Definition

You should always check the lecture note and the textbook for the definition, and make sure the definitions are consistents.

1. A **subspace** W of a vector space V over $\mathbb{F}$ is a <u>subset</u> of V together the same operations of V such that
   (a) $0 \in W$, where 0 is the zero vector of V
   (b) $\forall x, y \in W, x + y \in W$
   (c) $\forall x \in W, \forall c \in \mathbb{F}, c \cdot x \in W$
2. Let S $= \{v_1, \cdots, v_n\}$ be a subset of a vector space V, we say that S span W if $W = span(S)$ where $span(S) = \{a_1v_1 + \cdots + a_nv_n : a_1, \cdots, a_n \in \mathbb{F}\}$.

Questions

1. Let V be a vector space, determine whether or not W is a subspace of V
   (a) $V = \mathbb{P}_2(\mathbb{R})$, $W = \{f \in V : f(1) = 1\}$
   (b) $V = \mathbb{P}_3(\mathbb{R}), W := \{f \in \mathbb{P}_3(\mathbb{R}) : f(2) = 0\}$

(c) $V = \mathbb{R}^3$ and $W = \{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in V : x + y - z = 0\}$

(d) $V = \mathbb{R}^3$ and $W = \{ \begin{pmatrix} x \\ y \\ z \end{pmatrix} \in V : x^2 + y^2 - z^2 = 0\}$

Remark. *What is the difference between this and the previous question?*

2. Let $v_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, v_2 = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}, v_3 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$. Check whether or not $v_3 \in span(\{v_1, v_2\})$. How can you systematically solve this type of questions?

3. Let $f_1(x) = 1 + x, f_2(x) = x, f_3(x) = 2 + x, f_4(x) = x^3 + 1$. Check whether or not $g(x) = x^3 + 2x + 1 \in span(\{f_1, f_2, f_3, f_4\})$. Show all the details and justifications.

   Remark. *What does it mean to say that $g(x)$ is in the span?*

4. Find a spanning set for the subspace $W := \{f \in \mathbb{P}_3(\mathbb{R}) : f(2) = 0\}$ of $\mathbb{P}_3(\mathbb{R})$

   Remark. *Notice when proving a set is a spanning set, we are actually trying to prove a set equality. Therefore there are two directions to show.*