Definitions and Useful Formulas:

1. **Local/Absolute Maximum and Minimum**(14.7.1)
   (a) A function of two variables has a **local maximum** at (a,b) if $f(x,y) \leq f(a,b)$ when (x,y) is near (a,b). The number f(a,b) is called a **local maximum value**.
   (b) A function of two variables has a **local maximum** at (a,b) if $f(x,y) \geq f(a,b)$ when (x,y) is near (a,b). The number f(a,b) is called a **local maximum value**.
   (c) A function of two variables has a **absolute maximum** at (a,b) if $f(x,y) \leq f(a,b)$ for all (x,y) in the domain of f.
   (d) A function of two variables has a **absolute minimum** at (a,b) if $f(x,y) \geq f(a,b)$ for all (x,y) in the domain of f.
2. A point (a,b) is called a **critical point** (14.7.1) of f if $f_x(a,b) = 0$ and $f_y(a,b) = 0$, or if one of these partial derivatives does not exist.
3. **Second Derivatives Test**
   Suppose the second partial derivatives of f are continuous on a disk with center (a,b), and suppose that $f_x(a,b) = 0$ and $f_y(a,b) = 0$ [that is, (a,b) is a critical point of f]. Let
   $$D = D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - [f_{xy}(a,b)]^2$$
   (a) If $D > 0$ and $f_{xx}(a,b) > 0$, then f(a,b) is a local minimum.
   (b) If $D > 0$ and $f_{xx}(a,b) < 0$, then f(a,b) is a local maximum.
   (c) If $D < 0$, then f(a,b) is not a local maximum or minimum. The point (a,b) is called a saddle point of f.
   (d) If $D = 0$, the test gives no information.
   (e) In fact, D is the determinant $D = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{vmatrix}$
4. A boundary point of D is a point (a,b) such that every disk with center (a,b) contains points in D and also points not in D.
5. A **closed set** in $\mathbb{R}^2$ is one that contains all its boundary points.
6. A **bounded set** in $\mathbb{R}^2$ is one that is contained within some disk.
7. **Extreme Value Theorem for Functions of Two Variables**
   If f is continuous on a closed, bounded set D in $\mathbb{R}^2$, then f attains an absolute maximum value $f(x_1, y_1)$ and an absolute minimum value $f(x_2, y_2)$ at some points (x_1, y_1) and (x_2, y_2) in D.
8. **Lagrange Multiplier**: To find the maximum and minimum values of f(x,y,z) subject to the constraint g(x,y,z)=k, with $\nabla g \neq 0$, we have $\nabla f(x_0, y_0, z_0) = \lambda \nabla g(x_0, y_0, z_0)$, where (x_0, y_0, z_0) is a possible optimal solution.