MATB24 Quiz3, tut0022

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MATB24 Quiz3,...

MATB24 Quiz.3, TUT.0022

(1) [4 marks] Give a complete definition, or mathematical characterization of the word in red.

- The **Kernel** of a linear transformation T.

The kernel of a linear transformation $T: V \to W$

is $\{v \in V \mid T(v) = 0_w\}$

(If you defined kernel as $T^{-1}(0_w)$, then you need to define T^{-1})

(2) [4 marks] Give an example (with justification) of a mathematical object that satisfies all the described properties or explain why such an example does not exists.

- A linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^3$ with non-trivial kernel.

Example; Define $T: \mathbb{R}^2 \to \mathbb{R}^3$ by $(x,y) \mapsto (0,0,0)$

then $\ker(T) = \mathbb{R}^2 \neq \{0_v\}$

(3) [7 marks] Carefully prove the given statement:

- Let T: V -> W. Prove that if Ker(T) = $\{0_v\}$, then T is injective.

Assume $\ker(T) = \{0_v\}$

Let $T(x) = T(y)$ for some $x, y \in V$

then $T(x) - T(y) = 0$

$T(x-y) = 0$ b/c T is a linear transformation

then by def of kernel, $x - y \in \ker(T) = \{0_v\}$

s.t $x - y = 0$

and $x = y$

s.t T is injecte by definitions.