Bayesian Fama-French Portfolio Analysis

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Background and Motivation

- Interested in analyzing portfolio excess returns (alpha) as a Bayesian
- We use the Fama-French 5-factor model including long-term reversals and momentum
- Some limitations of Fama-French
  - Difficult to find statistically significant alpha for daily returns
  - Monthly returns generally require long lookback periods to ensure enough data points
- In literature, it is common to hear that small-cap, high-value portfolios tend to display significant alpha
- Our goal is to perform a more in-depth Bayesian analysis of alpha for a small-cap, high-value portfolio over the past 10 years (only 120 data points monthly)

Fama-French 5-Factor Model

$$R_{it} - R_{ft} = \alpha_{it} + \beta_1(R_{mt} - R_{ft}) + \beta_2 SML_t + \beta_3 VMG_t + \beta_4 RMW_t + \beta_5 CMA_t + \epsilon_t$$

Where,

R_{it} = total return of a portfolio i at time t

R_{ft} = risk free rate of return at time t

R_{mt} = total market portfolio return at time t

SML_t = size premium (small minus large)

VMG_t = value premium (value minus growth)

RMW_t = profitability premium reclassified as the quality premium

CMA_t = investment conservatism premium reclassified as the momentum premium

$\beta_{1,2,3,4,5}$ = factor coefficients

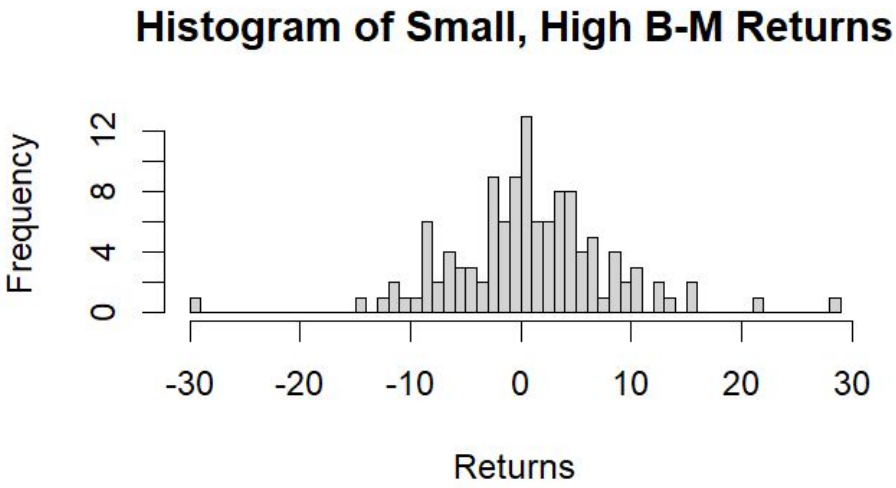
* Note in our data HML (high minus low) is the same as VMG here

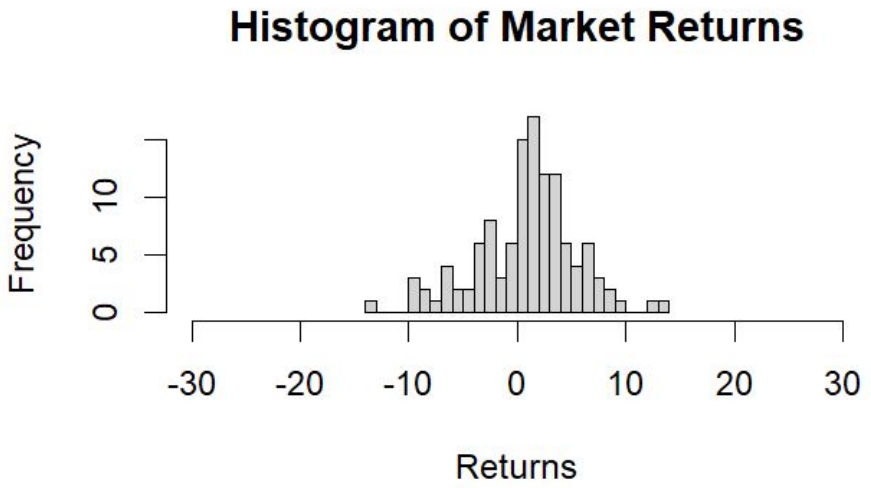
Data Retrieval

- All data is retrieved from the Kenneth French website
- The small, high book-to-market portfolio is selected due to its greater tendency to showcase significant positive alpha
- Returns and factors are monthly

