Intuition

Please check the tutorial note for intuition and useful formulas.

Question 1:

Find the limit, if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{5y^4 \cos^2 x}{x^4 + y^4}$$

- 1. Intuitively, does the limit exist or not? (Hint: What does $\cos^2 x$ contributes to the numerator part?)
- 2. If the limit exist, can you prove it and find the limit? If not, can you prove that the limit does not exist? (Hint: Can you combine x^4 and y^4 along some path?)

Question 2:

Find the limit, if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy}{\sqrt{x^2 + y^2}}$$

- 1. Intuitively, does the limit exist or not?
 (Hint: Which has the higher 'degree' intuitively, numerator or denominator? You can maybe think of xy be degree 2)
- 2. If the limit exist, can you prove it and find the limit? If not, can you prove that the limit does not exist? (Hint: Squeeze Theorem, relation between $x^2 + y^2$ and xy?)

Note

Please work on more questions in the section 14.2 Q5-22 of the textbook. You will see questions of many more different types, including trigonometry, exponential, etc.