

Learning Objective

Understand the structures of the linear programming problem in standard form and in canonical form. Be able to set up a linear programming model using suitable notation, and convert to different forms when needed. Understand how to reformulate a special type of piecewise linear programming problem into a standard linear programming problem.

Questions

1. For each of the following problems, determine whether or not it is a standard linear programming problem. If not, explain the reason and reformulate it to a standard linear programming problem using matrix notation, if it is not possible, explain why.

$$\begin{aligned} \text{(a) } \min z &= 4x + 5y \\ \text{subject to} \\ -4x &\leq 5 \\ x &\geq 0 \end{aligned}$$

$$\begin{aligned} \text{(b) } \max z &= 4x_1 + 5x_2 + 1 \\ \text{subject to} \\ 4x_1 &\leq 5 \\ x_1 \geq 0, x_2 \geq 0, 1 &\geq 0 \end{aligned}$$

2. Reformulate the following problem into a canonical linear programming problem using matrix notation.

$$\begin{aligned} \min z &= 4x + 5y \\ \text{subject to} \\ -4x &\leq 5 \\ x &\geq 0 \end{aligned}$$

3. Sketch the following on an xy plane

(a) $y = |x|$

(b) $\{(x, y) : y \geq x \text{ and } y \geq -x\}$

(c) Let x be an arbitrary fixed real number, so $|x|$ is also a real number now. Express $|x|$ as an element of the set $\{y : y \geq x \text{ and } y \geq -x\}$, and explain it graphically using part (a) and (b).

4. Consider the following problem:

$$\min z = 2|x_1| - 5x_2$$

subject to

$$x_1 + x_2 = -2$$

$$x_2 \leq 0$$

- (a) Without reformulating the problem, draw the feasible region and solve it directly using graphical method.
- (b) Reformulate the problem into a standard linear programming problem using the idea from question 3(c). Draw the feasible region and solve it graphically.
- (c) Verify whether you got the same answer before and after the reformulation.