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RE: EE 3220, Lab 5

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Summary

In this lab, we were tasked to design a band-pass filter using op-amps that would match the target Bode plot shown in fig. 1. We did this by estimating the frequency response by means of a straight-line approximation and comparing the approximation to the target using GNU Octave. We then used the estimated frequency response to determine the component values of a three-stage op-amp filter. Once the component values were determined, we compared our designed circuit's Bode plot by simulation and breadboard prototype. Finally, we designed and ordered a printed circuit board (PCB) of our final design.

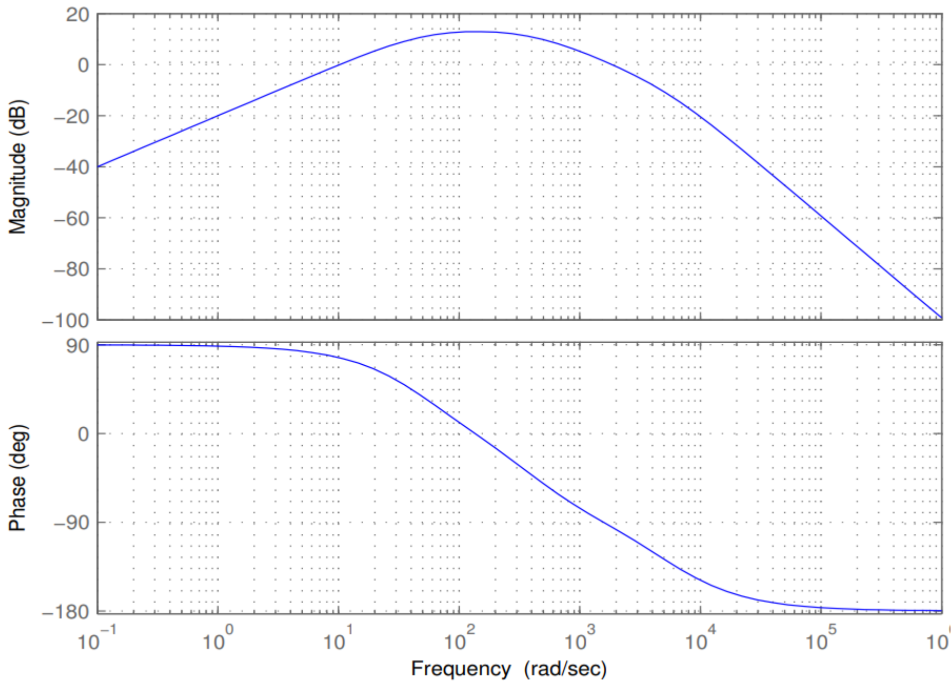

Fig. 1. Target Bode plot.

Design

The first step to designing the band-pass filter was determining the filters transfer function. We used a straight-line approximation to determine the locations of poles and zeros as seen in fig. 2. A $+20\ \frac{dB}{decade}$ slope corresponds to a zero. In our case, there was one zero that occurs at $\omega = 0$ since as the frequency goes to zero, the gain also goes to zero. A $-20\ \frac{dB}{decade}$slope corresponds to a pole. There were three poles that occur at $\omega_1 = 40\ \frac{rad}{s}$, $\omega_2 = 440\ \frac{rad}{s}$, and $\omega_3 = 5500\ \frac{rad}{s}$ since at each of these frequencies the slope decreases by an additional $20\ \frac{dB}{decade}$. Combining these poles and zeros, we generated the following transfer function:

$$H(s) = \frac{Gs}{(s+40)(s+440)(s+5500)}$$

where G is some constant used to control gain. G was found by first giving the transfer function unity gain:

$$H(s) = \frac{s \cdot 40 \cdot 440 \cdot 5500}{(s+40)(s+440)(s+5500)}$$

Then, we knew by looking at the target Bode plot, that the maximum gain should be approximately 15 dB which corresponds to a real gain of $5.6\ \frac{v}{v}$. The final transfer function of the filter was then:

$$H(s) = 5.6 \cdot \frac{s \cdot 40 \cdot 440 \cdot 5500}{(s+40)(s+440)(s+5500)}$$

