MATH 222 - APRIL 2012

Thank you for attending a PREP **101** session! Best of luck!

antoine.prep101@gmail.com

#1 $$\lim_{n\to\infty} \frac{\frac{1}{n}\left[\frac{(\ln n)^2}{n} - \sqrt{1+\frac{2}{n}}\right]}{\frac{1}{n}[1]} = -1 \to (b)$$

#2 $$\lim_{n\to\infty}\left|\frac{(n+1)!\,(n+1)!}{(2n+2)!}(x+2)^{n+1}\cdot\frac{(2n)!}{n!\,n!}\right| = \lim_{n\to\infty}\frac{|x+2|\,(n+1)(n+1)}{(2n+1)(2n+2)} = \frac{|x+2|}{4} < 1$$

$$\Rightarrow \quad -4 < x+2 < 4 \quad \Rightarrow \quad -6 < x < 2$$

$x=-6$ $\displaystyle\sum_{n=0}^{\infty}\frac{(n!)^2}{(2n)!}(-1)^n 4^n$ $\qquad x=2$ $\displaystyle\sum_{n=0}^{\infty}\frac{(n!)^2\,4^n}{(2n)!}$

$$\frac{(n!)^2}{(2n)!}4^n = \frac{1.2.\dots n \;.\;1.2.\dots n}{1.2\dots n\;(n+1)\cdots 2n}\cdot 4^n = \frac{1}{n+1}\cdot\frac{2}{n+2}\cdots\cdots\frac{n}{2n}\,4^n > \left(\tfrac{1}{2}\right)^n 4^n$$

$\therefore \dfrac{(n!)^2\,4^n}{(2n)!} > \dfrac{4^n}{2^n} = 2^n$ but $\displaystyle\sum_{n=0}^{\infty} 2^n$ is Divgt $\therefore \displaystyle\sum_{n=0}^{\infty}\frac{(n!)^2\,4^n}{(2n)!}$ Divgt

Also $\displaystyle\sum_{n=0}^{\infty}\frac{(-1)^n\,(n!)^2\,4^n}{(2n)!}$ Div. because $\displaystyle\lim_{n\to\infty}\frac{(n!)^2\,4^n}{(2n)!} > \lim_{n\to\infty} 2^n$ so $\neq 0$

$$I = (-6,2) \to (e)$$

#3 $\vec{d}_1 = \langle 1,1,1\rangle$ $\qquad \vec{d}_2 = \langle 2,2,3\rangle$ $\qquad \vec{d}_1\times\vec{d}_2 = \langle 1,-1,0\rangle$

Pt on L_1 $A(2,3,4)$ $\qquad$ Pt on L_2 $B(-2,1,0)$ $\qquad \vec{AB} = \langle -4,-2,6\rangle$

$$\text{dist} = \frac{|\vec{AB}\cdot(\vec{d}_1\times\vec{d}_2)|}{\|\vec{d}_1\times\vec{d}_2\|} = \frac{|-4+2+0|}{\sqrt{1+1+0}} = \frac{2}{\sqrt{2}} = \sqrt{2} \longrightarrow (c)$$

#4 $$\int_0^2\sqrt{16t+64+t^2}\,dt = \int_0^2 (t+8)\,dt = \frac{t^2}{2}+8t\,\Big|_0^2 = \frac{4}{2}+16 = 18 \to (a)$$

Thank you for attending a PREP **101** session! Best of luck!

antoine.prep101@gmail.com

#5 $f(x,y) = (x^2+y^2)e^{-xy}$

$f_x = 2x e^{-xy} - y(x^2+y^2)e^{-xy}$

$= e^{-xy}(2x - yx^2 - y^3)$

$f_y = 2y e^{-xy} - x(x^2+y^2)e^{-xy}$

$= e^{-xy}(2y - x^3 - xy^2)$

$f_x = 0 \Rightarrow 2x - yx^2 - y^3 = 0$ $\quad (1,1), (-1,-1), (0,0)$

$f_y = 0 \Rightarrow 2y - x^3 - xy^2 = 0$

$f_{xx} = -y e^{-xy}(2x - yx^2 - y^3) + e^{-xy}(2 - 2xy) = e^{-xy}(-4xy + y^2x^2 + y^4 + 2)$

$f_{yy} = e^{-xy}(-4xy + y^2x^2 + x^4 + 2)$

$f_{xy} = -y e^{-xy}(2y - x^3 - xy^2) + e^{-xy}(-3x^2 - y^2) = e^{-xy}(-3y^2 + yx^3 + xy^3 - 3x^2)$

	f_{xx}	f_{yy}	f_{xy}	D		
(0,0)	2	2	0	+	L. min	
(1,1)	0	0		−	saddle	→ (c)
(-1,-1)	0	0		−	saddle	

