Useful Definitions:

- (16.1.1) Let D be a set in $\mathbb{R}^2$. A **vector field** on $\mathbb{R}^2$ is a function **F** that assigns to each point (x,y) in D a two-dimensional vector **F**(x,y)
- (16.1.2) Let E be a subset of $\mathbb{R}^3$. A **vector field** on $\mathbb{R}^3$ is a function **F** that assigns to each point (x,y,z) in E a three-dimensional vector **F**(x,y,z)
- Examples of vector fields: Velocity field, gravitational field, force field, electric field, gradient vector field.
- If f is a scalar function of two variables, then its gradient $f(x,y) = f_x(x,y)\mathbf{i} + f_y(x,y)\mathbf{j}$ is a vector field on $\mathbb{R}^2$, and is called a **gradient vector field**.
- A vector field **F** is called a **conservative vector field** if it is the gradient of some scalar function, i.e. there exists a function f such that $\mathbf{F} = \nabla f$, and f is called a **potential function** for F.

Procedures to find the potential function

To find the potential function f of a conservative vector field F(x,y,z)=(F_1, F_2, F_3):

1. Assume there is a function f(x,y,z), such that $\nabla f(x,y,z) = F(x,y,z)$
2. Now you have a system of equations, remember your goal is to find the function f(x,y,z):
$$f_x(x,y,z) = F_1(x,y,z)$$
$$f_y(x,y,z) = F_2(x,y,z)$$
$$f_z(x,y,z) = F_3(x,y,z)$$
3. Choose one of the equations above to start, and integrate both sides with respect to the corresponding variable. Notice, you only found ”some” part of the function f, after every time you integrate.

Questions:

1. Let $F(x,y,z) = (y, x + 2ysinz, z + y^2cosz)$ be a conservative vector field. Find a potential function of F.
2. Evaluate $\iint_D sin(9x^2+4y^2)dA$, where D the region bounded by the ellipse $9x^2+4y^2 = 1$.