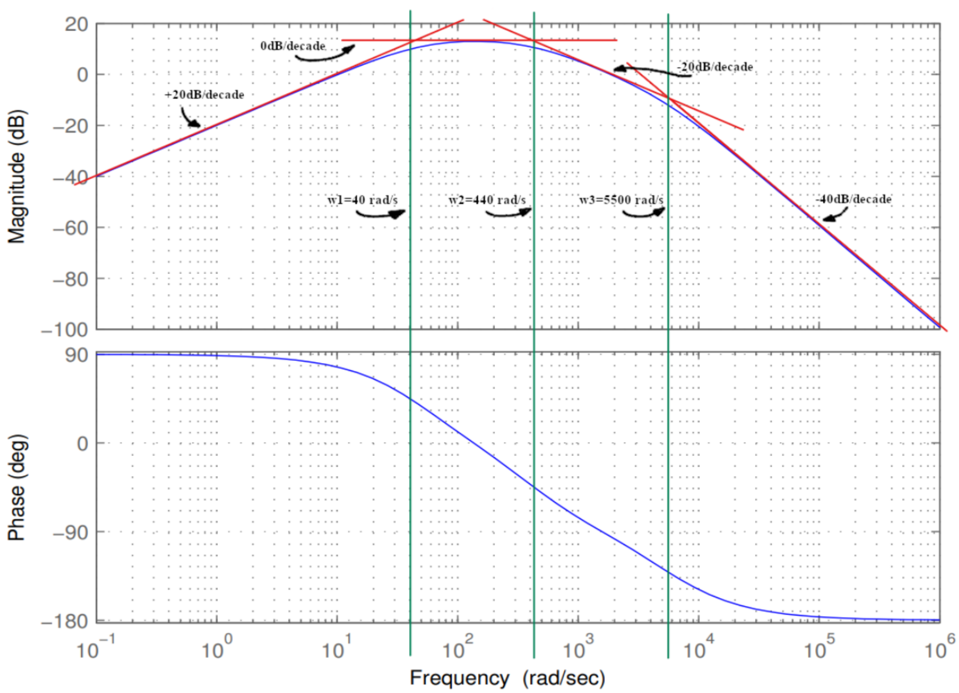

Fig. 2. Target Bode plot with overlayed straight-line approximation.

The second step to designing the band-pass filter was deciding how to realize the transfer function with an RC op-amp circuit. Using the given first-order low-pass and high-pass filter transfer functions,

$$H_l(s) = \frac{a_n}{s+\omega_n},$$

$$H_h(s) = \frac{s a_n}{s+\omega_n},$$

we decided to write the overall transfer function as a series combination of one high-pass filter and two low-pass filters:

$$H(s) = H_h(s) \cdot H_l^1(s) \cdot H_l^2(s) = 5.6 \cdot \frac{s}{s+40} \cdot \frac{440}{s+440} \cdot \frac{5500}{s+5500}$$

To prevent amplifying noise, we kept the high-pass filter gain at unity and split the 5.6 $\frac{v}{v}$ between the two low-pass filters:

$$H_h(s) = \frac{s}{s+40},$$

$$H_l^1(s) = \frac{2.2 \cdot 440}{s+440},$$

$$H_l^2(s) = \frac{2.68 \cdot 5500}{s + 5500}$$

Note that $2.2 \cdot 2.68 = 5.9$, not 5.6 as needed. This is due to physical constraints on choosing common resistor and capacitor values. Once we found our overall transfer function, we plotted the Bode using Octave as shown in fig. 3 and overlayed it (fig. 4) with the target Bode to compare. Overall, our approximated transfer function matches the target response very closely.

