MATB24 Quiz5, tut0022

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MATB24 Quiz5,...

MATB24 Quiz.5, TUT.0022

(1) [4 marks] Give a complete definition, or mathematical characterization of the word in *red*.

- An orthogonal set of vectors

let V be an inner product space equip with $\langle -,- \rangle$

then $\{v_1, \dots, v_n\}$ is an orthogonal set of vectors in V

if $v_i \in V$ and $\langle v_i, v_j \rangle = 0$ for all $i \neq j$, $1 \le i \le n$, $1 \le j \le n$.

(2) [3 marks] Give an example (with justification) of a mathematical object that satisfies all the described properties or explain why such an example does not exist.

- An inner product on $\mathbb{R}^3$ other than the dot product

$\langle (a_1, b_1, c_1), (a_2, b_2, c_2) \rangle := 2011a_2 + 2b_1b_2 + 201c_2$

Justification!!

domain and codomain

well-defined

① function

② positive definite.

(3) [8 marks] Carefully prove the following

- V = the space of infinite bounded sequences $a_1, a_2, \dots$ is an inner product space defined by $\langle f, g \rangle = \sum_i \frac{a_i b_i}{2^i}$
- Hint: There are four conditions to check if a given < , > is an inner product

③ conjugate symmetry $\langle f,g \rangle = \overline{\langle g,f \rangle}$

④ linearity (in the first component)

④. (1) why is $\langle -,- \rangle$ a valid function? ①.

(2) how can you check $\forall$ statements ~~pick arbitrary element.~~

(3) Does it make a difference if $\mathbb{F} = \mathbb{C}$ instead of $\mathbb{R}$? NO, counter example.

① codomain: Bounded, s.t $|a_i| < M$, $|b_i| < N$ for all i

converges. $\sum_i \frac{a_i b_i}{2^i} = \sum_i \frac{MN}{2^i}$, known $\sum \frac{1}{2^i}$ converges by geometric series with $r = \frac{1}{2} < 1$

s.t $MN \sum \frac{1}{2^i} = \boxed{\sum \frac{MN}{2^i}}$ also converges.

well-define: $(f_1, g_1) = (f_2, g_2) \Rightarrow \langle f_1, g_1 \rangle = \langle f_2, g_2 \rangle$

② positive definite: $\langle f, f \rangle = \sum \frac{a_i a_i}{2^i} = \sum \frac{a_i^2}{2^i} \geq 0$ b/c $a_i^2 \geq 0$.

③ conjugate symmetry

$\langle f, g \rangle = \sum \frac{a_i b_i}{2^i} = \sum \frac{b_i a_i}{2^i} = \sum \frac{\overline{b_i}\,\overline{a_i}}{2^i}$ b/c $a_i, b_i \in \mathbb{R}$.

$= \overline{\left(\sum \frac{b_i a_i}{2^i}\right)} = \overline{\langle g, f \rangle}$

④ Linearity: By def directly.

Question D①.

Assume $B = \{b_1, \dots, b_n\}$ basis of V $T: V \to W$.

basis of W.

WTS $A = \{T(b_i)\}_{1 \le i \le n}$ basis of W.

LI

$\sum a_i T(b_i) = 0$.

WTS $a_i = 0$.

$T(\sum a_i b_i) = 0$. T is 1-1.

$\sum a_i b_i \in \ker(T) = \{0\}$.

$\sum a_i b_i = 0$.

$\Rightarrow a_i = 0$.

span(A) = W.

$\subseteq$

$T(b_i) \in W$.

$\mathrm{span}(A) \subseteq W$.

$\supseteq$

let $w \in W$. WTS

$w \in \mathrm{span}(A) = \mathrm{span}(T(b_i))$

$w = \sum a_i T(b_i)$

|| — T is linear.

$w = T(\sum a_i b_i)$

|| — T is invertible

$T^{-1}(w) = \sum a_i b_i$.

$T^{-1}(w) = \sum a_i b_i$.

$\in V$.

$\{b_i\}$ basis of V.

D②: Give one example

and one non-example

Show false.