Pg 726, 17. In what directions at the point (2,0) does the function $f(x,y) = xy$ have the rate of change of -1? -3? -2?

$\vec{\nabla} f = \begin{bmatrix} y \\ x \end{bmatrix}$, $\vec{\nabla} f(2,0) = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$

let $\hat{u} = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$ st $\|\hat{u}\| = 1$.

We want $D_{\hat{u}} f = -1$

then $\vec{\nabla} f(2,0) \cdot \hat{u} = -1 \longrightarrow \begin{bmatrix} 0 \\ 2 \end{bmatrix} \cdot \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} = -1$

$\boxed{u_2 = -1/2}$ $\because$ $\|\hat{u}\| = 1$, $u_1^2 + u_2^2 = 1$

$\boxed{u_1 = \pm \frac{\sqrt{3}}{2}}$

For $D_{\hat{u}} f = -3$, we get $2u_2 = -3 \rightarrow u_2 = -3/2$ which does not satisfy $\|\hat{u}\| = 1$.

For $D_{\hat{u}} f = -2$ we get $\boxed{u_2 = -1}$, $\boxed{u_1 = 0}$

21. $T(x,y) = x^2-2y^2$, T is temperature.

a) Draw a contour diagram.

b) In what direction should we move (2,-1) to cool off quickest?

c) If we move in said direction at k, what rate do we experience decrease in temperature?

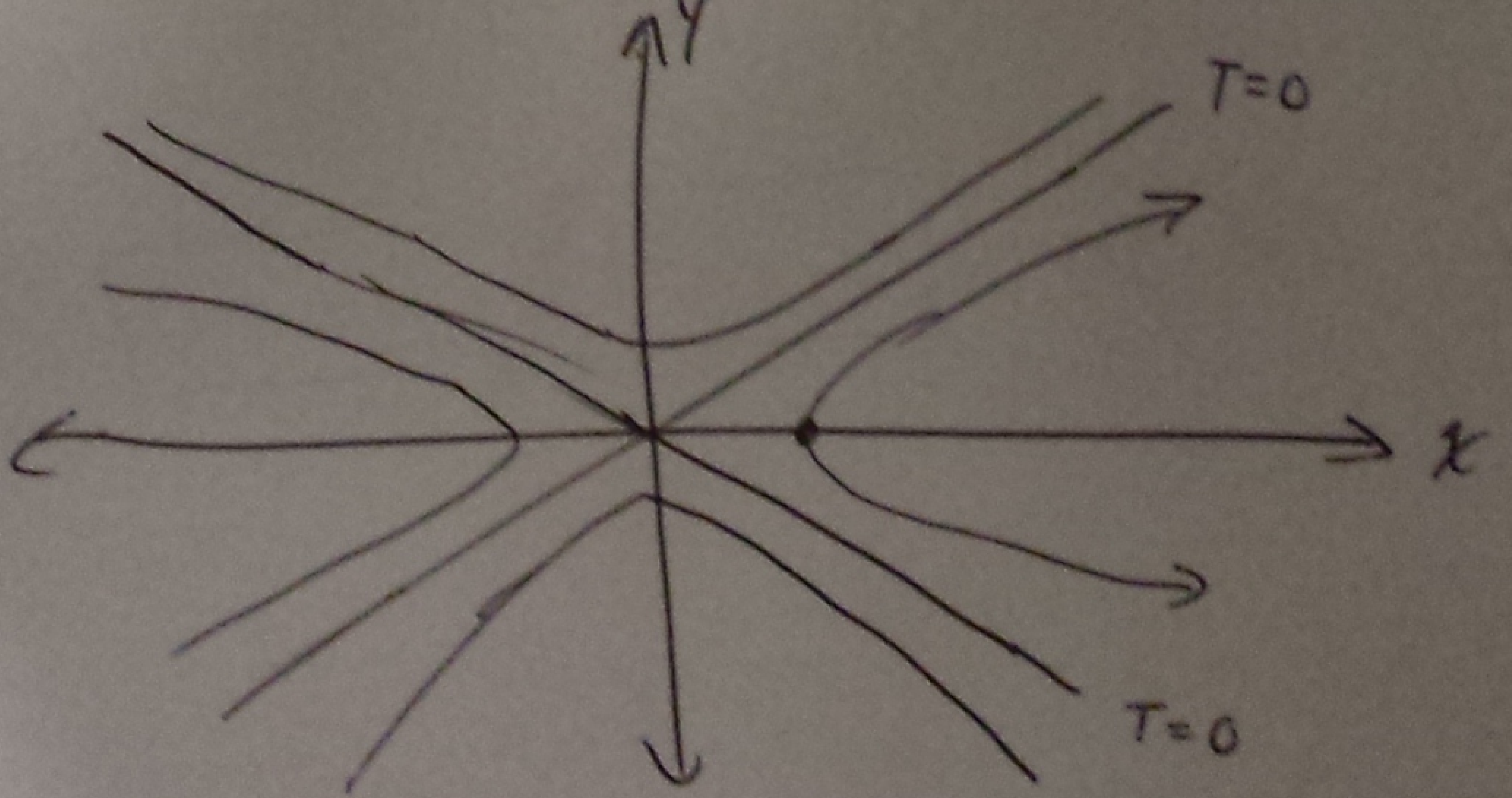
d) at what rate would we experience decrease in temp if we moved from (2,-1) @ speed k in the direction $-\hat{i}-2\hat{j}$?

