Integration of Various Types:

- <u>**Polar Coordinate**</u> **in a Double Integral:** If f is continuous on a polar rectangle R given by $0 \leq a \leq r \leq b$, $\alpha \leq \theta \leq \beta$, where $0 \leq \beta - \alpha \leq 2\pi$, then

$$\iint_R f(x,y)dA = \int_\alpha^\beta \int_a^b f(rcos, rsin\theta)rdrd\theta$$

- **Formula for triple integration in** <u>**cylindrical coordinate**</u>**:**

$$\iiint_E f(x,y,z)dV = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(rcos\theta, rsin\theta)}^{u_2(rcos\theta, rsin\theta)} f(rcos\theta, rsin\theta, z)rdzdrd\theta$$

- **Formula for triple integration in** <u>**spherical coordinate**</u>**:**

$$\iiint_E f(x,y,z)dV = \int_c^d \int_\alpha^\beta \int_a^b f(rsin\phi cos\theta, rsin\phi sin\theta, rcos\phi)r^2 sin\phi drd\theta d\phi$$

- **Strategy:** $Intuition/Graphing \rightarrow SetupModel \rightarrow Computation(Fubini, Polar, etc)$

1. Evaluate the integral $\int_0^2 \int_0^{\sqrt{2x-x^2}} \sqrt{x^2+y^2}dydx$
   - Whenever you see $\sqrt{x^2+y^2}$ (which is $= r$), polar coordinate may be useful.
   - Use the intuition/graphing at the beginning to find the corresponding ranges after changing coordinate. Don't forget the extra r when chanting coordinate.
2. Find the volume of the solid which is inside the sphere $x^2+y^2+z^2 = 16$ and outside the cylinder $x^2+y^2 = 4$
   - Setup integration model $\int_{xy-plane} zdA$ to solve a volume problem.
   - Finding the ranges of r which represents the radius and θ which represent angle.
3. The solid E is bounded by the cylinder $y^2+z^2 = 9$ and the planes $x = 3y$, $x = 0$, $z = 0$ in the first octant. Sketch the solid and set up an iterated integral for the volume of the solid.
   - What does each of the equations $y^2+z^2 = 9$ and $x = 3y$, $x = 0$, $z = 0$ represent?
4. Use a double integral to find the area enclosed by one loop of the four-leaved rose $r = cos2\theta$.
   - Area of the region D is given by $A(D) = \iint_D dA$
5. Evaluate $\iint_R (3x+4y^2)dA$, where R is the region in the upper half-plane bounded by the circles $x^2+y^2 = 1$ and $x^2+y^2 = 4$.
   - Using polar coordinate to represent the region R and simplify the question.