Definitions

Let $\mathbf{F}$ be a vector field on $\mathbb{R}^3$ with $\mathbf{F} = (F_1, F_2, F_3)$

- $\nabla = (\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z})$
- $curl\mathbf{F} = \nabla \times \mathbf{F}$
- $div\mathbf{F} = \nabla \cdot \mathbf{F}$

Let f be a scalar function of three variables

- $\nabla f = (\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z})$
- $\nabla^2 f = \nabla \cdot (\nabla f) = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$, where ∇^2 is called the Laplace operator

Questions

1. Find the $curl\mathbf{F}$ and $div\mathbf{F}$ given the vector field $\mathbf{F}(x, y, z) = (x + yz, y + xz, z + xy)$

2. Let $\mathbf{F}(x, y, z) = (x, y, z)$ be a vector field on $\mathbb{R}^3$, and let $f(x, y, z) = |\mathbf{F}(x, y, z)|$ be a scalar function. Verify the following identities using the above definitions

   (a) $div\mathbf{F} = 3$

(b) $div(f\mathbf{F}) = 4f$, where $(f\mathbf{F})(x,y,z) = f(x,y,z)\mathbf{F}(x,y,z) = (f(x,y,z)x, f(x,y,z)y, f(x,y,z)z)$ is a vector field on $\mathbb{R}^3$

(c) $\nabla^2 f^3 = 12f$

(d) $curl\mathbf{F} = (0, 0, 0)$

3. Sketch the vector fields $\mathbf{F}$ by drawing the diagram with some points.

    (a) $\mathbf{F}(x, y) = (3, 4)$

    (b) $\mathbf{F}(x, y) = (y, x + y)$

4. Prove the following identities assuming the partial derivatives exists and are continuous. Let $\mathbf{F}$, $\mathbf{G}$ be vector fields on $\mathbb{R}^3$, and let f be a scalar function with three variables.

   (a) $div(\mathbf{F} + \mathbf{G}) = div(\mathbf{F}) + div(\mathbf{G})$

   (b) $curl(\mathbf{F} + \mathbf{G}) = curl(\mathbf{F}) + curl(\mathbf{G})$

   (c) $div(f\mathbf{F}) = f div\mathbf{F} + \mathbf{F} \cdot \nabla f$