

TUT1_Set_Absolute_Value_and_Distance_(printing_friendly)

Friday, September 24, 2021 15:46

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Set, Absolute Value and Distance

MATH1510 TUT
Sep.24 2021

Questions

1. Translate the following English sentences using set notation.

(a) The set of all positive real numbers

$\{x \in \mathbb{R} \mid x > 0\}$ or $\{x \mid x \in \mathbb{R} \text{ and } x > 0\}$

(b) The set of all real numbers greater or equal to 2 and less than 10

$\{x \in \mathbb{R} \mid 2 \leq x < 10\} = [2, 10)$

(c) The set of all even numbers

Every even number is of the form $x = 2k$ for some integer k ($k \in \mathbb{Z}$)

$\{x \mid x = 2k \text{ for some } k \in \mathbb{Z}\}$

"for some" means "there exists at least one"

(d) The set of all real numbers that are less than 5 units away from 1

Distance between 2 points a and b in \mathbb{R} is given by $|b-a| = |-(a-b)| = |a-b|$

$\{x \in \mathbb{R} \mid |x-1| < 5\} = (-4, 6)$

$|x-1| < 5 \iff \begin{cases} x-1 < 5 \\ x-1 > -5 \end{cases} \iff \begin{cases} x < 6 \\ x > -4 \end{cases}$

1 unit, 2 units, 5 units away = $|6-1| = 5$

(e) The set of all real numbers that are greater or equal to 6 units away from 2

$\{x \in \mathbb{R} \mid |x-2| \geq 6\}$

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2. Write the descriptions of the following sets in English.

(a) $\{x \in \mathbb{R} \mid x \leq -3\}$

The set of all real numbers that are less than or equal to -3.

(b) $\{x \in \mathbb{R} \mid |x-3| < 2\}$

The set of all real numbers that are less than 2 units away from 3.

(c) $\{x \in \mathbb{R} \mid x = 2k+1 \text{ for some } k \in \mathbb{Z}\}$

The set of all real numbers that are of the form $2k+1$ for some integer k .

In fact, this is the set of all odd numbers.

(d) $\{x \in \mathbb{R} \mid |x-3| = 2\}$

The set of all real numbers that are 2 units away from 3.

In fact, this is the set that contains the numbers 1 and 5.

3. Perform the set operation and graph the set.

(a) $A = \mathbb{Z}, B = [1, 6]$. Find $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

The set of all integers.

The set of all real numbers that are greater or equal to 1 and less than or equal to 6.

$A \cap B = \{x \mid x \in \mathbb{Z} \text{ and } x \in \mathbb{R} \text{ and } 1 \leq x \leq 6\}$

$= \{1, 2, 3, 4, 5, 6\}$

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(b) $A = (-\infty, 0), B = [-2, 4]$. Find $A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

The set of all real numbers that are less than 0.

$A \cup B = (-\infty, 4]$

It is okay if $x \in A$ and $x \in B$ at the same time, as long as at least one of the conditions is met.

(c) $A = (-2, 0), B = [0, 4]$. Find $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

Does not include 0.

$A \cap B = \emptyset$

Empty set, a set with no elements in it.

(d) $A = (-4, 0), B = [-2, 5]$. Find $A \cap B$

$A \cap B = [-2, 0)$

Example: square bracket and circular bracket

$[-2, 0) = \{x \in \mathbb{R} \mid -2 \leq x < 0\}$

Inclusion

Exclusion

Always use circular bracket for $+\infty$ or $-\infty$.

Example: $(-\infty, 2)$, $[4, +\infty)$

(e) $A = \mathbb{Z}, B = (1, 5], C = \{1, 2, 3\}$. Find $C \cup (A \cap B)$

$A \cap B = \{x \mid x \in A \text{ and } x \in B\} = \{x \in \mathbb{Z} \mid 1 < x \leq 5\} = \{2, 3, 4, 5\}$

$C \cup (A \cap B) = \{1, 2, 3\} \cup \{2, 3, 4, 5\} = \{1, 2, 3, 4, 5\}$

Only consider the elements once even if it appears in both sets.

(f) $A = \emptyset, B = [0, 2]$. Find $A \cup B$ and $A \cap B$.

$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$

$= \{x \mid x \in \emptyset \text{ or } x \in [0, 2]\} = \{x \mid x \in [0, 2]\} = [0, 2] = B$

not going to give us any possible choice of x

$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$

$= \{x \mid x \in \emptyset \text{ and } x \in [0, 2]\} = \emptyset = A$

at the same time

have to satisfy both conditions