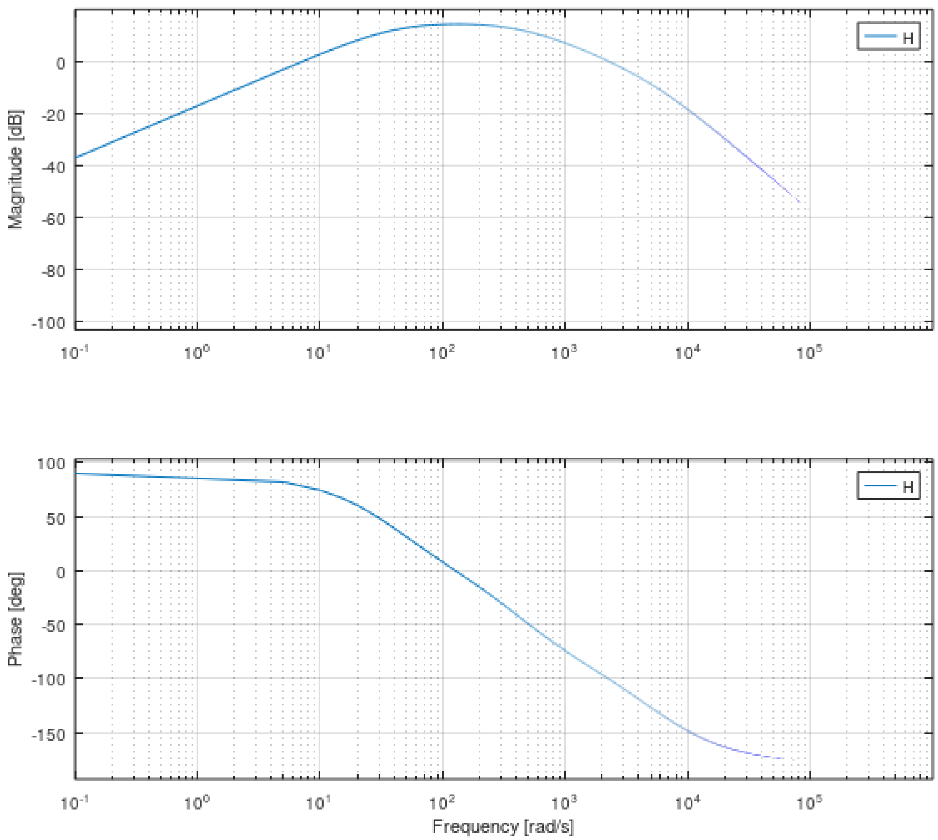

Fig. 3. Bode plot of our approximated transfer function. Note the line fading at $\omega = 10^5 \ \frac{rad}{s}$ is a visual bug.

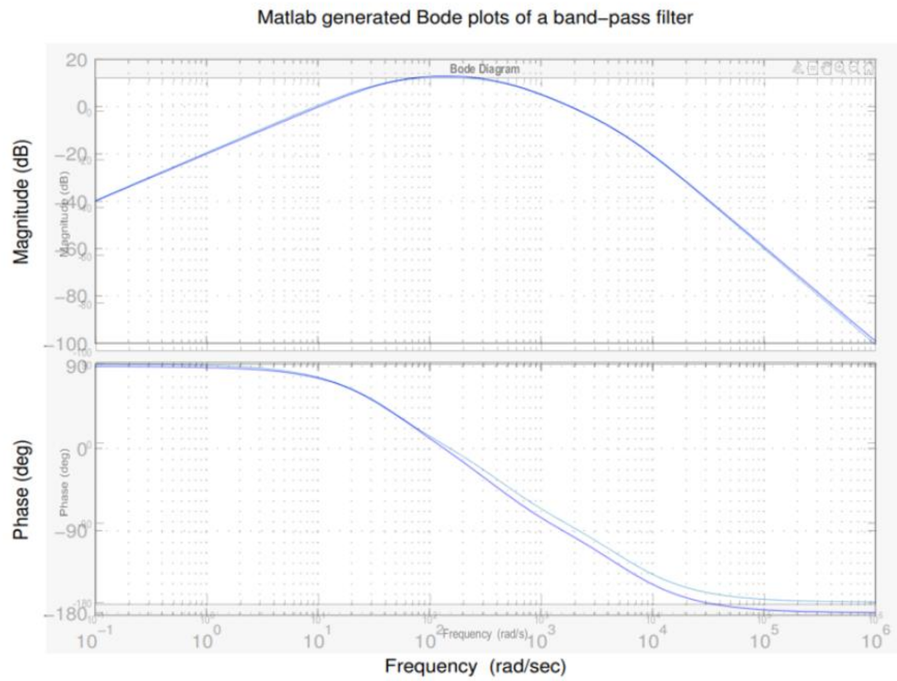

Fig. 4. Overlay of our approximated transfer function Bode plot and the target Bode plot.

The third step of the design involved choosing our resistor and capacitor values for each stage of the filter. We used

$$\left|\frac{v_{out}}{v_{in}}\right| = \frac{R_2}{R_1},$$

$$\omega_{l,p} = \frac{1}{CR_2},$$

$$\omega_{h,p} = \frac{1}{CR_1},$$

to determine the values of R_1, R_2, and C and for the corresponding schematics shown in fig. 5 and fig. 6.

