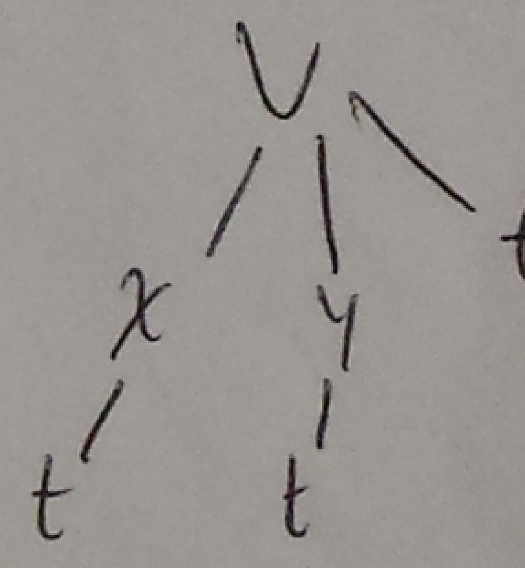
$$\vec{V} = \begin{bmatrix} 2x-4 \\ cy \\ 5t \end{bmatrix}$$ determine c if flow is incompressible. Calculate the acceleration of a fluid particle @ $(x,y) = (3,2)$.

$\vec{\nabla} \cdot \vec{V} = 0$ for incompressible flow.

$$2 + c = 0 \rightarrow c = -2 \quad \therefore \quad \vec{V} = \begin{bmatrix} 2x-4 \\ -2y \\ 5t \end{bmatrix}$$



$$\vec{a} = \frac{D\vec{V}}{Dt} = \frac{\partial \vec{V}}{\partial t} + \frac{\partial \vec{V}}{\partial x}\frac{dx}{dt} + \frac{\partial \vec{V}}{\partial y}\frac{dy}{dt}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix}(2x-4) + \begin{bmatrix} 0 \\ -2 \\ 0 \end{bmatrix}(-2y)$$

$$= \begin{bmatrix} 4x-8 \\ 4y \\ 5 \end{bmatrix}$$

$$\therefore \vec{a}(3,2) = \begin{bmatrix} 4 \\ 8 \\ 5 \end{bmatrix}$$

pg 252, 7. Find and Classify the critical points of $f(x,y)=x\sin y$

$\frac{\partial f}{\partial x}=\sin y \qquad \frac{\partial f}{\partial y}=\cancel{y}x\cos y$

$\frac{\partial f}{\partial x}=0=\sin y$

$y=n\pi\cancel{\ }, \; n\in\mathbb{Z}$

$\frac{\partial f}{\partial y}=0=x\cos y$

$x=0 \cap y=\frac{(2k+1)\pi}{2}, \; k\in\mathbb{Z}$

$y=n\pi \; \cancel{\ } \cap \; y=\frac{2k+1}{2}\pi$ DNE

$\therefore$ our critical pts @ $(0,n\pi)$

Recall: $D_1=f_{xx} \quad D_2=|H_{ij}|, \; H_{ij}=\frac{\partial^2 f}{\partial x_i \partial x_j}$

$\frac{\partial^2 f}{\partial x^2}=0=D_1 \quad |H|=\begin{vmatrix} 0 & \cos y \\ \cos y & -x\sin y \end{vmatrix}=-\cos^2 y$

We have $\cos^2 y<0$ for $y=n\pi$.

$\therefore D_2<0$, we have saddle points.

Pg 752, 19. Find the maximum and minimum values of $f(x,y) = xye^{-x^2-y^4}$.

Our $f(x,y)$ defined for $x, y \in \mathbb{R}$

$$\frac{\partial f}{\partial x} = xye^{-x^2-y^4}[-2x] + ye^{-x^2-y^4}$$

$$= (1-2x^2)ye^{-x^2-y^4}$$

$$\frac{\partial f}{\partial y} = xe^{-x^2-y^4} + xye^{-x^2-y^4}[-4y^3]$$

$$= [1-4y^4]xe^{-x^2-y^4}$$

by observation, we have critical points @

$(0,0)$, $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$, $\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$, $\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$

$f(0,0) = 0$

$f\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = f\left(\frac{-1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right) = \frac{1}{2}e^{-3/4}$

$f\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right) = f\left(\frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right) = \frac{-1}{2}e^{-3/4}$

$\because$ $f(x,y) \to 0$ as "$(x,y) \to (\infty, \infty)$",

we have minimum $\frac{-1}{2}e^{-3/4}$, maximum $\frac{1}{2}e^{-3/4}$

Find the maximum and minimum values of

$f(x,y) = xy - y^2$ on $x^2+y^2 \le 1$.

$\frac{\partial f}{\partial x} = y \qquad \frac{\partial f}{\partial y} = \cancel{4y}\; x - 2y$

By observation, we have a critical pt @ $(0,0)$.

$\boxed{f(0,0)=0}$

reparametrize using $x = \cos t$, $y = \sin t$, $0 \le t \le 2\pi$

$g(t) = f(\cos t, \sin t) = \cos t \sin t - \sin^2 t$

$= \frac{1}{2}[\sin 2t + \cos 2t - 1]$

$g'(t) = \cos 2t - \sin 2t$

set $g'(t) = 0$ then $\tan 2t = 1$ so

$2t = \frac{\pi}{4}$, $2t = \frac{5\pi}{4}$

$t = \frac{\pi}{8} \qquad t = \frac{5\pi}{8}$

$g(\frac{\pi}{8}) = \frac{1}{2}[\sin\frac{\pi}{4} + \cos\frac{\pi}{4} - 1]$

$\boxed{g(\frac{\pi}{8}) = \frac{1}{\sqrt{2}} - \frac{1}{2}}$

$\boxed{g(\frac{5\pi}{8}) = -\frac{1}{\sqrt{2}} - \frac{1}{2}}$

$\therefore$ max and min values are $\frac{1}{\sqrt{2}} - \frac{1}{2}$ and $-\frac{1}{\sqrt{2}} - \frac{1}{2}$ respectively.

5. Find maximum value of $f(x,y) = xy - x^3y^2$ over $0 \le x \le 1$, $0 \le y \le 1$.

$$\frac{\partial f}{\partial x} = y - 3x^2y^2 = y(1-3x^2y)$$

$$\frac{\partial f}{\partial y} = x - 2x^3y = x(1-2x^2y)$$

By observation, we have a critical point @ $(0,0)$.

On the sides $x=0$, or $y=0$, $f(x,y)=0$.

Consider $x=1$. $g(y) = f(1,y) = y - y^3$

$g'(y) = 1-2y$, $g'(y)=0$ @ $y=\frac{1}{2}$ $\therefore$ $(1,\frac{1}{2})$ critical.

Consider $y=1$. $h(y) = f(x,1) = x - x^3$

$h'(x) = 1-3x^2$, $h'(x)=0$ @ $x = \pm\frac{1}{\sqrt{3}}$ $\therefore$ $(\frac{1}{\sqrt{3}},1)$, $(\frac{-1}{\sqrt{3}},$

$f(1,\frac{1}{2}) = \frac{1}{2} - (\frac{1}{2})^2 = \frac{1}{4}$

$f(\frac{1}{\sqrt{3}},1) = \frac{1}{\sqrt{3}} - (\frac{1}{\sqrt{3}})^3 \longleftarrow$ max $\frac{2}{3\sqrt{3}}$

$f(\frac{-1}{\sqrt{3}},1) = \frac{-1}{\sqrt{3}} + (\frac{1}{\sqrt{3}})^3$