MATB24 Quiz6, tut0022

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MATB24 Quiz6,...

MATB24 Quiz.6, TUT.0022

(1) [4 marks] Give a <u>complete</u> definition, or mathematical characterization of the word in *red*.

- An eigenvalue of a matrix

λ is called an eigenvalue of a matrix A
if there exist a non-zero vector v,
s.t $Av = \lambda v$

(2) [5 marks] Determine whether the given statement is true or false. Justify your answer.

- Suppose v_1 and v_2 are eigenvectors corresponding to distinct eigenvalues λ_1 and λ_2, then v_1 and v_2 are linearly independent

Assume $T(v_1) = \lambda_1 v_1$, $T(v_2)\ \lambda_2 v_2$, assume $\lambda_1 \neq \lambda_2$

Assume $a_1v_1 + a_2v_2 = 0$ ① for some $a_1, a_2 \in \mathbb{F}$, WTS $a_1 = a_2 = 0$

$0 = T(0)$ by T being linear transformation

$= T(a_1v_1 + a_2v_2) = a_1T(v_1) + a_2T(v_2)$ by property of L.T

$= a_1\lambda_1 v_1 + a_2\lambda_2 v_2$ ②

①$\cdot\lambda_1 \Rightarrow a_1\lambda_1 v_1 + a_2\lambda_1 v_2 = 0$ ③

then ②-③ $a_2(\lambda_2 - \lambda_1)v_2 = 0$

since $\lambda_1 \neq \lambda_2$, s.t $\lambda_2 - \lambda_1 \neq 0$, and v_2 is eigenvector

s.t $v_2 \neq 0$, s.t $a_2 = 0$

sub to ①

$a_1 v_1 = 0$

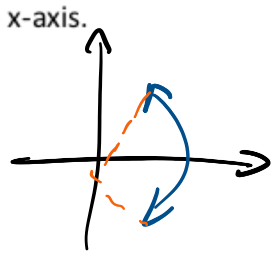
and v_1 is eigenvector

s.t $v_1 \neq 0$, s.t $a_1 = 0$ ∎

(3) [6 marks] For the following transformation, find an eigenvector using any methods you can think of, including basic geometry, if this is possible. What are the corresponding eigenvalues?

- $V = \mathbb{R}^2, T =$ reflection over the x-axis.

consider $w = \begin{pmatrix}1\\0\end{pmatrix} \in \mathbb{R}^2$



then $T\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}$

by geometric def of T

s.t $\begin{pmatrix}1\\0\end{pmatrix}$ is an eigenvector with eigenvalue $\lambda = 1$

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