Objectives

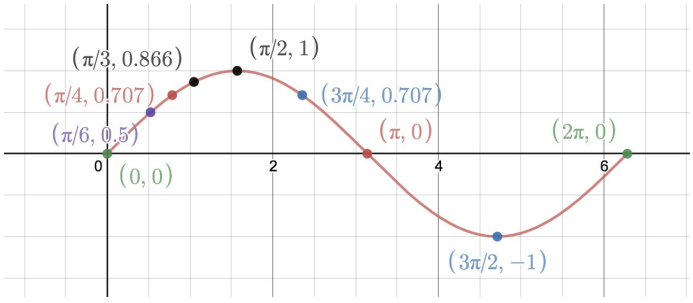
1. (Theoretical Intuition) Understand the basic definitions of the polar curve, including polar coordinate, polar equation, tangent, visualization, area and arc-length, etc
2. (Practical Visualization) Simple sketching of polar curve for modeling purpose
3. (Mathematical Model) Be able to set up mathematical model based on the intuition, mental visualization and/or simple sketching
4. (Computation) Compute the area and length given a polar equation. Find the tangent line of a polar curve.

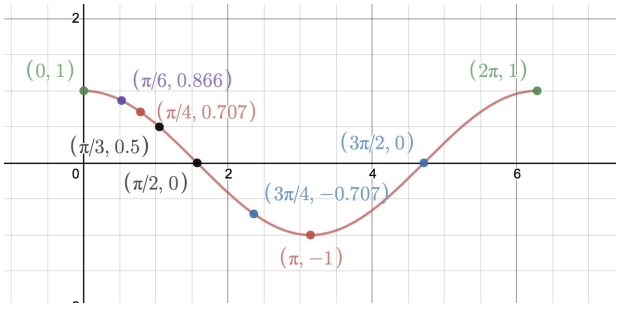
Useful Formulas

1. (10.4) **Area of Polar Curve:** $A = \int_a^b \frac{1}{2} r^2 d\theta$
2. (10.4) **Length of a Curve with Polar Equation:** $L = \int_a^b \sqrt{r^2 + (\frac{dr}{d\theta})^2} d\theta$
   Notice: This formula can be derived from the usual formula for the length of a curve
3. **Trigonometry Double-Angle Identities:** Increase/Reduce the power
   (a) $sin(2\theta) = 2sin(\theta)cos(\theta)$
   (b) $cos(2\theta) = 2cos^2(\theta) - 1 = 1 - 2sin^2(\theta) = cos^2(\theta) - sin^2(\theta)$
4. **Trigonometry Angle Identities:** Useful for computation
   (a) $sin(\theta) = -sin(\theta + \pi)$; $cos(\theta) = -cos(\theta + \pi)$
   (b) $sin(\theta) = sin(\theta - \pi)$; $cos(\theta) = -cos(\theta - \pi)$
   (c) $cos(\theta) = cos(-\theta)$; $sin(\theta) = -sin(-\theta)$

Sine&Cosine Graphs

Note: $sin(\frac{\pi}{3}) = \frac{\sqrt{3}}{2} = cos(\frac{\pi}{6})$; $sin(\frac{\pi}{4}) = \frac{1}{\sqrt{2}} = cos(\frac{\pi}{4})$





Sketch Polar Curve (Trigonometry)

Even though you can still follow the same method for sketching parametric curve that we talked about in the first tutorial, there is a faster way to sketch polar curve because of the symmetries, periods and smoothness of the sine and cosine graphs. Consider the polar equation r = f(t), such that (x,y)=(rcos(t),rsin(t)). (Example: Question2, $r = cos2\theta$)

1. Get intuition by sketching the r-t graph. Make your self a table that pay extra attentions to some convenient points: t=0, t=$\frac{\pi}{2}$, t=π,t=$\frac{3\pi}{2}$, as well as the 'gaps' between any of the two consecutive points
   Note: May need to add more points if the graphs are stretched or compressed. For example, need to consider t = $\frac{\pi}{4}$ (more than just this) as well for the graph r=cos(2t).

   For example:

θ	0	$\frac{\pi}{4}$	$\frac{2\pi}{4}$	$\frac{3\pi}{4}$	$\frac{4\pi}{4}$	$\frac{5\pi}{4}$	$\frac{6\pi}{4}$	$\frac{7\pi}{4}$	$\frac{8\pi}{4}$
r									
$\cos(\theta)$									
$\sin(\theta)$									
x									
y									

2. Using (10.3)symmetry to help you sketch faster!
   (a) If a polar equation is unchanged when θ is replaced by -θ, then the curve is symmetric about the polar axis.
   (b) If the equation is unchanged when r is replaced by -r, or when θ is replaced by $\theta + \pi$, then the curve is symmetric about the pole. (This means that the curve remains unchanged if we rotate it through 180 about the origin.)
   (c) If the equation is unchanged when θ is replaced by $\pi - \theta$, then curve is symmetric about the vertical line $\theta = \frac{\pi}{2}$.

Additional Problems and Reminders

1. Find the area enclosed by the polar curve $r = 1 + cos(\theta)$ and the y-axis for $x \leq 0$
2. Find the arc length of the polar curve in (1), $0 \leq \theta \leq \pi$

Note: These two questions are from your past test. Everything you need to solve these questions are all in this handout. Work hard and get ready for the quiz next week :)

Reminder: MAT235 Quiz1 : October.2 Wednesday Tutorial; Please take the quiz in the tutorial that you enrolled in Acorn. You will get a 0 for writing a quiz in different tutorial :(Always write something to solve the problem. You are guaranteed a 0 for writing nothing.