MATB24 Quiz4, tut0022

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(1) [4 marks] Give a complete definition, or mathematical characterization of the word in *red*.

- The coordinate vector of v in a vector space V with field F, relative to an ordered basis s = $\{v_1, \dots, v_n\}$ of the vector space V

$v = a_1 v_1 + \dots + a_n v_n$ b/c S is a basis of V

where $a_i \in F$

then the coordinate vector of v wrt S, $[v]_S = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}$

(2) [4 marks] True or false? Justify your answer. That is give an explicit counter example if you think the statement is false, or prove the statement if you think it is true.

- Every nxn matrix is a change-of-coordinate matrix relative to some basis of $\mathbb{R}^n$

let $A = \begin{pmatrix} 0 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 0 \end{pmatrix}$, this can not be the change-of-coordinate matrix, b/c if it is, for some $\alpha = \{v_1, \dots, v_n\}$ and $\beta = \{w_1, \dots, w_n\}$, then by the first column of A, $[v_1]_\beta = \begin{pmatrix} 0 \\ \vdots \\ 0 \end{pmatrix}$, i.e. $v_1 = 0w_1 + \dots + 0w_n = 0$

but α is a basis, and $v_1 = 0 \in \alpha$ makes it linearly dependent. contradiction.

(3) [7 marks] Carefully answer the following using the correct notations:

- Let V and W be n and m dimensional F-vectorspaces and Let B and A be bases for V and W respectively. Let $T_B: V \to F^n$ and $T_A: W \to F^m$ denote the coordinate isomorphisms. Let $S: V \to W$ be a linear transformation. Prove that $Null[S]_{B,A} = T_B(Ker S)$. Recall, $[S]_{B,A}$ is the matrix representation of S related to the bases B and A.

4/7 For only 1 direction.

Pf: ① WTS $Null[S]_{B,A} \subseteq T_B(\ker S)$

let $x \in Null[S]_{B,A}$, i.e. $[S]_{B,A}(x) = 0$, where $x \in F^n$.

b/c T_B is an isomorphism between V and F^n. , WTS $x \in T_B(\ker S)$.

s.t. T_B is a bijection, and T_B is onto, s.t. $\exists \alpha \in V$, $T_B(\alpha) = x$ (*).

i.e. $x = [\alpha]_B$

then $0_{F^m} = [S]_{B,A}(x) = [S]_{B,A}[\alpha]_B = [S(\alpha)]_A$

Notice T_A is an isomorphism, in particular, it is 1-1

s.t. $S(\alpha) = 0$, i.e. $\alpha \in \ker(S)$

then $x = T_B(\alpha)$ b/c (*), where $\alpha \in \ker(S)$.

s.t. $x \in T_B(\ker S)$ by definition.

② WTS $T_B(\ker S) \subseteq Null[S]_{B,A}$

let $x \in T_B(\ker S)$, then $x = T_B(\alpha)$ for some $\alpha \in \ker S$

B/c $\alpha \in \ker(S)$, s.t. $S(\alpha) = 0$ (**).

$0_{F^m} = [0_V]_A = [S(\alpha)]_A = [S]_{B,A}[\alpha]_B = [S]_{B,A}\, x.$

b/c T_A is 1-1 (**) (*). b/c $x = T_B(\alpha)$ i.e. $x = [\alpha]_B$.

s.t. $x \in Null[S]_{B,A}$ ∎