Test 2 Information (from Quercus)

Date: Friday, Nov.22 Regular sitting 16:15-17:45
Test Material: 13.1, 13.2, 13.3(no curvature), 13.4 (velocity, acceleration and projectile motion) and 14.1-14.4

Topics Checklist

How to use the checklist: First of all, understand every topic intuitively (Geometrical interpretation is usually very useful). Secondly, find an example for each of the topic. You should be able to find those in the tutorial problems, past tests, and suggested problems from the textbook. I did not make a review sheet containing every formulas and graphs as you might expect, because you will get more out of making one yourself. **Notice:** The list may not cover everything you will be tested on.

1. section 13.1 Vector Function and Space Curves
   (a) What is the definition of vector function? Can you find the domain of a vector function? What should you be careful when finding the domain?
   (b) What are the definitions of limit and continuity of a vector function? (i.e What does it mean by a vector function is continuous at a point?) Can you find the limit of a vector function?
   (c) What is the definition of a space curve?
   (d) Can you sketch a space curve? (e.g. $r(t) = cost\mathbf{i} + sint\mathbf{j} + t\mathbf{k}$) Can you indicate the direction of the curve as t increases?
   (e) Find a vector equation, parametric equation for a line segment joining two points.
   (f) Given an equation of a curve, can you check certain properties of the curve? (i.e. if it passes through some points? if it satisfies a given formula?)
2. section 13.2 Derivatives and Integrals of Vector Functions
   (a) What is the definition of derivative of a vector function? What is a tangent vector? How can you find the tangent line to a curve given by vector function at a point?
   (b) Given a vector, how can you change it to a unit vector?
   (c) Can you compute derivatives of vector functions?
   (d) What are the differentiation rules?
   (e) What is the definition of the definite integral of a vector function? Can you compute the definite integral?
   (f) Find the unit tangent vector $\mathbf{T}$(t) at the point with given value of the parameter.

3. section 13.3 Arc Length and Curvature(No curvature)
   (a) What is the definition of arc length of a curve?
   (b) Suggested problem in the textbook: Let C be the curve of intersection of the parabolic cylinder $x^2 = 2y$ and the surface $3z = xy$. Find the exact length of C from the origin to the point (6,18,36). Understanding the logic behind is much more important.
   (c) What is the definition of an arc length function s(t)? Can you find the arc length function given a vector function v(t)? Can you reparametrized the vector function r(t(s))? What is the formula for parametrization of a curve with respect to arc length? (NOTE: Make sure you check the tutorial problem on Quercus. I skipped this question in our tutorial.)
4. section 13.4 Motion in Space: Velocity and Acceleration, Projectile Motion
   (a) What are the definitions of position vector, velocity vector and acceleration?
   (b) What is the relations among position, velocity and acceleration?
   (c) Given the position vector, how can you find the velocity vector and acceleration?
   (d) Given the acceleration, how can you find the velocity and position vector? What extra information do you need comparing to (c) and why?
   (e) Can you visualize/draw a projectile motion?
   (f) What is the value of gravity and what is the direction?
   (g) What is the parametric equations of the trajectory of projectile motion in terms of initial velocity, angle and time?
   (h) Given the position vector, when is the speed a minimum, and why does this work(optimization?)? Considering a projectile motion, what is the maximum height of the projectile? What is the range of the projectile?
   (i) Can you draw the path of the particle given position function?
5. section 14.1 Functions of Several Variables
   (a) Given a function f of two variables, what are the domain and range? What are independent variables and dependent variables? What is the definition of graph of f? Can you sketch the graph?
   (b) What are the level curves of a function f of two variables? Can you sketch the level curves of a function? What is a contour map?
   (c) Given many functions, with their graphs and contour maps, can you match the graph and its contour map for each function?
   (d) Can you find and sketch the domain of the function? Can you sketch a graph of a two-variable function f(x,y)?

6. section 14.2 Limits and Continuity
   (a) What is the definition of limit and continuity?
   (b) Existence of Limits: How can you guess whether a limit exists or not before you do any work, just for intuition?
      i. If the limit does not exist:
         A. How can you tell if a limit does not exist?
         B. What are the possible paths that you can take to show that?
      ii. If the limit exists:
         A. If you can find two paths that lead to the same values, does it mean limit exist? **NO!!** Finding two such paths does not guarantee the existence of limit.
         B. Useful technique to prove limit exist: Squeeze Theorem
         C. Useful formula: $x^2 + y^2 \geq 2xy$ and $\frac{1}{|2xy|} \geq \frac{1}{x^2+y^2}$ (without the absolute value, this is not true!!)
      iii. Make sure you work on the questions 5-22 in section 14.2 of the textbook. The more you practice, the better you can be.
   (c) What does it mean to say a function is continuous at a point, and how can you show it? Can you tell if a piece-wise function is continuous? Can you find the points where the function is discontinuous?

7. section 14.3 Partial Derivatives
   (a) What is the definition of partial derivative and how can you evaluate a partial derivative? Can you compute a second partial derivative?
   (b) Is $f_{xy} = f_{yx}$ in general? (NO!)
   (c) Given an equation z(x,y). Can you find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ using implicit differentiation?
   (d) Can you match the graphs of the functions with its partial derivatives? Can you determine the sign of a partial derivative from the graph of the function?
   (e) What is the Laplace equation. Given a function, can you check whether it satisfies the Laplace equation?
   (f) Given a PDE equation, can you verify a funciton is a solution to the equation?
   (g) Can you repeats the above for real-life problems? (Check textbook)

8. section 14.4 Tangent Planes and Linear Approximations
   (a) What is the definition of tangent plane? How can you find an equation of the tangent plane to the surface z=f(x,y) at the point $P(x_0, y_0, z_0)$, given f has continuous partial derivatives?

(b) How can you find the linear approximation of a function f at a point? Can you verify a linear approximation at a point? Use linear approximation to estimate a value of the function, how about using tangent plane? Can you approaximate a value by definning a function? For example, $\frac{1}{(1.01)^2+(0.99)^2}$

(c) How can you tell if a function is differentiable at a point?

(d) What is the definition of differential? What is dz, where z=f(x,y)?

Trust yourself, you know more than you think you do
Never do tomorrow what you can do today
Good luck on your test :) -Nick