Objectives: Hello ∂f

The purpose of this tutorial is to understand the definitions of derivatives in higher dimension, in particular the partial derivative with intuition, and why do we care about partial derivative? Given a function, how can you find the partial derivatives by definition. Given an equation, can you use implicit differentiation to find the partial derivatives? After the tutorial, you should also checkout the applications of partial derivatives, such as partial differential equation (PDE). Make sure you spend some time working on the worksheet and questions from the textbook. The more you practise, the better you can be. :)

Understanding Partial Derivative ∂

Recall from first-year Calculus, we cared about derivatives because we are interested in the rate of change of a function. Similarly, the function with multiple variables are even more interesting. Even though in many real life applications, variables of a function are usually dependent, it is still worthy to see how the value of a function changes, while only one variable varies with others fixed. The rate of such change (only one variable varies and others fixed) is called the partial derivative. Let's consider a function f of two variables x and y:

1. **Partial Derivative of f with respect to x at (a,b):**

$$\frac{\partial f}{\partial x} = \lim_{h \to 0} \frac{f(a+h,b) - f(a,b)}{h}$$

2. **Partial Derivative of f with respect to y at (a,b):**

$$\frac{\partial f}{\partial y} = \lim_{h \to 0} \frac{f(a,b+h) - f(a,b)}{h}$$

3. Notations for partial derivatives: If z = f(x,y), we write

$$f_x = \frac{\partial f}{\partial x} = \frac{\partial z}{\partial x} = f_1 = D_1 f = D_x f$$

4. **Clairaut's Theorem:** Suppose f is defined on a disk D that contains the point (a,b). If the functions f_{xy} and f_{yx} are both continuous on D. Then, $f_{xy}(a,b) = f_{yx}(a,b)$
Caution: In general, $\frac{\partial^2 f}{\partial y \partial x} \neq \frac{\partial^2 f}{\partial x \partial y}$

Computing Partial Derivative by definition

Rule for finding partial derivatives of z = f(x,y):

1. To find $\frac{\partial f}{\partial x}$, consider y as a constant and differentiate f(x,y) with respect to x
2. To find $\frac{\partial f}{\partial y}$, consider x as a constant and differentiate f(x,y) with respect to y

Therefore everything(e.g Chain rule) from first-year Calculus about derivatives applies to the partial derivative, when regarding the other variable as constant.

Problem: Let

$$f(x,y) = \begin{cases} \frac{xy(x^2-y^2)}{x^2+y^2} & if(x,y) \neq (0,0) \\ 0 & if(x,y) = (0,0) \end{cases}$$

1. Find f_x
2. Find f_y
3. Prove that $f_{xy}(0,0) \neq f_{yx}(0,0)$

Problem: Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ for z=f(x)+g(y)

Implicit Differentiation and Partial Derivative $\frac{\partial}{\partial x}$

In fact, given an equation of three variables, where one is defined implicitly as a function of the other two. You can find the partial derivatives using implicit differentiation. Consider the equation $x^2 + 2y^2 + 3z^2 = 1$, where z is defined implicitly as a function of x and y. Use implicit differentiation with respect to x and y, to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
Problem: Use implicit differentiation to find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$: $x^2 - y^2 + z^2 - 2z = 4$ where z is defined implicitly as a function of x and y.

Reminder:

MAT235 Test2: November.22, Friday. 16:15-17:45 for regular sitting. There is no tutorial next week, but extra tutoring service in PG101 same as last time. (Please check Quercus for more details) I will again prepare the topic checklist by Wednesday of the test week. It is almost the end of the first semester. Fortunately, I have no conflict and hopefully can continues running this tutorial section next semester. If you have an academic conflict next semester, you are allowed to switch tutorial section. Please make sure you do so if you need to, since there will be a lot more quizzes in the second semester. It has been a great time with you all, and I wish you all the best and success in the future. -Nick :)