \frac{1}{2}(x^2 + 4y^2 + 9z^2)^{\frac{1}{2}}, 0)$ and $(0, 0, \frac{1}{9}(x^2 + 4y^2 + 9z^2)^{\frac{1}{2}})$.

I-4 p9 (modified) An object moves in the xy-plane in such a way that its position vector $\mathbf{r}$ is given by

$$\mathbf{r}(t) = a\cos(\omega t)\hat{\mathbf{i}} + b\sin(\omega t)\hat{\mathbf{j}}$$

(a) How far is the object from the origin at any time t?

(b) Find the velocity and acceleration as functions of time.

(c) Show that the object moves on the elliptical path

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1.$$

* (This problem is an optional exploration. It places the idea of the the derivative of a vector field in the context of differential equations. We will return to this idea later in the course) Recall the function $\mathbf{F}$ defined above

$$\mathbf{F}(x, y, z) := x(3 - x - y - z)\hat{\mathbf{i}} + y(6 - x - 2y - 3z)\hat{\mathbf{j}} + z(8 - 2x - 3y - 3z)\hat{\mathbf{k}}$$

The differential equation $\dot{\mathbf{p}} = \mathbf{F}(\mathbf{p})$ models three biological species competing for the same ecological niche. Here we have defined $\mathbf{p} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ with x, y and z denoting the populations of the three species.

(a) What does it mean in modeling terms that $\mathbf{F}(1, 1, 1) = \mathbf{0}$.

(b) What do the eigenvalues (and eigenvectors if you'd like to be more detailed) of the matrix $A(1, 1, 1)$ tell you about the model?

(c) The eigenvalues of $A(1, 1, 1)$ give us much more information about the model than the eigenvalues of $A(2, 3, 2)$. Why?