Surface Area of Spheres - the Wok Problem

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1   Introduction

Since the first introduction to geometry in elementary school, students have investigated the volume, surface area, and many other geometric properties of 2-Dimensional,as well as 3-Dimensional shapes. Yet one area the students do not completely understand is circles and spheres. As a part of the curriculum of second year Calculus course, today we look at one of these problems and approach from a Calculus point of view:

One such problem states that a semi-spherical wok exists with diameter of 16 cm and height 9 cm. We are to paint the outside and inside of 5000 woks with 0.5 mm thick of paint each. Calculate the volume of paint needed in liters.
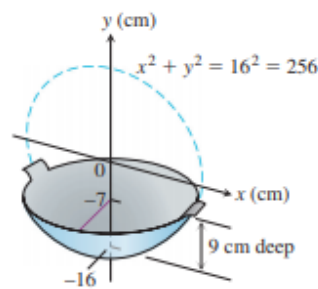

Figure 1: Wok Dimensions

2   Interpolating the Problem

The problem states that a wok needs to be painted on both its inside and outside. This meant the area to paint is the surface area of the wok's inside and outside. Additionaly the paint is painted 0.5 mm thick, which meant the volume of paint needed on each side is the surface area multiplied by the height of the paint, which is 0.5 mm. This meant the real question this question is asking is what is the surface area of part of a sphere, since height is known.

Realizing this we can use the formula $A = \int 2\pi y \sqrt{1^2 + \frac{dy}{dx})^2} dx$ to calculate surface area This indeed meant the calculation must be done in terms of dx, which will be addressed.

Thus we need to find $f'(x)$ with given equation

$$x^2 + y^2 = 256$$

Furthermore since the wok represents part of a sphere, we can naturally rotate the sphere shown in Figure 1 onto the x-axis for easier calculation with dx instead of dy. Thus simplifying we get:

$$y = \sqrt{256 - x^2}$$

3   Calculations

Using equations we got from section 2:

$$\frac{dy}{dx} = \frac{1}{2}(256 - x^2)^{-1/2} 2x$$

Plugging in the equation and the bounds:

$$A = \int_{7}^{16} 2\pi \sqrt{256 - x^2} \sqrt{1^2 + (\frac{1}{2}(256 - x^2)^{-1/2} 2x)^2} \, dx$$

s Expanding each term:

$$A = 2\pi \int_{7}^{16} \sqrt{256 - x^2} \sqrt{1 + (256 - x^2)^{-1} x^2} dx$$

Changing negative exponent to a fraction:

$$A = 2\pi \int_{7}^{16} \sqrt{256 - x^2} \sqrt{1 + \frac{x^2}{256 - x^2}} dx$$

Rewriting 1 with common denominator:

$$A = 2\pi \int_7^{16} \sqrt{256 - x^2} \sqrt{\frac{256 - x^2 + x^2}{256 - x^2}} dx$$

Canceling out similar terms and pulling out constant 256:

$$A = 2\pi \int_7^{16} 16\sqrt{256 - x^2} \sqrt{\frac{1}{256 - x^2}} dx$$

Moving the constants out of the integral and simplifying the integrand:

$$A = 32\pi \int_7^{16} (256 - x^2)^{1/2} (256 - x^2)^{-1/2} dx$$

Combining similar terms:

$$A = 32\pi \int_7^{16} (256 - x^2)^{1/2 - 1/2} dx$$

Simplifying the integrand:

$$A = 32\pi \int_7^{16} 1 dx$$

Taking the integral:

$$A = 32\pi \left[x\right]_7^{16}$$

Plugging in the bounds:

$$A = 32\pi (16 - 7) = 288\pi$$

Thus we have found the surface area of one side of the wok in cm^2. To calculate the total volume of paint needed, we multiply the surface area. In order to convert final result to liters, we use dimensional analysis:

$$V = \frac{288\pi cm^2}{1side} \cdot \frac{0.5mm}{1side} \cdot \frac{1cm}{10mm} \cdot \frac{1side}{1wok} \cdot 5000wok \cdot \frac{1liter}{1000cm^3} = 72\pi/side$$

Thus to paint 5000 woks of this dimension we need 72π liters of paint for each side of the wok.