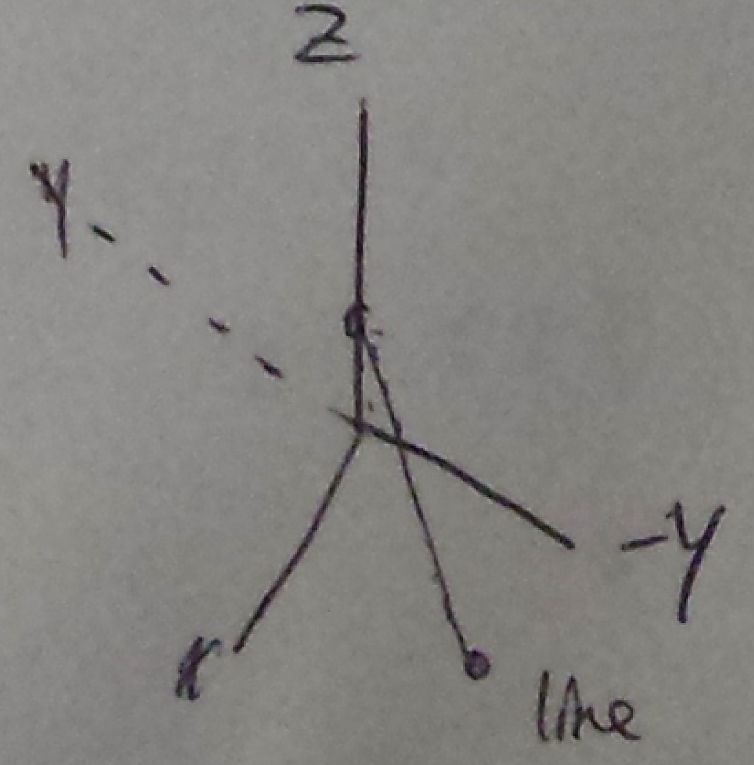
Pg 629, 5. $\vec{r} = \begin{bmatrix} t^2 \\ -t^2 \\ 1 \end{bmatrix}$ $\quad \vec{v} = \begin{bmatrix} 2t \\ -2t \\ 0 \end{bmatrix}$ $\quad \vec{a} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$

$$|\vec{v}| = \sqrt{(2t)^2 + (2t)^2}$$
$$= \sqrt{8}\,t$$



$$\boxed{\begin{matrix} x = -y & x > 1 \\ z = 1 & -y > 1 \end{matrix}}$$

13. $\vec{r} = \begin{bmatrix} e^{-t}\cos(e^t) \\ e^{-t}\sin e^t \\ -e^t \end{bmatrix}$ $\quad \vec{v} = \begin{bmatrix} -e^{-t}\cos(e^t) + \cancel{e^{-t}e^{t}}\sin(e^t) \\ -e^{-t}\sin(e^t) + \cos(e^t) \\ -e^t \end{bmatrix}$