High-pass at $\omega = 40\ \frac{rad}{s}$:

$$\omega_{h,l} = 40 = \frac{1}{CR_1} = \frac{1}{1uF \cdot 25k\Omega}$$

$$\left|\frac{v_{out}}{v_{in}}\right| = 1 = \frac{R_2}{R_2} = \frac{25k\Omega}{25\Omega}$$

$$\therefore R_1 = 25k\Omega$$

$$\therefore R_2 = 25k\Omega$$

$$\therefore C = 1uF$$

Low-pass at $\omega = 440\ \frac{rad}{s}$:

$$\omega_{l,p} = 440 = \frac{1}{CR_2} \approx \frac{1}{100nF \cdot 22k\Omega}$$

$$\left|\frac{v_{out}}{v_{in}}\right| = 2.2 = \frac{R_2}{R_1} = \frac{22k\Omega}{10k\Omega}$$

$$\therefore R_1 = 10k\Omega$$

$$\therefore R_2 = 22k\Omega$$

$$\therefore C = 100nF$$

Low-pass at $\omega = 5500\ \frac{rad}{s}$:

$$\omega_{l,p} = 5500 = \frac{1}{CR_2} \approx \frac{1}{10nF \cdot 18.2k\Omega}$$

$$\left|\frac{v_{out}}{v_{in}}\right| = 2.68 = \frac{R_2}{R_1} = \frac{18.2k\Omega}{6.8k\Omega}$$

$$\therefore R_1 = 6.8k\Omega$$

$$\therefore R_2 = 18.2k\Omega$$

$$\therefore C = 10nF$$

The final schematic of the band-pass filter is shown in fig. 7.

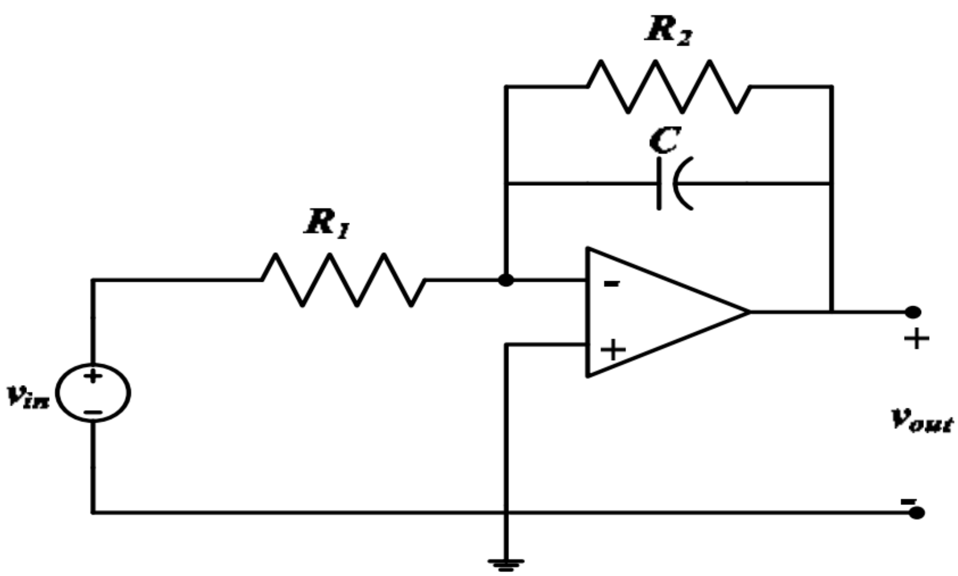

Fig. 5. Schematic of a first-order low-pass filter.

