Check the definitions for yourself from the theorem citing list. I will briefly remind you the definitions when working on the questions. We will focus on the discussion of eigenvalue and eigenvector today.

Questions

1. Determine whether the given linear mappings are diagonalizable, and if so, find a basis of the appropriate vector space consisting of eigenvectors:

(a)
$$T: \mathbb{R}^3 \to \mathbb{R}^3$$
 given by
$$\begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 2 \\ 1 & 2 & 1 \end{pmatrix}$$

(b)
$$T: \mathbb{R}^4 \to \mathbb{R}^4$$
 given by
$$\begin{pmatrix} 4 & 2 & 7 & -1 \\ -1 & 1 & 3 & 2 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 1 & 1 \end{pmatrix}$$

(c) $T: \mathbb{P}_3(\mathbb{R}) \to \mathbb{P}_3(\mathbb{R})$ given by T(p)(x) := 3p(x) + xp''(x)

2. Find all values a, b, c for which

$$\begin{pmatrix} 1 & a & b \\ 0 & 3 & c \\ 0 & 0 & 3 \end{pmatrix}$$

is diagonalizable