This list of questions is for the students in MATB24, TUT0005 summer 2021 at the University of Toronto Scarborough. You should not use any of the facts in this document as a reference in the midterm test. Everything covered in this document has been talked about in the lecture, tutorial or the textbook. The test may cover materials that are not included in this document. You should refer to the professor's email for the coverage of the test. Good luck in the midterm!

True or false

Answer the following true/false questions. If the statement is true, give a brief justification, otherwise give an explicit counter example. The following true/false questions can be found in the textbook Linear Algebra Done Wrong.

1. Every vector space contains a zero vector.

2. A vector space can have more than one zero vector.

3. If f and g are polynomials of degree n, then $f + g$ is also a polynomial of degree n

4. If f and g are polynomials of degree at most n, then $f + g$ is also a polynomial of degree at most n

5. Any set containing a zero vector is linearly dependent

6. A basis must contain 0

7. subsets of linearly dependent sets are linearly dependent

8. subsets of linearly independent sets are linearly independent

9. If $a_1v_1 + a_2v_2 + ... + a_nv_n = 0$ then all scalars a_k are zero

10. Every vector space that is generated by a finite set has a basis

11. Every vector space has a (finite) basis

12. A vector space cannot have more than one basis

13. If a vector space has a finite basis, then the number of vectors in every basis is the same

14. The dimension of $\mathbb{P}_n(\mathbb{R})$ is n

15. The dimension on $M_{m\times n}$ is $m+n$

16. If vectors $v_1, v_2, ..., v_n$ generate (span) the vector space V , then every vector in V can be written as a linear combination of vector $v_1, v_2, ..., v_n$ in only one way

17. Every subspace of a finite-dimensional space is finite-dimensional

18. If V is a vector space having dimension n, then V has exactly one subspace of dimension 0 and exactly one subspace of dimension n

19. Any system of linear equations has at least one solution

20. Any system of linear equations has at most one solution

21. Any homogeneous system of linear equations has at least one solution

22. Any system of n linear equations in n unknowns has at least one solution

23. Any system of n linear equations in n unknowns has at most one solution

24. If the homogeneous system corresponding to a given system of a linear equations has a solution, then the given system has a solution

25. If the coefficient matrix of a homogeneous system of n linear equations in n unknowns is

invertible, then the system has no non-zero solution

26. The solution set of any system of m equations in n unknowns is a subspace in $\mathbb{R}_n$

27. The solution set of any homogeneous system of m equations in n unknowns is a subspace in $\mathbb{R}_n$

28. Every change of coordinate matrix is square

29. Every change of coordinate matrix is invertible

30. The rank of a matrix is equal to the number of its non-zero columns

31. The rank of an $n \times n$ matrix is at most n

32. Every linear operator in an n-dimensional vector space has n distinct eigenvalue

33. If a matrix has one eigenvector, it has infinitely many eigenvectors, assuming a real vector space.

34. There exists a square real matrix with no real eigenvalues

35. Similar matrices always have the same eigenvalues

36. Similar matrices always have the same eigenvectors

37. A non-zero sum of two eigenvectors of a matrix A is always an eigenvector

38. A non-zero sum of two eigenvectors of a matrix A corresponding to the same eigenvalue λ is always an eigenvector

39. A^T has the same eigenvalues as A

40. A^T has the same eigenvectors as A

41. If A is diagonalizable, then so is A^T