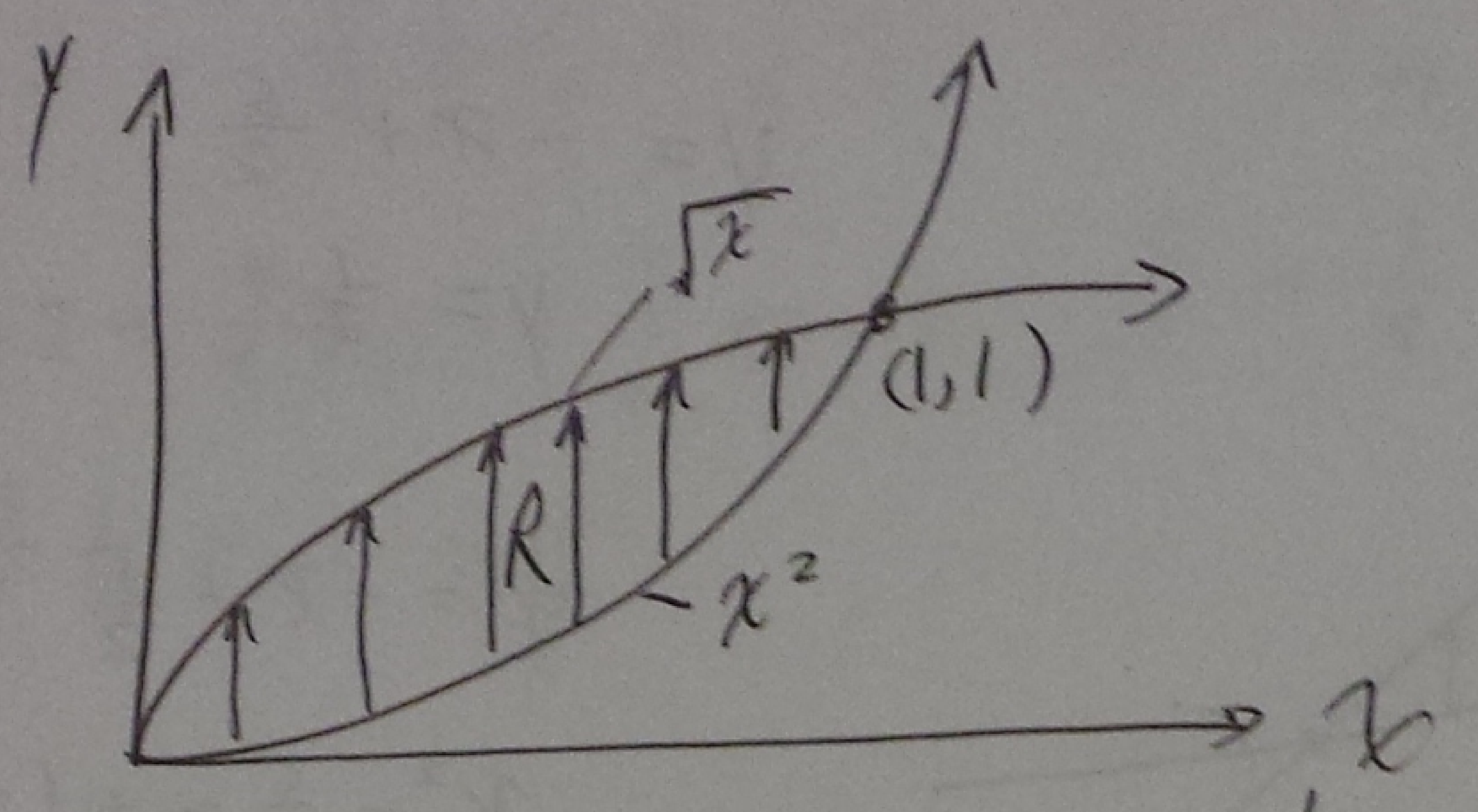
pg 819, 9. Compute $\iint_R xy^2\,dA$, R is in the first quadrant and bounded by $y=x^2$, $x=y^2$