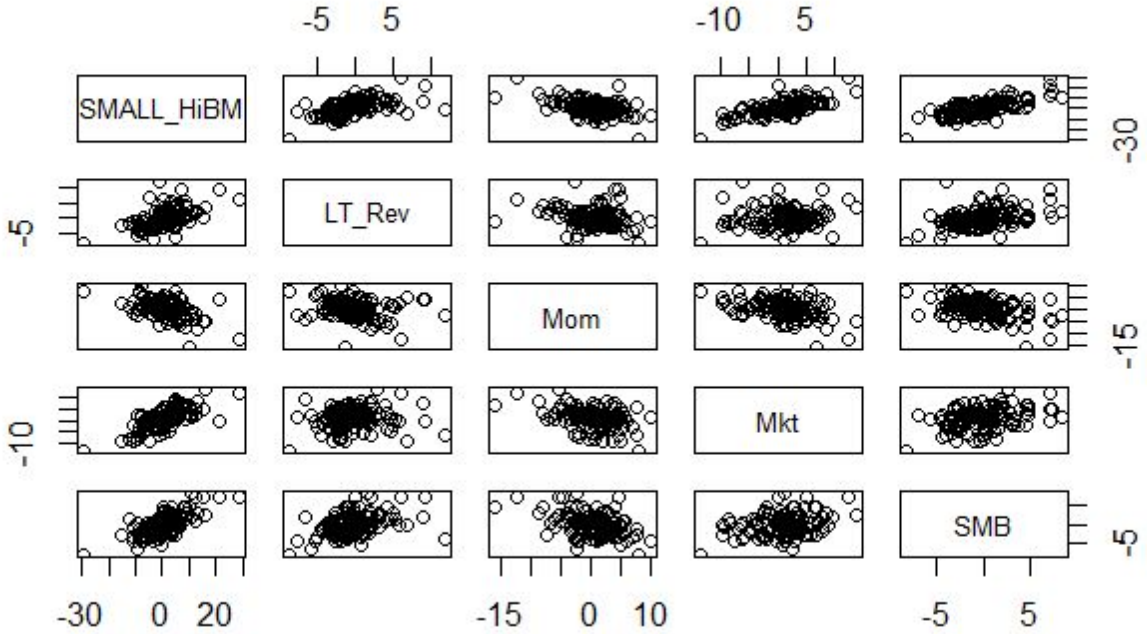
	Date	SMALL_HiBM	LT_Rev	Mom	Mkt	SMB	HML	RMW	CMA
1	2015-01	-3.3593	-3.34	3.84	-3.11	-0.92	-3.59	1.61	-1.65
2	2015-02	3.6986	-2.04	-2.82	6.13	0.32	-1.86	-1.12	-1.82
3	2015-03	1.7875	-2.52	2.74	-1.12	3.07	-0.38	0.09	-0.52

