MATB24 Quiz7, tut0022

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MATB24 Quiz7,…

MATB24 QUIZ.7 TUT0022

(1) [4 marks] In each part, give a complete definition, or mathematical characterization of the word in red

- An isometry

The linear transformation $T: V \to V$ is an isometry

iff for all v, w in V,

$$\langle T(v), T(w) \rangle = \langle v, w \rangle$$

(2) [5 marks] Give an example of the described object or explain why such an example does not exist.

- A unitary matrix that is not in $M_{2\times 2}(\mathbb{R})$

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \notin M_{2\times 2}(\mathbb{R})$$

$$I_3^* = \overline{I_3}^T = I_3$$

s.t. $I_3^* I_3 = I_3$ which shows I_3 is unitary.

(3) [6 marks] Answer the following question:

- Find the complex fourth roots of -1

Let $z = r(\cos\theta + i\sin\theta)$, s.t. $z^4 = -1$

then $z^4 = r^4(\cos 4\theta + i\sin 4\theta)$

where $r^4 = |-1| = 1 \Rightarrow r = 1$

and $\begin{cases} \cos 4\theta = -1 \\ \sin 4\theta = 0 \end{cases} \Rightarrow 4\theta = \pi + 2\pi k,\ k \in \mathbb{Z}$

s.t. $\theta = \frac{\pi + 2\pi k}{4},\ k \in \mathbb{Z}$

s.t. when $k=0$, $z_0 = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4}$

$k=1$, $z_1 = \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}$

$k=2$, $z_2 = \cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}$

$k=3$, $z_3 = \cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}$

are the complex fourth roots of -1

by the fundamental thm of Algebra. ∎