McGill University April 2012

Faculty of Science Final examination

VERSION number 1

Calculus 3

Math 222

Tuesday, April 24, 2011

Time: 6pm-9pm

Examiner: Prof. J. Loveys Associate Examiner: Prof. W. Jonsson

Student name (last, first)	**Student number (McGill ID)**

INSTRUCTIONS

This is a closed book exam. Calculators are not permitted. Use of a regular dictionary is not permitted. Use of a translation dictionary is permitted.
PART 1 of this exam consists of 10 multiple choice problems. They are to be answered on the machine readable sheets (scantrons) provided.
PART 2 consists of 4 problems to be answered on the exam itself. If you require the extra pages at the end of the exam (which may also be used for scrap work), please indicate there which problem(s) you are continuing.
This exam comprises the cover page, three pages of 10 multiple choice questions, numbered 1 to 10, 4 pages of written questions, and 2 extra pages.
The Examination Security Monitor Program detects pairs of students with unusually similar answer patterns on multiple-choice exams. Data generated by the program can be used as admissible evidence, either to initiate or corroborate an investigation or a charge of cheating under Section 16 of the Code of Student Conduct and Disciplinary Procedures.

(Part 2) Problem	1	2	3	4	Total	Multiple choice
Mark						
Out of	12.5	12.5	12.5	12.5	50	50

PART 1 (Multiple choice.)
BEFORE EVEN LOOKING AT THESE PROBLEMS, MAKE SURE YOU HAVE COMPLETELY AND ACCURATELY FILLED IN THE NECESSARY INFORMATION ON YOUR SCANTRON. IN PARTICULAR, BE SURE YOU HAVE FILLED IN THE CORRECT VERSION NUMBER IN BOTH APPROPRIATE PLACES, AND YOUR STUDENT NUMBER IN BOTH APPROPRIATE PLACES. IF THIS IS NOT DONE CORRECTLY, YOU WILL RECEIVE 0% FOR THIS PART OF THE EXAM. THIS IS VERSION 1.
Each of these question is worth 5 marks.

1. Let (a_n) be the sequence defined by
$$a_n = \frac{(\ln n)^2 - \sqrt{n^2+2n}}{n}.$$
The limit $\lim_{n\to\infty} a_n$
(a) is -2. (b) is -1. (c) is 0. (d) is 1. (e) does not exist.

2. Let
$$a_n = \frac{(n!)^2}{(2n)!}$$
for $n = 0, 1, 2, \ldots$. The interval of convergence of
$$\sum_{n=0}^{\infty} a_n(x+2)^n \text{ is}$$
(a) $(-2, 6)$. (b) $[-6, 2)$. (c) $[-2, 6)$. (d) $[-6, 2]$. $(e)(-6, 2)$.

3. Let $\ell_1$and ℓ_2 be two lines in 3-space, defined by
$$\ell_1 : (x, y, z) = (2, 3, 4) + t\langle 1, 1, 1\rangle \quad \ell_2 : (x, y, z) = (-2, 1, 0) + s\langle 2, 2, 3\rangle.$$
The shortest distance between these lines is

(a) $\sqrt{6}$. (b) 0. (c) $\sqrt{2}$. (d) 6. (e) 2.

4. The arc length from $(0, 0, 0)$ to $(\frac{16}{3}\sqrt{2}, 16, 2)$ along the curve defined by
$$\vec{r}(t) = \frac{8}{3}t^{\frac{3}{2}}\vec{i} + 8t\vec{j} + \frac{1}{2}t^2\vec{k} \text{ is}$$
(a) 18. (b) 10. (c) $\sqrt{\frac{2852}{9}}$. (d) $\frac{488}{3}$ (e) 40.

5. Let

$$f(x,y) = (x^2 + y^2)e^{-xy}.$$

The graph of f has

(a) 1 local maximum, no local minimums, and 2 saddle points.