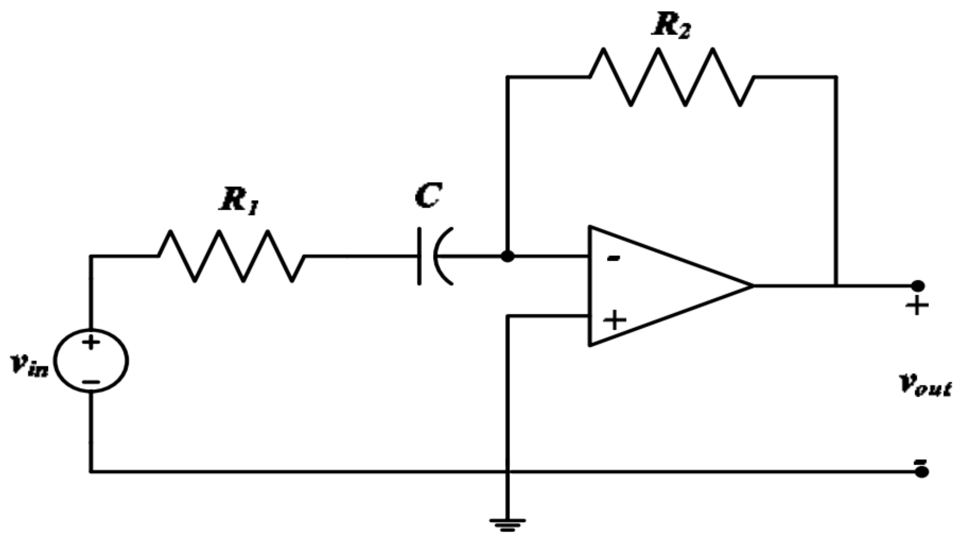

Fig. 6. Schematic of a first-order high-pass filter.

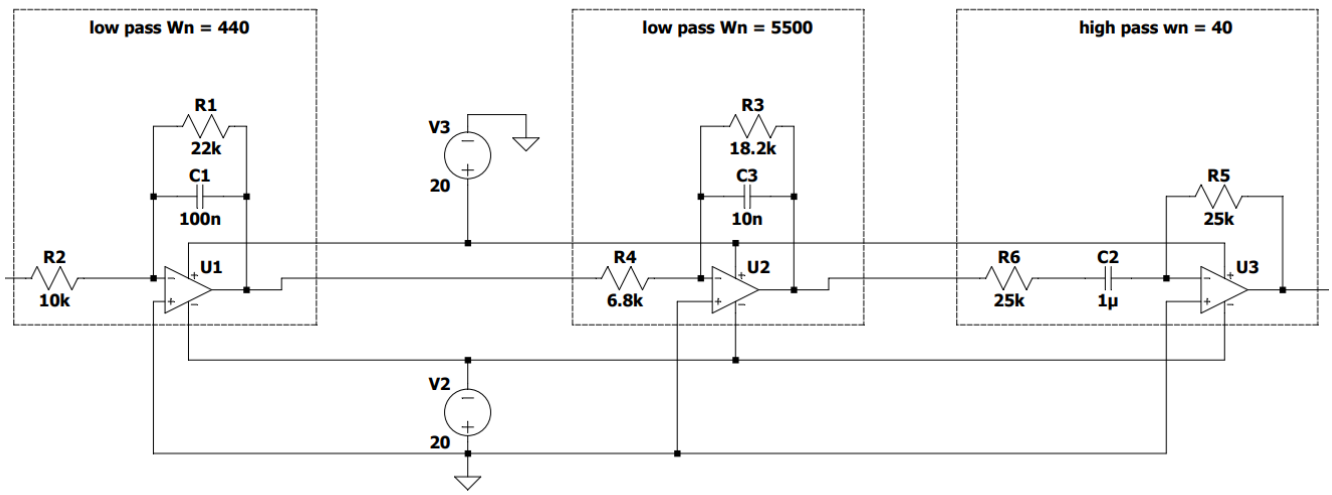

Fig. 7. Final schematic of the band-pass filter.

Testing

We tested our band-pass filter in two ways. We first used LTspice to simulate the circuit and generate a Bode plot. Once the simulated Bode plot shown in fig. 8 matched very closely to the target Bode, we moved on to build the circuit on a breadboard. The breadboard prototype was tested using an Analog Discovery Kit (ADK). The Bode plot from the prototype is shown in fig. 9 which also matches very closely to the target Bode. Note that at around 20 kHz the magnitude and phase start to diverge from the ideal line due to bandwidth limitations of the op-amps used. When we were happy with the simulated and prototype Bode plots, we moved onto designing and ordering a PCB version of our band-pass filter.

