

Worksheet2

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MATB61 TUT03/04

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Graphing and Proof of a Convex Set Jan.28 2021 Worksheet2

Learning Objective

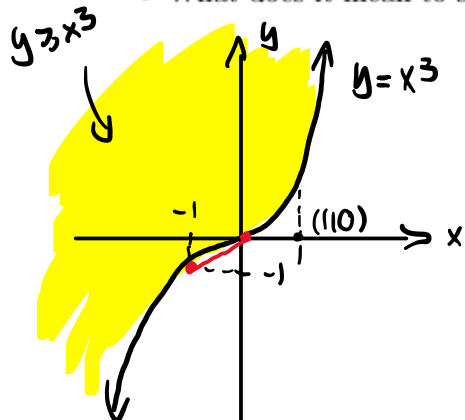
Understand the definition of a convex set graphically and by definition. Understand how to convert the intuition from graphing to a proof.

Questions

1. Show that the graph of $y \geq x^3$ in the xy plane is not a convex set.

(a) by drawing the graph and a line segment between two points of the set.

- What does it mean to say that a set S is convex graphically?



- Notice $(1,0)$ is not in the graph of $y \geq x^3$, b/c $0 \not\geq 1^3$, so the shaded region is the graph of $y \geq x^3$ (including the line $y = x^3$)
- Notice $(-1, -1)$, $(0,0)$ are points in the graph of $y \geq x^3$, but the graph does not contain the whole line segment joining those points, so

(b) directly from the definition of convex set. The graph is not a convex set.

- What does it mean to say that a set S is not convex by definition?
- How is your proof related to part(a)?

Let $S = \{(x,y) \in \mathbb{R}^2 \mid y \geq x^3\}$ this is the graph of $y \geq x^3$

- we say that S is convex, if for all $z_1, z_2 \in S$, and any $\lambda \in \mathbb{R}$ with $0 \leq \lambda \leq 1$, we have $\lambda z_1 + (1-\lambda)z_2 \in S$ as well.

WTS S is not convex.

choose some choice and recognize that it does not satisfy all choices!

Notice for $z_1 = (0,0)$, $z_2 = (-1, -1) \in \mathbb{R}^2$

① WTS $z_1, z_2 \in S$

→ verify your choice is a valid choice

$0 \geq 0^3$ and $-1 \geq (-1)^3$, so $z_1, z_2 \in S$

② Pick $\lambda = \frac{1}{2}$ and it is clear that $0 \leq \lambda \leq 1$

Define $z = \lambda z_1 + (1-\lambda)z_2 = \frac{1}{2}(0,0) + \frac{1}{2}(-1,-1) = (-\frac{1}{2}, -\frac{1}{2})$

but $(-\frac{1}{2})^3 = -\frac{1}{8} > -\frac{1}{2}$ ← " $x^3 > y$ "

so $z \notin S$

hence S is not convex by definition. \square

what does it by not containing the whole line by definition?

At least one point on the line is not in the graph!

here, this z point is not in the graph.