This worksheet contains various examples of LPP with various situations that you might see when using the two-phase method. The final answers are provided, so you can check the solution yourself, but the intermediate steps are more important than the final answer when doing the practices. The sub-questions are designed to guide and help you with the theory of the method. If there is any step which you are not sure why and how it works, please feel free to send me an email with your question and I would be happy to help.
Notation: The two-phase method that we would be using follows the notation used in the lecture.

Questions

1. Solve the following LPP using the two-phase method and answer the sub-questions.

   min $z = -2x_1 - x_2$

   subject to

   $x_1 + x_2 \geq 2$

   $-x_1 + x_2 \leq -6$

   $x_1, x_2 \geq 0$

   (a) Write the original problem into canonical from using matrix notation.

   (b) Introduce the artificial variable x_0 and write the corresponding auxilliary

   (c) Explain how you pick the pivot when introducing x_0 as the basic variable at the beginning when you solve the auxilliary problem.

   (d) Clearly indicate the phase 1 and phase 2, as well as the current BFS and current cost for each tableauxes corresponding to the LPP that you are solving, except for the starting tableaux for the auxilliary problem.

2. Consider the following LPP

   $\max z = c^T x$

   subject to

   $Ax = b$

   $x \geq 0$

   Prove that if its auxilliary problem (minimization problem) has optimal cost $w = x_0 \neq 0$, then the original LPP given above has no feasible solution. In other words, there is no such x that satisfies $Ax = b$ and $x \geq 0$. (Hint: Proof by contrapositive and construct an optimal solution for the auxilliary problem)

3. Given the LPP, answer the following questions.

   max $z = x_1 + 2x_2$

   subject to

   $2x_1 + x_2 \leq 2$

   $-4x_1 + x_2 \leq -5$

   $x_1, x_2 \geq 0$

   (a) Draw the feasible region of the above LPP on the x_1x_2 plane. Use the graph to solve the LPP with justification.

   (b) Use the two-phase method to solve the LPP by introducing the artifitial variable x_0. What is the optimal solution of the auxilliary problem Q? What can you conclude? (Hint: Question 2)

4. This is a practice on the simplex method. Consider the following tableaux which occurs at a current BFS when running the simplex algorithm.

$$
\begin{array}{ccccccc|c}
x_1 & x_2 & x_3 & x_4 & x_5 & x_6 & x_7 & \\
0 & J & 0 & A & 1 & 0 & 3 & B \\
H & 0 & 1 & -2 & C & D & -1 & 2 \\
1 & 0 & 0 & 0 & -1 & 1 & 1 & 3 \\
\hline
0 & 0 & 0 & E & -3 & F & G & I
\end{array}
$$

Give a choice of $A, B, C, D, E, F, G, H, I$ and J for each of the following statement, such that the statement is true without ambiguity following the notations that we have. If it is not possible, explain why.

(a) The current basic variables are x_1, x_2, x_3, and the other variables are non-basic variables.

(b) The current BFS is an optimal solution with current cost 32, and the basic variables x_2, x_3, x_6.

(c) The LPP has no optimal solution and the cost is infinite followed from the above tableaux.

(d) The current BFS is not an optimal solution, and we will get a solution with higher cost by introducing the variable x_4 as basic variable.

(e) The current BFS is an optimal solution, but the optimal solution is not unique. Explicitly give two distinct optimal solutions.

(f) The current BFS is not an optimal solution and we are not sure whether the given LPP has an optimal solution or not yet, but we would be able to conclude that the LPP has no optimal solution after one iteration of the simplex method.

(g) The current BFS is not an optimal solution. We could introduce x_6 to be the basic variable, and x_3 will become non-basic, to get a BFS with a higher cost.

(h) This tableaux occurs at a BFS when x_2 is a basic varible and $B < 0$.