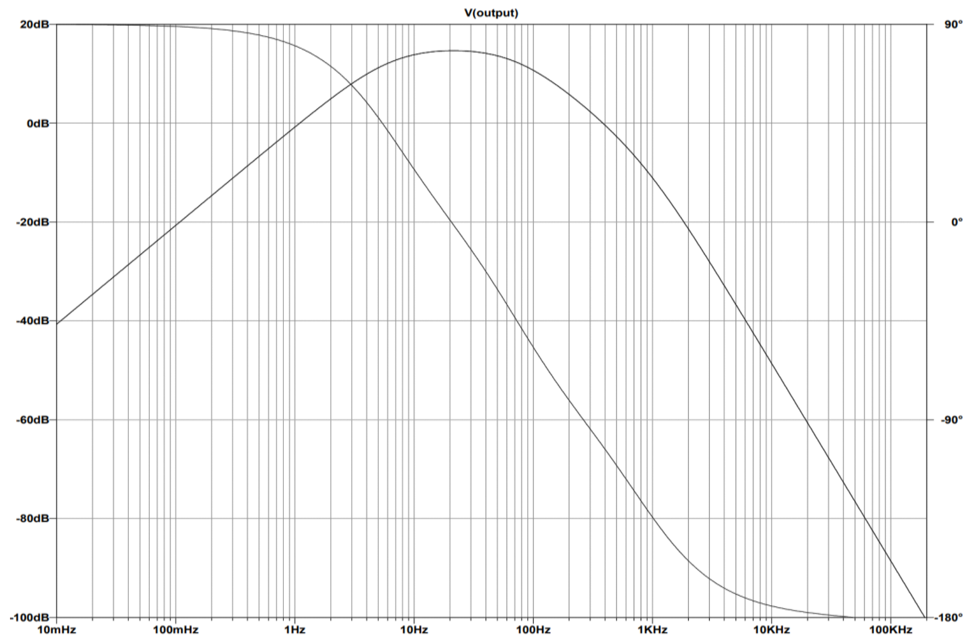

Fig. 8. Simulated Bode plot of the band-pass filter using LTspice. Note the frequency axis is in hertz not $\frac{rad}{s}$.

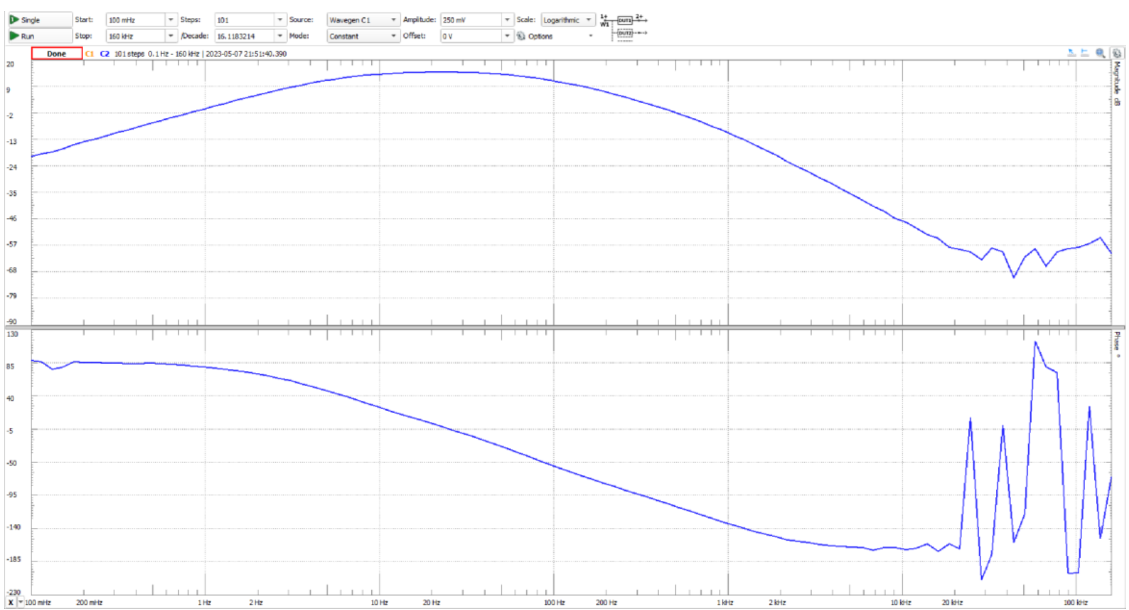

Fig. 9. Bode plot generated using the ADK. Note the frequency starts at 100 mHz hence the starting magnitude is only $\sim -20\ dB$ instead of $-40\ dB$.

PCB

The PCB design is functionally equivalent to our breadboard prototype with the major difference being that we used a LM2902 4-in-one op-amp IC instead of 3 discrete LM741 op-amps to lower our costs and reduce the BOM. The passive components are standard 0603 thick film resistors and ceramic capacitors. We used KiCad to design the hardware and had OshPark manufacture the boards. The board was assembled using manually applied (no stencil) solder paste and a hot-air gun. Once the board was assembled, we used an ADK to generate the Bode plot which is shown in fig. 13. Overall, the Bode plot matches the target Bode plot very closely, as we expected. Just like with the breadboard prototype, the PCB has a limited bandwidth and falters around 20 kHz.

