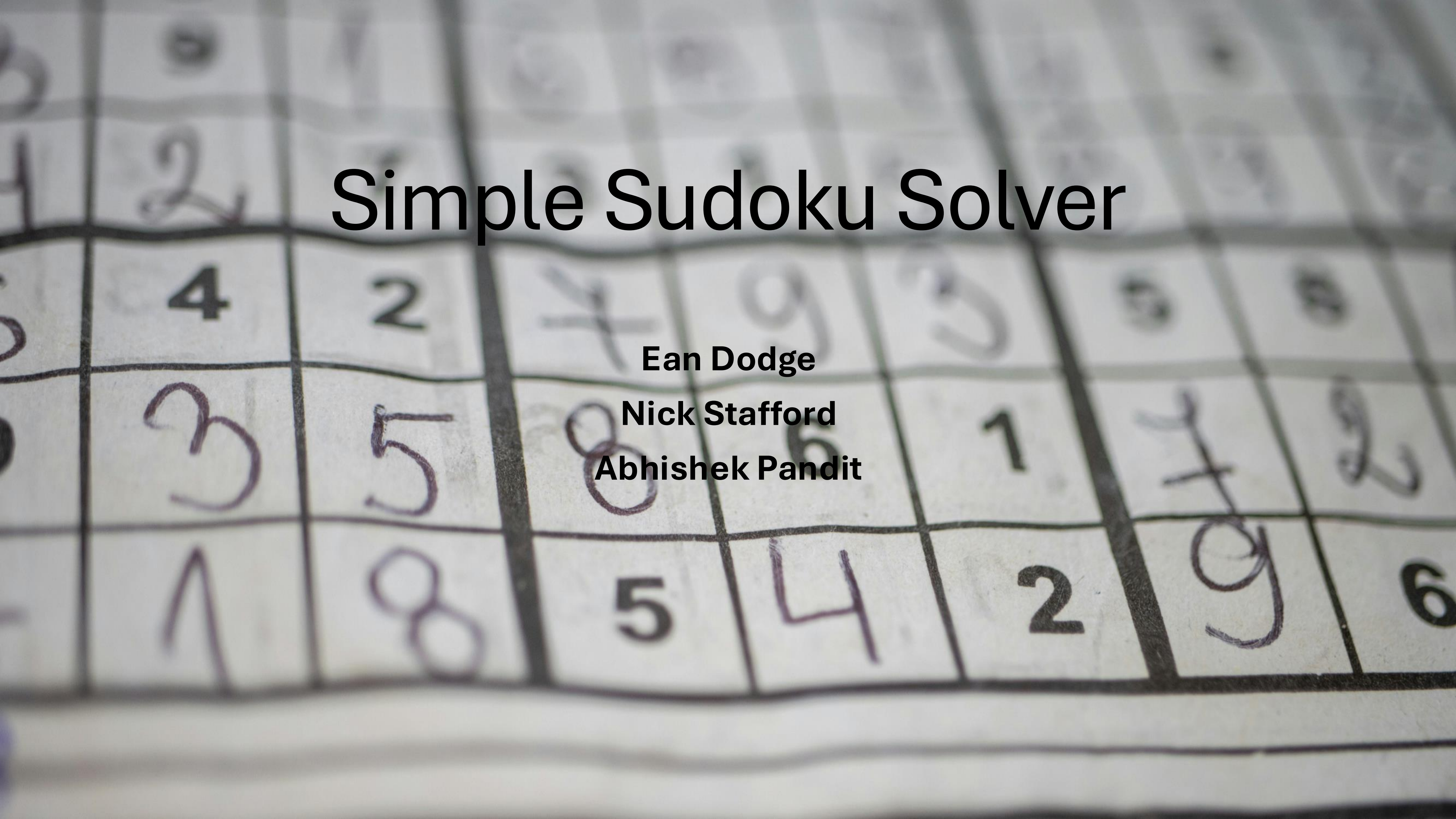

Simple Sudoku Solver

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Creating the Sudoku Puzzle

- Can accept a custom grid from the user, or automatically generate one
- Partially completed solutions auto generated with py-sudoku

```
def get_puzzle():
    if (
        input(
            "Do you want to enter custom sudoku board or automatically generated board?\n (press 'c' for custom): "
        )
        == "c"
    ):
        custom_grid = _get_custom_puzzle()
        return Sudoku(3, board=custom_grid)
    else:
        # Auto generate a solvable soduku
        difficulty = _get_difficulty()
        return Sudoku(3).difficulty(difficulty)
```