Data Exploration: Distribution

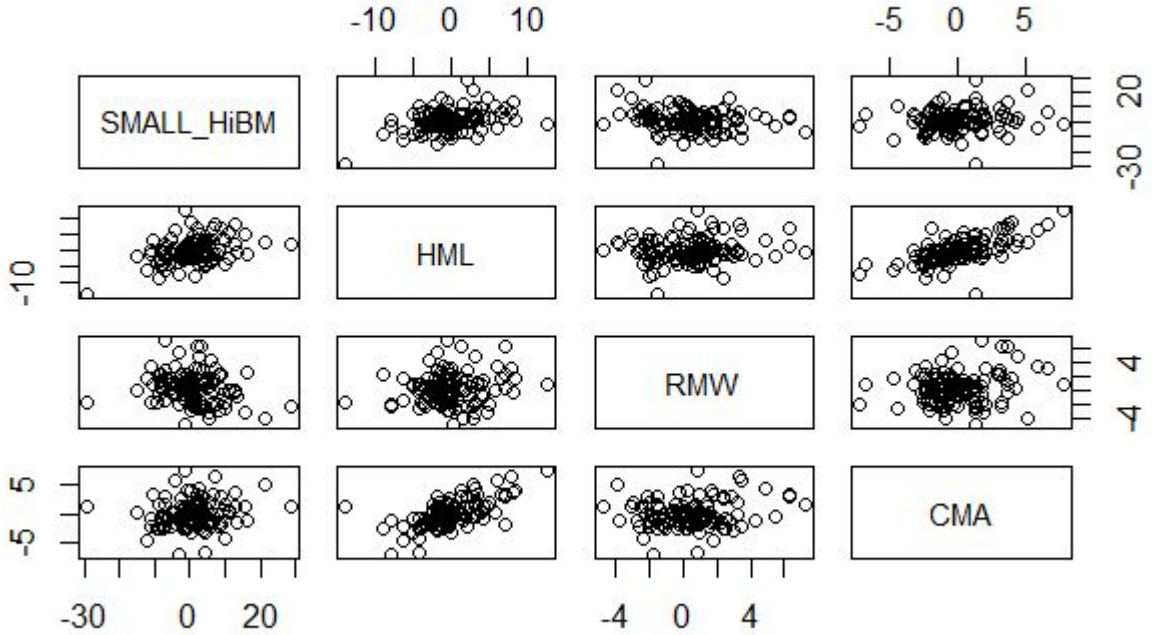




Data Exploration: Pairs Plot



Data Exploration: Pairs Plot Continued



Outline

- Provide frequentist linear regression as baseline
- Use Stochastic Search Variable Selection (SSVS) in Bayesian linear regression
- Investigate time-varying alpha by implementing a random walk for the alpha prior
- Bayesian LASSO is adopted whenever possible (double exponential beta priors) for more robust alpha estimates
- Time-varying beta is considered (random walk for beta priors)
- K-means clustering is applied to separate data into 4 types of market regimes and then random effects for the regimes are added

Frequentist Linear Model and SSVS

	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	0.30491	0.25503	1.196	0.234394
LT_Rev	0.47845	0.12776	3.745	0.000287***
Mom	-0.07107	0.07403	-0.960	0.339081
Mkt	0.89493	0.06269	14.275	< 2e-16***
SMB	1.04225	0.11116	9.376	9.12e-16***
HML	0.02645	0.10728	0.247	0.805700
RMW	-0.09605	0.14010	-0.686	0.494387
CMA	0.33589	0.16535	2.031	0.044589*

Residual standard error: 2.658 on 112 degrees of freedom

Multiple R-squared: 0.8761, Adjusted R-squared: 0.8684

F-statistic: 113.2 on 7 and 112 DF, p-value: < 2.2e-16

	Inc_Prob	50%	5%	95%
alpha	1.00	0.27	-0.14	0.69
LT_Rev	1.00	0.58	0.35	0.78
Mom	0.12	0.00	-0.09	0.00
Mkt	1.00	0.88	0.79	0.98
SMB	1.00	1.06	0.90	1.22
HML	0.14	0.00	0.00	0.12
RMW	0.14	0.00	-0.11	0.00
CMA	0.61	0.18	0.00	0.51

Models Investigated

- Linear Model after SSVS
- Time-Varying Alpha LASSO Model
- Time-Varying Alpha and Beta Model
- Time-Varying Alpha LASSO Model with Regime Random Effects
- Time-Varying Alpha and Beta Model with Regime Random Effects

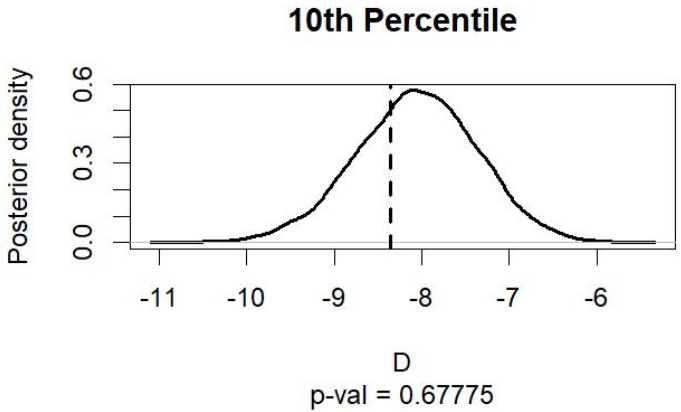
Market Regime Random Effects

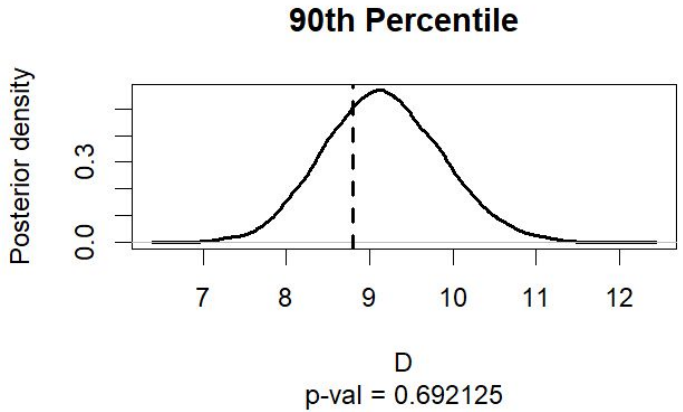
- Applied K-means clustering to separate data into 4 market regimes
- The features used to cluster the data are derived from the market factor:
  - Current market return
  - 3-month rolling market volatility
  - 3-month momentum
- Market regimes:
  - Volatile bull
  - Non-volatile bull
  - Volatile bear
  - Non-volatile bear

