QUIZ3.extreme_pts_tut4

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QUIZ3.extreme_pts_tu...

MATB61 TUT04

Nick Huang **Quiz3 Extreme point and extreme point theorem** Mar.4 2021 Week 8

Instuction: You will have 30 minutes to finish quiz and will have 5 minutes to submit the quiz to Crowdmark after the quiz has finished. You will need to keep your cameras on during the quiz and submission times. One hand-held calculator is allowed. Electronic devices, online calculators, notes and other aids are not allowed. Violation of the instruction can be considered as an academic misconduct, and will be reported to the instructor and the department immediately.

Question

Consider the following linear programming problem.

$$\max\ z = 2x_1 + x_2 - x_3$$

at least -6, given $0 \le x_2 \le 2$

subject to

$$2x_1 - 3x_2 + 6x_3 \le 12$$
$$x_2 + 2x_3 \le 2$$
$$x_1, x_2, x_3 \ge 0$$

Find all the extreme points and find the optimal solution.

- **Note:** In order to receive full marks, you will need to do the computation correctly and justify every steps. In particular, explain why your solution is the optimal solution.

max 1 points

① Feasible region is non-empty and bounded, $0 \le x_1 \le 9$, $0 \le x_2 \le 2$, $0 \le x_3 \le 1$

At optimal solution exists at one of the extreme point by extreme point theorem, Equivalently at one of the basic feasible solution.

② canonical form:

max 2 points

$\max\ z = c^T x$

s.t. $Ax = b$

$x \ge 0$

where $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ u \\ v \end{pmatrix}$, $A = \begin{pmatrix} 2 & -3 & 6 & 1 & 0 \\ 0 & 1 & 2 & 0 & 1 \end{pmatrix}$,

$b = \begin{pmatrix} 12 \\ 2 \end{pmatrix}$, $c = \begin{pmatrix} 2 \\ 1 \\ -1 \\ 0 \\ 0 \end{pmatrix}$

max 5 points

−1 if consider this pair

$\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ are linearly dependent, so we skip this pair here.

③ Find basic solutions:

−1 for each missing one

(1) $\left(\begin{array}{cc|c} 2 & -3 & 12 \\ 0 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 9 \\ 0 & 1 & 2 \end{array}\right)$, $x = \begin{pmatrix} 9 \\ 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

(2) $\left(\begin{array}{cc|c} 2 & 6 & 12 \\ 0 & 2 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 1 \end{array}\right)$, $x = \begin{pmatrix} 3 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

(3) $\left(\begin{array}{cc|c} 2 & 0 & 12 \\ 0 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & 2 \end{array}\right)$, $x = \begin{pmatrix} 6 \\ 0 \\ 0 \\ 0 \\ 2 \end{pmatrix}$

(4) $\left(\begin{array}{cc|c} -3 & 6 & 12 \\ 1 & 2 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & -1 \\ 0 & 1 & \frac{3}{2} \end{array}\right)$, $x = \begin{pmatrix} 0 \\ -1 \\ \frac{3}{2} \\ 0 \\ 0 \end{pmatrix}$

(5) $\left(\begin{array}{cc|c} -3 & 1 & 12 \\ 1 & 0 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 18 \end{array}\right)$, $x = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 18 \\ 0 \end{pmatrix}$

(6) $\left(\begin{array}{cc|c} -3 & 0 & 12 \\ 1 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & -4 \\ 0 & 1 & 6 \end{array}\right)$, $x = \begin{pmatrix} 0 \\ -4 \\ 0 \\ 0 \\ 6 \end{pmatrix}$

(7) $\left(\begin{array}{cc|c} 6 & 1 & 12 \\ 2 & 0 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 6 \end{array}\right)$, $x = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 6 \\ 0 \end{pmatrix}$

(8) $\left(\begin{array}{cc|c} 6 & 0 & 12 \\ 2 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -2 \end{array}\right)$, $x = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 0 \\ -2 \end{pmatrix}$

(9) $\left(\begin{array}{cc|c} 1 & 0 & 12 \\ 0 & 1 & 2 \end{array}\right)$, $x = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 12 \\ 2 \end{pmatrix}$

max 2 points

④ Basic feasible solutions: check $x \ge 0$, get the following

$\boxed{x = \begin{pmatrix} 9 \\ 2 \\ 0 \\ 0 \\ 0 \end{pmatrix} \text{ with } z = 20}$ $\qquad$ $x = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 18 \\ 0 \end{pmatrix}$ with $z = 2$

$x = \begin{pmatrix} 3 \\ 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ with $z = 5$ $\qquad$ $x = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 6 \\ 0 \end{pmatrix}$ with $z = -1$

$x = \begin{pmatrix} 6 \\ 0 \\ 0 \\ 0 \\ 2 \end{pmatrix}$ with $z = 12$ $\qquad$ $x = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 12 \\ 2 \end{pmatrix}$ with $z = 0$

Therefore, given we have a maximization problem, an optimal solution is $x = \begin{pmatrix} 9 \\ 2 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ with optimal value $z = 20$ ∎