Pg689, 13. Find equations of the tangent plane and normal line to the graph $f(x,y) = x^2 - y^2$ @ $(-2,1)$.

$$\frac{\partial f}{\partial x} = 2x \qquad \frac{\partial f}{\partial x}(-2,1) = -4$$

$$\frac{\partial f}{\partial y} = -2y \qquad \frac{\partial f}{\partial y}(-2,1) = -2$$

recall:

$$z = \underset{\uparrow}{f(a,b)} + \underbrace{f_x(a,b)(x-a) + f_y(a,b)(y-b)}_{\text{linear combination spans the plane}}$$

height of point

$$\therefore \quad z = 3 + -4(x+2) + -2(y-1)$$

$$z = -3 - 4x - 2y$$

recall:

$$\frac{x-a}{f_x(a,b)} = \frac{y-b}{f_y(a,b)} = \frac{z - f(a,b)}{-1}$$

$$\frac{x+2}{-4} = \frac{y-1}{-2} = \frac{z-3}{-1}$$

pg 689, 23. $z = x^4 - 4xy^3 + 6y^2 - 2$

$$\frac{\partial z}{\partial x} = 4x^3 - 4y^3 = 4(x-y)(x^2+xy+y^2)$$

$$\frac{\partial z}{\partial y} = -12xy^2 + 12y = 12y(1-xy)$$

We require $\frac{\partial z}{\partial x} = 0$ <u>and</u> $\frac{\partial z}{\partial y} = 0$ for horizontal tangent plane.

If we take $x = y \cup y = 0$ then we have (0

If we take $x = y \cup xy = 1$, then $x^2 = 1$,

then we have $(0,0), (1,1), (-1,1)$.

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pg 694, 9. $f(x,y) = 3x^2y - y^3$

we need $\nabla^2 f = 0$.

$$\frac{\partial f}{\partial x} = 6xy \quad \frac{\partial^2 f}{\partial x^2} = 6y \quad \frac{\partial f}{\partial y} = -3y^2 \quad \frac{\partial^2 f}{\partial y^2} = -6y$$

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = 6y - 6y = 0 \quad \text{QED}$$

pg 709, 3. Evaluate $\frac{\partial z}{\partial u}$ if $z = g(x,y)$, $y = f(x)$, $x = h(u,v)$.

$$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial y}\frac{\partial y}{\partial x} + \frac{\partial z}{\partial x}$$

$$= \frac{\partial z}{\partial y}\frac{\partial y}{\partial x}\frac{\partial x}{\partial u} + \frac{\partial z}{\partial x}\frac{\partial x}{\partial u}$$

15. $z = f(x,y)$, $x = 2s+3t$, $y = 3s-2t$

a) $$\frac{\partial^2 z}{\partial s^2} = \frac{\partial}{\partial s}\left[\frac{\partial z}{\partial s}\right] = \frac{\partial}{\partial s}\left[\frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s}\right]$$

$$= \frac{\partial}{\partial s}\left[\frac{\partial f(x,y)}{\partial x} 2 + \frac{\partial f(x,y)}{\partial y} 3\right]$$

$$= 2\frac{\partial^2 f}{\partial s \partial x} + 3\frac{\partial^2 f}{\partial s \partial y}$$

$$= 2\left[\frac{\partial^2 f}{\partial x \partial x}\frac{\partial x}{\partial s} + \frac{\partial^2 f}{\partial y \partial x}\frac{\partial y}{\partial s}\right] + 3\left[\frac{\partial^2 f}{\partial y \partial y}\frac{\partial y}{\partial s} + \frac{\partial^2 f}{\partial x \partial y}\frac{\partial x}{\partial s}\right]$$

$$= 4\frac{\partial^2 f}{\partial x^2} + 6\frac{\partial^2 f}{\partial y \partial x} + 9\frac{\partial^2 f}{\partial y^2} + 6\frac{\partial^2 f}{\partial x \partial y}$$

b) $\dfrac{\partial^2 z}{\partial s \partial t} = \dfrac{\partial}{\partial s}\left[\dfrac{\partial z}{\partial t}\right] = \dfrac{\partial}{\partial s}\left[\dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial t}\right]$

$= \dfrac{\partial}{\partial s}\left[\dfrac{\partial f}{\partial x}3 + \dfrac{\partial f}{\partial y}(-2)\right]$

$= 3\dfrac{\partial^2 f}{\partial x^2}\dfrac{\partial x}{\partial s} + 3\dfrac{\partial^2 f}{\partial x \partial y}\dfrac{\partial y}{\partial s} - 2\dfrac{\partial^2 f}{\partial y^2}\dfrac{\partial y}{\partial s} - 2\dfrac{\partial^2 f}{\partial x \partial y}\dfrac{\partial x}{\partial s}$

$\cancel{= 9\dfrac{\partial^2 f}{\partial x^2} - 6\dfrac{\partial^2 f}{\partial x \partial y} + 4\dfrac{\partial^2 f}{\partial y^2}}$

$= 6\dfrac{\partial^2 f}{\partial x^2} + 9\dfrac{\partial^2 f}{\partial x \partial y} - 6\dfrac{\partial^2 f}{\partial y^2} - 4\dfrac{\partial^2 f}{\partial x \partial y}$

$= 6\dfrac{\partial^2 f}{\partial x^2} + 5\dfrac{\partial^2 f}{\partial x \partial y} - 6\dfrac{\partial^2 f}{\partial y^2}$

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c) $\dfrac{\partial^2 z}{\partial t^2} = \dfrac{\partial}{\partial t}\left[\dfrac{\partial z}{\partial t}\right] = \dfrac{\partial}{\partial t}\left[\dfrac{\partial z}{\partial x}\dfrac{\partial x}{\partial t} + \dfrac{\partial z}{\partial y}\dfrac{\partial y}{\partial t}\right]$

$= \dfrac{\partial}{\partial t}\left[\dfrac{\partial f}{\partial x}3 + \dfrac{\partial f}{\partial y}(-2)\right]$

$= 3\dfrac{\partial^2 f}{\partial x^2}\dfrac{\partial x}{\partial t} + 3\dfrac{\partial^2 f}{\partial x \partial y}\dfrac{\partial y}{\partial t} - 2\dfrac{\partial^2 f}{\partial y^2}\dfrac{\partial y}{\partial t} - 2\dfrac{\partial^2 f}{\partial x \partial y}\dfrac{\partial x}{\partial t}$

$= 9\dfrac{\partial^2 f}{\partial x^2} - 6\dfrac{\partial^2 f}{\partial x \partial y} + 4\dfrac{\partial^2 f}{\partial y^2} - 6\dfrac{\partial^2 f}{\partial x \partial y}$

$= 9\dfrac{\partial^2 f}{\partial x^2} - 12\dfrac{\partial^2 f}{\partial x \partial y} + 4\dfrac{\partial^2 f}{\partial y^2}$