MATB24 Quiz8, tut0022

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MATB24 Quiz8,...

MATB24 QUIZ.8 TUT0022

(1) [4 marks] In each part, give a complete definition, or mathematical characterization of the word in red

- An unitary diagonalizable complex matrix

$A \in M_n(\mathbb{C})$ is unitary diagonalizable if there exists a diagonal matrix $D \in M_n(\mathbb{C})$ and an unitary matrix $U \in M_n(\mathbb{C})$ s.t. $A = UDU^*$

complex matrix

(2) [5 marks] Give an example of the described object (with justification) or explain why such an example does not exist.

- A unitarily diagonalizable complex matrix with real eigenvalues

$A^* = A$

consider the identity matrix I

① $I \in M_n(\mathbb{C})$, because $\mathbb{R} \subseteq \mathbb{C}$

② The eigenvalue of I is $1 \in \mathbb{R}$

③ Notice $I^* = \bar{I}^T = I$, and $I^* I = II = I$, s.t. I is unitary.

s.t. $I = I^* I I$ ← unitary, hence I is unitarily diagonalizable

↑ Diagonal

∎

∘ In general, a Hermitian matrix will work follows from the spectral thm.

(3) [6 marks] Answer the following question:

- Prove that if A is unitarily diagonalizable, and it has real eigenvalues, then A is Hermitian

Let A be an unitarily diagonalizable matrix with real eigenvalues

then $A = U^* D U$ where U is unitary and D is diagonal where its entries are the eigenvalues of A, s.t. $D \in M_n(\mathbb{R})$

WTS A is Hermitian, i.e. $A^* = A$

$$A^* = (U^* D U)^* = \overline{(U^* D U)}^T = \overline{(U^T D^T (U^*)^T)}$$

$$= \bar{U}^T \bar{D}^T (\overline{U^*})^T = U^* D U = A, \text{ as required}$$

where $\bar{D}^T = D$, because D is diagonal, s.t. $D^T = D$ and $D \in M_n(\mathbb{R})$, s.t. $\bar{D} = D$

and $(\overline{U^*})^T = (\overline{\bar{U}^T})^T = ((\bar{\bar{U}})^T)^T = \bar{\bar{U}} = U$ ∎