

3. The time/space complexity of Bubble Sort is simple. Because the algorithm uses nested for-loops to help place everything, the time is very slow for Bubble Sort having  $O(n^2)$  as it's runtime. This means that Bubble Sort is effectively slower than Merge Sort or other sorts similar that use  $O(n \log n)$  runtimes. Bubble Sort's space uses 3 different variables, being the 2 looping variables and a Boundary variable. This means that effectively, Bubble Sort does not require a large amount of registers to be available, making it one of the least space required sorts. Also, because the algorithm just swaps positions one-by-one it is also a stable form of sorting. Overall, given the poor run-time, but low amount of required space. Bubble Sort could be most effectively used with a small amount of data. It's lack of moving parts allows for smaller arrays to quickly be sorted into either ascending or descending order without having a large algorithm to complete each step, and a lack of recursion to keep testing until the array returns sorted.

4.

| Line Number | Code                               | Actual Test                   | Variables   |
|-------------|------------------------------------|-------------------------------|---|
|             |                                    |                               | A = [13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11   |
| 2           | P = A[left] q = A[right]           | P = 13, Q = 21                | A = [13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21                             |
| 3           | If p>q Swap p and q                | 13>21                         | A = [13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21                             |
| 4           | L = left+1<br>G = right-1<br>k = l | L = 0 +1<br>G = 11-1<br>K = 1 | A = [13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 1<br>G = 10<br>K = 1 |
| 5           | While k <= g                       | 1<10                          | A = [13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13                                       |

|    |              |       |   |
|----|--------------|-------|---|
|    |              |       | Q = 21<br>L = 1<br>G = 10<br>K = 1  |
| 6  | If A[k] < p  | 19<13 | A = [13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 1<br>G = 10<br>K = 1 |
| 9  | else         | else  | A = [13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 1<br>G = 10<br>K = 1 |
| 10 | If A[k] > q  | 19>21 | A = [13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 1<br>G = 10<br>K = 1 |
| 20 | K = k+1      | K = 2 | A = [13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 1<br>G = 10<br>K = 2 |
| 5  | While k <= g | 2<=10 | A = [13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13                                       |

|    |                |                       |   |
|----|----------------|-----------------------|---|
|    |                |                       | Q = 21<br>L = 1<br>G = 10<br>K = 2  |
| 6  | If (A[k]<p)    | 9<13                  | A = [13, 19, 9, 5, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 1<br>G = 10<br>K = 2 |
| 7  | Swap A[k] A[L] | A[2] = 19<br>A[1] = 9 | A = [13, 9, 19, 5, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 1<br>G = 10<br>K = 2 |
| 8  | L = l+1        | L = 2                 | A = [13, 9, 19, 5, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 2<br>G = 10<br>K = 2 |
| 20 | K = k+1        | K = 3                 | A = [13, 9, 19, 5, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 2<br>G = 10<br>K = 3 |
| 5  | While k<=g     | 3<=10                 | A = [13, 9, 19, 5, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13                                       |

|    |                |                       |   |
|----|----------------|-----------------------|---|
|    |                |                       | Q = 21<br>L = 2<br>G = 10<br>K = 3  |
| 6  | If A[k] < p    | 5<13                  | A = [13, 9, 19, 5, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 2<br>G = 10<br>K = 3 |
| 7  | Swap A[k] A[L] | A[3] = 19<br>A[2] = 5 | A = [13, 9, 5, 19, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 2<br>G = 10<br>K = 3 |
| 8  | L = l+1        | L = 3                 | A = [13, 9, 5, 19, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 3<br>G = 10<br>K = 3 |
| 20 | K = k+1        | K = 4                 | A = [13, 9, 5, 19, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 3<br>G = 10<br>K = 4 |
| 5  | While k<=g     | 4<=10                 | A = [13, 9, 5, 19, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13                                       |

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|----|----------------|------------------------|---|
|    |                |                        | Q = 21<br>L = 3<br>G = 10<br>K = 4  |
| 6  | If (A[k] < p)  | 12<13                  | A = [13, 9, 5, 19, 12, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 3<br>G = 10<br>K = 4 |
| 7  | Swap A[k] A[l] | A[4] = 19<br>A[3] = 12 | A = [13, 9, 5, 12, 19, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 3<br>G = 10<br>K = 4 |
| 8  | L = l+1        | L = 4                  | A = [13, 9, 5, 12, 19, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 4<br>G = 10<br>K = 4 |
| 20 | K = k+1        | K = 5                  | A = [13, 9, 5, 12, 19, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 4<br>G = 10<br>K = 5 |
| 5  | While k<=g     | 5<=10                  | A = [13, 9, 5, 12, 19, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13                                       |

|    |                |                       |   |
|----|----------------|-----------------------|---|
|    |                |                       | Q = 21<br>L = 4<br>G = 10<br>K = 5  |
| 6  | If (A[k] < p)  | 8 < 13                | A = [13, 9, 5, 12, 19, 8, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 4<br>G = 10<br>K = 5 |
| 7  | Swap A[k] A[l] | A[5] = 19<br>A[4] = 8 | A = [13, 9, 5, 12, 8, 19, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 4<br>G = 10<br>K = 5 |
| 8  | L = l+1        | L = 5                 | A = [13, 9, 5, 12, 8, 19, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 5<br>G = 10<br>K = 5 |
| 20 | K = k+1        | K = 6                 | A = [13, 9, 5, 12, 8, 19, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 5<br>G = 10<br>K = 6 |
| 5  | While k <= g   | 6 <= 10               | A = [13, 9, 5, 12, 8, 19, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13                                       |