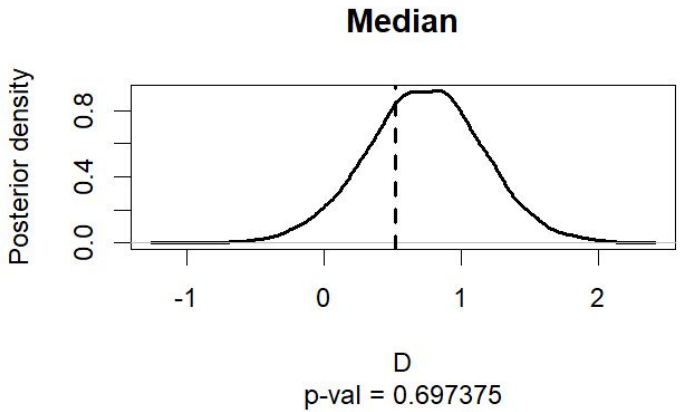
Model Comparison

Model	Penalty term for Watanabe-Akaike Information Criterion (WAIC)	Watanabe-Akaike Information Criterion
Linear with SSVS	8.058412	583.1164
Time-Varying Alpha LASSO	14.96284	584.2297
Time-Varying Alpha and Beta	47.097	563.321
Time-Varying Alpha LASSO with Regime Random Effects	16.432	574.651
Time-Varying Alpha and Beta with Regime Random Effects	48.02771	538.1327

