1 Eigenvalue and eigenvector

Let $T : V \to V$ be a linear transformation. Assume that $V \neq \{0\}$. Show that if T is nilpotent, then $\sigma(T) = \{0\}$

2 Diagonalizable matrix: Computational example

Consider the matrix

$$A = \begin{pmatrix} 2 & 6 & -6 \\ 0 & 5 & -2 \\ 0 & 0 & 4 \end{pmatrix}$$

determine whether A is diagonalizable.

3 Eigenvalue and eigenvector of a linear transformation

Consider the linear transformation $T : \mathbb{P}_2(\mathbb{R}) \to \mathbb{P}_2(\mathbb{R})$ defined by $T(f) = f'$

1. Find $T(a_0 + a_1x + a_2x^2)$ and find the matrix representation of T with respect to the standard basis of $\mathbb{P}_2(\mathbb{R})$
2. Find the eigenvalues and eigenvectors of T by finding the eigenvalues and eigenvectors of the matrix A. Check whether T is diagonalizable.
3. Find the eigenvalues and eigenvectors of T by finding and verifying the candidates using the definitions instead.