This worksheet contains various examples of LPP with various situations that you might see when using the simplex method. The final answers are provided, so you can check the solution yourself, but the intermediate steps are more important than the final answer when doing the practices.

Questions

1. Solve the following LPP using the simplex method. Clearly indicate the canonical form, the current BFS at each stage, optimal solution and optimal cost.

   min $z = -2x_1 - x_2$

   subject to

   $x_1 - x_2 \leq 2$

   $x_1 + x_2 \leq 6$

   $x_1, x_2 \geq 0$

   Answer. *Optimal solution* $x = (4, 2, 0, 0)$ *with optimal cost* $z = -10$

2. Solve the following LPP using the simplex method. Clearly indicate the canonical form, the current BFS at each stage, optimal solution and optimal cost.

   max $z = 2x_1 + 3x_2 + x_3 + x_4$

   subject to

   $x_1 - x_2 - x_3 \leq 2$

   $-2x_1 + 5x_2 - 3x_3 - 3x_4 \leq 10$

   $2x_1 - 5x_2 + 3x_4 \leq 5$

   $x_1, x_2, x_3, x_4 \geq 0$

   Answer. *No optimal solution, the cost can be infinite.*

3. Solve the following LPP using the simplex method. Clearly indicate the canonical form, the current BFS at each stage, optimal solution and optimal cost.

   max $z = 200x_1 + 60x_2 + 206x_3$

   subject to

   $3x_1 + x_2 + 5x_3 \leq 8$

   $5x_1 + x_2 + 3x_3 \leq 5$

   $x_1, x_2, x_3 \geq 0$

   Answer. *Optimal solution* $x = (0, \frac{3}{2}, \frac{1}{2}, 0, 0)$ *with optimal cost* 339. *Be careful choosing the pivot entries when using the simplex method.*

4. Consider the following complex tableaux when solving a LPP using simplex method.

x_1	x_2	x_3	x_4	x_5	x_6	
1	-1	1	1	0	0	1
0	1	2	0	1	0	3
0	1	1	1	0	1	2
0	0	-14	-3	0	0	-28

(a) What is the current BFS given the simplex tableaux? What are the current cost?

(b) Looking at the tableaux, do we know the optimal solution and optimal cost now, why?

(c) If such optimal solution exists, can you find another optimal solution by introducing a new basic variable? (Hint: Introduce x_2, consider the x_2 column and convert the pivot as usual)

(d) If you can find two different optimal solutions from the previous questions, called them x and y, prove that any convex combination of x and y is also an optimal solution.