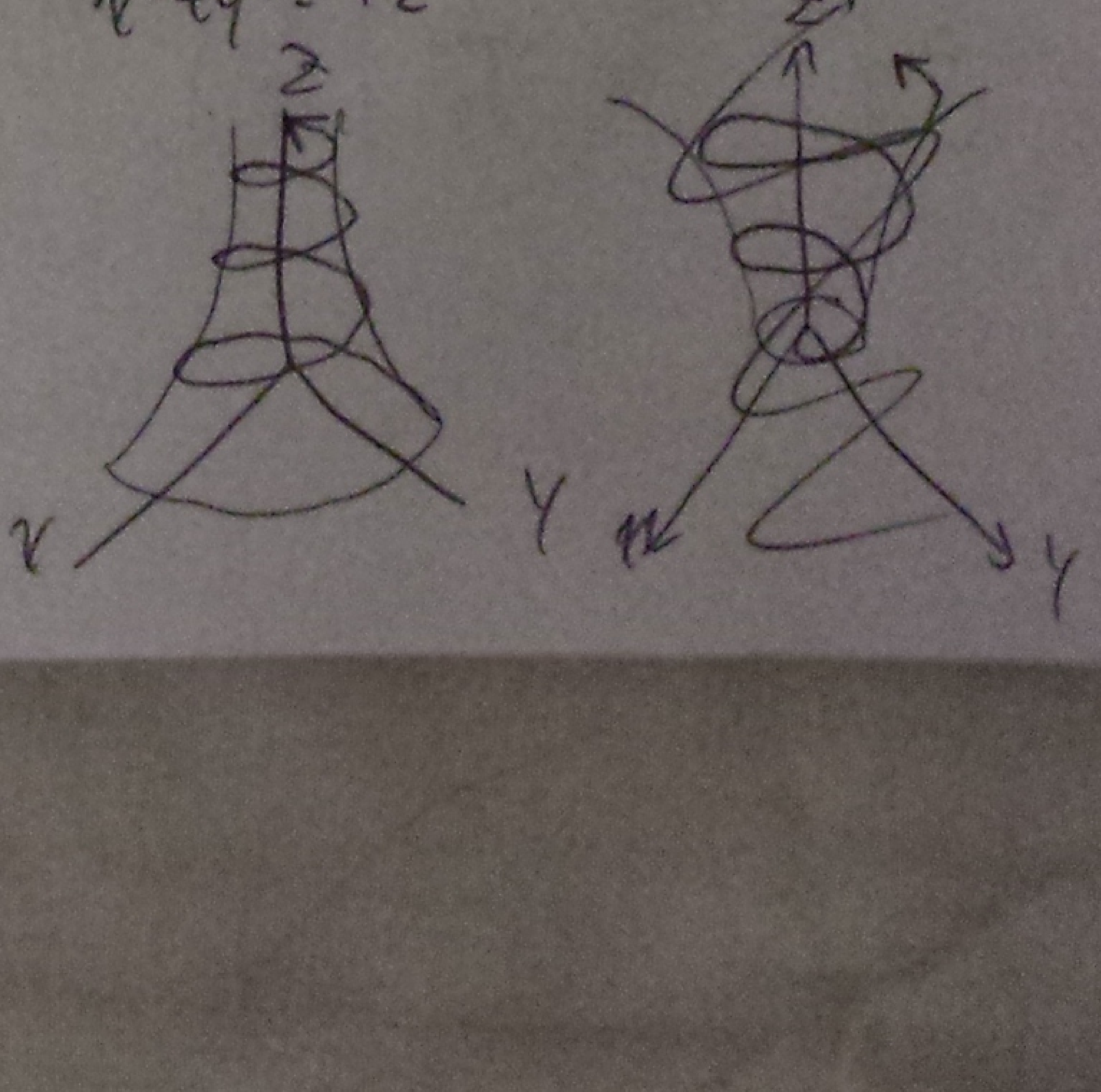
$$|\vec{v}| = \sqrt{e^{-2t}\cos^2 e^t \cancel{\pm 2e^{-t}\sin(e^t)\cos(e^t)} + \sin^2 e^t + e^{-2t}\sin^2 e^t \cancel{- 2e^{-t}\cos e^t \sin e^t} + \cos^2 e^t + e^{2t}}$$

