Second order linear equations

When solving the general solution of the heat equation using the method of separation of variables, we have two linear equations $T' + \lambda kT = 0$ and $X'' + \lambda X = 0$ where $\lambda > 0$. Using the knowdledge of ordinary differential equations, we solved that

$$T(t) = Ae^{-\lambda kt}$$

and

$$X(x) = Ccos(\sqrt{\lambda}x) + Dsin(\sqrt{\lambda}x)$$

The details are as follow: To solve a second order differential equation

$$ay'' + by' + cy = 0$$

start with the guess $y = e^{rx}$ for some constant r. Plug into the differential equation, and ...