QUIZ3.extreme_pts_tut3

Thursday, March 4, 2021 18:34

QUIZ3.extre me_pts_tu...

Instuction: You will have 30 minutes to finish quiz and will have 5 minutes to submit the quiz to Crowdmark after the quiz has finished. You will need to keep your cameras on during the quiz and submission times. One hand-held calculator is allowed. Electronic devices, online calculators, notes and other aids are not allowed. Violation of the instruction can be considered as an academic misconduct, and will be reported to the instructor and the department immediately.

Question

Consider the following linear programming problem.

max $z = x_1 + 2x_2 - x_3$

subject to

$2x_1 - 6x_2 + 3x_3 \le 12$

$2x_2 + x_3 \le 2$

$x_1, x_2, x_3 \ge 0$

Find all the extreme points and find the optimal solution.

Plot $(x_1, x_2, x_3) = (0,0,0)$

- **Note:** In order to receive full marks, you will need to do the computation correctly and justify every steps. In particular, explain why your solution is the optimal solution.

max 1 points

① Feasible region is bounded and non-empty, because $0 \le x_1 \le 9$, $0 \le x_2 \le 1$, $0 \le x_3 \le 2$, and by extreme point theorem, optimal solution exists at an extreme point which is a basic feasible solution.

② Canonical form: max $z = C^T x$ s.t. $Ax = b$, $x \ge 0$ where $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ u \\ v \end{pmatrix}$, $c = \begin{pmatrix} 1 \\ 2 \\ -1 \\ 0 \\ 0 \end{pmatrix}$, $A = \begin{pmatrix} 2 & -6 & 3 & 1 & 0 \\ 0 & 2 & 1 & 0 & 1 \end{pmatrix}$, $b = \begin{pmatrix} 12 \\ 2 \end{pmatrix}$

max 2 points

we don't consider $\begin{pmatrix} 2 \\ 0 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ because they are linearly dependent.

-1 if consider this

③ Basic Solution

max 5 points

(1) $\left(\begin{array}{cc|c} 2 & -6 & 12 \\ 0 & 2 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 9 \\ 0 & 1 & 1 \end{array}\right)$, $x = \begin{pmatrix} 9 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

(2) $\left(\begin{array}{cc|c} 2 & 3 & 12 \\ 0 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 2 \end{array}\right)$, $x = \begin{pmatrix} 3 \\ 0 \\ 2 \\ 0 \\ 0 \end{pmatrix}$

(3) $\left(\begin{array}{cc|c} 2 & 0 & 12 \\ 0 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 6 \\ 0 & 1 & 2 \end{array}\right)$, $x = \begin{pmatrix} 6 \\ 0 \\ 0 \\ 0 \\ 2 \end{pmatrix}$

(4) $\left(\begin{array}{cc|c} -6 & 3 & 12 \\ 2 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & 3 \end{array}\right)$, $x = \begin{pmatrix} 0 \\ -\frac{1}{2} \\ 3 \\ 0 \\ 0 \end{pmatrix}$

-1 if each missing or incorrectly calculate any one of these

(5) $\left(\begin{array}{cc|c} -6 & 1 & 12 \\ 2 & 0 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & 18 \end{array}\right)$, $x = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 18 \\ 0 \end{pmatrix}$

(6) $\left(\begin{array}{cc|c} -6 & 0 & 12 \\ 2 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & 6 \end{array}\right)$, $x = \begin{pmatrix} 0 \\ -2 \\ 0 \\ 0 \\ 6 \end{pmatrix}$

(7) $\left(\begin{array}{cc|c} 3 & 1 & 12 \\ 1 & 0 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & 6 \end{array}\right)$, $x = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 6 \\ 0 \end{pmatrix}$

(8) $\left(\begin{array}{cc|c} 3 & 0 & 12 \\ 1 & 1 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & -2 \end{array}\right)$, $x = \begin{pmatrix} 0 \\ 0 \\ 4 \\ 0 \\ -2 \end{pmatrix}$

(9) $\left(\begin{array}{cc|c} 1 & 0 & 12 \\ 0 & 1 & 2 \end{array}\right)$, $x = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 12 \\ 2 \end{pmatrix}$

max 2 points

④ Basic feasible solution, check $x \ge 0$

$x = \begin{pmatrix} 9 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ with $z = 11$, $x = \begin{pmatrix} 3 \\ 0 \\ 2 \\ 0 \\ 0 \end{pmatrix}$ with $z = 1$, $x = \begin{pmatrix} 6 \\ 0 \\ 0 \\ 0 \\ 2 \end{pmatrix}$ with $z = 6$

$x = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 18 \\ 0 \end{pmatrix}$ with $z = 2$, $x = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 6 \\ 0 \end{pmatrix}$ with $z = -2$, $x = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 12 \\ 2 \end{pmatrix}$ with $z = 0$

Therefore, optimal solution exists at $x = \begin{pmatrix} 9 \\ 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$ with optimal value $z = 11$ to the given maximization problem, by reasoning in ①.