

# MATB24 Quiz1, tut0022

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MATB24  
Quiz1,...

## MATB24 Quiz.1, TUT.0022

(1) [3 marks] Give a complete definition, or mathematical characterization of the word in bold:

- A **subspace** of a vector space  $V$

$W$  is a subspace of  $V$  if  $W \subseteq V$  and  $W$  is a vector space with same addition and scalar multiplication for  $V$

practical method to verify.

$W$  is subspace of  $V$  if  $\vec{0} \in W$  and  $W$  is closed under addition and scalar multiplication,  $W \subseteq V$ .

(2) [4 marks] Give an example of a mathematical object that satisfies all the described properties or explain why such an example does not exist.

- An invertible element in a set with a binary operation that has an identity

The invertible element is

$0 \in (\mathbb{R}, +)$ , where the inverse of  $0$  is  $0$ , the  $0 + 0 = 0$

usual addition

identity

(3) [8 marks] Carefully prove the given statement:

- Let  $V$  be a vector space. Prove that, if  $v \in V$  and  $r$  is a scalar and  $rv = 0$ , then either  $r = 0$  or  $v = 0$

WTS  $r=0$  or  $v=0$

consider the following cases:

①  $r=0$ , then we are done.

②  $r \neq 0$ , WTS  $v=0$

known  $rv=0$ , where  $r$  is a scalar in some field corresponding to vector space  $V$

and since  $r \neq 0$ , it  $r^{-1}$  exists in the field (Every nonzero element in the field has inverse).

$r^{-1}rv = r^{-1} \cdot 0$  (Multiply  $r^{-1}$  from the left)

$(r^{-1}r)v = 0$  by associativity.

$1 \cdot v = 0$  by def of inverse.

$v = 0$  by def of identity 1

Explanation

WTS

by existence of inverse in  $F$

② for existence of  $r^{-1}$

This will be a 0

①  $0v=0$   
②  $v \neq 0, v^{-1}$  exists