Tut7: Answers to Questions

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① max $Z = x_1 + 2x_2 - x_3$
subject to $2x_1 - 6x_2 + 3x_3 \le 12$
$2x_2 + x_3 \le 2$
$x_1, x_2, x_3 \ge 0$

Sln (1) convert to canonical form in matrix notation

max $Z = C^T x$
s.t. $Ax = b$
$x \ge 0$

where $C = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \\ 0 \end{pmatrix}$, $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$

$b = \begin{pmatrix} 12 \\ 2 \end{pmatrix}$ $A = \begin{pmatrix} 2 & -6 & 3 & 1 & 0 \\ 0 & 2 & 1 & 0 & 1 \end{pmatrix}$

(2) starting BFS: $x = (0,0,0,12,2)$, construct simplex tableaux and use the simplex method.

Alternatively, can start with this and get same result

	x_1	x_2	x_3	x_4	x_5	
0	2	-6	3	1	0	12
0	0	2	1	0	1	2
-1	1	2	-1	0	0	0

Basic variables x_4, x_5
BFS $(0,0,0,12,2)$

Negative of current cost

0	2	0	6	1	3	18
0	0	1	1/2	0	1/2	1
-1	1	0	-2	0	-1	-2

Basic variables x_2, x_4
BFS $(0,1,0,18,0)$

0	1	0	3	1/2	3/2	9
0	0	1	1/2	0	1/2	1
-1	0	0	-8	-1	-4	-11

Basic variables x_1, x_2
BFS $(9,1,0,0,0)$

Negative of current cost

No positive entries in objective row, we reach optimal solution $\left(\begin{array}{cc|c} 1 & 0 & 9 \\ 0 & 1 & 1 \end{array}\right)$ gives $x_1 = 9$, $x_2 = 1$

s.t. optimal solution is $(9,1,0,0,0)$ with

optimal cost 11

② max $Z = 2x_1 + x_2 - x_3$
subject to
$2x_1 - 3x_2 + 6x_3 \le 12$
$x_2 + 2x_3 \le 2$
$x_1, x_2, x_3 \ge 0$

Sln (1) convert to Canonical form in matrix notation

max $Z = C^T x$
s.t. $Ax = b$
$x \ge 0$

where $C = \begin{pmatrix} 2 \\ -1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$, $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix}$, $b = \begin{pmatrix} 12 \\ 2 \end{pmatrix}$

$A = \begin{pmatrix} 2 & -3 & 6 & 1 & 0 \\ 0 & 1 & 2 & 0 & 1 \end{pmatrix}$

(2) Starting BFS: $(0,0,0,12,2)$, construct the simplex tableaux and solve the LPP using simplex method

z	x_1	x_2	x_3	x_4	x_5	
0	2	-3	6	1	0	12
0	0	1	2	0	1	2
-1	2	1	-1	0	0	0

Basic variables: x_4, x_5
BFS $(0,0,0,12,2)$

Negative of current cost

0	1	-3/2	3	1/2	0	6
0	0	1	2	0	1	2
-1	0	4	-7	-1	0	-12

Basic variables: x_1, x_5
BFS $(6,0,0,0,2)$

0	1	0	6	1/2	3/2	9
0	0	1	2	0	1	2
-1	0	0	-15	-1	-4	-20

Basic variables: x_1, x_2
BFS $(9,2,0,0,0)$

← Negative of current cost

There are no positive entries in the objective row,
s.t. we reach the optimal solution. $\left(\begin{array}{cc|c} 1 & 0 & 9 \\ 0 & 1 & 2 \end{array}\right) \Rightarrow x_1 = 9, x_2 = 2$

Optimal solution at $(9,2,0,0,0)$ with optimal

cost 20