1 [illegible] 17.

Evaluate the limit of the following sequence: $a_n = (-1)^n \frac{n}{n^3+1}$

Consider $\lim_{n\to\infty} (-1)^n \frac{n}{n^3+1}$

$$= \underbrace{\lim_{n\to\infty} (-1)^n \frac{1}{n^2+\frac{1}{n}}}$$

$$\lim_{n\to\infty} \frac{-1}{n^2+\frac{1}{n}} \le \quad \le \lim_{n\to\infty} \frac{1}{n^2+\frac{1}{n}}, \text{ for } n>1$$

$\therefore$ by squeeze theorem, $\lim_{n\to\infty} a_n = 0$.

Other Way

Theorem: If $\lim_{n\to\infty} |a_n| = 0$ then $\lim_{n\to\infty} a_n = 0$

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30. Let $a_1 = 1$, and $a_{n+1} = \sqrt{1+2a_n}$ $(n = 1, 2, 3 \dots)$. Show that $\{a_n\}$ is increasing and bounded above. Hint: Show that 3 is an upper bound. Hence, conclude that the sequence converges, and find its limit.

$a_1 = 1 \quad a_2 = \sqrt{3}$

$$a_{k+2} = \sqrt{1+2a_{k+1}} > \underbrace{\sqrt{1+2a_k}}_{a_{k+1}} \quad \therefore \{a_n\} \text{ is increasing}$$

Suppose $a_k < 3$, then

$$a_{k+1} = \sqrt{1+2a_k} < \sqrt{1+2\cdot 3} = \sqrt{7}.$$

$\therefore$ $a_n < 3$. $\{a_n\}$ is increasing and bounded above (not necessarily by 3!)

Let $\lim_{n\to\infty} a_n = a = \lim_{n\to\infty} \sqrt{1+2a_{n-1}}$

$$a = \sqrt{1+2a}$$

$$a^2 = 1+2a$$

$$a^2 - 2a - 1 = 0$$

$$a = \frac{2 \pm \sqrt{4-4(1)(-1)}}{2}$$

$$= 1 \pm \sqrt{2}$$

$$\boxed{a = 1+\sqrt{2}}$$

pst 0, 11.

Find the sum or show that it diverges.

$$\sum_{n=1}^{\infty} \frac{1}{n(n+2)} \qquad \frac{A}{n} + \frac{B}{n+2}$$

$An + 2A + Bn = 1$

$A = 1/2$

$B = -1/2$

$$= \sum_{n=1}^{\infty} \frac{1/2}{n} - \frac{1/2}{n+2}$$

$$= \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+2}$$

$$= \frac{1}{2}\left[\left(1 - \cancel{\tfrac{1}{3}}\right) + \left(\frac{1}{2} - \cancel{\tfrac{1}{4}}\right) + \left(\cancel{\tfrac{1}{3}} - \frac{1}{5}\right) + \left(\cancel{\tfrac{1}{4}} - \frac{1}{6}\right) + \dots \right]$$

$$= \frac{1}{2}\left[1 + 1/2\right]$$

$$= 3/4$$

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21. a ball bounces to 3/4 of the height from which it fell. The ball is dropped from 2m, what is the total distance?

$$2 + 2\left[2 \cdot 3/4 + 2 \cdot (3/4)^2 + \dots\right]$$

this is a geometric series

$$= 2 + \left[3 + 3 \cdot \tfrac{3}{4} + 3 \cdot \left(\tfrac{3}{4}\right)^2 + \dots\right]$$

$$= 2 + \sum_{n=1}^{\infty} 3\,(3/4)^{n-1}$$

recall: $\sum_{n=1}^{\infty} ar^{n-1}$

has $S_n = \dfrac{a(1-r^n)}{1-r}$

so for $n \to \infty$ we have

$$= 2 + \frac{3}{1-(3/4)}$$

$$= 2 + 12 = 14\text{ m}$$

$$\sum_{n=2}^{\infty} \frac{1}{n \ln n} \qquad \text{find conv/div}$$

Consider $f(x) = \dfrac{1}{x \ln x}$

$$\int_2^{\infty} \frac{1}{x\ln x}\,dx = \lim_{t\to\infty} \int_2^t \frac{1}{x \ln x}\,dx \qquad \begin{aligned} u &= \ln x \\ du &= x^{-1}dx \end{aligned}$$

$$= \lim_{t\to\infty} \int_{\ln 2}^{t} u^{-1}\,du$$

$$= \lim_{t\to\infty} \ln u \Big|_{\ln 2}^{t} = \infty \quad \therefore \text{ diverges}$$

Determine conv/div

$$\sum_{n=1}^{\infty} \frac{n^2+2}{n^4+5}$$

$$< \sum_{n=1}^{\infty} \frac{n^2+2}{n^4 \cancel{+5}} = \sum_{n=1}^{\infty} \frac{1}{n^2} + \sum_{n=1}^{\infty} \frac{2}{n^4}$$

$\uparrow$ conv $\qquad \uparrow$ conv

$\therefore \sum_{n=1}^{\infty} \frac{n^2+2}{n^4+5}$ conv by comparison test.

Determine conv/div

$$\sum_{n=0}^{\infty} \frac{1}{3^n-n}$$

Consider $\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n$ a conv geo seri

$$\lim_{n\to\infty} \frac{(1/3)^n}{\frac{1}{3^n-n}}$$

$$= \lim_{n\to\infty} \frac{3^n-n}{3^n}$$

$$= \lim_{n\to\infty} 1 - \frac{n}{3^n}$$

$$= 1 - \lim_{n\to\infty} \frac{n}{3^n}$$

$$= 1 - \lim_{n\to\infty} \frac{1}{3^n \ln 3} = 1$$

$$\cancel{\lim_{n\to\infty} \frac{1/(3^n-n)}{1/3^n} = \lim_{n\to\infty} \frac{3^n}{3^n-n}}$$

$\therefore$ by limit compa test both series converge.