Useful Definitions:

- (16.2.3 Line Integral of Function) If f is defined on a smooth curve C given by equation $x = x(t), y = y(t), a \le t \le b$, then the line integral of f along C is $\int_C f(x,y)ds = \int_a^b f(x(t),y(t))\sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2}dt$
- (16.2.13 Line Integral of Vector Field) Let F be a continuous vector field defined on a smooth curve C given by a vector function r(t), $a \le t \le b$. Then the line integral of F along C is $\int_C F \cdot dr = \int_a^b F(r(t)) \cdot r'(t)dt$
- $\int_C F \cdot dr = \int_C Pdx + Qdy + Rdz$, where $F = Pi + Qj + Rk$
- Common Parametrizations:
  - Circle of Radius r center at (0,0) positive oriented: $x = rcost, y = rsint$ where $0 \le t \le 2\pi$
  - The line segment from r_0 to r_1: $r(t) = (1-t)r_0 + tr_1$, where $0 \le t \le 1$
- (16.3.2 Fundamental Theorem for Line Integral:) Let C be a smooth curve given by the vector function r(t), $a \le t \le b$. Let f be a differentiable function of two or three variables whose gradient vector ∇f is continuous on C. Then $\int_C \nabla f \cdot dr = f(r(b)) - f(r(a))$
- (16.3.6) Let $F = Pi + Qj$ be a vector field on an open simply-connected region D. Suppose that P and Q have continuous first-order partial derivatives and $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ throughout D, then F is conservative.
- (16.4 Green's Theorem) Let C be a positive oriented, piecewise-smooth, simple closed curve in the plane and let D be the region bounded by C. If P and Q have continuous partial derivatives on an open region that contain D, then

$$\int_C Pdx + Qdy = \iint_D (\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y})dA$$

Problems:

1. Evaluate the line integral $\int_C F \cdot dr$, where $F = e^x lny - \frac{e^y}{x}, y + \frac{e^x}{y} - e^y lnx$ and C: $r = t, tcos(\frac{t}{3})$, $1 \le t \le \pi$
2. Evaluate the line integral $\int_C F \cdot dr$, where $F = xy^2 i + cosxj$ and C is the boundary of the region bounded by the curves $y = |x|$ and $y = 1$. Counterclockwise
3. Evaluate the line integral $\int_C (tan^{-1}x + 4xy)dx + (2x^2 + 5x + 1 + ln(y^2+1))dy$ where C is the boundary of the region enclosed by the circle $x^2 + y^2 = 25$ (counter clockwise) and the circle $x^2 + y^2 = 1$ (clockwise).