# MATB44_TUT_4

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## Reduction of Order:(D'Alembert Method)

Suppose that we know one solution $y_1(t)$, not everywhere zero, of

$$(*) \quad y'' + p(x)y' + q(t)y = 0$$

If $y_2(t) = c y_1(t)$ where $c$ is a constant then $y_1, y_2$ not form fundamental set of soln.

$y = c_1 y_1 + c_2 y_1$

To find a second solution, let

$$\boxed{y = v(t)y_1(t);}$$ Try to find y

then

$$y' = v'(t)y_1(t) + v(t)y_1'(t)$$

and

$$y'' = v''(t)y_1(t) + 2v'(t)y_1'(t) + v(5)y_1''(t)$$

(*),

$$\boxed{\begin{cases} y_1(t) = e^{rt} \\ y_2(t) = te^{rt}. \end{cases}}$$

If you are given 2 solutions and not sure if they form fundamental set of soln. → wronskian.

Substituting for $y, y'$, and $y''$ in equation and collecting terms, we find that

equal to 0 b/c $y_1$ is a solution.

$$(*) \quad y_1 v'' + (2y_1' + py_1)v' + \boxed{(y_1'' + py_1' + qy_1)v} = 0$$

Since $y_1$ is a solution of the equation, the coefficient of v in equation is zero so that equation becomes

$$\overset{f(t)}{y_1}\overset{w'}{v''} + \overset{g(t)}{(2y_1' + py_1)}\overset{w}{v'} = 0$$

$v' = w$

which is a first-order differential equation for the function v'. Solve for v and find the second solution.

## Question:

$$\boxed{xy'' - y' + 4x^3y = 0,}\ y_1 = sinx^2 \quad (*)$$

$y = \overset{v(x)}{v} y_1 = v\sin x^2$

$xy'' - y' + 4x^3 y = 0$

$x(vy_1)'' - (vy_1)' + 4x^3(vy_1) = 0$

$x(v''y_1 + 2v'y_1' + vy_1'') - (v'y_1 + vy_1') + 4x^3 v y_1 = 0$

$v(\underbrace{xy_1'' - y_1' + 4x^3 y_1}_{0}) + xv''y_1 + 2xv'y_1' - v'y_1 = 0$

b/c $y_1$ is a solution



$x(v''y_1 + 2v'y_1') - v'y_1 = 0$

$x(v''\sin x^2 + 2v' \cdot \cos x^2 \cdot 2x) - v'\sin x^2 = 0$

$xv''\sin x^2 + 4x^2 v'\cos x^2 - v'\sin x^2 = 0$

$v''(x\sin x^2) + v'(4x^2\cos x^2 - \sin x^2) = 0$

let $w = v'$,

$w'(x\sin x^2) = w(\sin x^2 - 4x^2\cos x^2)$

$w' = \frac{dw}{dx}$ $\quad \int \frac{dw}{w} = \int \frac{\sin x^2 - 4x^2\cos x^2}{x\sin x^2}\, dx$

$\log w = \int \frac{1}{x} - \boxed{4x \cdot \frac{\cos x^2}{\sin x^2}\, dx}$

$u = \sin x^2$

$du = \cos x^2 \cdot 2x\, dx$

$\int 4x \cdot \frac{\cos x^2}{\sin x^2}\, dx = 2\int \frac{1}{u}\, du$

$= 2\log(u)$

$\log w = \log x - 2\log(\sin x^2) + C$

choose $C = 0$

$w = e^{\log x - 2\log(\sin x^2)}$

$= e^{\log x} \cdot e^{-2\log(\sin x^2)}$

$= x \cdot \frac{1}{(\sin x^2)^2} = \frac{x}{(\sin x^2)^2}.$

$w = \frac{x}{(\sin x^2)^2}$

$v' = \frac{x}{(\sin x^2)^2}$

$v = \int \frac{x}{(\sin x^2)^2}\, dx = -\frac{1}{2}\cot x^2.$

$y_2 = vy_1 = -\frac{1}{2}\cot x^2 \cdot \sin x^2 = -\frac{1}{2}\cos x^2.$

st $y_1 = \sin x^2$, $\boxed{y_2 = -\frac{1}{2}\cos x^2}$ form a fundamental set of soln of the (*)

$y_2 = \cos x^2$

check using wronskian.