Check the definitions for yourself from the theorem citing list. I will briefly remind you the definitions when working on the questions. We will focus on the discussion of eigenvalue and eigenvector today.

Questions

1. For each of the following linear maps T find all its eigenvalues, and for each eigenvalue λ, find a basis for the eigenspace E_λ.

   (a) $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by the matrix $\begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix}$

(b) $T : \mathbb{R}^4 \to \mathbb{R}^4$ given by the matrix $\begin{pmatrix} 1 & 2 & -3 & 1 \\ 2 & 1 & 7 & 2 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{pmatrix}$

(c) $T : \mathbb{P}_2(\mathbb{R}) \to \mathbb{P}_2(\mathbb{R})$ given by $T(a+bx+cx^2) = (a+2b+3c)+(4a+5b+6c)x+(7a+8b+9c)x^2$

Remark. *As a further practice, prove for yourself that $T : V \to V$ has an eigenvalue λ if and only if $[T]_\alpha^\alpha$ has an eigenvalue λ for some basis α. In other words, this method does not depends on the choice of the basis.*

2. Find the eigenvalues and eigenspaces of the transposition map

$$T : M_n(\mathbb{R}) \to M_n(\mathbb{R})$$

given by $T(A) := A^T$

3. Prove that a matrix $A \in M_3(\mathbb{R})$ has at least one real eigenvalue.

4. Prove that if A, B are $n \times n$ matrices, then $trace(AB) = trace(BA)$. Conclude that similar matrices have the same trace.