Learning Objective

Understand the structures of the linear programming problem in standard form and in canonical form. Be able to set up a linear programming model using suitable notation, and convert to different forms when needed. Understand how to reformulate a special type of piecewise linear programming problem into a standard linear programming problem.

Questions

1. For each of the following problems, determine whether or not it is a standard linear programming problem. If not, explain the reason and reformulate it to a standard linear programming problem using matrix notation, if it is not possible, explain why.

   (a) min $z = 4x + 5y$
   subject to
   $-4x \leq 5$
   $x \geq 0$

   (b) max $z = 4x_1 + 5x_2 + 1$
   subject to
   $4x_1 \leq 5$
   $x_1 \geq 0,\ x_2 \geq 0,\ 1 \geq 0$

2. Reformulate the following problem into a canonical linear programming problem using matrix notation.

   min $z = 4x + 5y$
   subject to
   $-4x \leq 5$
   $x \geq 0$

3. Sketch the following on an xy plane

   (a) $y = |x|$

   (b) $\{(x, y) : y \geq x \text{ and } y \geq -x\}$

   (c) Let x be an arbitrary fixed real number, so $|x|$ is also a real number now. Express $|x|$ as an element of the set $\{y : y \geq x \text{ and } y \geq -x\}$, and explain it graphically using part (a) and (b).

4. Consider the following problem:

   $$\min z = 2|x_1| - 5x_2$$

   subject to

   $$x_1 + x_2 = -2$$
   $$x_2 \leq 0$$

   (a) Without reformulating the problem, draw the feasible region and solve it directly using graphical method.

   (b) Reformulate the problem into a standard linear programming problem using the idea from question 3(c). Draw the feasible region and solve it graphically.

   Hint: Replace(why can you do this?) $|x_1|$ with a new variable with additional constraints, then reformulate the question so that it becomes a question of two variables only instead of three, so you can draw the feasible region in a two dimensional space.

   (c) Verify whether you got the same answer before and after the reformulation.