

## Intuition

1. There are points, lines and planes in three dimensional space, and they can be represented in many different ways. For example, the line  $y=x$  can be given by an equation  $x-y=0$ , or a parametric equation  $f(t)=(t,t)$  where  $t \in \mathbb{R}$ . Similarly, there are many different equations to represent a plane as well. (See tutorial 4)
2. Let's consider a particular equation of a plane:  $x + y + z = 1$  First of all, why does this equation give you a plane, in particular two different directions? Notice that this region is all the points  $(x,y,z)$  that satisfies the equation  $x + y + z = 1$ . Therefore we can rewrite  $(x, y, z) = (1 - y - z, y, z) = (1, 0, 0) + (-1, 1, 0)y + (-1, 0, 1)z$  Notice that we have two different(non-parallel) directions  $(-1, 1, 0)$  and  $(-1, 0, 1)$ , which determine a plane.
3. Since we have two different directions for a plane, when considering the intersection of it with others, which direction should we pay more attention to? They are equally important! This is why we introduce the normal vector, because the normal vector is orthogonal to both directions.
4. If two planes intersect, what is the angle between the planes? In fact, this is the same as the angle between the corresponding normal vectors. Why? Because they both 'rotates' 90 degrees, which preserves the angle in between.

## Example

1. Find the angle between the planes  $x+y+z=1$  and  $x-2y+3z=1$
2. Find symmetric equations for the line of intersection  $L$  of these two planes.
3. Find a parametric equation for the line of intersection  $L$  of these two planes directly from the two equations, without using the symmetric equation.
4. Compare the symmetric equation and the parametric equation, and show they are equivalent. (You can change from one to another, and how?)

## Note

By the end of this worksheet, you should be able to find the angle between two planes, find the parametric/symmetric equations of the intersection of planes, etc. Notice this worksheet is just an extra practice of the concept that we already talked about in the tutorials. Therefore please make sure you are comfortable with these problems, since they may occur in your next quiz.