TUT8_More_about_simplex_tableaux

Thursday, March 18, 2021 09:03

TUT8_More_about_si...

Introduction

Last week, we talked about the simplex method and how it can be used to solve a LPP with $b \geq 0$. This week, we will take a closer look at the simplex tableaux and think about what is happening behind geometrically when we are running the simplex algorithm.

Questions

1. Consider the following LPP:

   max $z = x_1 + x_2$

   subject to

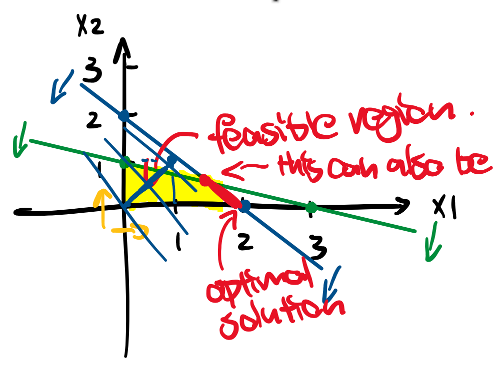
   $x_1 + x_2 \leq 2$

   $x_1 + 3x_2 \leq 3$

   $x_1, x_2 \geq 0$

   Answer the following questions.

   (a) Draw the feasible region in the x_1x_2 plane and solve the LPP using the graphical method. Find all the optimal solutions and what is the optimal cost?



$\nabla z = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

Intersection of two lines can be optimal solution, that is $(x_1, x_2) \in \mathbb{R}^2$ st

$\begin{cases} x_1 + x_2 = 2 & (1) \\ x_1 + 3x_2 = 3 & (2) \end{cases}$

$(2) - (1)$

$2x_2 = 1$

$x_2 = \frac{1}{2} \Rightarrow x_1 = \frac{3}{2}$

st $(x_1, x_2) = (\frac{3}{2}, \frac{1}{2})$ is an optimal sln with cost 2 ∎

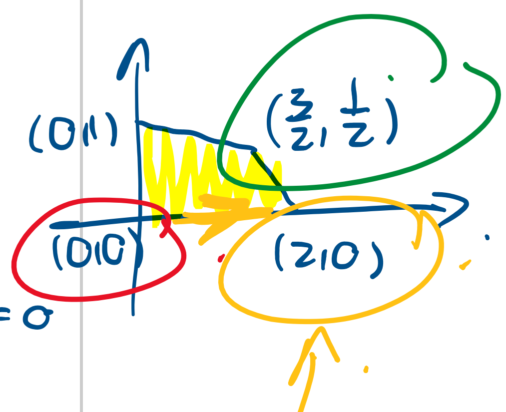
   (b) Looking at the graph of the feasible region, find all the extreme points using geometrical approach then solve the LPP using the extreme point theorem.

The feasible region is bonded and non-empty, st by extreme pt thm, optimal solution exists, and at least one of the extreme points is optimal solution.

The extreme points are $(2,0)$, $(0,1)$ and $(\frac{3}{2}, \frac{1}{2})$, $(0,0)$ corresponding to $z = 2, 1,$ and $2, 0$ respectively.

and hence $(2,0)$ and $(\frac{3}{2}, \frac{1}{2})$ are both optimal solutions.

   (c) Use the simplex method to solve the LPP. Clearly indicate the current BFSs, current cost, and identify the current BFSs in the graph of the feasible region, at every stage.



max $z = c^T x$

st $Ax = b$

$x \geq 0$

where $c = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$, $x = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}$

$A = \begin{pmatrix} 1 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{pmatrix}$, $b = \begin{pmatrix} 2 \\ 3 \end{pmatrix} \geq 0$

starting BFS: $x = \begin{pmatrix} 0 \\ 0 \\ 2 \\ 3 \end{pmatrix}$, current cost = 0

$$\begin{array}{cccc|c} x_1 & x_2 & x_3 & x_4 & \\ 1 & 1 & 1 & 0 & 2 \\ 1 & 3 & 0 & 1 & 3 \\ \hline 1 & 1 & 0 & 0 & 0 \end{array} \qquad \begin{array}{l} \to 2/1 \\ \to 3/1 \end{array}$$

x_1, x_4 basic variables.

$$\leadsto \begin{array}{cccc|c} 1 & 1 & 1 & 0 & 2 \\ 0 & 2 & -1 & 1 & 1 \\ \hline 0 & 0 & -1 & 0 & -2 \end{array}$$

x_2, x_3 are non-basic variables.

