Pg765, 5. Use the Lagrange method to find greatest and least distance from $P(2,1,-2)$ to $x^2+y^2+z^2=1$.

Let $d^2=(x-2)^2+(y-1)^2+(z+2)^2$ be the square of the distance. Extremities of function d^2 are extremeties of function d.

$$L=(x-2)^2+(y-1)^2+(z+2)^2+\lambda(x^2+y^2+z^2-1)$$

$$\frac{\partial L}{\partial x}=2(x-2)+2\lambda x=0 \longrightarrow x=\frac{2}{1+\lambda}$$

$$\frac{\partial L}{\partial y}=2(y-1)+2\lambda y=0 \longrightarrow y=\frac{1}{1+\lambda}$$

$$\frac{\partial L}{\partial z}=2(z+2)+2\lambda z=0 \longrightarrow z=\frac{-2}{1+\lambda}$$

$$\frac{\partial L}{\partial \lambda}=x^2+y^2+z^2-1=0$$

$$\frac{4+1+4}{(1+\lambda)^2}=1$$

$$9=(1+\lambda)^2$$

$$\boxed{1+\lambda=\pm 3}$$

For $1+\lambda=3$

$Q(2/3, 1/3, -2/3)$

$d(Q)=2$

$d(R)=4$

For $1+\lambda=-3$

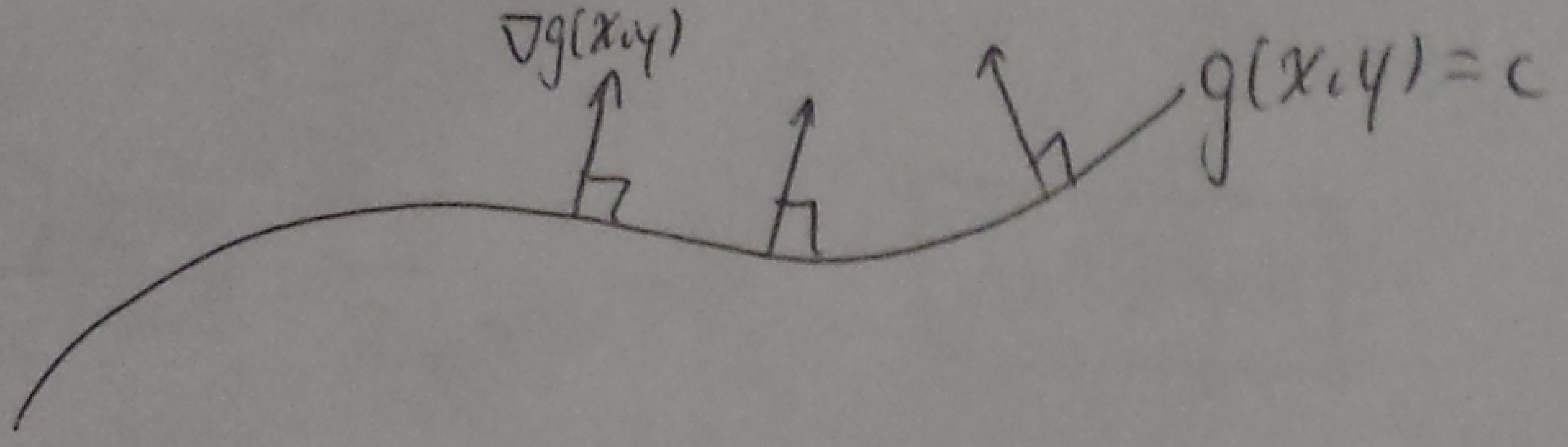
$R(-2/3, -1/3, 2/3)$

Why does the Lagrange Method Work?

We want to maximize $f(x,y)$ constrained by $g(x,y)=c$.

$\nabla f(x,y)$ tells the direction to move for maximum change in $f(x,y)$. If we move $\perp$ to this, we do not change $f(x,y)$.

Why would we ever move $\perp$ to $\nabla f(x,y)$? b/c we are constrained by $g(x,y)=c$. $\nabla g(x,y)$ always $\perp$ to $g(x,y)=c$.



If $\nabla f(x_0,y_0)$ // to $\nabla g(x_0,y_0)$, then @ point (x_0,y_0) we are moving $\perp$ to $\nabla f(x_0,y_0)$ due to our restrai $g(x,y)=c$, indicating an extremity.

pg 265, 11. Find the distance from the origin to $xy^2z^4 = 32$.

$$L = \underbrace{x^2+y^2+z^2}_{d^2} + \lambda[xy^2z^4 - 32],$$

$$\frac{\partial L}{\partial x} = 2x + \lambda y^2 z^4 = 0 \longrightarrow -\lambda = \frac{2x}{y^2z^4}$$

$$\frac{\partial L}{\partial y} = 2y + 2\lambda xyz^4 \longrightarrow -\lambda = \frac{2y}{2xyz^4} = \frac{1}{xz^4}$$

$$\frac{\partial L}{\partial z} = 2z + 4\lambda xy^2z^3 \longrightarrow -\lambda = \frac{2z}{4xy^2z^3} = \frac{1}{2xy^2z^2}$$

$$\frac{2x}{y^2z^4} = \frac{1}{xz^4} \longrightarrow \boxed{2x^2 = y^2}$$

$$\frac{1}{2xy^2z^2} = \frac{1}{xz^4} \longrightarrow 2xy^2z^2 = xz^4 \longrightarrow \boxed{4x^2 = z^2}$$

$$x(2x^2)(4x^2)^2 = 32 \longrightarrow x^7 = 1 \longrightarrow \boxed{x=1} \quad y = \pm\sqrt{2}, \quad z = \pm 2$$

$$d = \sqrt{x^2+y^2+z^2} = \sqrt{7}$$

5. Find max/min values of $f(x,y,z) = 4-z$ on the ellipse formed by the intersection of $x^2+y^2=8$ and $x+y+z=1$.

Ellipse? Hilarious.

$L = 4 - z + \lambda(x^2+y^2-8) + \mu(x+y+z-1)$

$\frac{\partial L}{\partial x} = 2\lambda x + \mu = 0$

$\frac{\partial L}{\partial y} = 2\lambda y + \mu = 0$

$\frac{\partial L}{\partial z} = \cancel{2\lambda} -1 + \mu = 0 \longrightarrow \boxed{\mu = 1}$

$\frac{\partial L}{\partial \lambda} = x^2+y^2-8 = 0$

then $2\lambda x = -1$, $2\lambda y = -1$

$\lambda(x-y) = 0$

$\frac{\partial L}{\partial \mu} = x+y+z-1 = 0$

$x - y = 0$ $\boxed{x = y}$

then $2x^2 = 8$

$\boxed{x = \pm 2} = y$

$z = 1 - 2x$

so we have $P(2,2,-3)$, $Q(-2,-2,5)$

$f(P) = 7$ $f(Q) = -1$.