(b) no saddle points, no local maximums, and 1 local minimums.

(c) 2 saddle points, 1 local minimum, and no local maximums.

(d) 2 saddle points, no local minimums, and 1 local maximum.

(e) 1 local maximum, 1 local minimum, and 1 saddle point.

6. Consider the function $z = f(x,y)$ defined by

$$z = \sqrt[3]{x^2 + y^2}.$$

The linearization (equation of the tangent plane) of this function when $(x,y) = (2,2)$ is

(a) $z = \frac{1}{3}x + \frac{1}{3}y - \frac{4}{3}$.

(b) $z = x + y - 2$.

(c) $z = x + y$.

(d) $z = \frac{1}{3}x + \frac{1}{3}y + \frac{2}{3}$.

(e) $z = 3x + 3y + 1$.

7. Consider the function $z = z(x,y)$ defined implicitly by

$$xy + x^2 z + \ln(x^2 + z^2) = 3.$$

At the point $(1,3,0)$, the derivative z_x is

(a) not defined. (b) $\frac{1}{5}$. (c) $-\frac{1}{5}$. (d) 5. (e) -5.

8. Suppose the density δ of the solid region where $1 \leq x^2 + y^2 + z^2 \leq 4$ is given by $\delta = \sqrt{x^2 + y^2 + z^2}$. The mass of the solid is

(a) 16π. (b) 15π. (c) $\frac{14}{3}\pi^2$. (d) $3\pi^2$. (e) $\frac{28}{3}\pi$.

9. We consider points on the sphere defined by

$$x^2 + y^2 + z^2 = 1$$

which are closest to and furthest from the point $(2, 1, -2)$. Which of the following is true?

(a) The distance to the closest point is 3, and the distance to the furthest point is 4.

(b) The distance to the closest point is 2, and the distance to the furthest point is 3.

(c) The distance to the closest point is $\sqrt{6}$, and the distance to the furthest point is $\sqrt{14}$.

(d) The distance to the closest point is 2, and the distance to the furthest point is 4.

(e) The distance to the closest point is $\sqrt{8}$, and the distance to the furthest point is $2 + \sqrt{8}$.

10. The normal $\vec{N}(t)$to the curve defined by

$$\vec{r}(t) = \frac{8}{3}t^{\frac{3}{2}}\vec{i} + 8t\vec{j} + \frac{1}{2}t^2\vec{k}$$

at the point where $t = 4$ is
(a) $\langle \frac{1}{3}, -\frac{2}{3}, \frac{2}{3} \rangle$. (b) $\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle$. (c) $\langle 1, 0, 1 \rangle$. (d) $\frac{1}{\sqrt{2}}\langle 1, 0, 1 \rangle$. (e) $\langle 4, -8, 8 \rangle$.

END OF PART 1 (MULTIPLE CHOICE.)

(PART 2)

1. (12.5 points total) Find the absolute maximum and absolute minimum, and the points where these occur, of the function

$$f(x,y) = x^2 - 4x + y^2 - 8y$$

on the disc defined by $x^2 + y^2 \leq 45$.

2. Evaluate

$$\int_0^1 \int_{\arcsin(x)}^{\frac{\pi}{2}} \sqrt{1+\cos x}dydx.$$

3. Find the surface area of the surface defined by

$$z = \frac{2}{3}(x^{\frac{3}{2}} + y^{\frac{3}{2}}) \quad 0 \leq x \leq 1, \quad 0 \leq y \leq 1.$$

4. Find the volume of the solid which is inside both the cylinder defined by $x^2 + y^2 = 4$ and the sphere defined by $x^2 + y^2 + z^2 = 9$.

This page is for the continuation of problem . (It may also be used for rough work.) DO NOT REMOVE THIS PAGE OR ANY OTHER PAGE FROM THE EXAM. YOU WILL LOSE 3 MARKS FOR EACH PAGE YOU TEAR OUT OF THE EXAM.

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