Objectives

1. (Theoretical Intuition) Review the basic definitions and properties of vectors, understand the higher dimensions, why do we care about vectors, as well as how to think about the world in vector spaces.
2. (Practical Visualization) Convert parametric equations into vector equation.
3. (Computation) Be able to compute dot product by definition, and cross product using determinant. Verify if two lines are parallel/orthogonal using dot/cross products.

Useful Formulas

1. IMPORTANT! Many of you forgot this in the first quiz. That's actually sad :(
Strategy to solve mat235 problems: Graph/Intuition + Setup Model + Computation!
2. (12.3.3) If θ is the angle between the nonzero vectors $\mathbf{a}$ and $\mathbf{b}$, then $\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}|cos\theta$
3. (12.3.7) Two vectors $\mathbf{a}$ and $\mathbf{b}$ are orthogonal if and only if $\mathbf{a} \cdot \mathbf{b}$
4. (12.3) **Projections**
    (a) Scalar projection of $\mathbf{b}$ onto $\mathbf{a}$: $comp_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$
    (b) Vector projection of $\mathbf{b}$ onto $\mathbf{a}$: $proj_{\mathbf{a}}\mathbf{b} = (\frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|})\frac{\mathbf{a}}{|\mathbf{a}|} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\mathbf{a}$
5. (12.4.9) If θ is the angle between $\mathbf{a}$ and $\mathbf{b}$ (so $0 \leq \theta \leq \pi$), then $|\mathbf{ab}| = |\mathbf{a}||\mathbf{b}|sin\theta$
6. (12.4.10) Two nonzero vectors $\mathbf{a}$ and $\mathbf{b}$ are parallel if and only if $\mathbf{ab} = \mathbf{0}$
7. (12.5) **Skew lines** are lines that do not intersect and are not parallel (and therefore do not lie in the same plane).
8. (12.5.1) **Vector Equation:** $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$
9. (12.5.3) **Symmetric Equation:** $\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$
where $r = (x, y, z), r_0 = (x_0, y_0, z_0), v = (a, b, c),\ a, b, c \neq 0$
10. (12.5.6) **Vector Equation of the Plane:** $\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$
11. (12.5.7) **Scalar equation of the plane through** $P_{0(}x_0, y_0, z_0)$ **with normal vector** $n = (a, b, c)$**:** $a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$
12. (12.4.4) **Cross Product:** $\mathbf{a}\times\mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$

Parametric and Vector Equations

1. Intuition: Pick a corner of the classroom to be the origin and imagine the world is made out of points, then everywhere in the classroom has a coordinate (x,y,z). How about the graph of the parabola $y = x^2$? In fact, it is just a collection of points (x, x^2). How about the graph of the equation $x + 2y + sinz = e^3$? Similarly, it is just the collection of points (x,y,z) that satisfies the equation.

2. Vector Equation: $\mathbf{r} = \mathbf{r}_0 + t\mathbf{v}$

   (a) Why does this equation make sense?
   Imagine an arbitrary line in $\mathbb{R}^2$, as in the intuition, the line is just a collection of points that satisfy the equation. Pick any such point $r_0 = (x_0, y_0, z_0)$, so that you can 'walk' from the origin r_0 by following the coordinate. Now, how can you get to the other points on the line? Follow the direction of the line! The direction is given by v, and t decides how far you want to 'walk' along that direction.

   (b) Given an usual equation of a line, how can I change it to vector equation?

      i. Pick any point that satisfies the equation, call it r_0 (You are on the line!)

      ii. Pick another point that satisfies the equation, say r_1, then the 'direction' $v = r_1 - r_0$ works

      iii. Things get more complicated in higher dimensions.

   (c) Given an parametric equation, how can I change it to vector equation? ($\mathbb{R}^3$)

      i. Known that every points must be of the form (x,y,z) = (x(t),y(t),z(t)), where x(t),y(t),z(t) are the components of the parametric equation

      ii. Now t is the one you want in the vector equation, separate the terms with t and without t in the vector (x(t),y(t),z(t)), see what happen!

   (d) Given a general equation ax+by+cz=k, how can I change it to vector equation?

      i. Same as (c), but now think one of x,y or z as parameter t. If you pick x, and $a \neq 0$, then (x,y,z)=(x(y,z),y,z), where x is a function of y,z. Separate the terms with y only, with z only, and without any of those two, and see what happen!

   (e) Given more than one of those equations as in (d), how can I change it to vector equation? This is when linear algebra becomes useful, but you should still be able to do the same thing as in (d) without linear algebra.

3. Why does the symmetric equation make sense?

4. Draw a diagram to convince yourself that the vector equation of the plane makes sense. ($\mathbf{n} \cdot \mathbf{r} = \mathbf{n} \cdot \mathbf{r}_0$ is the same as $\mathbf{n} \cdot (\mathbf{r} - \mathbf{r}_0) = 0$)