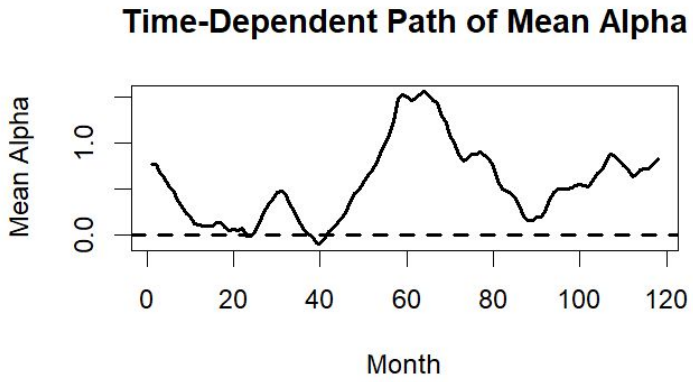
Selected Model: Goodness of Fit

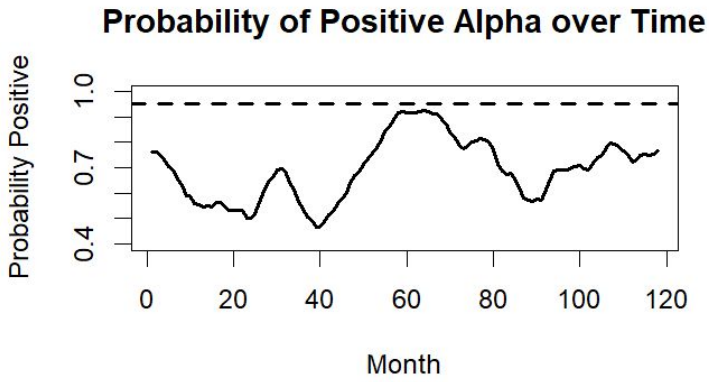


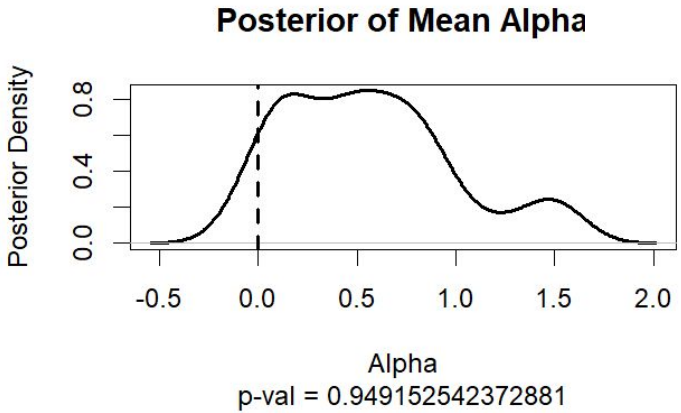




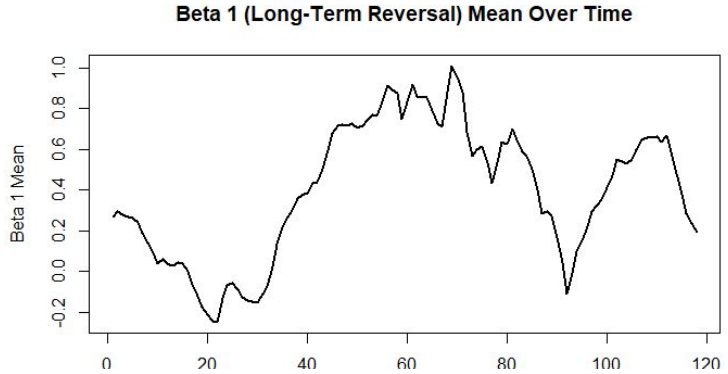
Selected Model: Alpha Analysis

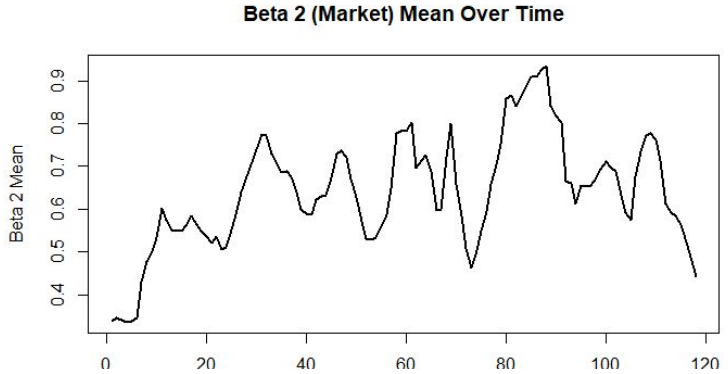


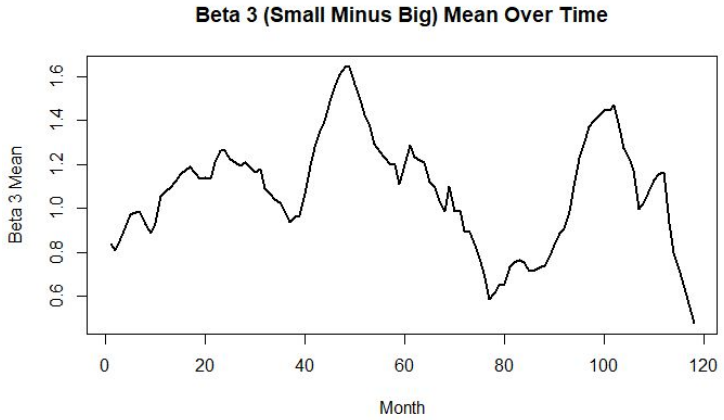


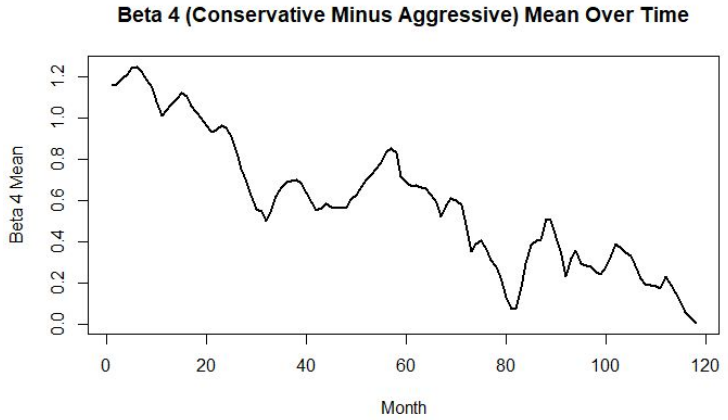


Selected Model: Beta Analysis









Implications

- Portfolio exposures are not constant over time and neither are excess returns
  - Portfolio exposures are likely mean-reverting
- Our Bayesian time-varying analysis provides more valuable insight
  - Probability of positive alpha for each month
  - Posterior distribution of alpha
  - Time-dependent path of alpha on average

Further Research

- Although Fama-French factors tend to display linear relationships, adding polynomial or interaction terms could be a logical extension
- Daily returns could be analyzed instead of monthly returns
- Ornstein-Uhlenbeck process for betas since they appear mean-reverting
- More informative beta priors; e.g. Normal(1, 0.01) for the market factor
- Time-series analysis of alpha path; e.g. ARIMA to forecast alpha or other models to conduct stress tests on alpha
- Hidden Markov Model (HMM) could be trained on past data to determine market regimes

Thank you!