Notes 1

Yuhang Cai, Tianyi Wu

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Here we present some notes about the full model in [Li et al., 2019]. The model considers a population of neurons connected by local circuitry and the population contains N_E excitatory (E) neurons and N_I inhibitory (I) neurons.

1 Model description

Firstly we introduce the elements in our model.

- We assume that N_E excitatory neurons are labeled $1, 2, ..., N_E$ and N_I inhibitory neurons are labeled $N_E + 1, N_E + 2, ..., N_E + N_I$.
- The **membrane potential** of a neuron i, denoted V_i, takes values in

$$\Gamma := \{-M_r, -M_r + 1, \dots, -1, 0, 1, 2, \dots, M\} \cup \{\mathcal{R}\},$$

  where M represents the threshold for spiking, $-M_r$ represents the inhibitory reversal potential and $\mathcal{R}$ represents the refractory state.
- Each neuron receives synaptic input from an **external source** in the form of Poisson kicks; these kicks are independent from neuron to neuron.

Secondly, we introduce the connections and interactions between them.

- **External drive to neurons**: The action from external drive to neurons are inputs delivered in the form of impulsive kicks, arriving at random (Poissonian) times and the Poisson processes are independent from neuron to neuron. We assume there are two parameters $\lambda^E, \lambda^I > 0$, representing the rate of the Poisson kicks. When a kick arrives and $V_i \neq \mathcal{R}$, V_i jumps by 1.Kicks received by neuron i when $V_i = \mathcal{R}$ has no effect.
- **Spikes of neurons**: When the membrane potential of a neuron i reaches M, the neuron spikes immediately, goes to a refractory state and remains there for an exponential distributed random time with parameter $\tau_{\mathcal{R}}$.
- **Connections of neurons**: We assume the connectivity in our model is random and time-dependent, so that every time a neuron spikes, a random set of postsynaptic neurons is chosen anew. More precisely, for $Q, Q' \in \{E, I\}$, we let $P_{Q,Q'} \in [0, 1]$ be the probability that a neuron of type Q is postsynaptic when a neuron of type Q' spikes, and the set of postsynaptic neurons is determined by a coin flip with these probabilities following each spike.

- **Effects of kicks**: Firstly, we assume an kick received by an neuron takes effect at a random time after its arrival. This delay is given by an exponential random variable with mean τ^E for the excitatory kick and τ^I for the inhibitory kick. We let the H_i^E, H_i^I to denote the number of E-kicks and I-Kicks received by neuron i. Thus the state of neuron i at any moment in time can be described by the triplet (V_i, H_i^E, H_i^I).

  Then we will introduce the effects of two different kicks.

  - **Effect of E-kicks**: Each E-kick received by neuron i carries an independent exponential clock. When this clock rings, if $V_i = \mathcal{R}$, then it's unchanged. If $V_i \neq \mathcal{R}$, V_i is modified instaneously according to $S_{Q,Q'}$, $Q, Q' \in \{E, I\}$, where $S_{Q,Q'}$ denotes the synaptic coupling when a neuron of type Q' synapses on a neuron of type Q. If $S_{Q,Q'}$ is an integer, V_i jumps up $S_{Q,Q'}$.For non-integer values of $S_{Q,Q'}$, let $p = \lfloor S_{Q,Q'} \rfloor$ be the greatest integer less than or equal to $S_{Q,Q'}$. Then $S_{Q,Q'} = p + u$ where u be a Bernoulli random variable taking values in {0,1} with $\mathbb{P}[u = 1] = S_{Q,Q'} - p$ independent of all other random variables in the model.
  - **Effect of I-kicks**: The rule is similar to that for E-kicks, except that V_i will jump down by an amount proportional to $V_i + M_r$. More specifically, the size of the jump is

$$S_{Q,I}(V_i) := (V_i + M_r) / (M + M_r) * S_{Q,I}.$$

This completes the description of the model. We can observe the parameters are

$$\{N_I, N_E, M_r, M, \lambda^E, \lambda^I, \tau_{\mathcal{R}}, P_{Q,Q'}, \tau^E, \tau^I, S_{Q,Q'}\}.$$

2 Implementations

Here we collect their settings for 3 models.Firstly, some parameters are fixed.

$$N_E = 300, N_I = 100, P_{EE} = 0.15, P_{IE} = P_{EI} = 0.5, P_{II} = 0.4,$$
$$S_{EE} = 5, S_{IE} = 2, S_{EI} = S_{II} = 4.91, \tau_{\mathcal{R}} = 2.5, M = 100, M_r = 66.$$

The 3 models are:

- **"Homegeneous"**:

$$\tau^{EE} = 4, \quad \tau^{IE} = 1.2, \quad \tau^I = 4.5 \quad \text{(in ms)}$$

- **"Regular"**:

$$\tau^{EE} = 2.0, \quad \tau^{IE} = 1.2, \quad \tau^I = 4.5 \quad \text{(in ms)}$$

- **"Synchronized"**:

$$\tau^{EE} = 1.3, \quad \tau^{IE} = 0.95, \quad \tau^I = 4.5 \quad \text{(in ms)}$$

And λ^E, λ^R are set as variables. In Figure 2, they set $\lambda^E = \lambda^I = 7000$ spikes/s.

References

Yao Li, Logan Chariker, and Lai-Sang Young. How well do reduced models capture the dynamics in models of interacting neurons? *Journal of mathematical biology*, 78(1-2):83–115, 2019.