MATB61 TUT3/4 Final Review
Identifying the Mistakes and Consequences

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This document is for the students in MATB61, TUT0003 and TUT0004 winter 2021 at the University of Toronto Scarborough. You should not use this document as your reference in the final exam. Everything covered in this document have been talked about in the lectures or in the textbook. The purpose of this document is for students to do more practices at various types of questions that they have seen in class. Also some questions are designed for students to detect the mistakes that the questions have by the definitions of the concepts. This document may not covered all materials that will appear in the final exam.

The next few pages contain some common mistakes from your previous quizzes. Please identify the mistakes and the consequences results from those mistakes. It is very common and natural to make mistakes at the beginning of the learning process, but we want to learn from those mistakes.

1. Question: Solve the following LPP using the two-phase method.

$$\max z = -x - y$$

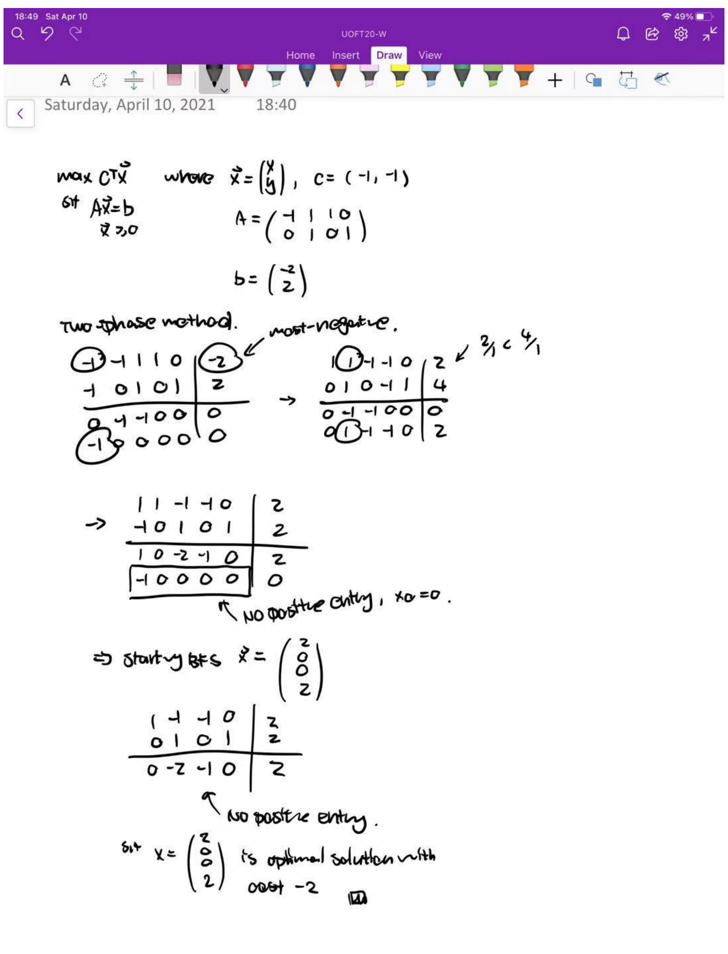
subject to

$$-x + y \leq -2$$

$$y \geq 2$$

$$x, y \geq 0$$

Answer:



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max $c^T\vec{x}$ where $\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$, $c = (-1, -1)$

s.t. $A\vec{x} = b$

$\vec{x} \geq 0$

$A = \begin{pmatrix} -1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$

$b = \begin{pmatrix} -2 \\ 2 \end{pmatrix}$

Two-phase method. most-negative.

$$\begin{array}{cccccc|c} -1 & -1 & 1 & 1 & 0 & & -2 \\ -1 & 0 & 1 & 0 & 1 & & 2 \\ \hline 0 & -1 & -1 & 0 & 0 & & 0 \\ -1 & 0 & 0 & 0 & 0 & & 0 \end{array} \rightarrow \begin{array}{cccccc|c} 1 & 1 & -1 & -1 & 0 & & 2 \\ 0 & 1 & 0 & -1 & 1 & & 4 \\ \hline 0 & -1 & -1 & 0 & 0 & & 0 \\ 0 & 1 & -1 & -1 & 0 & & 2 \end{array}$$

$2/1 < 4/1$

$$\rightarrow \begin{array}{ccccc|c} 1 & 1 & -1 & -1 & 0 & 2 \\ -1 & 0 & 1 & 0 & 1 & 2 \\ \hline 1 & 0 & -2 & -1 & 0 & 2 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{array}$$

No positive entry, $x_0 = 0$.

