

Objectives

1. (Theoretical Intuition) Understand the basic definitions and properties of vectors and their operations, and be able to identify different types of conic sections
2. (Practical Visualization) Sketching the conic section given the parametric equation
3. (Computation) Compute the dot product and review integration methods (substitution and integration by parts).

Useful Formulas

1. (10.5) Standard Equations of Ellipses and Hyperbolas

- (a) Ellipses: $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a \geq b > 0$ has foci $(\pm c, 0)$, where $c^2 = a^2 - b^2$
- (b) Hyperbola: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, with asymptotes $y = \pm(\frac{b}{a})x$, vertices $(\pm a, 0)$

Exercise: Sketch the ellipses and hyperbolas given by the standard equations. There should be two different types each (10.5), even though only one each is provided above.

2. (12.3) **Dot product:** $a \cdot b = \sum a_i b_i$
3. (12.3) **Properties of the Dot Product:**

- (a) $a \cdot a = |a|^2$
- (b) $a \cdot (b + c) = (a \cdot b) + (a \cdot c)$
- (c) $a \cdot b = |a||b|\cos\theta$, where θ is the angle between the vectors a and b

Question: What can we conclude from (c) to compare the values of $a \cdot b$ and $|a||b|$?

4. **Integration by parts:** $\int v du = uv - \int u dv$

Questions

1. **Problem 1** Identify the type of the following conic section and sketch it: $3x^2 - 6x - 4y^2 + 8y = 13$
2. **Problem 2** Show that $|\vec{a} + \vec{b}| \leq |\vec{a}| + |\vec{b}|$
3. **Problem 3** Let θ be the acute angle between the lines $x + 2y = 7$ and $5x - y = 2$. Find $\cos\theta$
4. **Problem 4** Find the area of the region bounded by the parametric equation $x = \sin t, y = e^t, -\pi \leq t \leq 0$ and y-axis

Additional Problems and Reminders

The following are equations for either an ellipse or a parabola or a hyperbola. Please name each of them beside the equation.

1. $x^2 + 1 = y^2 - y$

2. $y^2 - 2y + 2x = 7$

3. $1 - x^2 - y^2 = x + y$

Reminder: MAT235 Quiz2 : October.9 Wednesday Tutorial; Please take the quiz in the tutorial that you enrolled in Acorn. You will get a 0 for writing a quiz in different tutorial :(Always write something to solve the problem. You are guaranteed a 0 for writing nothing.