Learning Objective

Understand the structures of the linear programming problem in standard form and in canonical form. Be able to set up a linear programming model using suitable notation, and convert to different forms when needed. Understand how to reformulate a special type of piecewise linear programming problem into a standard linear programming problem.

Questions

1. For each of the following problems, determine whether or not it is a standard linear programming problem. If not, explain the reason and reformulate it to a standard linear programming problem using matrix notation, if it is not possible, explain why.

   (a) min $z = 4x + 5y$
   subject to
   $-4x \leq 5$
   $x \geq 0$

   (b) max $z = 4x_1 + 5x_2 + 1$
   subject to
   $4x_1 \leq 5$
   $x_1 \geq 0,\ x_2 \geq 0,\ 1 \geq 0$

2. Reformulate the following problem into a canonical linear programming problem using matrix notation.

   min $z = 4x + 5y$
   subject to
   $-4x \leq 5$
   $x \geq 0$

3. Sketch the following on an xy plane

   (a) $y = |x|$

   (b) $\{(x, y) : y \geq x \text{ and } y \geq -x\}$

   (c) Let x be an arbitrary fixed real number, so $|x|$ is also a real number now. Express $|x|$ as an element of the set $\{y : y \geq x \text{ and } y \geq -x\}$, and explain it graphically using part (a) and (b).

4. Consider the following problem:

   min $z = 2|x_1| - 5x_2$

   subject to

   $x_1 + x_2 = -2$

   $x_2 \leq 0$

   (a) Without reformulating the problem, draw the feasible region and solve it directly using graphical method.

   (b) Reformulate the problem into a standard linear programming problem using the idea from question 3(c). Draw the feasible region and solve it graphically.

   (c) Verify whether the feasible regions are the same before and after the reformulation. Verify whether you got the same answer.