Objectives

1. Understand the basic definitions of parametric equation, and be able to sketch parametric equation with intuition and steps.
2. Compute the first and second derivatives with respect to different variables.
3. Compute the arc length of a curve given by a parametric equation and understand the intuition of the formula using.

Useful Formulas

1. (10.2)Computing first derivative $\frac{\partial y}{\partial x} = \frac{(\frac{\partial y}{\partial t})}{(\frac{\partial x}{\partial t})}$ if $\frac{\partial x}{\partial t} \neq 0$
2. (10.2)Computing second derivative $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}(\frac{dy}{dx})}{\frac{dx}{dt}}$
3. Trigonometry
   (a) $sin(-x) = -sin(x)$
   (b) $cos(x) = cos(-x)$
   (c) $sin(\frac{\pi}{4}) = cos(\frac{\pi}{4})$
   (d) $sin^2(x) + cos^2(y) = 1$
4. (10.2)If a curve C is described by the parametric equations x = f(t), y = g(t), $\alpha \leq t \leq \beta$, where f' and g' are continuous on $[\alpha,\beta]$ and C is traversed exactly once as t increases from α to β, then **the length of C** is

$$L = \int_{\alpha}^{\beta} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt$$

Sketch Parametric Curve

1. Get intuition by sketching the x-t and y-t graphs
2. Find initial point and terminal point
3. Compute the first derivatives to determine the rate of change and find the critical points. **Example:** as t increasing(moving along the curve), x increases or decreases?
4. Compute the second derivatives to determine the concavity(concave up/down) of the graph and find the inflection points. **Caution:** Concavity in x-y or y-x graph?
5. Sketch! and label all the 'special' points for clarification :)

Additional Problems

1. Find the dy/dx and d^2y/dx^2. For what values of t is the curve concave upward?
   (a) $x = t^2 + 1, y = t^2 + t$
   (b) $x = cost, y = sin2t$
2. Find the exact length of the curve
   (a) $x = 1 + 3t^2, y = 4 + 2t^3, 0 \leq t \leq 1$
   (b) $x = tsint, y = tcost, 0 \leq t \leq 1$