

This list of questions is for the students in MATB24, TUT0005 summer 2021 at the University of Toronto Scarborough. You should not use any of the facts in this document as a reference in the midterm test. Everything covered in this document has been talked about in the lecture, tutorial or the textbook. The test may cover materials that are not included in this document. You should refer to the professor's email for the coverage of the test. Good luck in the midterm!

True or false

Answer the following true/false questions. If the statement is true, give a brief justification, otherwise give an explicit counter example. The following true/false questions can be found in the textbook Linear Algebra Done Wrong.

1. Every vector space contains a zero vector.
2. A vector space can have more than one zero vector.
3. If f and g are polynomials of degree n , then $f + g$ is also a polynomial of degree n
4. If f and g are polynomials of degree at most n , then $f + g$ is also a polynomial of degree at most n
5. Any set containing a zero vector is linearly dependent
6. A basis must contain 0
7. subsets of linearly dependent sets are linearly dependent
8. subsets of linearly independent sets are linearly independent
9. If $a_1v_1 + a_2v_2 + \dots + a_nv_n = 0$ then all scalars a_k are zero
10. Every vector space that is generated by a finite set has a basis
11. Every vector space has a (finite) basis
12. A vector space cannot have more than one basis
13. If a vector space has a finite basis, then the number of vectors in every basis is the same
14. The dimension of $\mathbb{P}_n(\mathbb{R})$ is n
15. The dimension on $M_{m \times n}$ is $m + n$
16. If vectors v_1, v_2, \dots, v_n generate (span) the vector space V , then every vector in V can be written as a linear combination of vector v_1, v_2, \dots, v_n in only one way
17. Every subspace of a finite-dimensional space is finite-dimensional
18. If V is a vector space having dimension n , then V has exactly one subspace of dimension 0 and exactly one subspace of dimension n

19. Any system of linear equations has at least one solution
20. Any system of linear equations has at most one solution
21. Any homogeneous system of linear equations has at least one solution
22. Any system of n linear equations in n unknowns has at least one solution
23. Any system of n linear equations in n unknowns has at most one solution
24. If the homogeneous system corresponding to a given system of a linear equations has a solution, then the given system has a solution
25. If the coefficient matrix of a homogeneous system of n linear equations in n unknowns is invertible, then the system has no non-zero solution
26. The solution set of any system of m equations in n unknowns is a subspace in \mathbb{R}_n
27. The solution set of any homogeneous system of m equations in n unknowns is a subspace in \mathbb{R}_n
28. Every change of coordinate matrix is square
29. Every change of coordinate matrix is invertible
30. The rank of a matrix is equal to the number of its non-zero columns
31. The rank of an $n \times n$ matrix is at most n
32. Every linear operator in an n -dimensional vector space has n distinct eigenvalue
33. If a matrix has one eigenvector, it has infinitely many eigenvectors, assuming a real vector space.
34. There exists a square real matrix with no real eigenvalues
35. Similar matrices always have the same eigenvalues
36. Similar matrices always have the same eigenvectors
37. A non-zero sum of two eigenvectors of a matrix A is always an eigenvector
38. A non-zero sum of two eigenvectors of a matrix A corresponding to the same eigenvalue λ is always an eigenvector
39. A^T has the same eigenvalues as A
40. A^T has the same eigenvectors as A
41. If A is diagonalizable, then so is A^T