$\Rightarrow$ Starting BFS $\vec{x} = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 2 \end{pmatrix}$

$$\begin{array}{cccc|c} 1 & -1 & -1 & 0 & 2 \\ 0 & 1 & 0 & 1 & 2 \\ \hline 0 & -2 & -1 & 0 & 2 \end{array}$$

No positive entry.

s.t. $x = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 2 \end{pmatrix}$ is optimal solution with cost -2 ◻

2. Question: Solve the following LPP using the two-phase method.

$$\max z = -x - y$$
$$\text{subject to}$$
$$-x + y \leq -2$$
$$y \geq 2$$
$$x, y \geq 0$$

Answer:

$\max c^T\vec{x}$ where $\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$, $c = (-1, -1)$

s.t. $A\vec{x} = b$

$\vec{x} \geq 0$

$$A = \begin{pmatrix} -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$$

$$b = \begin{pmatrix} -2 \\ -2 \end{pmatrix}$$

Two-phase method.

most-negative.

$$\left(\begin{array}{ccccc|c} -1 & -1 & 1 & 1 & 0 & -2 \\ -1 & 0 & -1 & 0 & 1 & -2 \\ \hline 0 & -1 & -1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{array}\right) \rightarrow \left(\begin{array}{ccccc|c} 1 & 1 & -1 & -1 & 0 & 2 \\ 0 & 1 & -2 & -1 & 1 & 0 \\ \hline 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & -1 & -1 & 0 & 2 \end{array}\right)$$

$$\rightarrow \left(\begin{array}{ccccc|c} 1 & 1 & -1 & -1 & 0 & 2 \\ -1 & 0 & -1 & 0 & 1 & -2 \\ \hline 1 & 0 & -2 & -1 & 0 & 2 \\ -1 & 0 & 0 & 0 & 0 & 0 \end{array}\right)$$

No positive entries, $x_0 = 0$

Starting BFS $x = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \\ -2 \end{pmatrix}$

$$\rightarrow \left(\begin{array}{cccc|c} 1 & -1 & -1 & 0 & 2 \\ 0 & -1 & 0 & 1 & -2 \\ \hline 0 & 2 & -1 & 0 & 2 \end{array}\right)$$

No positive entries

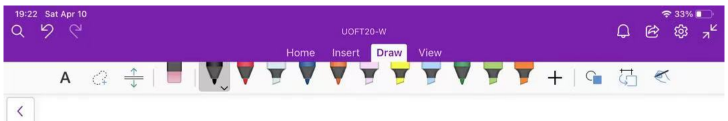
s.t. $x = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \\ -2 \end{pmatrix}$ is optimal solution

with cost $= -2$

3. Question: Solve the following LPP using the two-phase method.

$$\begin{aligned} \max\ z &= -4x - 3y \\ \text{subject to} \\ x + y &\geq 2 \\ 2x - y &\geq 1 \\ x, y &\geq 0 \end{aligned}$$

Answer:



$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$, $c = \begin{pmatrix} -4 \\ -3 \\ 0 \\ 0 \end{pmatrix}$, $A = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 2 & -1 & 0 & 1 \end{pmatrix}$, $b = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$

consider $\max\ c^T x$
s.t. $Ax = b$
$x \geq 0$

$$\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 2 \\ 2 & -1 & 0 & 1 & 1 \\ \hline -4 & -3 & 0 & 0 & 0 \end{array}$$

