

QUIZ2.Convexity_of_a_set_tut4-sample-quiz

Thursday, February 11, 2021

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Quiz2 Convexity of a Set

MATB61 TUT04
Feb.11 2021 Week 5

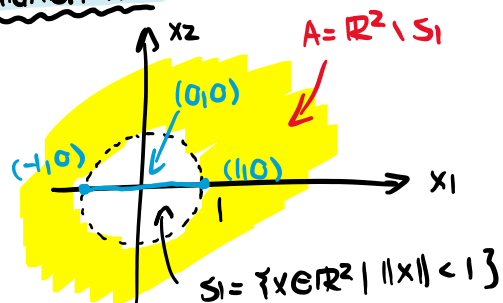
Instuction: You will have 30 minutes to finish quiz and will have 5 minutes to submit the quiz to Crowdmark after the quiz has finished. You will need to keep your cameras on during the quiz and submission times. One hand-held calculator is allowed. Electronic devices, online calculators, notes and other aids are not allowed. Violation of the instruction can be considered as an academic misconduct, and will be reported to the instructor and the department immediately.

Question

Let $A = \mathbb{R}^n \setminus \{\vec{x} \in \mathbb{R}^n \mid \|\vec{x}\| < 1\}$. Check whether or not the set A is convex. If yes, give a proof directly using the definition of convex set. Give a counter example otherwise, to support your argument. A graph is not a proof but may be given partial marks depending on each cases. Make sure your argument holds in \mathbb{R}^n , not just \mathbb{R}^2 .

- **Note:** In order to get full marks in the quiz, you will need to be precise with your explanations, including no lack of justification, correct and consistent notations, correct definitions, clear and organized proof structure (assumption and want to show), and well-defined symbols.

Intuition in \mathbb{R}^2



marking scheme

0/10 confirm that A is convex

-1 for each minor mistakes in the proof, up to 5 points

2/10 Graphical proof only

Proof in \mathbb{R}^n

Let $S_1 = \{\vec{x} \in \mathbb{R}^n \mid \|\vec{x}\| < 1\}$

WTS A is not convex

pick $\vec{x} = (1, 0, \dots, 0) \in \mathbb{R}^n$
 $\vec{y} = (-1, 0, \dots, 0) \in \mathbb{R}^n$

pick $\lambda = \frac{1}{2} \in (0, 1)$

notice $\|\vec{x}\| = \sqrt{1^2 + 0^2 + \dots + 0^2} = \sqrt{1} = 1 \notin S_1$

$\|\vec{y}\| = \sqrt{(-1)^2 + 0^2 + \dots + 0^2} = \sqrt{1} = 1 \notin S_1$

so $\vec{x}, \vec{y} \notin S_1$, and hence $\vec{x}, \vec{y} \in A$

notice $\vec{z} := \lambda \vec{x} + (1-\lambda) \vec{y} = \frac{1}{2}(1, 0, \dots, 0) + \frac{1}{2}(-1, 0, \dots, 0)$
 $= (0, 0, \dots, 0) \in \mathbb{R}^n$

and $\|\vec{z}\| = \sqrt{0^2 + 0^2 + \dots + 0^2} = \sqrt{0} = 0 < 1$

so $\vec{z} \in S_1$ and hence $\vec{z} \notin A$

so A is not convex.