Creating the Sudoku Puzzle

This is the puzzle that is created when automatically generated with a difficulty of .6

```
Do you want to enter custom sudoku board or automatically generated board?
 (press 'c' for custom):
What difficulty would you like the puzzle to be?(0-1): .6
Puzzle has multiple solutions
+-------+-------+-------+
|   1   |   7   | 2     |
| 4     |   1   | 3   7 |
|     7 |       |   5 6 |
+-------+-------+-------+
|   2 1 |       |   7   |
| 7 5 4 |       |     3 |
| 8     |       | 4 6   |
+-------+-------+-------+
| 3 4   |   5 7 | 6   8 |
|       |   4   |       |
| 9 8 5 | 2   1 |     4 |
+-------+-------+-------+
```

Naive solution with backtracking

- Brute force
- Going through every cell to put a value and make sure its valid.
- Recursively calls itself for backtracking.
- Recursive ends if there are no more 0's

```python
def brute_force(board: list[list[int]]) -> bool:
    """
    A brute-force backtracking solution to solve Sudoku.
    """
    for row in range(9):
        for col in range(9):
            if board[row][col] == 0:
                # Try every number from 1 to 9
                for num in range(1, 10):
                    if solver.is_valid(board, row, col, num):
                        board[row][col] = num
                        # print(f"Placing {num} at ({row},{col})...")
                        if brute_force(board):
                            return True
                        board[row][col] = 0
                return False
    return True
```

Heuristics: most constrained

- Finding the most constrained variables
- Solving cells that are most constrained
- Tied cells returned

```
What difficulty would you like the puzzle to be?(0-1): 0.3
Puzzle has exactly one solution
+-------+-------+-------+
| 5 6 7 |   8 1 | 4 2   |
| 8 1   | 2   4 |   3   |
| 2 4 3 | 5     | 6 1   |
+-------+-------+-------+
|       | 8 4 7 | 1 9 6 |
|     4 | 6 1 3 |     2 |
| 1 8 6 | 9 2   | 3 4   |
+-------+-------+-------+
| 4 5 2 | 1 9 6 |   7 3 |
| 9   1 | 7   8 | 2     |
| 6 7 8 |   3 2 | 9 5 1 |
+-------+-------+-------+

this code is working
[(0, 3), (0, 8), (1, 2), (2, 4), (2, 5), (3, 0), (3, 2), (4, 0), (4, 1), (4, 7), (5, 5), (6, 6), (7,
 1), (7, 4), (7, 7), (7, 8), (8, 3)] 1
```

```python
def most_constrained_variables(board: list[list[int]]) -> list[tuple[int, int]]:
    tied_cells: list[tuple[int, int]] = []
    min_valid_values = 10  # Start with a value larger than the max (9)

    for r in range(9):
        for c in range(9):
            if board[r][c] == 0:
                valid_values = [
                    num for num in range(1, 10) if is_valid(board, r, c, num)
                ]
                if len(valid_values) == min_valid_values:
                    # ties are allowed, if two cells are equally constrained
                    tied_cells.append((r, c))
                elif len(valid_values) < min_valid_values:
                    min_valid_values = len(valid_values)
                    # remove old cells, since this cell is more constrined.
                    tied_cells = [(r, c)]

    return tied_cells
```

Heuristics: most constraining

- Find the most constraining variables
- Return the most constraining cell

```python
def most_constraining_variable(
    board: list[list[int]], tied_cells: list[tuple[int, int]]
) -> tuple[int, int]:
    """From a list of tied cells, find the cell that imposes the most constraints on its
    neighbors. if there is a tie, return random of the maxs"""
    max_constraints = -1
    most_constraining_cell = tied_cells[0]
    for r, c in tied_cells:
        constraints = 0

        # counting unassigned cells in the same row and column
        for i in range(9):
            if board[r][i] == 0:
                constraints += 1
            if board[i][c] == 0:
                constraints += 1

        # counting unassigned cells in the subgrid
        start_row, start_col = 3 * (r // 3), 3 * (c // 3)
        for i in range(start_row, start_row + 3):
            for j in range(start_col, start_col + 3):
                if board[i][j] == 0:
                    constraints += 1

        # updating the most constraining variable
        if constraints > max_constraints:
            max_constraints = constraints
            most_constraining_cell = (r, c)

    return most_constraining_cell
```

