TUT2_Simplifying_and_factoring_(printing_friendly)

Friday, October 1, 2021 16:07

TUT2_Simplifying_and...

Challenge Question from Tutorial #1 : Describe the set of all integers that are divisible by 3 using the set notation.

$\{x \in \mathbb{Z} \mid x = 3k \text{ for some } k \in \mathbb{Z}\}$

Questions

1. Perform the operators if any, and simplify

(a) $\frac{6x^3}{2x}$

$= 3x^2$

(b) $(4x-1)^2 - (2x-3)^2$ known $(a+b)^2 = a^2 + 2ab + b^2$

$= [(4x)^2 + 2(4x)(-1) + (-1)^2] - [(2x)^2 + 2(2x)(-3) + (-3)^2]$

$= (16x^2 - 8x + 1) - (4x^2 - 12x + 9) = 16x^2 - 8x + 1 - 4x^2 + 12x - 9$

$= 12x^2 + 4x - 8$

(c) $\frac{24}{6}\frac{x^{-3}y^4}{y^{-3}}$

$= 4x^{-3} y^4 \cdot x^{-2} y^{-(-3)} = 4x^{-3-2} y^{4+3}$ known $x^a x^b = x^{a+b}$

$= 4x^{-5}y^7$

(d) $\sqrt[3]{16a^4b^9c^8}$ known $\sqrt[n]{x} = x^{\frac{1}{n}}$, in particular $\sqrt[3]{x} = x^{\frac{1}{3}}$

$= (16a^4b^9c^8)^{\frac{1}{3}} = (2^4 a^4 b^9 c^8)^{\frac{1}{3}} = 2^{\frac{4}{3}} a^{\frac{4}{3}} b^{\frac{9}{3}} c^{\frac{8}{3}} = (2 \cdot 2^{\frac{1}{3}})(a a^{\frac{1}{3}}) b^3 (c^2 c^{\frac{2}{3}}) = 2ab^3c^2 \sqrt[3]{2ac^2}$

$2^4 = 2 \cdot 2 \cdot 2 \cdot 2 = 16$

$c^{\frac{8}{3}} = c^{\frac{6}{3}+\frac{2}{3}} = c^{\frac{6}{3}} c^{\frac{2}{3}} = c^2 c^{\frac{2}{3}}$

(e) $\frac{4x}{2x-3} + \frac{5x}{x-5}$ Find the common denominator: $(2x-3)(x-5)$

$= \frac{4x(x-5)}{(2x-3)(x-5)} + \frac{5x(2x-3)}{(x-5)(2x-3)}$ Not unique, but this is the most straightforward one.

$= \frac{4x^2-20x}{2x^2-10x-3x+15} + \frac{10x^2-15x}{2x^2-10x-3x+15} = \frac{4x^2-20x+10x^2-15x}{2x^2-13x+15}$

$= \frac{14x^2-35x}{2x^2-13x+15}$ Try to see whether we could simplify further $= \frac{7x(2x-5)}{(x-3)(2x-5)}$

$(a+b)(c+d) = ac + ad + bc + bd$

Nothing can be cancelled

Final answer

2. Factor

(a) $5x + 20$

$= 5(x+4)$

Cross method: Given $x^2 + kx + p$

Goal is to write the equation into the form $(x+a)(x+b)$, know that

$(x+a)(x+b) = x^2 + bx + ax + ab$

$= x^2 + (a+b)x + ab$

$\Rightarrow \begin{cases} k = a+b \\ p = ab \end{cases}$

Group method

(b) $x^2 + 7x + 12$

$= x^2 + 3x + 4x + 12$

$= x(x+3) + 4(x+3)$

$= (x+4)(x+3)$

(c) $x^2 + 6x + 5$

$= x^2 + x + 5x + 5$

$= x(x+1) + 5(x+1)$

$= (x+5)(x+1)$

check and it work by the cross method

why call this the cross method, here is an example

$x^2 + 7x + 12$

$\Rightarrow x^2 + 7x + 12 = (x+3)(x+4)$

$4x + 3x = 7x$

(d) $(x-3)(a+b) + (x-3)(a+2b)$ where a, b are constants

$= (x-3)(a+b+a+2b) = (x-3)(2a+3b)$

(e) $6x^2 + xy - 40y^2$

$= (3x+8y)(2x-5y)$

Using the cross method:

$6x^2 + xy - 40y^2$

$3x \quad 8y$

$2x \quad -5y$

$-15xy + 16xy = xy$

Factor the GCF from every group

(f) $3x^3 + x^2 + 6x + 2$

$= x^2(3x+1) + 2(3x+1)$

$= (x^2+2)(3x+1)$

Group method

(g) $x^4 + x^2 - 20$

$= (x^2+5)(x^2-4)$

$= x^4 + 5x^2 - 4x^2 - 20$

$= x^2(x^2+5) - 4(x^2+5)$

$= (x^2+5)(x^2-4)$

Cross method

$x^4 + x^2 - 20$

$x^2 \quad 5$

$x^2 \quad -4$

$5x^2 + (-4x^2) = x^2$

Group method

(h) $4x^5 - 4x^3 - 8x$

$= 4x(x^4 - x^2 - 2)$

$= 4x(x^4 - 2x^2 + x^2 - 2)$

$= 4x(x^2(x^2-2) + (x^2-2))$

$= 4x(x^2+1)(x^2-2)$

Cross method

$x^4 - x^2 - 2 = (x^2-2)(x^2+1)$

$x^2 \quad -2$

$x^2 \quad 1$

$\Rightarrow -x^2$