e) Along what curve through (2,-1) should we move to experience maximum rate of cooling?

a) $T=0 \rightarrow x^2=2y^2 \rightarrow y=\pm\frac{1}{\sqrt{2}}x$

$T=2 \rightarrow x^2-2y^2=2 \rightarrow y=\pm\sqrt{\frac{x^2-2}{2}}$

$T=-2 \rightarrow x^2-2y^2=-2 \rightarrow y=\pm\sqrt{\frac{x^2+2}{2}}$



1. b) $\vec{\nabla T} = \begin{bmatrix} 2x \\ -4y \end{bmatrix}$ $\vec{\nabla T}(2,-1) = \begin{bmatrix} 4 \\ 4 \end{bmatrix}$

To get the greatest magnitude for $D_{\hat{u}} \vec{\nabla T}$, take $\hat{u}$ to be // to $\vec{\nabla T}$, $\therefore$ $\hat{u} = \begin{bmatrix} -1 \\ -1 \end{bmatrix} / \sqrt{2}$.

c) $\vec{\nabla T}(2,-1) \cdot \hat{u} \cdot k = \begin{bmatrix} 4 \\ 4 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \end{bmatrix} / \sqrt{2} \cdot k = -4\sqrt{2}\,k$

d) let $\vec{v} = \begin{bmatrix} -1 \\ -2 \end{bmatrix}$ then $\hat{v} = \begin{bmatrix} -1 \\ -2 \end{bmatrix} / \sqrt{5}$

$D_{\hat{v}} \vec{\nabla T} \cdot k = \begin{bmatrix} 4 \\ 4 \end{bmatrix} \begin{bmatrix} -1 \\ -2 \end{bmatrix} / \sqrt{5}\, k = \frac{-12}{\sqrt{5}} k$

N.B. $D_{\hat{u}} \vec{\nabla T} > D_{\hat{v}} \vec{\nabla T}$ as expected!

e) Let $r(t)$ be the curve, which is always tangent to $\vec{\nabla T}$.

Write this as $\begin{bmatrix} \frac{dx}{dt} \\ \frac{dy}{dt} \end{bmatrix} = \lambda \begin{bmatrix} 2x \\ -4y \end{bmatrix}$. $\frac{dx}{dt} = \lambda 2x$ ①
$\frac{dy}{dt} = -\lambda 4y$ ②

From ①
$\lambda = \frac{1}{2x} \frac{dx}{dt}$

From ②
$\lambda = \frac{-1}{4y} \frac{dy}{dt}$

$\therefore \frac{1}{2x} \frac{dx}{dt} = \frac{-1}{4y} \frac{dy}{dt}$

$$\frac{1}{y}\frac{dy}{dt} = -\frac{2}{x}\frac{dx}{dt}$$

integrate wrt dt

$$\ln|y(t)| = -2\ln|x(t)| + \ln c = \ln|x(t)^{-2}\cdot c|$$

$$e^{y(t)} = e^{c\,x(t)^{-2}}$$

$$y(t) = c\,x(t)^{-2}$$

$$y(t)\,x(t)^{2} = c$$

We must pass through $(2,-1)$.

~~$x(t)^2 = c = 2$.~~ $-1\cdot 2^2 = c = 4$

$\therefore\ yx^2 = -4$ or $\boxed{y = -\frac{4}{x^2}}$ is our path.

26.23. Find an equation in xy-plane that passes through (2,-1) and intersects every curve of the form $x^2y^3 = K$ @ right angles.

Let our curve be $y = f(x)$.

Then @ x, y its normal is $\begin{bmatrix} dy/dx \\ -1 \end{bmatrix}$. This result from grade 9 math class.

The curve $x^2y^3 = K$ has normal $\begin{bmatrix} 2xy^3 \\ 3x^2y^2 \end{bmatrix}$. Recall the gradient is orthogonal to the level surface.

These curves will intersect at right angle if their normals are $\perp$.

write as: $$2xy^3 \frac{dy}{dx} - 3x^2y^2 = 0$$

$$\frac{dy}{dx} = \frac{3x}{2y}$$

$$2y\,dy = 3x\,dx$$

$$y^2 = 3/2\, x^2 + C$$

sub (2,-1)

$$1 = 3/2\; 2^2 + C \rightarrow C = -5$$

$$\therefore \boxed{y^2 = 3/2\, x^2 - 5}$$

pg 726, 27. Find a vector tangent to the curve of intersection of $x+y+z=6$ and $x^2+y^2+z^2=14$ ~~at~~ at the point $(1,2,3)$.

$\vec{n}_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\vec{n}_2 = \begin{bmatrix} 2x \\ 2y \\ 2z \end{bmatrix}$ $\vec{n}_2\Big|_{(1,2,3)} = \begin{bmatrix} 2 \\ 4 \\ 6 \end{bmatrix}$

$\vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 4 & 6 \end{vmatrix} = \begin{bmatrix} 2 \\ -4 \\ 2 \end{bmatrix}$