|    |                |                       |   |
|----|----------------|-----------------------|---|
|    |                |                       | Q = 21<br>L = 5<br>G = 10<br>K = 6  |
| 6  | If (A[k] < p)  | 7 < 13                | A = [13, 9, 5, 12, 8, 19, 7, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 5<br>G = 10<br>K = 6 |
| 7  | Swap A[k] A[l] | A[6] = 19<br>A[5] = 7 | A = [13, 9, 5, 12, 8, 7, 19, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 5<br>G = 10<br>K = 6 |
| 8  | L = l+1        | L = 6                 | A = [13, 9, 5, 12, 8, 7, 19, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 6<br>G = 10<br>K = 6 |
| 20 | K = k+1        | K = 7                 | A = [13, 9, 5, 12, 8, 7, 19, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 6<br>G = 10<br>K = 7 |
| 5  | While k <= g   | 7 <= 10               | A = [13, 9, 5, 12, 8, 7, 19, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13                                       |

|    |                |                       |   |
|----|----------------|-----------------------|---|
|    |                |                       | Q = 21<br>L = 6<br>G = 10<br>K = 7  |
| 6  | If (A[k] < p)  | 4<13                  | A = [13, 9, 5, 12, 8, 7, 19, 4, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 6<br>G = 10<br>K = 7 |
| 7  | Swap A[k] A[l] | A[7] = 19<br>A[6] = 4 | A = [13, 9, 5, 12, 8, 7, 4, 19, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 6<br>G = 10<br>K = 7 |
| 8  | L = l+1        | L = 7                 | A = [13, 9, 5, 12, 8, 7, 4, 19, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 7<br>G = 10<br>K = 7 |
| 20 | K = k+1        | K = 8                 | A = [13, 9, 5, 12, 8, 7, 4, 19, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 7<br>G = 10<br>K = 8 |
| 5  | While k<=g     | 8<=10                 | A = [13, 9, 5, 12, 8, 7, 4, 19, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13                                       |



|    |                |                        |   |
|----|----------------|------------------------|---|
|    |                |                        | Q = 21<br>L = 7<br>G = 10<br>K = 8  |
| 6  | If (A[k] < p)  | 11<13                  | A = [13, 9, 5, 12, 8, 7, 4, 19, 11, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 7<br>G = 10<br>K = 8 |
| 7  | Swap A[k] A[l] | A[8] = 19<br>A[7] = 11 | A = [13, 9, 5, 12, 8, 7, 4, 11, 19, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 7<br>G = 10<br>K = 8 |
| 8  | L = l+1        | L = 8                  | A = [13, 9, 5, 12, 8, 7, 4, 11, 19, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 8<br>G = 10<br>K = 8 |
| 20 | K = k+1        | K = 9                  | A = [13, 9, 5, 12, 8, 7, 4, 11, 19, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 8<br>G = 10<br>K = 9 |
| 5  | While k<=g     | 9<=10                  | A = [13, 9, 5, 12, 8, 7, 4, 11, 19, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13                                       |

|    |                |                       |  |
|----|----------------|-----------------------|--|
|    |                |                       | Q = 21<br>L = 8<br>G = 10<br>K = 9   |
| 6  | If (A[k] < p)  | 2<13                  | A = [13, 9, 5, 12, 8, 7, 4, 11, 19, 2, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 8<br>G = 10<br>K = 9  |
| 7  | Swap A[k] A[l] | A[9] = 19<br>A[8] = 2 | A = [13, 9, 5, 12, 8, 7, 4, 11, 2, 19, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 8<br>G = 10<br>K = 9  |
| 8  | L = l+1        | L = 9                 | A = [13, 9, 5, 12, 8, 7, 4, 11, 2, 19, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 9<br>G = 10<br>K = 9  |
| 20 | K = k+1        | K = 10                | A = [13, 9, 5, 12, 8, 7, 4, 11, 2, 19, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 9<br>G = 10<br>K = 10 |
| 5  | While k<=g     | 10<=10                | A = [13, 9, 5, 12, 8, 7, 4, 11, 2, 19, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13  |

|    |                |                        |   |
|----|----------------|------------------------|---|
|    |                |                        | Q = 21<br>L = 9<br>G = 10<br>K = 10   |
| 6  | If (A[k] < p)  | 6<13                   | A = [13, 9, 5, 12, 8, 7, 4, 11, 2, 19, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 9<br>G = 10<br>K = 10  |
| 7  | Swap A[k] A[l] | A[10] = 19<br>A[9] = 6 | A = [13, 9, 5, 12, 8, 7, 4, 11, 2, 6, 19, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 9<br>G = 10<br>K = 10  |
| 8  | L = l+1        | L = 10                 | A = [13, 9, 5, 12, 8, 7, 4, 11, 2, 6, 19, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 10<br>G = 10<br>K = 10 |
| 20 | K = k+1        | K = 11                 | A = [13, 9, 5, 12, 8, 7, 4, 11, 2, 19, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 9<br>G = 10<br>K = 11  |
| 5  | While k<=g     | 11<=10                 | A = [13, 9, 5, 12, 8, 7, 4, 11, 2, 19, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13   |

|    |                    |                 |  |
|----|--------------------|-----------------|--|
|    |                    |                 | Q = 21<br>L = 9<br>G = 10<br>K = 11  |
| 22 | L = l-1<br>g = g+1 | L = 8<br>G = 11 | A = [13, 9, 5, 12, 8, 7, 4, 11, 2, 19, 6, 21]<br>Left = 0<br>Right = 11<br>P = 13<br>Q = 21<br>L = 8<br>G = 11<br>K = 11 |