"sweep" y, then "sweep" x

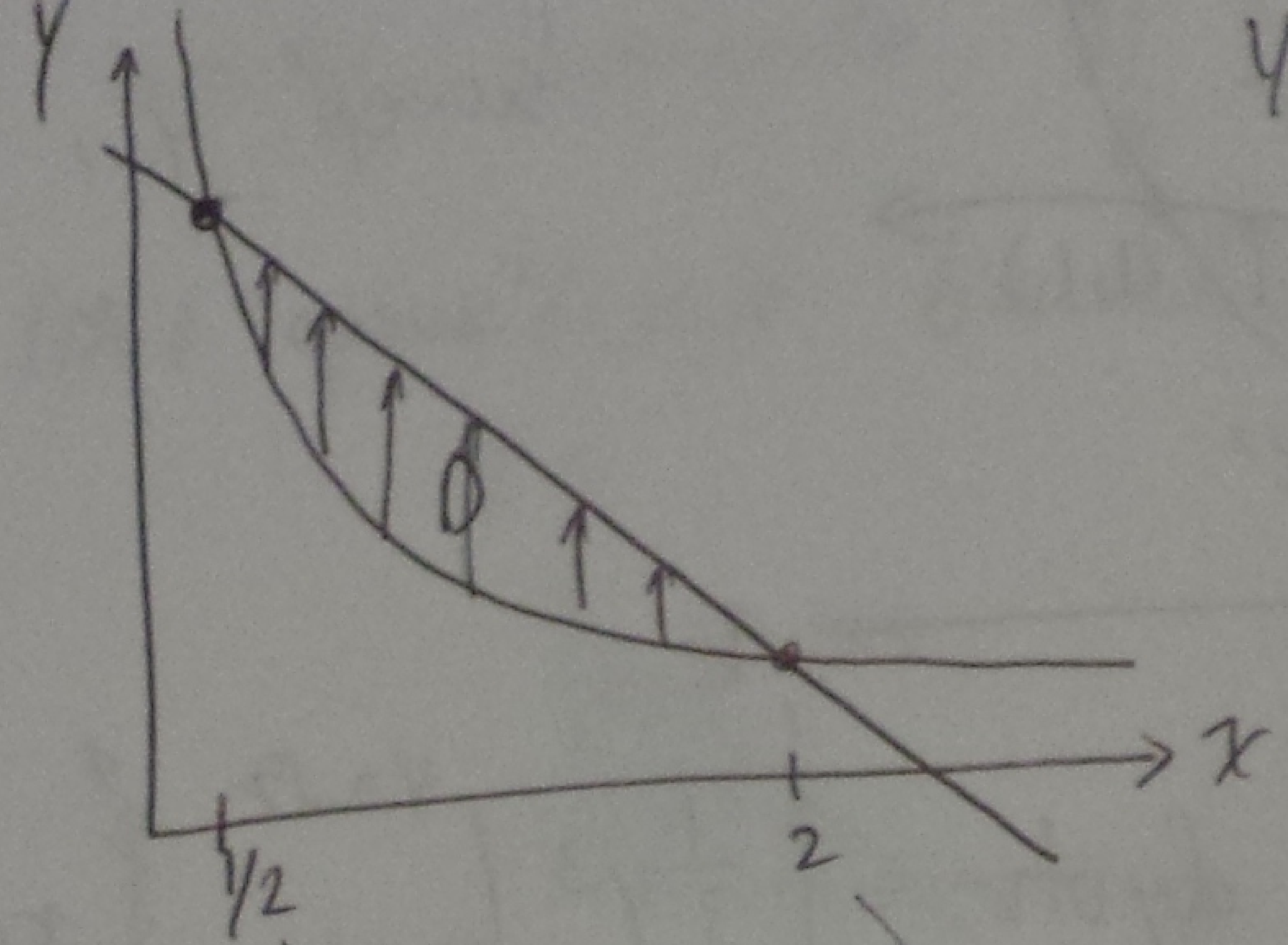
$$\iint_R xy^2\,dA = \int_0^1\int_{x^2}^{\sqrt{x}} xy^2\,dy\,dx = \int_0^1 \frac{1}{3}y^3\Big|_{y=x^2}^{y=\sqrt{x}} \, x\,dx$$

$$= \frac{1}{3}\int_0^1 x^{5/2} - x^7\,dx$$

$$= \frac{1}{3}\left[\frac{2}{7}x^{7/2} - \frac{1}{8}x^8\right]_0^1$$

$$= \frac{1}{3}\left[\frac{2}{7} - \frac{1}{8}\right] = \frac{3}{56}$$

11. $\iint_D \ln x \, dA$, D in first quadrant bounded by $2x+2y=5$ and $xy=1$.