Heuristics: least constraining

- Find the least constraining variable

```
def least_constraining_values(board: list[list[int]], row: int, col: int) -> list[int]:
    """Get the possible values of a cell, ordered by their least constraining effect"""
    candidates: list[tuple[int, int]] = []
    for num in range(1, 10):
        if is_valid(board, row, col, num):
            # counting how many other cells this value would restrict
            constraint_count = 0
            for r in range(9):
                if board[r][col] == 0 and is_valid(board, r, col, num):
                    constraint_count += 1
            for c in range(9):
                if board[row][c] == 0 and is_valid(board, row, c, num):
                    constraint_count += 1
            candidates.append((num, constraint_count))

    # Sort by the number of constraints (ascending)
    candidates.sort(key=lambda x: x[1])
    return [x[0] for x in candidates]
```

Testing

- Ensure updates don't break code.
- Parameters used to run different tests.

```
(aigroupproject-py3.12) →  aigroupproject git:(master) × python tests.py -h
usage: tests.py [-h] [-lt] [-bf] [-v] [-p] [epochs]

Run multiple Sudoku tests.

positional arguments:
  epochs               Number of test boards to run (default is 1000)

options:
  -h, --help           show this help message and exit
  -lt, --lookup_table  Run the tests with a lookup table
  -bf, --brute_force   Run the tests with the brute force algorithm
  -v, --verbose        Print extra information to the console.
  -p, --parallel       Run the tests in parallel
(aigroupproject-py3.12) →  aigroupproject git:(master) × python tests.py
```

Testing

- Tests algorithm speed through many tests
- Brute force < CSP < CSP with lookup table

```
(aigroupproject-py3.12) ➜  aigroupproject git:(master) ✗ python tests.py --brute_force
Testing 1000 test batch
Successfully passed 1000 tests.
Heuristic solving time: 13.571531 seconds.
(aigroupproject-py3.12) ➜  aigroupproject git:(master) ✗ python tests.py
Testing 1000 test batch
Successfully passed 1000 tests.
Heuristic solving time: 4.515533 seconds.
(aigroupproject-py3.12) ➜  aigroupproject git:(master) ✗ python tests.py --lookup_table
Testing 1000 test batch
Successfully passed 1000 tests.
Heuristic solving time: 2.770192 seconds.
```

Solving backtracking

- Only make moves if they are legal.
- Check how each state change affects other cells before commiting to new state
- Implemented with either a lookup table, or by checking all affected cells first.

