MATB24 Quiz0, tut0022

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MATB24 Quiz0,...

What you should learn from today's tutorial:

- Recognize the importance of understanding the definitions
- Be able to find the identity of a given operation, if it exists
- Start to write precise explanations

Nick

MATB24 Quiz.0, TUT.0022

(1) [4 marks] Give a complete definition, or mathematical characterization of the word: a binary operation # on a set S

is a function from $S \times S$ to S

$\#: S \times S \to S$

$(a,b) \mapsto a \# b$ for all $a, b \in S$

(2) [4 marks] Give an example of a mathematical object that satisfies all the described properties or explain why such an example does not exists.

- A binary operation * on set $M_n(\mathbb{R})$, the set of nxn matrices with real coefficients.

① $*: M_n(\mathbb{R}) \times M_n(\mathbb{R}) \to M_n(\mathbb{R})$

defined as $A * B \mapsto 0$, where $0 = \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{pmatrix}$. is the zero matrix in $M_n(\mathbb{R})$

for all $A, B \in M_n(\mathbb{R})$

② Matrix addition

$*: M_n(\mathbb{R}) \times M_n(\mathbb{R}) \to M_n(\mathbb{R})$

$A * B \mapsto C$, for all $A = (a_{ij})_{1 \le i \le n,\ 1 \le j \le n}$, $B = (b_{ij})_{1 \le i \le n,\ 1 \le j \le n}$, where $C = (a_{ij} + b_{ij})_{1 \le i \le n,\ 1 \le j \le n}$ in $M_n(\mathbb{R})$

b/c $a_{ij} + b_{ij} \in \mathbb{R}$ for all i, j

(3) [7 marks] Carefully prove the given statement: Every element in $\mathbb{R}^2$ has an inverse with respect to standard vector addition.

Let $(x,y) \in \mathbb{R}^2$ be arbitrary, s.t. $x \in \mathbb{R}$ and $y \in \mathbb{R}$

known $x + (-x) = 0$, $y + (-y) = 0$, where $-x, -y \in \mathbb{R}$

※ Do not write $x - x = 0$

Therefore $(-x, -y) \in \mathbb{R}^2$, notice that $(x,y) + (-x,-y) = (x + (-x), y + (-y))$

$= (0,0)$

$(-x,-y) + (x,y) = (0,0)$

$(x,y) + (0,0) = (0,0) + (x,y) = (x,y)$ for all $(x,y) \in \mathbb{R}^2$

s.t. $(0,0)$ is the identity

and hence the inverse of (x,y) is $(-x,-y) \in \mathbb{R}^2$

s.t. every element in $\mathbb{R}^2$ has an inverse w.r.t. standard vector addition

QED