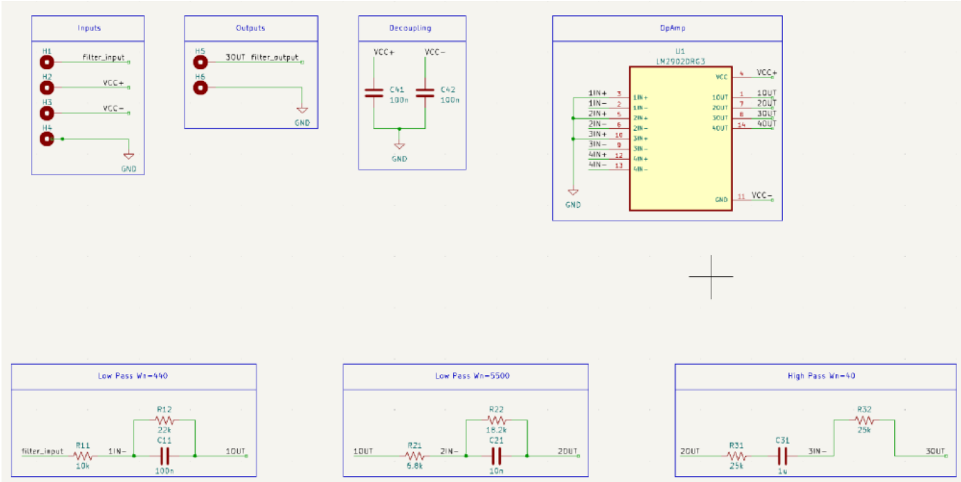

Fig. 10. PCB schematic drawn in KiCad.

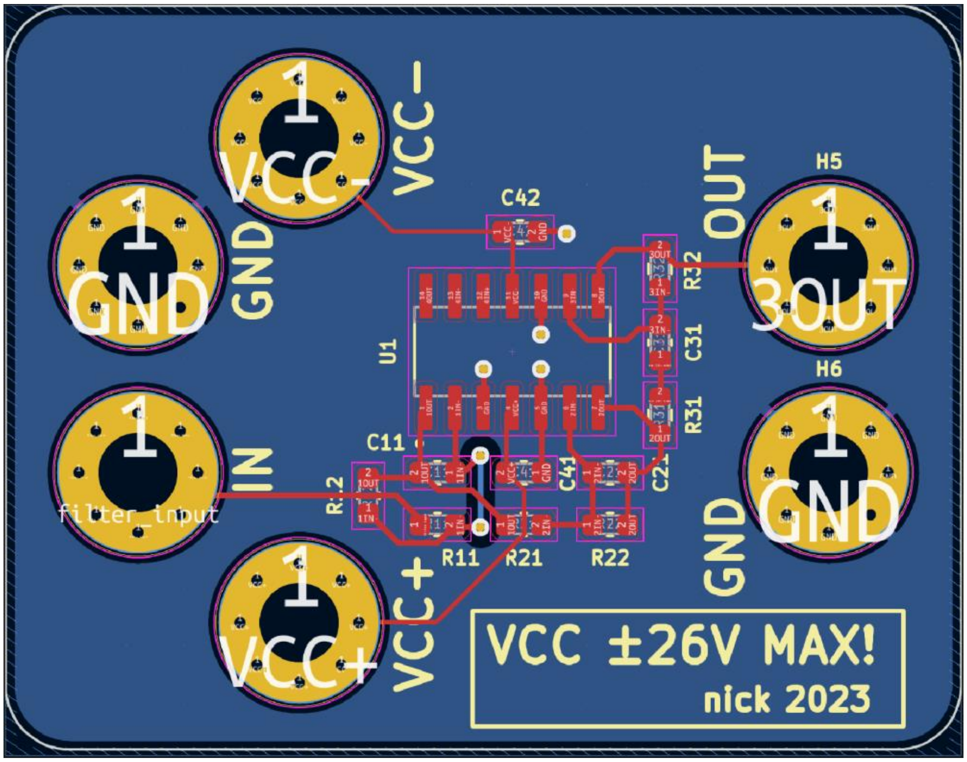

Fig. 11. PCB board layout showing solder mask and copper layers drawn in KiCad.