sudoku_solver.py

```python
            # counting how many other cells this value w
            constraint_count = 0
            for r in range(9):
                if board[r][col] == 0 and is_valid(board, r, col, num):
                    constraint_count += 1
            for c in range(9):
                if board[row][c] == 0 and is_valid(board, row, c, num):
                    constraint_count += 1
            candidates.append((num, constraint_count))

    # Sort by the number of constraints (ascending)
    candidates.sort(key=lambda x: x[1])
    return [x[0] for x in candidates]


def solve_heuristics(board: list[list[int]], depth: int = 0) -> bool:
    tab = " " * depth
    print(f"{tab}Depth={depth}")
    # Finding the most constrained variable(s)
    tied_cells: list[tuple[int, int]] = most_constrained_variables(board)
    # print(f"{tab}{len(tied_cells)} most constrained")

    # If the board is solved
    if not tied_cells:
        return True

    # If there's a tie, use Most Constraining Variable to break it
    if len(tied_cells) > 1:
        row, col = most_constraining_variable(board, tied_cells)
    else:
        row, col = tied_cells[0]

    # trying the least constraining values for the selected cell
    for num in least_constraining_values(board, row, col):
        board[row][col] = num
        # print(f"Trying {num} at ({row}, {col})")
        if solve_heuristics(board, depth + 1):
            print(f"{tab}Backtracking Success")
            return True
        print(f"{tab}Failed")
        board[row][col] = 0
        print(f"Backtracking at ({row}, {col})")

    print(f"{tab}Backtracking Fail")
    return False
```

```
Depth=3
Depth=4
Depth=5
Depth=6
Depth=7
Depth=8
Depth=9
Depth=10
Depth=11
Depth=12
Depth=13
Depth=14
Depth=15
Depth=16
Depth=17
Depth=18
Depth=19
Depth=20
Depth=21
Depth=22
Depth=23
Depth=24
Depth=25
Depth=26
Depth=27
Depth=28
Depth=29
Depth=30
Depth=31
Depth=32
Depth=33
Depth=34
Depth=35
Depth=36
Depth=37
Depth=38
Depth=39
Depth=40
Backtracking Success
Backtracking Success
Backtracking Success
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Backtracking Success
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Backtracking Success
Backtracking Success
```
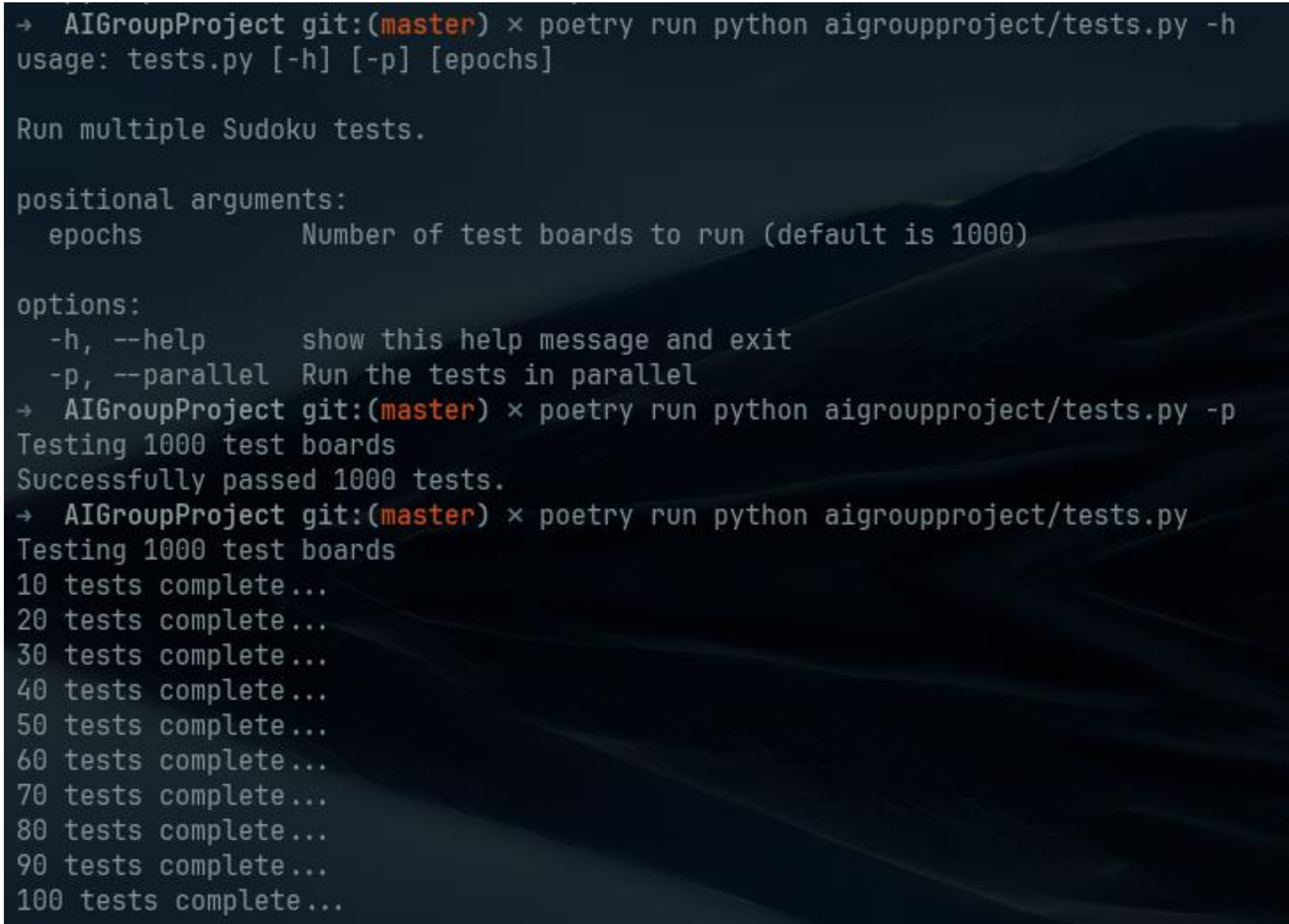

Parallel Programming

- Multithreading performance increases scale with number of cpu cores
- enabled with the "-p" parameter on cli.