$y = -x + \frac{5}{2}$

$y = \frac{1}{x}$

$\frac{1}{x} = -x + \frac{5}{2}$

$x^2 - \frac{5}{2}x + 1 = 0$

$(x-2)(x-1/2) = 0$

$$\int_{1/2}^{2} \int_{1/x}^{-x+5/2} \ln x \, dy \, dx$$

recall:

$$\int \frac{\ln x}{x} dx = \frac{1}{2}(\ln x)^2 + c$$

$$= \int_{1/2}^{2} \ln x \left[-x + 5/2 - \frac{1}{x} \right] dx$$

$$= \int_{1/2}^{2} \underbrace{\ln x}_{u} \underbrace{\left[5/2 - x \right] dx}_{dv} - \frac{1}{2}(\ln x)^2 \Big|_{1/2}^{2}$$

$$= \Big[\underbrace{\ln x}_{u} \underbrace{\left(\frac{5}{2}x - \frac{1}{2}x^2 \right)}_{v} \Big]_{1/2}^{2} - \int_{1/2}^{2} \underbrace{\frac{1}{x}}_{du} \underbrace{\left(\frac{5}{2}x - \frac{1}{2}x^2 \right)}_{v} dx - \frac{1}{2}(\ln x)^2 \Big|_{1/2}^{2}$$

$$= \ln 2\,(5-2) - \ln\tfrac{1}{2}\left(\tfrac{5}{4}-\tfrac{1}{8}\right) - \int_{1/2}^{2} \tfrac{5}{2} - \tfrac{1}{2}x\,dx - \overset{\to 0}{\cancel{\tfrac{1}{2}(\ln 2)^2 + \tfrac{1}{2}(\ln\tfrac{1}{2})^2}}$$

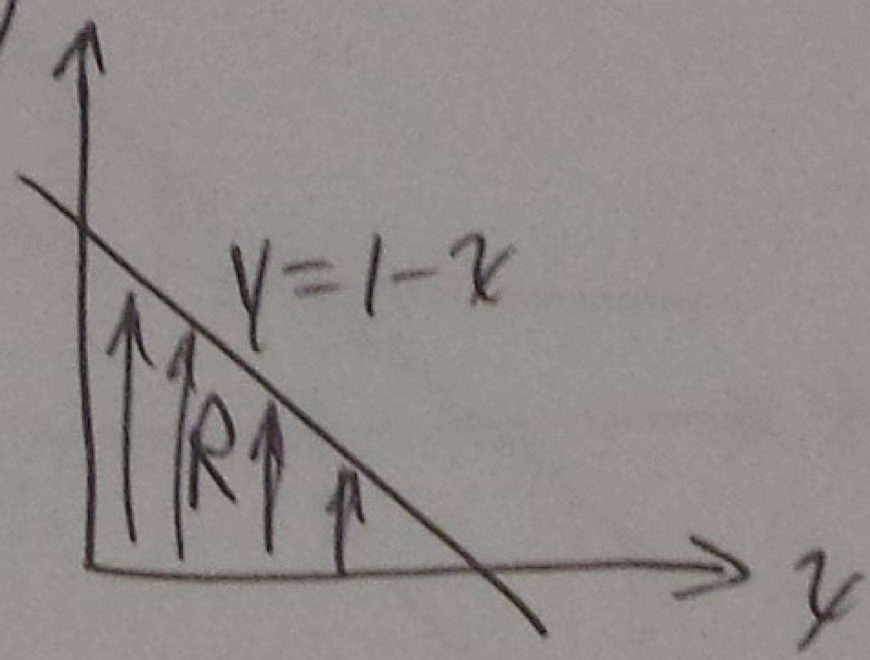
$$= 3\ln 2 - \tfrac{9}{8}\ln\tfrac{1}{2} - \left[\tfrac{5}{2}x - \tfrac{1}{4}x^2\right]_{1/2}^{2}$$

$$= 3\ln 2 + \tfrac{9}{8}\ln 2 - \left[5 - 1 - \tfrac{5}{4} + \tfrac{1}{8}\right]$$

$$= \tfrac{33}{8}\ln 2 - \tfrac{45}{16}$$

21. Under $z=1-x^2-y^2$, above $x\geq 0$, $y\geq 0$, $x+y\leq 1$.

In other words, compute $\iint_R 1-x^2-y^2\,dA$, R in the first quadrant bounded by $x+y=1$.



$$V=\int_0^1\int_0^{1-x}\int_0^{1-x^2-y^2} dz\,dy\,dx$$

$$=\int_0^1\int_0^{1-x} 1-x^2-y^2\,dy\,dx$$

$$=\int_0^1 y-x^2y-\frac{1}{3}y^3\Big|_{y=0}^{y=1-x}\,dx$$

$$=\int_0^1 1-x-x^2(1-x)-\frac{1}{3}(1-x)^3\,dx$$

$$=\int_0^1 \frac{2}{3}-2x^2+\frac{4}{3}x^3\,dx=\frac{2}{3}-\frac{2}{3}x^3+\frac{1}{3}x^4\Big|_0^1=\frac{1}{3}$$