Definition

A dual pair is a pair of LPPs (1, 2), where the first LPP 1 is called the primal, and the second LPP 2 is called the dual. The forms of the primal and dual are as follows:
1
max $c^T x$
subject to
$Ax \leq b$
$x \geq 0$
2
min $b^T y$
subject to
$A^T y \geq c$
$y \geq 0$

Strong and Weak Duality Theorems

1. **Strong Duality Theorem**

   If an LPP has an optimal solution, then so does its dual, and the optimal cost in their respective problems are equal.

2. **Weak Duality Theorem**

   If x is a feasible solution to the primal which is a maximization problem, and y is a feasible solution to the dual, then
   $$c^T x \leq b^T y$$

Questions

1. Find the dual of the following LPP

   (a) max $z = x_1 + 2x_2$
   subject to
   $$x_1 + x_2 \leq 1$$
   $$2x_1 + 2x_2 \leq 3$$
   $$x_1, x_2 \geq 0$$

   (b) min $z = x_1 + 2x_2$
   subject to
   $$x_1 + x_2 = 1$$
   $$2x_1 + 2x_2 \leq 3$$
   $$x_2 \geq 0$$

2. Use the duality theorems to check whether the following situations are possible given that the primal is a maximization problem.

Primal \ Dual	Finite optimal	Unbounded	Infeasible
Finite optimal			
Unbounded			
Infeasible			

Explain why using the duality theorems or give explicit examples if it is possible to be true and also possible to be false sometimes

3. Let A be a symmetric $n \times n$ matrix. Consider the following LPP:

   min $c^T x$
   subject to
   $Ax \geq c$
   $x \geq 0$

   Prove that if x_0 satisfies $Ax_0 = c$ and $x_0 \geq 0$, then x_0 is an optimal solution. (Hint: Duality theorems)