Portfolio Selection with Approximate Dynamic Programming

Fischer, Kuo, Marschall, Mirsadeghi

Barcelona Graduate School of Economics

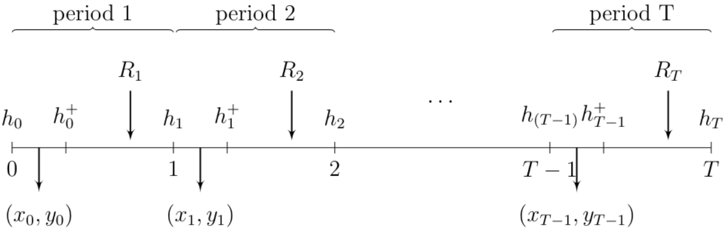
4th March 2018

Overview

Motivation

- **Maximizing portfolio return**
  Through optimal strategy, maximizing the portfolio return, while minimizing risk
- **Deadline**
  Introduction of deadline for investment to adjust for variance differences in possible stock options
- **Dynamic programming approach**
  Using DP possible to capture the risk aversion aspect towards the end of the time horizon by rebelancaing successive periods

Timeline


period 1
period 2
period T
R_1
R_2
R_T
h_0
h_0^+
h_1
h_1^+
h_2
$\cdots$
$h_{(T-1)}$
h_{T-1}^+
h_T
0
1
2
$T-1$
T
(x_0, y_0)
(x_1, y_1)
(x_{T-1}, y_{T-1})

States, Actions & Uncertainty

- **States**
  Holdings of each stock n at time t with the **post-decision state**: $\mathbf{h_t^+} = (h_{0t}^+, h_{1t}^+, ..., h_{Nt}^+) \in \mathbb{R}_+^{N+1}$.

- **Actions**
  At each period, choose a vector of buyings:
  $\mathbf{x_t} = (x_{1t}, ..., x_{Nt}) \in \mathbb{R}_+^N$ and a similiar vector of sellings $\mathbf{y_t}$

- **Uncertainty**
  Vector of returns at each t is uncertain:
  $\mathbf{R_t} = (R_{0t}, R_{1t}, ..., R_{Nt}) \in \mathbb{R}_+^{N+1}$

Constraints

- **Holding constraints**
  All asset holdings must be positive

$$-x_{it} + y_{it} \leq h_{it} \quad \forall i \in (1, ..., N)$$

- **Budget constraints**
  Proportional **transaction costs** θ, equal for buying and selling

$$(1+\theta)\sum_{i=1}^{N} x_{it} - (1-\theta)\sum_{i=1}^{N} y_{it} \leq h_{0t}$$

- **Positive buying and selling vectors**
  We require $x_{it} \geq 0$ and $y_{it} \geq 0$ for all i and t.

Dynamics

For $1 \leq t \leq T-1$:

$$\begin{cases} h^+_{0t} = R_{0t}\left(h^+_{0(t-1)} - (1+\theta)\sum_{i=1}^N x_{it} + (1-\theta)\sum_{i=1}^N y_{it}\right) \\ h^+_{it} = R_{it}h^+_{i(t-1)} + x_{it} - y_{it}, \quad i \in (1, ..., N) \end{cases}$$

For $t = 0$,

$$\begin{cases} h^+_{00} = h^+_{00} - (1+\theta)\sum_{i=1}^N x_{i0} + (1-\theta)\sum_{i=1}^N y_{i0} \\ h^+_{i0} = h^+_{i(0)} + x_{i0} - y_{i0}, \quad i \in (1, ..., N) \end{cases}$$

For $t = T$,

$$h^+_{iT} = R_{iT}h^+_{i(T-1)}, i \in (0, 1, ..., N)$$

Rewards

$$\gamma \sum_{i=0}^{N} h_{iT}^{+} - (1-\gamma) CVaR(h_T^{+})$$

with:

CVaR: as the expected return given that we stay within the $1-\beta$ worst cases

γ : risk aversion parameter of the investor

Initialization and Recursion

- Initialization:

$$V_T^+(h_T^+) = \gamma \sum_{i=0}^{N} h_{iT}^+ - (1-\gamma) CVaR(h_T^+)$$

- Recursion:

  Last period h_{T-1}^+:

$$V_{T-1}^+(h_{T-1}^+) = \mathbb{E}_{R_T}[V_T^+(h_T^+) | h_{T-1}^+]$$

  For $1 \leq t \leq T-2$,

$$V_{t-1}^+(h_{t-1}^+) = \mathbb{E}_{R_t}[\max_{(x_t, y_t)} V_t^+(h_t^+) | h_{t-1}^+]$$

  For $t = 0$, we do not have returns available. Hence

$$V_0(h_0) = \max_{(x_0, y_0)} V_0^+(h_0^+)$$

"Monte Carlo" approximation

Instead of:

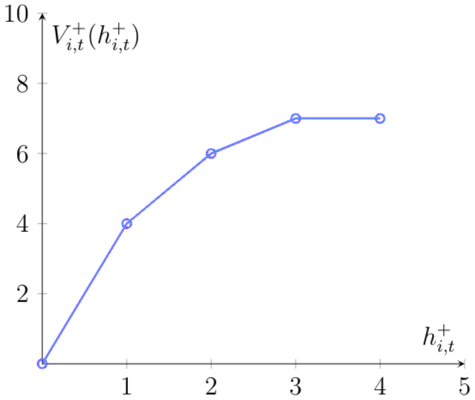
$$V_t^+ = \max_{(x_t, y_t)} \mathbb{E}_{R_t}[V_{t+1}^+ | h_t^+]$$

We use:

$$V_t^+ = \max_{(x_t, y_t)} [V_{t+1}^+ | h_t^+, R_t^s]$$

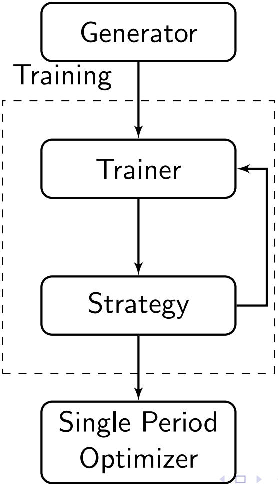
where R_t^s is redrawn for each training iteration s.

Piecewise linear approximation

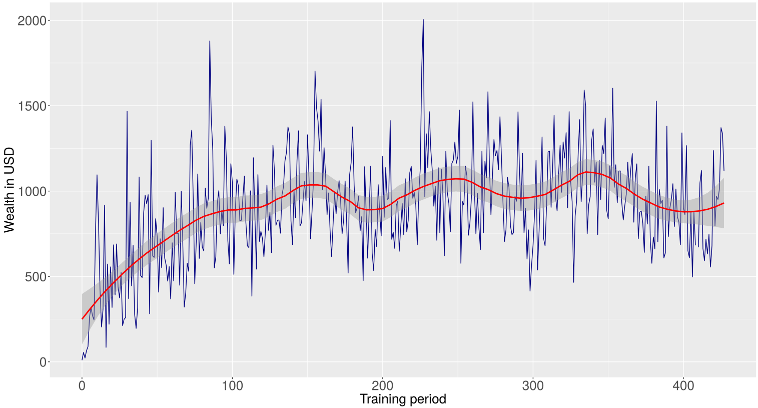


Slopes = (4,2,1,0)
Breakpoints = (0,1,2,3)

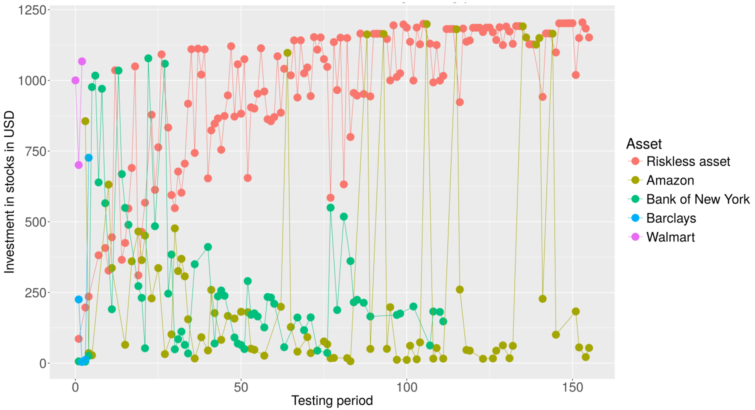
Implementation



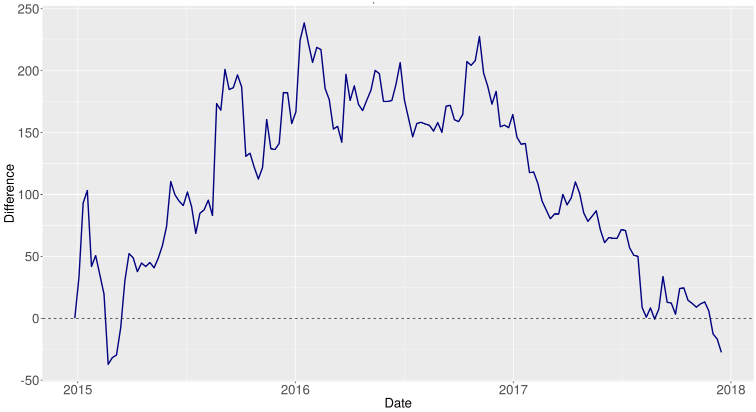
Training Process



Diversification of our Portfolio



Difference between Portfolio and Index investment



Conclusion

- Dynamic Programming useful tool for solving finite-horizon problems
- Approximations necessary to make problem computationally tractable

Improvements

- Train on more data
  1. Obtain more historical data, or
  2. Fit model to historical data, use it to generate training data
- Extend search space for value functions
  1. Include non-linear functions

References

- Edouard Berthe, *Stochastic Dynamic Programming for Portfolio Selection Problem applied to CAC40*, Jun 2017
- Dimitrios Karamanis, *Stochastic Dynamic Programming Methods for the Portfolio Selection Problem*, 2013