Look up table

```
+-----------------------------------+
| 25679      25679      3           |
| 245679     8          1           |
| 245679     245679     245679      |
+-----------------------------------+
| 123456789  12345679   2456789     |
| 123456789  12345679   2456789     |
| 123456789  12345679   2456789     |
+-----------------------------------+
| 123456789  12345679   2456789     |
| 123456789  12345679   2456789     |
| 123456789  12345679   2456789     |
+-----------------------------------+
```

```
+--------------------------------+--------------------------------+--------------------------------+
| 25679     25679     3          | 256789    4         256789     | 256789    1         256789     |
| 245679    8         1          | 235679    235679    235679     | 2345679   2345679   2345679    |
| 245679    245679    245679     | 12356789  12356789  12356789   | 23456789  23456789  23456789   |
+--------------------------------+--------------------------------+--------------------------------+
| 123456789 12345679  2456789    | 123456789 12356789  123456789  | 123456789 23456789  123456789  |
| 123456789 12345679  2456789    | 123456789 12356789  123456789  | 123456789 23456789  123456789  |
| 123456789 12345679  2456789    | 123456789 12356789  123456789  | 123456789 23456789  123456789  |
+--------------------------------+--------------------------------+--------------------------------+
| 123456789 12345679  2456789    | 123456789 12356789  123456789  | 123456789 23456789  123456789  |
| 123456789 12345679  2456789    | 123456789 12356789  123456789  | 123456789 23456789  123456789  |
| 123456789 12345679  2456789    | 123456789 12356789  123456789  | 123456789 23456789  123456789  |
+--------------------------------+--------------------------------+--------------------------------+

+--------------------------------+--------------------------------+--------------------------------+
| 25679     25679     3          | 26789     4         26789      | 256789    1         256789     |
| 24679     8         1          | 5         23679     23679      | 234679    234679    234679     |
| 245679    245679    245679     | 1236789   1236789   1236789    | 23456789  23456789  23456789   |
+--------------------------------+--------------------------------+--------------------------------+
| 123456789 12345679  2456789    | 12346789  12356789  123456789  | 123456789 23456789  123456789  |
| 123456789 12345679  2456789    | 12346789  12356789  123456789  | 123456789 23456789  123456789  |
| 123456789 12345679  2456789    | 12346789  12356789  123456789  | 123456789 23456789  123456789  |
+--------------------------------+--------------------------------+--------------------------------+
| 123456789 12345679  2456789    | 12346789  12356789  123456789  | 123456789 23456789  123456789  |
| 123456789 12345679  2456789    | 12346789  12356789  123456789  | 123456789 23456789  123456789  |
| 123456789 12345679  2456789    | 12346789  12356789  123456789  | 123456789 23456789  123456789  |
+--------------------------------+--------------------------------+--------------------------------+

+--------------------------------+--------------------------------+--------------------------------+
| 25679     25679     3          | 2789      4         2789       | 256789    1         256789     |
| 2479      8         1          | 5         2379      6          | 23479     23479     23479      |
| 245679    245679    245679     | 123789    123789    123789     | 23456789  23456789  23456789   |
+--------------------------------+--------------------------------+--------------------------------+
| 123456789 12345679  2456789    | 12346789  12356789  12345789   | 123456789 23456789  123456789  |
| 123456789 12345679  2456789    | 12346789  12356789  12345789   | 123456789 23456789  123456789  |
| 123456789 12345679  2456789    | 12346789  12356789  12345789   | 123456789 23456789  123456789  |
+--------------------------------+--------------------------------+--------------------------------+
| 123456789 12345679  2456789    | 12346789  12356789  12345789   | 123456789 23456789  123456789  |
| 123456789 12345679  2456789    | 12346789  12356789  12345789   | 123456789 23456789  123456789  |
| 123456789 12345679  2456789    | 12346789  12356789  12345789   | 123456789 23456789  123456789  |
+--------------------------------+--------------------------------+--------------------------------+

+--------------------------------+--------------------------------+--------------------------------+
| 25679     25679     3          | 2789      4         2789       | 256789    1         256789     |
| 2479      8         1          | 5         279       6          | 3         2479      2479       |
| 245679    245679    245679     | 123789    123789    123789     | 2456789   2456789   2456789    |
+--------------------------------+--------------------------------+--------------------------------+
| 123456789 12345679  2456789    | 12346789  12356789  12345789   | 12456789  23456789  123456789  |
| 123456789 12345679  2456789    | 12346789  12356789  12345789   | 12456789  23456789  123456789  |
| 123456789 12345679  2456789    | 12346789  12356789  12345789   | 12456789  23456789  123456789  |
+--------------------------------+--------------------------------+--------------------------------+
| 123456789 12345679  2456789    | 12346789  12356789  12345789   | 12456789  23456789  123456789  |
| 123456789 12345679  2456789    | 12346789  12356789  12345789   | 12456789  23456789  123456789  |
| 123456789 12345679  2456789    | 12346789  12356789  12345789   | 12456789  23456789  123456789  |
+--------------------------------+--------------------------------+--------------------------------+
```