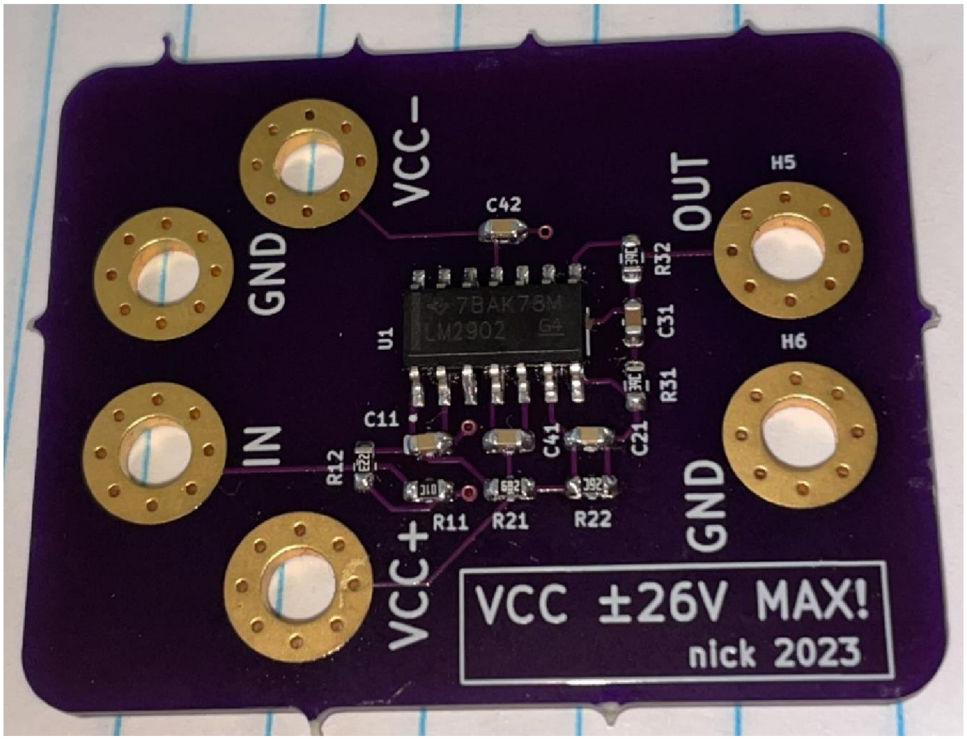

Fig. 12. Assembled PCB.

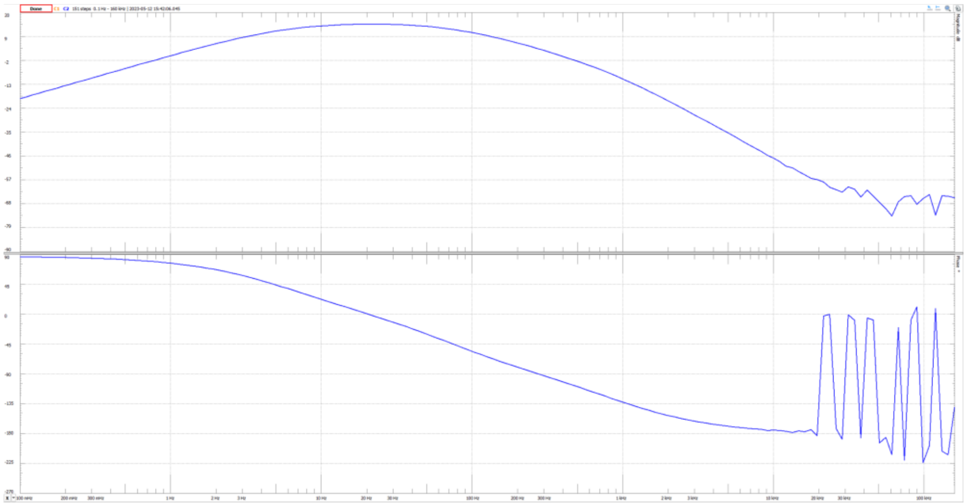

Fig. 13. Bode plot of the PCB filter generated using an ADK.

Conclusion

We designed a band-pass filter to match a given Bode plot by finding the transfer function, prototyping a filter, and finally getting our filter manufactured. Through each step our experimental and target data matched very closely. Our finished PCB band-pass filter matched the frequency response of the target Bode plot very closely.