Definition

Recall, let V be a vector space. We say that $\{v_1, \cdots, v_n\} \subset V$ is linearly independent if

$$a_1v_1 + \cdots + a_nv_n = 0 \text{ for some } a_1, \cdots, a_n \in \mathbb{F} \implies a_1 = \cdots = a_n = 0$$

Otherwise, we call them linearly dependent.

Remark. *Notice the definition of linearly independent is an implication, instead of an 'and' statement. Therefore, given a linearly independent set, we can not make any conclusion until the hypothesis of the implication is satisfied.*

Questions

1. Let V, W be vector spaces over $\mathbb{F}$. Let $T : V \to W$ be an invertible linear transformation. Let $I = \{v_1, \cdots, v_n\}$ be a linearly independent set of vectors in V. Prove that,

   (a) Show that $T(I) = \{T(v_1), \cdots, T(v_n)\}$ is also linearly independent

2. Let V, W be vector spaces over $\mathbb{F}$. Let $T : V \to W$ be a linear transformation. Assume that $S = \{T(v_1), \cdots, T(v_n)\}$ are linearly independent for some $v_1, \cdots, v_n \in \mathbb{F}$. Show that $v_1, \cdots, v_n$ are linearly independent.