Test 1 Information (from Quercus)

Date: Friday, Oct.18 Regular sitting 18:15-19:45
Test Material: 10.1-10.5, 12.1-12.6

Topics Checklist

How to use the checklist: First of all, understand every topic intuitively (Geometrical interpretation is usually very useful). Secondly, find an example for each of the topic. You should be able to find those in the tutorial problems, past tests, and suggested problems from the textbook. I did not make a review sheet containing every formulas and graphs as you might expect, because you will get more out of making one yourself. **Notice:** The list may not cover everything you will be tested on.

1. Problem solving strategy: Sketching/Intuition + Setup Model + Computation!
2. Something useful for the computation part:
   (a) Trigonometry identities: Double angle identity!
   (b) Given the value of $sin\theta$, can you find the value of $cos\theta$? How about $tan\theta$? You are not allowed to use calculator!
   (c) Integration method: Substitution, integration by parts
   (d) Familiar with the sine and cosine graphs: What are the values on $x = \pi/2, \pi, 3\pi/2, 2\pi$? When does $sin(x) = cos(x)$? What is the relations between $cos(x)$ and $cos(-x)$? How about $sin(x)$ and $sin(-x)$?
3. Section 10.1 Curves Defined by Parametric Equations
   (a) Definition of parametric equation, what are the initial, terminal points, and direction. (See note from tutorial 1)
   (b) How to sketch a parametric curve in general? Special points, 1st and 2nd derivatives?(See note from tutorial 1)
   (c) Example of parametric curve: Cycloid (See textbook section 10.1), circle, etc.
4. Section 10.2 Calculus with Parametric Curves
   (a) Compute the tangent dy/dx given a parametric equation. Given a parametric equation, can you find the tangent line at a given point? Can you identify when the curve has horizontal or vertical tangents?
   (b) What does it mean to say two lines are parallel? What does it tell us about their tangents(or slope)?

(c) Given the tangent, can you tell when does y increases with respect to x? (i.e. increase at the same time) What does it mean by increasing/decreasing? (Sign?) (See tutorial problem 1)

(d) Can you tell the concavity(concave up/down) of a curve? (2nd Derivative)

(e) Compute the area under a parametric curve. Example: What is the area enclosed by one arc of the cycloid? How can you setup the model?

(f) Compute the arc length of a parametric curve. What is the formula? Practice more, and you will notice that double-angle identity is very useful in here. :)

(g) Compute the surface area. What is the formula?

5. Section 10.3 Polar Coordinates

(a) Definition of polar coordinate system. How to change from the Cartesian coordinate to polar coordinate? i.e. given (x,y), how can we change it to (r, θ)?

(b) What is the polar equation of the circle with radius r?

(c) How to sketch a curve given by a polar equation? Make a table, what are the values that you should pay more attention to? (See note from tutorial 2)

(d) Compute the tangent to polar curve. What is the formula? (Chain rule from 1st year calculus is used when deriving the formula)

6. Section 10.4 Areas and Lengths in Polar Coordinates

(a) Compute area of polar region. Can you find the area bounded by a polar region? Can you find the area outside a polar region and inside another one? (Sketch first!) How should you set up the model?

(b) Compute arc length of the curve with polar equation, there is a derived formula from the general one for polar equation in the textbook/or note!

7. Section 10.5 Conic Section

(a) Identify the type of conic section given an equation. (Can you complete the square in an equation?) (See tutorial problem 3)

(b) Sketch conic sections: Sketch the standard one, shifted conics.

(c) In my opinion, the best way to review for this section is to make yourself a summary including all the equations and graphs of different conic sections. There might be one online available, but still better to make your own.

8. Section 12.1 Three-Dimensional Coordinate Systems

(a) Compute the distance between points in three dimension

(b) What is the distance between point and point, between point and line, between line and line, between point and plane, between line and plane?

(c) Equation of a sphere: Find the radius of the sphere given equation. Where is the center of the sphere? Try with a circle first!

9. Section 12.2 Vectors

(a) Operation on vectors: Vector addition, scalar multiplication, properties of vector operation? What does it mean by $|x - y|$ geometrically?

(b) What is a unit vector? How do you change a vector to a unit vector preserving the same direction?

10. Section 12.3 The Dot Product

(a) Compute dot product, what is the formula?

(b) Properties of dot product, in particular, the relation between dot product and length of a vector?

(c) Formula for calculating dot product: $a \cdot b = |a||b|cos\theta$

(d) Classical example: Show that $|a + b| \leq |a| + |b|$ (See tutorial problem 3, there is also a similar one in the suggested problems from textbook)

(e) Verify orthogonality of two vectors using dot product, and why this works?

(f) Given two vectors, can you find the value of cosine of the angle in between? (See tutorial problem 3)

(g) Compute scalar projection and vector project

(h) Would you be able to related every operations in this section? For example, can you find $|a \cdot b|$ given the $|a|$ and $|proj_a b|$?

(i) Can you find the acute angle between two lines? Find $cos(\theta)$? (See tutorial problem 2)

11. Section 12.4 The Cross Product

(a) Compute cross product using determinant

(b) Find orthogonal vector using cross product. Can you find a vector that is orthogonal to two non-parallel vectors? Can you find a vector that is orthogonal to a plane? Is it possible to find a vector that is orthogonal to x,y,z axis in $\mathbb{R}^3$?

(c) Formula for calculating the length of cross product: $|a \times b| = |a||b|sin\theta$, why do we care about this? (related to parallelogram) Can you find the area of a triangle given its vertices in 3-dimension?

(d) Verify whether two vectors are parallel using cross product, and why this works?

(e) Properties of cross product, what property is different between dot and cross product? (i.e. What property holds for dot product, but not cross product?)

12. Section 12.5 Equations of Lines and Planes

(a) Equations of lines: Vector equation, parametric equations, symmetric equation, etc. How can you find them and when are they useful? Can you change a parametric equation(section10) to a vector equation? Can you change a vector equation back to a parametric equation?

(b) Verify if two lines are parallel, orthogonal, if they intersect or are skew, given the parametric equations? (See tutorial problem 4)

(c) How can you check if two lines intersect? (See tutorial problem 3) Caution: Even though the two parametric equations have the same parameter t, does it mean they are the exact same t? What does the parameter t means in vector equation?

(d) Can you find an equation of a plane that contains a line, and/or parallel to another line? (See tutorial problem 4)

(e) Equation of line segment, and why does it work?

(f) Definition of skew lines. Geometrically, what does it mean to be skew lines?

(g) Equations of plane: Normal vector, vector equation of the plane, scalar equation of the plane, etc. Why does the vector equation of the plane work? i.e. why does $n \cdot (r - r_0) = 0$?

(h) Verify whether two planes are parallel using their normal vectors.

13. Section 12.6 Cylinders and Quadric Surfaces

(a) Graphs and equations of quadric surfaces: Ellipsoid, cone, elliptic paraboloid, hyperboloid, etc (See table 1 in section 12.6 of the textbook)

You are better than what you think you are
A number does not define you
Trust yourself, and good luck on your test
-Nick :)