$$|\vec{v}| = \sqrt{e^{-2t} + e^{2t} + 1}$$

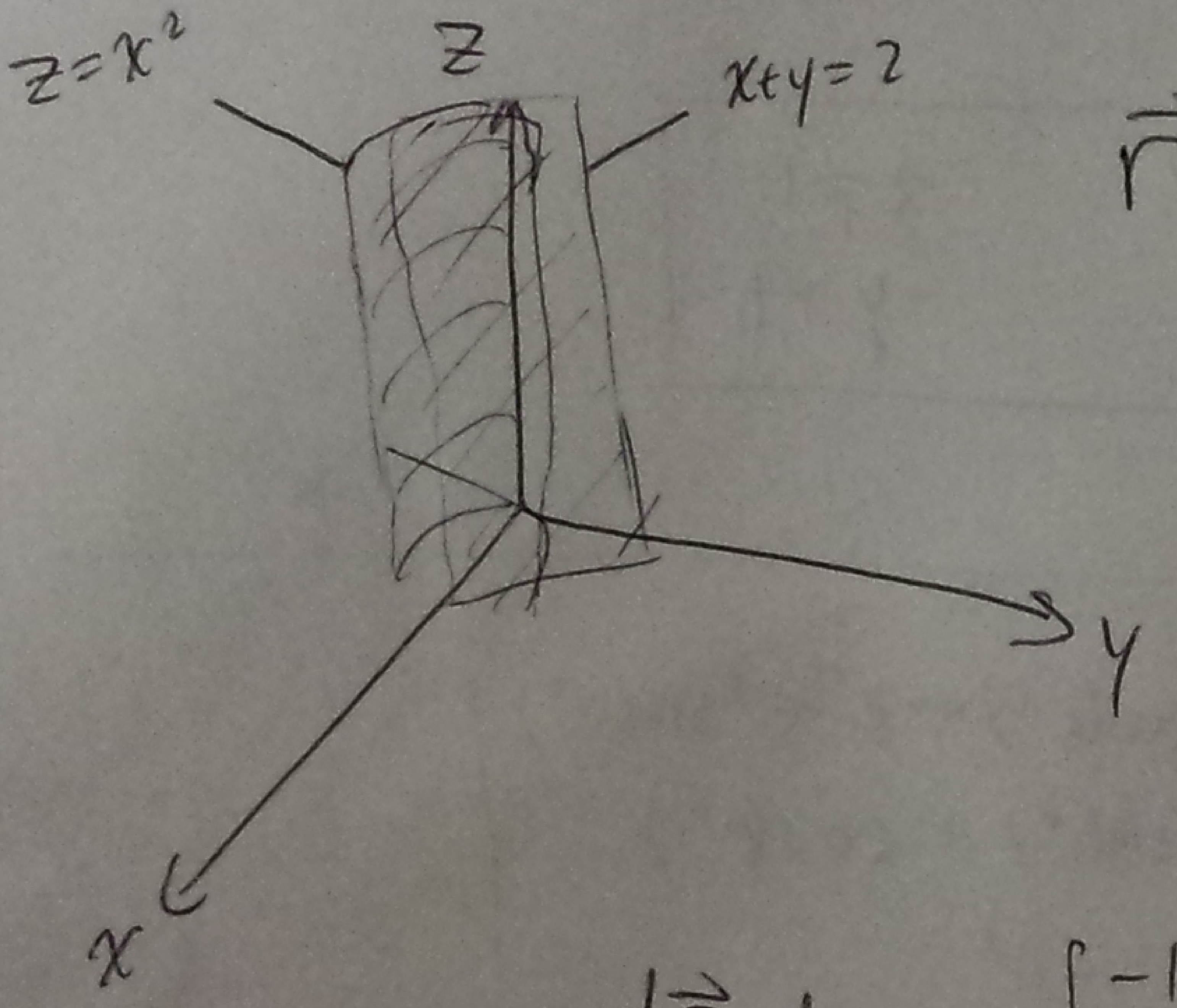
imagine $x^2 z^2 = 1$ then rotate the 360°

$$\vec{a} = \begin{bmatrix} e^{-t}\cos e^t + \sin e^t + e^t \cos e^t \\ e^{-t}\sin e^t - \cos e^t - e^t \sin e^t \\ -e^t \end{bmatrix}$$

$x^2 + y^2 = +z^{-2}$



17. a point P moves along the curve of intersection of the cylinder $z=x^2$ and the plane $x+y=2$ in the direction of increasing y w/ constant speed $|\vec{v}|=3$. Find the velocity of P when at (1,1,1).