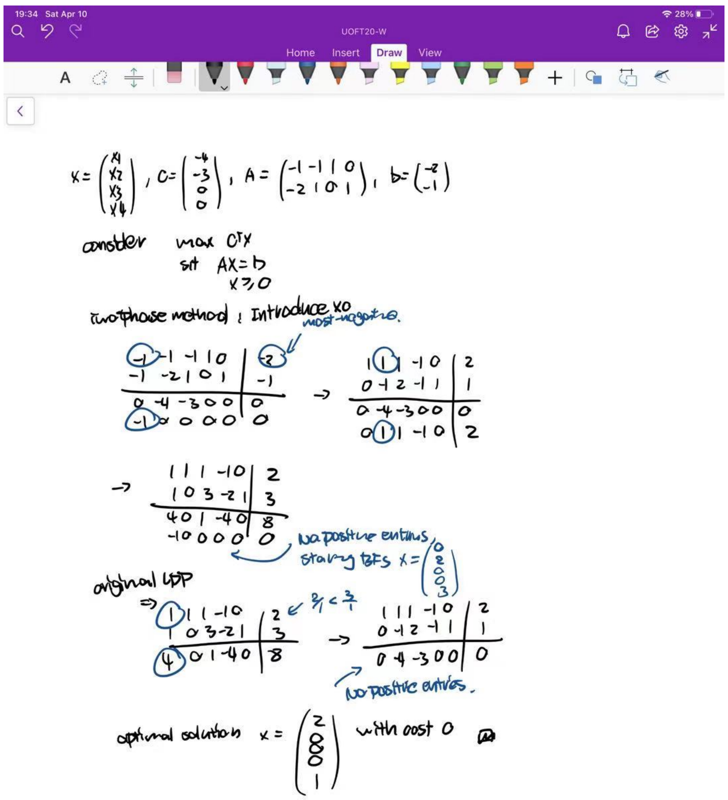
no positive entries here!

optimal solution is $\begin{pmatrix} 0 \\ 0 \\ 2 \\ 1 \end{pmatrix}$ with cost 0

4. Question: Solve the following LPP using the two-phase method.

$$\max\ z = -4x - 3y$$
subject to
$$x + y \geq 2$$
$$2x - y \geq 1$$
$$x, y \geq 0$$

Answer:



$x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$, $c = \begin{pmatrix} -4 \\ -3 \\ 0 \\ 0 \end{pmatrix}$, $A = \begin{pmatrix} -1 & -1 & 1 & 0 \\ -2 & 1 & 0 & 1 \end{pmatrix}$, $b = \begin{pmatrix} -2 \\ -1 \end{pmatrix}$

consider max $c^T x$
s.t. $Ax = b$
$x \geq 0$

two-phase method: Introduce x_0

most-negative.

$$\left(\begin{array}{cccccc|c} -1 & -1 & -1 & 1 & 0 & & -2 \\ -1 & -2 & 1 & 0 & 1 & & -1 \\ \hline 0 & -4 & -3 & 0 & 0 & & 0 \\ -1 & 0 & 0 & 0 & 0 & & 0 \end{array}\right) \to \left(\begin{array}{cccccc|c} 1 & 1 & 1 & -1 & 0 & & 2 \\ 0 & -1 & 2 & -1 & 1 & & 1 \\ \hline 0 & -4 & -3 & 0 & 0 & & 0 \\ 0 & 1 & 1 & -1 & 0 & & 2 \end{array}\right)$$

$$\to \left(\begin{array}{cccccc|c} 1 & 1 & 1 & -1 & 0 & & 2 \\ 1 & 0 & 3 & -2 & 1 & & 3 \\ \hline 4 & 0 & 1 & -4 & 0 & & 8 \\ -1 & 0 & 0 & 0 & 0 & & 0 \end{array}\right)$$

No positive entries, starting BFS $x = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 0 \\ 3 \end{pmatrix}$

original LPP

$$\Rightarrow \left(\begin{array}{ccccc|c} 1 & 1 & 1 & -1 & 0 & 2 \\ 1 & 0 & 3 & -2 & 1 & 3 \\ \hline 4 & 0 & 1 & -4 & 0 & 8 \end{array}\right) \to \left(\begin{array}{ccccc|c} 1 & 1 & 1 & -1 & 0 & 2 \\ 0 & -1 & 2 & -1 & 1 & 1 \\ \hline 0 & -4 & -3 & 0 & 0 & 0 \end{array}\right)$$

$2/1 < 3/1$

No positive entries.

optimal solution $x = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ with cost 0 ∎