current BFS $x = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ with current cost 2

No positive entries in the objective row, st we reach optimal solution and optimal sln is $x = \begin{pmatrix} 2 \\ 0 \\ 0 \\ 1 \end{pmatrix}$ with optimal value 2 ∎

   (d) Can you find a different optimal solution using the simplex tableaux that you end at question c?

$$\begin{array}{cccc|c} 1 & 1 & 1 & 0 & 2 \\ 0 & 2 & -1 & 1 & 1 \\ \hline 0 & 0 & -1 & 0 & -2 \end{array} \quad \begin{array}{l} 2/1 \\ 1/2 \end{array} \leadsto \begin{array}{cccc|c} 1 & 0 & \frac{3}{2} & -\frac{1}{2} & \frac{3}{2} \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \hline 0 & 0 & -1 & 0 & -2 \end{array}$$

current BFS $x = \begin{pmatrix} \frac{3}{2} \\ \frac{1}{2} \\ 0 \\ 0 \end{pmatrix}$

cost = 2

No positive, st optimal solution!

2. Consider the following complex tableaux that happens at one intermediate step when solving a LPP using simplex algorithm.

x_1	x_2	x_3	x_4	x_5	x_6	
1	−1	1	1	0	0	1
0	1	2	0	1	0	3
0	1	1	1	0	1	2
0	0	−14	−3	0	0	−28

The original $Ax = b$ is not necessary this, but is equivalent to this, since we were doing row operations in the simplex algorithm. (※)

   (a) What is the current BFS? What is the current cost? Is this an optimal solution?

Current BFS $x = (1, 0, 0, 0, 3, 2)$, current cost 28

This is the optimal cost b/c there is no positive entry in objective row.

   (b) Can you find a worse BFS with a lower cost using the simplex tableaux above? Verify that the answer you got is actually a BFS.

Basic variables

$$\begin{array}{cccccc|c} 1 & -1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 0 & 3 \\ 0 & 1 & 1 & 1 & 0 & 1 & 2 \\ \hline 0 & 0 & -14 & -3 & 0 & 0 & -28 \end{array} \to \begin{array}{cccccc|c} 1 & -1 & 1 & 1 & 0 & 0 & 1 \\ -2 & 3 & 0 & -2 & 1 & 0 & 1 \\ -1 & 2 & 0 & 0 & 0 & 1 & 1 \\ \hline 14 & -14 & 0 & 11 & 0 & 0 & -14 \end{array}$$

$x = (0, 0, 1, 0, 1, 1)$

check the equivalent conditions to $Ax = b$ on (※), for example

$1 \cdot x_1 - 1 \cdot x_2 + 1 \cdot x_3 + 1 \cdot x_4 + 0 \cdot x_5 + 0 \cdot x_6$

$= 1 \cdot 0 - 1 \cdot 0 + 1 \cdot 1 + 1 \cdot 0 + 0 \cdot 1 + 0 \cdot 1 = 1$

and also $x \geq 0$, st x is a BFS.

   (c) Can you find a worst BFS with the lowest cost using the simplex tableaux above?

$$\begin{array}{cccccc|c} 1 & -1 & 1 & 1 & 0 & 0 & 1 \\ -2 & 3 & 0 & -2 & 1 & 0 & 1 \\ -1 & 2 & 0 & 0 & 0 & 1 & 1 \\ \hline 14 & -14 & 0 & 11 & 0 & 0 & -14 \end{array} \to \begin{array}{cccccc|c} \frac{1}{3} & 0 & 1 & \frac{1}{3} & \frac{1}{3} & 0 & \frac{4}{3} \\ -\frac{2}{3} & 1 & 0 & -\frac{2}{3} & \frac{1}{3} & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 & \frac{4}{3} & -\frac{2}{3} & 1 & \frac{1}{3} \\ \hline \frac{14}{3} & 0 & 0 & \frac{5}{3} & \frac{14}{3} & 0 & -\frac{28}{3} \end{array}$$

$x = (0, \frac{1}{3}, \frac{4}{3}, 0, 0, \frac{1}{3})$ is the "worst" BFS with lowest cost $\frac{28}{3}$, since there is no negative entries in objective row to make lower cost

   (d) Can you find another optimal solution that is different from the current one, using the simplex tableaux above?

→ see worksheet 4, Question 4C.