$$\vec{r} = \begin{bmatrix} 2-y \\ y \\ (2-y)^2 \end{bmatrix}$$

$$\vec{v}(t) = \frac{d\vec{r}}{dy}\frac{dy}{dt} = \begin{bmatrix} -1 \\ 1 \\ -2(2-y) \end{bmatrix} \frac{dy}{dt}$$

sub $y=1$

$$\vec{v} = \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix} \frac{dy}{dt} \qquad |\vec{v}| = 3 = \frac{dy}{dt}\sqrt{1+1+4} = \frac{dy}{dt}\sqrt{6}$$

$$\frac{dy}{dt} = 3/\sqrt{6}$$

$$\therefore \left.\vec{v}\right|_{(1,1,1)} = \frac{3}{\sqrt{6}} \begin{bmatrix} -1 \\ 1 \\ -2 \end{bmatrix}$$

P 643, 5.

$Z = x^2$, $Z = 4y^2$, passes through $(2, -1, 4)$, $t = y$ as a parameter

$\boxed{Z = 4y^2}$

$4y^2 = x^2$

$x = \pm 2y$, but we have $(2, -1, 4)$

so $x = -2y$.

$\therefore \quad \vec{r}(t) = \begin{bmatrix} -2t \\ t \\ 4t^2 \end{bmatrix}$

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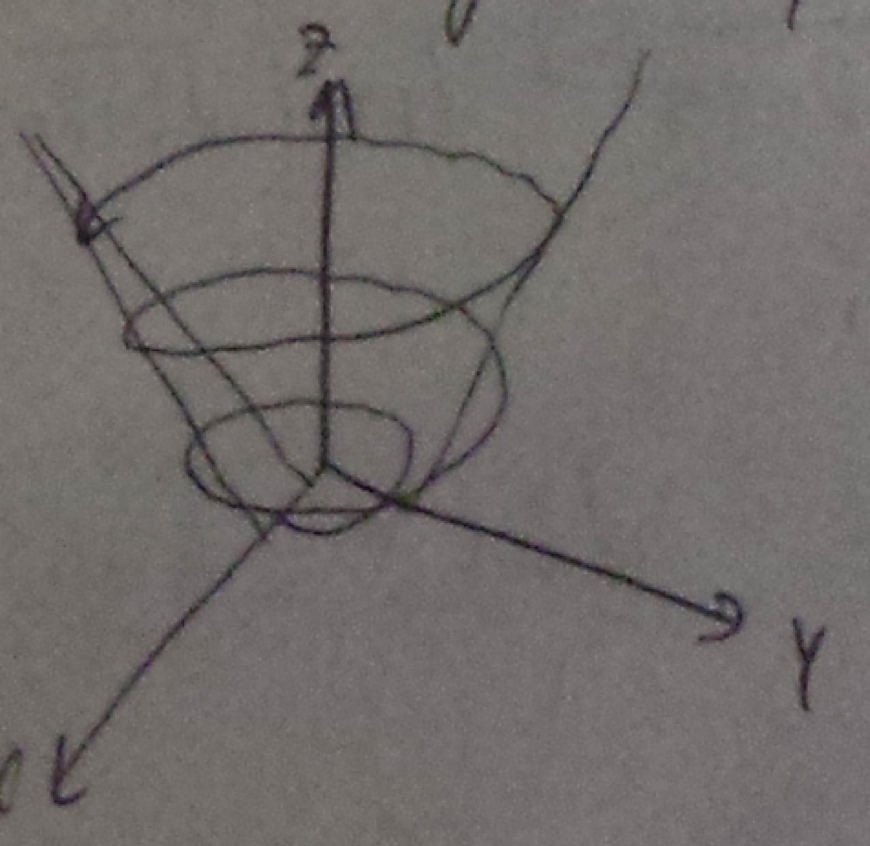
17. Find the length of the conical helix given by

$\vec{r} = \begin{bmatrix} t\cos t \\ t\sin t \\ t \end{bmatrix} \quad (0 \le t \le 2\pi)$



$\vec{v} = \begin{bmatrix} \cos t - t\sin t \\ \sin t + t\cos t \\ 1 \end{bmatrix}$