Lookup Table

- Same function, custom data structure.
- Use a set in each cell which has possible legal values
- Reference to see what values are possible.
- Update after each cell is placed.

```python
class State:
    table: list[list[set[int]]]
    is_set: set[tuple[int, int]]
    is_not_set: set[tuple[int, int]]
    verbose: bool = False

    def __init__(self, grid: list[list[int]], verbose: bool = False) → None:
        self.table = []
        self.is_set = set()
        self.is_not_set = set()
        self.verbose = verbose
        for r in range(9):
            row: list[set[int]] = []
            for c in range(9):
                row.append(set(range(1, 10)))
                self.is_not_set.add((r, c))
            self.table.append(row)

        self.init_table(grid)

    def is_valid(self, row: int, col: int, num: int) → bool:
        return (row, col) in self.is_not_set and num in self.table[row][col]

    def init_table(self, grid: list[list[int]]):
        for row in range(9):
            for col in range(9):
                cell = grid[row][col]
                if cell ≠ 0:
                    self.constrain(row, col, cell)
```

```python
    def constrain(self, row: int, col: int, val: int, verbose: bool = False):
        for i in range(9):
            self.table[row][i].discard(val)
            self.table[i][col].discard(val)

        start_row, start_col = row - row % 3, col - col % 3
        for i in range(3):
            for j in range(3):
                self.table[i + start_row][j + start_col].discard(val)
        self.table[row][col] = set([val])
        self.is_set.add((row, col))
        self.is_not_set.remove((row, col))
        if verbose or self.verbose:
            print(self.format_as_string(row, col))

    def constrain_trivial_cells(self) → bool:
        did_update = False
        to_update = deepcopy(self.is_not_set)
        for r, c in to_update:
            if len(self.table[r][c]) == 1:
                to_update.add((r, c))
                did_update = True
                val = next(iter(self.table[r][c]))
                self.constrain(r, c, val)
        return did_update
```

Trivial State Changes

- Optional addition for algorithm optimization
- Faster implementation of most constrained.
- If cell has only one possible value, commit that change. Repeat until no more trivial changes exist.

```python
def constrain_trivial_cells(self) → bool:
    did_update = False
    to_update = deepcopy(self.is_not_set)
    for r, c in to_update:
        if len(self.table[r][c]) == 1:
            to_update.add((r, c))
            did_update = True
            val = next(iter(self.table[r][c]))
            self.constrain(r, c, val)
    return did_update

def is_finished(self):
    return all(
        len(self.table[r][c]) ≤ 1
        for r, c in self.is_not_set
    )
```

```python
while state.constrain_trivial_cells():
    if state.is_finished():
        return state
```

