Understanding the definition should be the first step of the process of learning and doing mathematics. Over the four years of studying at the University of Toronto majoring in mathematics, I always ask myself the definition whenever I see a new concept or a question to work on. From my past experiences teaching mathematics to students varies from grade five children to third-year university students, one of the most common sentences that I heard is that "I don't know what this means".

In my opinion, understanding the definition of the concept should be one of the main focus of teaching mathematics. There are many aspects for what it means to understand the definitions. In my teaching, I focus on making sure that students can prove that their arguments are valid with references of definitions and theorems.

"Every child can learn math and love it"

I believe that everyone can learn mathematics with the appropriate guidance from the teachers after taking the JUMP Math teaching approach course at the University of Toronto by Prof. Mighton and Prof. Solomon. While the JUMP Math approach focuses on teaching students in elementary schools, I believe the same approach can be applied to teaching at the University. I remember coming to a linear algebra course with passion and excitement in learning mathematics on the first day of my university life. According to the JUMP Math approach, teacher should carefully design their lessons to help students learning mathematics with the positive attitudes by scaffolding, continue assessment and building excitement. The main idea of scaffolding is to introduce the concepts with gradually increasement for the level of the difficulties to help students understand the materials. One interesting example to demonstrate the importance of scaffolding is the rule of flip-and-multiply which is often memorized without being able to explain the reason behind. With the idea of scaffolding, the teacher can present the students with a few concrete examples using graphing before they build the abstract sense of how the process of flipping and multiplying really work. From my previous experience teaching linear optimization, students found it helpful to carefully work on the simpler examples that requires only one iteration of the algorithm before solving the harder question which might requires a combination of different algorithms. More importantly, students recognized the importance of consolidating the understanding of the previous materials before learning the new and harder concepts that might build on the previous ones. This type of lack of understanding can cause the learning difficulty which could further result a negative attitude in mathematics and possibly result an invalid argument when they are solving a problem. From my previous experiences, students did not recognize the mistakes they made when using the algorithm of the simplex method which they memorized but did not fully understand, even though they had a non-sense final answer at the end. On the other hands, students who show their understanding of the method by making the unnecessary but helpful indications tend to recognize the potential mistakes they would make. In my opinion, it is important for the teachers to carefully design the lesson in order to help the students learn and stay with the positive attitude. Making mistakes are common and natural in the process of learning mathematics, and it is important for the teachers to provide appropriate guidance with the observations from the continuous assessments, so that students can learn from doing mathematics.

Understanding-Connecting-Proving-Review

"University mathematics is so much different from what I have learned in high school." This is a sentence which was said by many universities' students, including myself, who were studying Calculus and linear algebra back in our first years at the university. Different from high school math which students can pass a test by memorizing a formula or algorithm to a problem, without truly understanding the question, university math does require a deeper understanding of the concept and justification of what you write on a test. This level of understanding and reasoning will have to be much higher when solving an abstract mathematical problem, which cannot be easily visualized most of the times, such as a problem in linear algebra or group theory. The four-phase method introduced by G. Polya in his book "*How to Solve It*" is a good strategy in approaching to solve a math problem. I summarized the four phases of the method as "Understanding-Connecting-Proving-Review" which is applied in my teaching in order to help students learn.

The first phase of the method is understanding the question. In particular, students should understand the hypothesis and the conclusion of the given question. In my opinion, students should be able to identify those keys of the statement and write down the equivalent definitions in which they could find a reference to support their argument. One of the biggest challenges that the students have found is that they are not sure how to express their idea correctly using suitable notations. On the other hands, it is equivalently important to not just being able to write down the definition but understand what the definition is saying. I believe that there is no best intuitive understanding of a given concept because someone may find the symbolic definition is enough, while other students may prefer to understand the definition using a concrete example. In my opinion, there is always one question that can and should be asked by the student, regardless of how confident they are in approaching to solve the question, "what is the definition?" which is also a question that I will always ask the student whenever they ask for a hint.

The second phase of the method is building the connections of the terms in order to devise a plan in solving the given question. There are multiple ways to achieve such a plan as mentioned by Polya, such as reminding themselves a similar problem that they have seen, dig into the definitions in the first phase, etc. It is easy and natural to get lost in the middle of this process. In teaching, I will encourage the students the explicitly write down the given assumptions and the conclusion which is the final goal that they want to reach and is sometimes called the "want-to-show".

The third phase of the method is to carry out the plan from the second phase, and I called this phase, proving. At this stage, students should have good understandings of the questions and some ideas to solve the problems with a plan. As mentioned by Polya, "The plan gives a general outline, and we have to convince ourselves that the details fit into the outline". It is very important to carefully distinguish between "seeing" and "proving" at this stage. While having a global understanding of the plan is helpful in solving the problem with the correct idea, it is important to make sure that the answer makes sense and contains enough justifications to convince the readers. I think of the third phase as building a tower. The type of tower that is asked to build and the materials have been given in the first phase. The diagram showing how to build the tower using the given materials has been given in the second phase. The third phase will be the time when we actually build the tower. In this process, we want to make sure the upper floors are stable and supported by the lower floors, otherwise the tower will fall, and it is not a good tower. The idea is very similar when writing a proof or doing computations. It is important to make sure that every argument that is made, is supported by the previous valid arguments, otherwise such a proof is untenable and will sometimes lead to an invalid argument at the end. In teaching, I believe finding counter examples is a good approach to demonstrate the importance of being careful in the third phase, because the students can see what will go wrong, not just someone else saying that would be wrong. In fact, being careful in the third phase can help students understand the concepts in a deeper way. From my previous experience teaching linear algebra, there was a question said "Let V be a vector space. Prove that, if $v \in V$, r is a scalar and $rv = 0$, then $r = 0$ or $v = 0$". Students thought this statement was trivial because they remembered back in high school that "If $ab = 0$, then $a = 0$ or $b = 0$", and they had no idea how they could prove such trivial statement. In fact, if the students dig into how multiplication works in a vector space, they will recognize that the question was testing whether they could remember the axioms from the definition of vector space. On the other hand, the question could be extremely hard if the students ignore the reasoning behind each step and pretend every step follows without a reason.

The fourth phase of the method is reviewing the argument from the third phase. Students should confirm the validity of their answers by carefully going through their writing using logical reasoning and give themselves a big picture of what they have learned from the question and whether they can extend the knowledge by applying the result to another question. From my previous experiences teaching linear optimization, I always tell the students not to just do the computations to solve the given problem but think about why they can do such computations. With this in mind, this is when algorithms, which they should understand the reasons behind and the required conditions, occur.

"I have no idea how to start?" is a sentence which is usually said by the students when trying to solve a math problem. Using the four-phase method, students should have something to start which is to understand what the question is asking, and at the very least one can even focus on every single words and symbols in the questions.

Conclusion

I enjoy teaching mathematics and I am working to adapt the four-phase method when teaching. "What is the definition?" is the first question I will always ask when looking at a mathematical problem. Learning mathematics requires the efforts from both the student and the teacher. The teacher will offer paths to students and keep them highly motivated on the way, but the students should be responsible for their own studies at the same time. One thing that I love about teaching is that we often also learn from the students while teaching, and I am always in the process of learning to be a better and better educator.