In this tutorial, we will solve some questions about determinant with the properties summarized as in section 3.6, chapter 3 of the textbook. In particular, we will not need the cofactor expansion of determinant to do the calculation.

Questions

1. If A is an $n \times n$ matrix, how are the determinants $detA$ and $det5A$ related?

2. How are the determinants $detA$ and $detB$ related if

   (a) $A = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$, $B = \begin{pmatrix} 2a_1 & 3a_2 & 5a_3 \\ 2b_1 & 3b_2 & 5b_3 \\ 2c_1 & 3c_2 & 5c_3 \end{pmatrix}$

   (b) $A = \begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix}$, $B = \begin{pmatrix} 3a_1 & 4a_2 + 5a_1 & 5a_3 \\ 3b_1 & 4b_2 + 5b_1 & 5b_3 \\ 3c_1 & 4c_2 + 5c_1 & 5c_3 \end{pmatrix}$

3. Using column or row operations compute the determinant of $\begin{pmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ 2 & 3 & 0 \end{pmatrix}$

4. A square matrix is called nilpotent if $A^k = 0$ for some positive integer k. Show that for a nilpotent matrix A, $detA = 0$.

5. Prove that if the matrices A and B are similar, then $detA = detB$.

6. A real square matrix Q is called orthogonal if $Q^T Q = I$. Prove that if Q is an orthogonal matrix, then $det Q = \pm 1$.

7. Show that $\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} = (z-x)(z-y)(y-x)$