Tut3 Quadratic Equations

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A quadratic equation is of the form $ax^2+bx+c=0$
for some $a,b,c\in\mathbb{R}$, and $a\neq 0$

Questions: (key: Factoring, and known if $ab=0$ then $a=0$ or $b=0$)

① $x^2-5x-14=0$
$(x-7)(x+2)=0$
then $x-7=0$ or $x+2=0$
hence $x=7$ or -2
"x will have one of these values"
Therefore the solutions are
$x_1=7$ and $x_2=-2$
"we have 2 possible solutions"
"this and that"

② $x^2+15x=-50$
$x^2+15x+50=0$
$(x+5)(x+10)=0$
then $x+5=0$ or $x+10=0$
hence $x=-5$ or $x=-10$
Therefore the solutions are
$x_1=-5$ and $x_2=-10$

③ $x^2=11x-28$
$x^2-11x+28=0$
$(x-4)(x-7)=0$
hence $x=4$ or 7
Therefore the solutions are
$x_1=4$ and $x_2=7$

④ $12x^2=25x$
$12x^2-25x=0$
$x(12x-25)=0$
hence $x=0$ or $12x-25=0$
that is $x=0$ or $\frac{25}{12}$
Therefore the solutions are
$x_1=0$ and $x_2=\frac{25}{12}$

Questions: (solve a Quadratic Equation given a condition)

① $\frac{x^2-10}{x+2}+x-4=x-3$

sln: $\frac{x^2-10}{x+2}=(x-3)-(x-4)$

$\frac{x^2-10}{x+2}=x-3-x+4=1$

where $x+2\neq 0$, that is $x\neq -2$

$x^2-10=x+2$
$x^2-x-12=0$
$(x-4)(x+3)=0$
hence $x=4$ or -3
Therefore the solutions are
$x_1=4$ and $x_2=-3$

② $\frac{4x}{x+1}+\frac{5}{x}=\frac{6x+5}{x^2+x}$ (☆)

sln $\frac{(4x)x}{(x+1)x}+\frac{5(x+1)}{x(x+1)}=\frac{6x+5}{x^2+x}$

$\frac{4x^2+5x+5}{x^2+x}=\frac{6x+5}{x^2+x}$

where $x^2+x\neq 0$, that is $x(x+1)\neq 0$
that is $x\neq 0$ and $x\neq -1$ (☆☆)
These two have to be true at the same time

$4x^2+5x+5=6x+5$
$4x^2-x=0$
$x(4x-1)=0$
then $\boxed{x=0 \text{ or } x=\frac{1}{4}}$
These are the solutions to the quadratic eqn $4x^2-x=0$ but not (☆)
Known $x\neq 0$ by (☆☆), hence the only solution to the original equation (☆) is $x=\frac{1}{4}$

③ Exercise: $x+1=\frac{2x-7}{x+5}-\frac{5x+8}{x+5}$ (similar idea to ②)

Questions: (Quadratic Formula) Always work given a quadratic equation

Given a quadratic eqn $ax^2+bx+c=0$ where $a,b,c\in\mathbb{R}$ and $a\neq 0$, then the solutions are

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

possibly two different (or same) solutions

① $2x^2+x-4=0$
Notice $a=2$, $b=1$, $c=-4$
then the solutions are

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-1\pm\sqrt{1-4(2)(-4)}}{4}=\frac{-1\pm\sqrt{33}}{4}$$

So the solutions are
$x_1=\frac{-1+\sqrt{33}}{4}$ and $x_2=\frac{-1-\sqrt{33}}{4}$

② $2x^2+x+4=0$
where $a=2$, $b=1$, $c=4$
then by quadratic formula,

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}=\frac{-1\pm\sqrt{1-32}}{4}=\frac{-1\pm\sqrt{-31}}{4}$$

complex number

$$=\frac{-1\pm i\sqrt{31}}{4}$$

So the solutions are $x_1=\frac{-1+i\sqrt{31}}{4}$ and $x_2=\frac{-1-i\sqrt{31}}{4}$

Thinking Question:

Here is a proof that someone wrote.
What is wrong with the proof?

Let $a=b$
then $a^2=ab$
$a^2+a^2=ab+a^2$
$2a^2=ab+a^2$
$2a^2-2ab=ab+a^2-2ab$
$2a^2-2ab=a^2-ab$
$2(a^2-ab)=(a^2-ab)$
$\frac{2(a^2-ab)}{a^2-ab}=\frac{a^2-ab}{a^2-ab}$
$2=1$

Here is the problem!! we are only allowed to divided a non-zero number, but $a^2-ab=a^2-a^2=0$ b/c $a=b$