#6 $z = \sqrt[3]{x^2+y^2}$ $\quad z_x = \frac{1}{3}(x^2+y^2)^{-2/3}(2x)$ $\quad z_y = \frac{1}{3}(x^2+y^2)^{-2/3}(2y)$

at (2,2) $\quad f(2,2) = \sqrt[3]{8} = 2$, $\quad z_x = \frac{4}{3(4)} = \frac{1}{3}$ $\quad z_y = \frac{1}{3}$

$L(x,y) = 2 + \frac{1}{3}(x-2) + \frac{1}{3}(y-2)$

$= \frac{1}{3}x + \frac{1}{3}y + \frac{2}{3} \rightarrow$ (d)

#7 $y + 2xz + x^2 z_x + \dfrac{2x + 2z z_x}{x^2+z^2} = 0$ $\quad$ at (1,3,0)

$3 + 0 + z_x + 2 = 0 \rightarrow z_x = -5 \rightarrow$ (e)

Thank you for attending a PREP **101** session! Best of luck!

antoine.prep101@gmail.com

#8 $\iiint_V \sqrt{x^2+y^2+z^2}\, dV$

$$\int_0^{2\pi}\int_0^{\pi}\int_1^{2} \rho\, \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

$$= 2\pi(-\cos\phi)\Big|_0^{\pi} \ \frac{\rho^4}{4}\Big|_1^2 = 2\pi[1+1]\left[4-\frac{1}{4}\right] = 4\pi\left[\frac{15}{4}\right] = 15\pi \rightarrow (b)$$

#9 max, min $(x-2)^2+(y-1)^2+(z+2)^2$ s.t. $x^2+y^2+z^2=1$

$2(x-2)=\lambda(2x) \Rightarrow x-2=\lambda x \quad x=\frac{2}{1-\lambda}$

$2(y-1)=\lambda(2y) \Rightarrow y-1=\lambda y \quad y=\frac{1}{1-\lambda}$ into

$2(z+2)=\lambda(2z) \Rightarrow z+2=\lambda z \quad z=\frac{-2}{1-\lambda}$

$x^2+y^2+z^2=1$

$\frac{4+1+4}{(1-\lambda)^2}=1 \Rightarrow (1-\lambda)^2=9 \Rightarrow (1-\lambda)=3$ or -3

$\Rightarrow \lambda=-2$ or $\lambda=4$

$\lambda=-2 \rightarrow x=\frac{2}{3},\ y=\frac{1}{3},\ z=-\frac{2}{3}$ $\quad\left(\frac{2}{3},\frac{1}{3},\frac{-2}{3}\right)$ dist $=\sqrt{\left(\frac{-4}{3}\right)^2+\left(\frac{-2}{3}\right)^2+\left(\frac{4}{3}\right)^2}=2$

$\lambda=4 \rightarrow$ $\quad\left(\frac{-2}{3},\frac{-1}{3},\frac{2}{3}\right)$ dist $=\sqrt{\left(\frac{-10}{3}\right)^2+\left(\frac{-4}{3}\right)^2+\left(\frac{10}{3}\right)^2}=4$

$\rightarrow$ (d)

#10 $\vec{r}'(t)=\langle 4\sqrt{t}, 8, t\rangle$

$\vec{r}''(t)=\langle \frac{2}{\sqrt{t}}, 0, 1\rangle$

at $t=4$ $\quad \vec{r}'(4)=\langle 8,8,4\rangle \quad \|\vec{r}'\|=12$

$\vec{r}''(4)=\langle 1,0,1\rangle$

$\vec{r}'\times\vec{r}''=\langle 8,-4,-8\rangle \quad \|\vec{r}'\times\vec{r}''\|=12$

$\vec{T}(4)=\langle \frac{2}{3},\frac{2}{3},\frac{1}{3}\rangle$

$\vec{B}(4)=\langle \frac{2}{3},\frac{-1}{3},\frac{-2}{3}\rangle$

$\vec{N}(4)=\vec{B}(4)\times\vec{T}(4)$

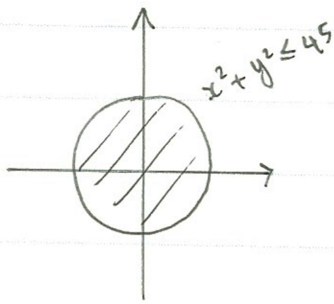
$=\langle \frac{1}{3},\frac{-2}{3},\frac{2}{3}\rangle \rightarrow$ (a)

