QUIZ2.Convexity_of_a_set_tut3

Thursday, February 11, 2021 18:33

QUIZ2.Convexity_of_a...

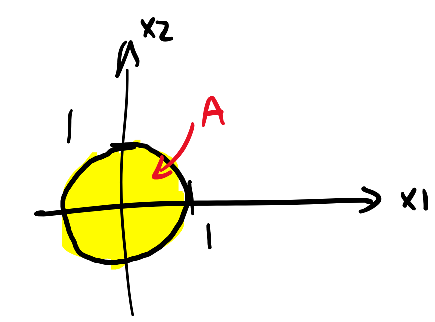
Instuction: You will have 30 minutes to finish quiz and will have 5 minutes to submit the quiz to Crowdmark after the quiz has finished. You will need to keep your cameras on during the quiz and submission times. One hand-held calculator is allowed. Electronic devices, online calculators, notes and other aids are not allowed. Violation of the instruction can be considered as an academic misconduct, and will be reported to the instructor and the department immediately.

Question

Let $A = \{\vec{x} \in \mathbb{R}^n \mid \|\vec{x}\| \le 1\}$. Check whether or not the set A is convex. If yes, give a proof directly using the definition of convex set. Give a counter example otherwise, to support your argument. A graph is not a proof but may be given partial marks depending on each cases. Make sure your argument holds in $\mathbb{R}^n$, not just $\mathbb{R}^2$.

- **Note:** In order to get full marks in the quiz, you will need to be precise with your explanations, including no lack of justification, correct and consistent notations, correct definitions, clear and organized proof structure (assumption and want to show), and well-defined symbols.

Intuition in $\mathbb{R}^2$



Proof in $\mathbb{R}^n$:

Let $x, y \in A$, let $\lambda \in [0,1]$ Assumption

WTS $\lambda x + (1-\lambda) y \in A$ want to show

① Notice $x \in A$, s.t. $x \in \mathbb{R}^n$ and $\|x\| \le 1$ what does each given assumption means

② Notice $y \in A$, s.t. $y \in \mathbb{R}^n$ and $\|y\| \le 1$

Let $z = \lambda x + (1-\lambda) y$, WTS $z \in A$, i.e. $z \in \mathbb{R}^n$ and $\|z\| \le 1$ what your want to show mean.

Since $x, y \in \mathbb{R}^n$, $\lambda \in \mathbb{R}$, s.t. $z = \lambda x + (1-\lambda) y \in \mathbb{R}^n$ ✓

and $\|z\| = \|\lambda x + (1-\lambda) y\|$

$\le \|\lambda x\| + \|(1-\lambda) y\|$ by triangle inequality of $\|\cdot\|$ in $\mathbb{R}^n$

$= \lambda \|x\| + (1-\lambda)\|y\|$ b/c $\lambda \in [0,1]$, $\lambda \ge 0$, $1-\lambda \ge 0$

$\le \lambda \cdot 1 + (1-\lambda) \cdot 1$, b/c $\|x\| \le 1$, $\|y\| \le 1$, $\lambda \ge 0$, $1-\lambda \ge 0$

$= 1$

s.t. $\|z\| \le 1$, and hence $z \in A$

s.t. A is convex by definition ∎

marking scheme:

0/10 confirm that A is not convex

-1 for each minor mistakes in the proof, up to 5 points

2/10 graphical proof only

☆ You will not be taken marks off for not verifying that $z \in \mathbb{R}^n$ this time, but make sure that you know why it is necessary to include this in your proof, especially when we talk about other spaces than $\mathbb{R}^n$. — Nick