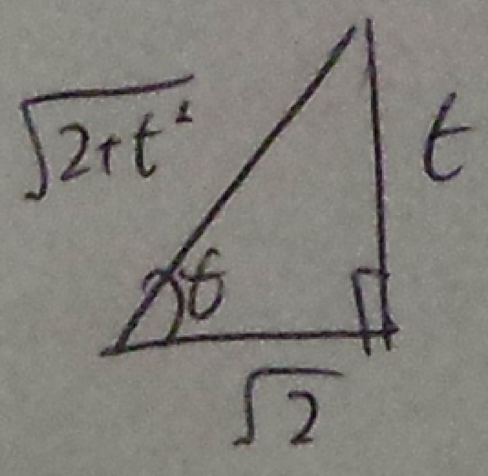
$|\vec{v}| = \sqrt{\cos^2 t - \cancel{2t\cos t\sin t} + t^2\sin^2 t + \sin^2 t + \cancel{2t\cos t\sin t} + t^2\cos^2 t + 1}$

$|\vec{v}| = \sqrt{2 + t^2}$

$\longrightarrow$

$|\vec{v}| = \sqrt{2+t^2}$

$L = \int_0^{2\pi} \sqrt{2+t^2}\, dt$



$\tan\theta = \frac{t}{\sqrt{2}}$

$\sec\theta = \frac{\sqrt{2+t^2}}{\sqrt{2}}$

$= 2\int \sec^3\theta\, d\theta$

This will be given on the exam.

$\boxed{\sqrt{2+t^2} = \sqrt{2}\sec\theta}$

$= \left[\sec\theta\tan\theta + \ln(\sec\theta + \tan\theta)\right]$

$t = \sqrt{2}\tan\theta$

$\boxed{dt = \sqrt{2}\sec^2\theta\, d\theta}$

$= \frac{\sqrt{2+t^2}}{\sqrt{2}} \cdot \frac{t}{\sqrt{2}} + \ln\left(\frac{\sqrt{2+t^2}}{\sqrt{2}} + \frac{t}{\sqrt{2}}\right)\Big|_0^{2\pi}$

$= \frac{2\pi\sqrt{2+4\pi^2}}{2} + \ln \frac{\sqrt{2+4\pi^2}+2\pi}{\sqrt{2}}$

$= \pi\sqrt{2+4\pi^2} + \ln\left(\sqrt{1+2\pi^2} + \sqrt{2}\pi\right)$ units

- recall $\text{dist} = \int_0^t v(t)\, dt$

- proof for $\int \sec^3 x\, dx$ on next page.

$\int \sec^3 x \, dx$

First we need $\int \sec x \, dx$

$= \int \sec x \frac{\sec x + \tan x}{\sec x + \tan x} dx$ $\qquad u = \sec x + \tan x$

$\qquad du = \sec x \tan x + \sec^2 x \, dx$

$= \int u^{-1} du$

$= \ln u + c$

$= \ln|\sec x + \tan x| + c$ $\qquad$ QED.

OK, $\underset{u \; dv}{\int \sec^3 x \, dx}$ $\qquad u = \sec x$ $\qquad v = \tan x$

$\qquad du = \sec x \tan x \, dx$ $\qquad dv = \sec^2 x \, dx$

$= \underset{uv \; - \; \int v du}{\sec x \tan x - \int \tan^2 x \sec x \, dx}$

$\cos^2 x + \sin^2 x = 1$

$= \sec x \tan x - \int \sec^3 x - \sec x \, dx$ $\qquad 1 + \tan^2 x = \sec^2 x$

$\qquad \tan^2 x = \sec^2 x - 1$

$= \sec x \tan x - \int \sec^3 x \, dx + \int \sec x \, dx$

$= -\int \sec^3 x \, dx + \sec x \tan x + \ln|\sec x + \tan x| + c$

$= \frac{1}{2}[\sec x \tan x + \ln|\sec x + \tan x|] + c$