McGill University	April 2014
Faculty of Science	Final Examination

Calculus 3

Math 222

Tuesday April 29, 2014
Time: 14:00 - 17:00

Examiner: Dmitry Jakobson   Associate Examiner: Vojkan Jaksic

Student name (last, first)	**Student number (McGill ID)**

INSTRUCTIONS

1. Please write your answers clearly in the space provided.
2. This exam is a total of 60 marks.
3. This is a closed book exam.
4. Translation dictionary is permitted.
5. Non-programmable calculators are permitted.

This exam comprises the cover page, and 2 pages of questions.

Problem 1 (6 points)
Let $\mathbf{r}(t) = (t, \cos^2 t, \sin^2 t)$.

i. *Find the velocity* $\mathbf{r}'(t)$ *and the acceleration* $\mathbf{r}''(t)$.

ii. *Find the tangential and normal components of the acceleration*

Hint: *you can use the formulas* $\sin(2\alpha) = 2\sin\alpha \cdot \cos\alpha$ *and* $\cos(2\alpha) = \cos^2\alpha - \sin^2\alpha$.

Problem 2 (6 points)

i. *Write the Taylor series about* $x = 0$ *of* $\ln(1+x)$. *You do not need to justify your answer.*

ii. *Use part i. to write the Taylor series about* $x = 0$ *of* $\ln(1+x^3)$.

iii. *Write the Taylor series about* $x = 0$ *of* $f(x) = \int_0^x \ln(1+t^3)dt$.

Problem 3 (6 points)
Let $f(x, y, z) = x^2\cos(y)z^3$.

i. *Find the gradient* $\nabla f(p)$ *at the point* $p = (1, \pi, -1)$. *In which direction does the function* f *increase the most?*

ii. *Find the directional derivative* $D_u f(p)$ *where* u *is the vector* $(1, 2, 1)$.

iii. *Let* S *be the level surface of* f *passing through* $(1, \pi, -1)$. *Find an equation for the tangent plane to* S *at* $(1, \pi, -1)$.

Problem 4 (6 points)
Let x, y, z *satisfy an equation:* $x^2 - y^2 + z^2 - 2z = 4$. *Find* $\partial z/\partial x$ *and* $\partial z/\partial y$.

Problem 5 (6 points)
Find all local maxima, local minima, and saddle points of the function

$$f(x, y) = x\cos y - x^3/3$$

in the region $\{(x, y) : |y| < \pi\}$.

Problem 6 (6 points)
Use the method of **Lagrange multipliers** *to find the minimum value of the function*

$$f(x, y) = (x^3 + y^3)/3$$

subject to the constraint $g(x, y) = xy = 4$.

Problem 7 (6 points)
Use **polar coordinates** *to find the volume of the region bounded by the paraboloids* $z = 3x^2 + 3y^2$ *and* $z = 4 - x^2 - y^2$.

Problem 8 (6 points)
Let D be the bounded region of the plane which is enclosed by the curves $y = 0, y = x^2$ and $x = 1$. Evaluate the following double integral:

$$\iint_D x \sin y \, dA.$$

Problem 9 (6 points)
Use **spherical coordinates** *to evaluate the triple integral*

$$\iiint_D xyz \, dV,$$

where D is the region lying between the spheres of radius $\rho = 2$ and $\rho = 4$, and above the cone $\phi = \pi/3$.

Problem 10 (6 points)
Use the transformation $x = u/v, y = v$ to evaluate the double integral

$$\iint_D xy \, dA,$$

where D is the region in the first quadrant bounded by the lines $y = x$ and $y = 3x$ and the hyperbolas $xy = 1$ and $xy = 3$.