Learning Objective

Understand the definitions and useful formulas to solve a wave equation.

Definition

The partial differential equation

$$u_{tt} = c^2 u_{xx}$$

is called the **wave equation** where c is a constant and $u(x,t)$ is a function of x, the position variable and t, the time variable. Suppose that we want to find the solution $u(x,t)$ on $0 < x < l$, with the given conditions:

- **Boundary conditions** $u(0,t) = 0$ and $u(l,t) = 0$
- **Initial conditions** $u(x,0) = \phi(x)$ and $u_t(x,0) = \psi(x)$

Using the method of separation of variables, the general solution is given by

$$u(x,t) = \sum_{n=1}^{\infty} sin(\frac{n\pi x}{l})(A_n cos(\frac{n\pi ct}{l}) + B_n sin(\frac{n\pi ct}{l}))$$

where the coefficients are given by

$$A_n = \frac{2}{l}\int_0^l \phi(x) sin(\frac{n\pi x}{l})$$

and

$$B_n = \frac{2}{n\pi c}\int_0^l \psi(x) sin(\frac{n\pi x}{l})$$

Useful Formulas

- Orthogonal Relations for Fourier Series

$$\int_0^l sin(\frac{n\pi x}{l}) sin(\frac{m\pi x}{l}) = \begin{cases} 0 & m \neq n \\ \frac{l}{2} & m = n \end{cases}$$

$$\int_0^l cos(\frac{n\pi x}{l}) cos(\frac{m\pi x}{l}) = \begin{cases} 0 & m \neq n \\ \frac{l}{2} & m = n (\neq 0) \end{cases}$$

$$\int_{-l}^l sin(\frac{n\pi x}{l}) cos(\frac{m\pi x}{l}) = 0$$

- Integration by parts $\int udv = uv - \int vdu$

Questions

Solve the questions from hw1.