NxN solutions

```
(aigroupproject-py3.12) → aigroupproject git:(master) ✗ python nxn_sudoku.py 5
Running Sudoku solver for a 25 grid
Verbose mode: False
+-----------------------+-----------------------+-----------------------+-----------------------+-----------------------+
| 4   16  10  5   13    | 2   9   20  19  23    | 17  14  3   12  7     | 25  11  18  22  24    | 21  6   1   15  8     |
| 22  11  17  3   2     | 1   16  12  21  7     | 8   23  19  10  25    | 6   9   15  14  13    | 18  24  5   20  4     |
| 25  7   14  9   6     | 5   17  8   18  15    | 4   24  13  21  22    | 1   12  10  3   20    | 23  16  11  19  2     |
| 21  24  20  23  15    | 25  3   6   13  10    | 11  18  1   5   2     | 4   16  7   19  8     | 22  17  12  9   14    |
| 8   1   18  19  12    | 22  4   14  24  11    | 9   6   15  16  20    | 21  2   5   23  17    | 13  3   10  25  7     |
+-----------------------+-----------------------+-----------------------+-----------------------+-----------------------+
| 5   3   24  16  10    | 19  7   9   17  20    | 6   2   11  23  18    | 22  14  8   1   25    | 4   12  15  21  13    |
| 12  2   25  22  4     | 10  13  18  23  6     | 15  20  17  8   19    | 11  21  9   16  7     | 1   14  24  5   3     |
| 14  20  6   15  17    | 24  11  1   25  21    | 13  7   10  22  12    | 18  5   3   4   19    | 9   2   23  8   16    |
| 1   23  9   11  18    | 16  12  15  8   22    | 14  4   21  3   5     | 24  13  17  2   10    | 20  7   19  6   25    |
| 13  8   19  7   21    | 14  5   3   4   2     | 16  25  24  9   1     | 12  23  6   20  15    | 11  22  17  10  18    |
+-----------------------+-----------------------+-----------------------+-----------------------+-----------------------+
| 17  13  22  12  3     | 7   20  24  16  19    | 5   9   18  14  6     | 15  10  2   21  1     | 25  4   8   11  23    |
| 9   25  15  18  16    | 8   2   21  1   5     | 22  17  23  13  11    | 7   20  4   12  14    | 3   19  6   24  10    |
| 19  5   23  20  1     | 15  14  4   9   13    | 25  16  12  24  10    | 17  6   11  8   3     | 2   18  22  7   21    |
| 7   21  8   10  11    | 3   25  23  6   18    | 20  1   4   2   15    | 9   22  19  24  16    | 12  13  14  17  5     |
| 24  4   2   6   14    | 11  22  17  10  12    | 3   8   7   19  21    | 5   18  25  13  23    | 16  15  20  1   9     |
+-----------------------+-----------------------+-----------------------+-----------------------+-----------------------+
| 15  9   4   24  19    | 13  6   2   22  17    | 10  5   25  20  14    | 8   3   23  18  12    | 7   1   21  16  11    |
| 16  14  13  8   22    | 23  15  11  5   25    | 7   3   2   1   9     | 19  17  24  6   21    | 10  20  4   18  12    |
| 23  10  7   1   25    | 21  19  16  14  4     | 12  22  8   18  17    | 20  15  13  11  5     | 6   9   3   2   24    |
| 3   17  11  2   5     | 20  18  10  12  24    | 21  13  6   15  4     | 16  7   1   9   22    | 14  8   25  23  19    |
| 6   18  12  21  20    | 9   8   7   3   1     | 23  19  16  11  24    | 2   25  14  10  4     | 15  5   13  22  17    |
+-----------------------+-----------------------+-----------------------+-----------------------+-----------------------+
| 2   15  5   13  7     | 18  1   19  11  3     | 24  10  14  25  16    | 23  4   22  17  6     | 8   21  9   12  20    |
| 20  22  1   17  9     | 4   21  13  2   8     | 18  15  5   7   23    | 3   19  12  25  11    | 24  10  16  14  6     |
| 11  19  21  14  24    | 6   10  25  7   16    | 2   12  22  4   8     | 13  1   20  5   9     | 17  23  18  3   15    |
| 10  6   16  4   8     | 12  23  5   15  9     | 1   11  20  17  3     | 14  24  21  7   18    | 19  25  2   13  22    |
| 18  12  3   25  23    | 17  24  22  20  14    | 19  21  9   6   13    | 10  8   16  15  2     | 5   11  7   4   1     |
+-----------------------+-----------------------+-----------------------+-----------------------+-----------------------+

Heuristic solving time: 1.340499 seconds.
Solution is solved and legal.
```