PART II :

#1 $f_x = 2x - 4 = 0 \Rightarrow x = 2$

$f_y = 2y - 8 = 0 \Rightarrow y = 4$

cr. pt $(2,4)$ $\quad f(2,4) = 4 - 8 + 16 - 32 = -20$



Max/Min $\quad x^2 - 4x + y^2 - 8y \quad$ s.t. $x^2 + y^2 = 45$

$$2x - 4 = \lambda 2x \rightarrow x - \lambda x = 2 \rightarrow x = \frac{2}{1-\lambda}$$

$$2y - 8 = \lambda 2y \rightarrow y - \lambda y = 4 \rightarrow y = \frac{4}{1-\lambda}$$

$$\frac{4+16}{(1-\lambda)^2} = 45 \Rightarrow (1-\lambda)^2 = \frac{20}{45} = \frac{4}{9} \Rightarrow (1-\lambda) = \pm \frac{2}{3}$$

$1 - \lambda = \frac{2}{3} \Rightarrow x = \frac{2}{2/3} = 3 \quad y = \frac{4}{2/3} = 6 \quad f(3,6) = 9 - 12 + 36 - 48 = -15$

$1 - \lambda = -\frac{2}{3} \Rightarrow x = \frac{2}{-2/3} = -3 \quad y = -6 \quad f(-3,-6) = 9 + 12 + 36 + 48 = 105$

$\Rightarrow$ Abs max $= 105$ at $(-3,-6)$

Abs min $= -15$ at $(3,6)$

#2 $$\int_0^1 \int_{\arcsin x}^{\pi/2} \sqrt{1+\cos x}\, dy\, dx = \int_0^1 y\sqrt{1+\cos x}\,\Big|_{\arcsin x}^{\pi/2} dx$$

$$= \int_0^1 \left(\frac{\pi}{2} - \arcsin x\right)\sqrt{1+\cos x}\, dx$$ (?? , maybe a Typo)

#3 $Z_x = x^{1/2}$ $\quad Z_y = y^{1/2}$

antoine.prep101@gmail.com

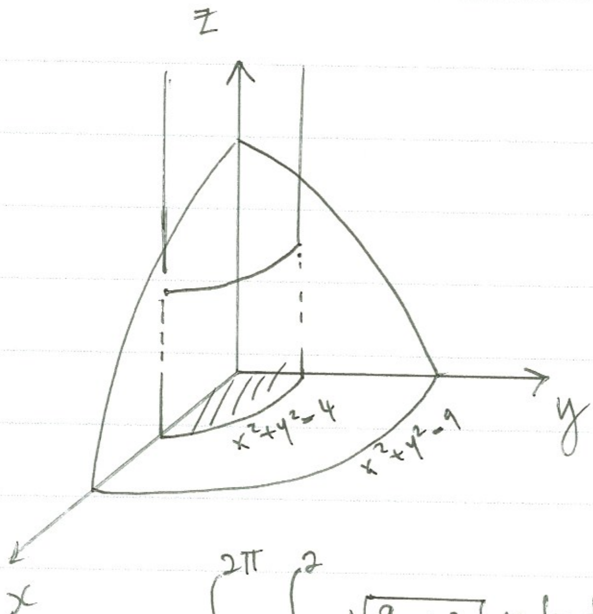
$$S = \int_0^1 \int_0^1 \sqrt{x+y+1}\; dy\,dx$$

$$= \int_0^1 \frac{2}{3}(x+y+1)^{3/2} \Big|_0^1 dx = \int_0^1 \left[\frac{2}{3}(x+2)^{3/2} - \frac{2}{3}(x+1)^{3/2}\right] dx$$

$$= \frac{2}{3}\cdot\frac{2}{5}(x+2)^{5/2} - \frac{2}{3}\frac{2}{5}(x+1)^{5/2} \Big|_0^1$$

$$= \frac{4}{15}\left(3^{5/2}\right) - \frac{8}{15}(2)^{5/2} - \frac{4}{15}.$$

#4



$$Z = \sqrt{9-x^2-y^2}$$

$$\int_0^{2\pi}\int_0^2 \sqrt{9-r^2}\; r\,dr\,d\theta = 2\pi\left[-\frac{1}{2}\frac{2}{3}(9-r^2)^{3/2}\right]_0^2$$

$$= 2\pi\left[-\frac{1}{3}(5)^{3/2} + \frac{1}{3}(9)^{3/2}\right].$$