Thursday, March 25, 2021 09:10



MATB61 TUT03/04 Nick Huang Two-phased Method Mar.25 2021 Week 11 Introduction Recall, we require the condition that $b \geq 0$ (in other words, $b_i \geq 0$ for all i) when running the simplex algorithm, because we have a way to choose the pivot (positive $a_i j$ with smallest ratio) following from the interpretation when $b \geq 0$. However, if one of the $b_i < 0$, then we are not sure how to choose the pivot without violating Ax = b. Besides, if $b_i < 0$, then we do not have a starting BFS simply by making all slack variables to be the basic variables. In this tutorial, we would introduce a method called the two-phased method which allows us to solve a LPP with some $b_i < 0$ by introducing an artificial variable x_0 . The goal of the two-phased method is to find a starting BFS for the original LPP when using the simplex algorithm. Notation: The two-phased method that we would be using is different from the one introduced in the textbook or other resources, however the ideas are equivalently the same. Besides, the notation for the simplex method follows from the one given in lectures. That is, we have c^T as the objective row, and the bottom right corner represents the negative of current cost. Covenical form

Max Z=CTX

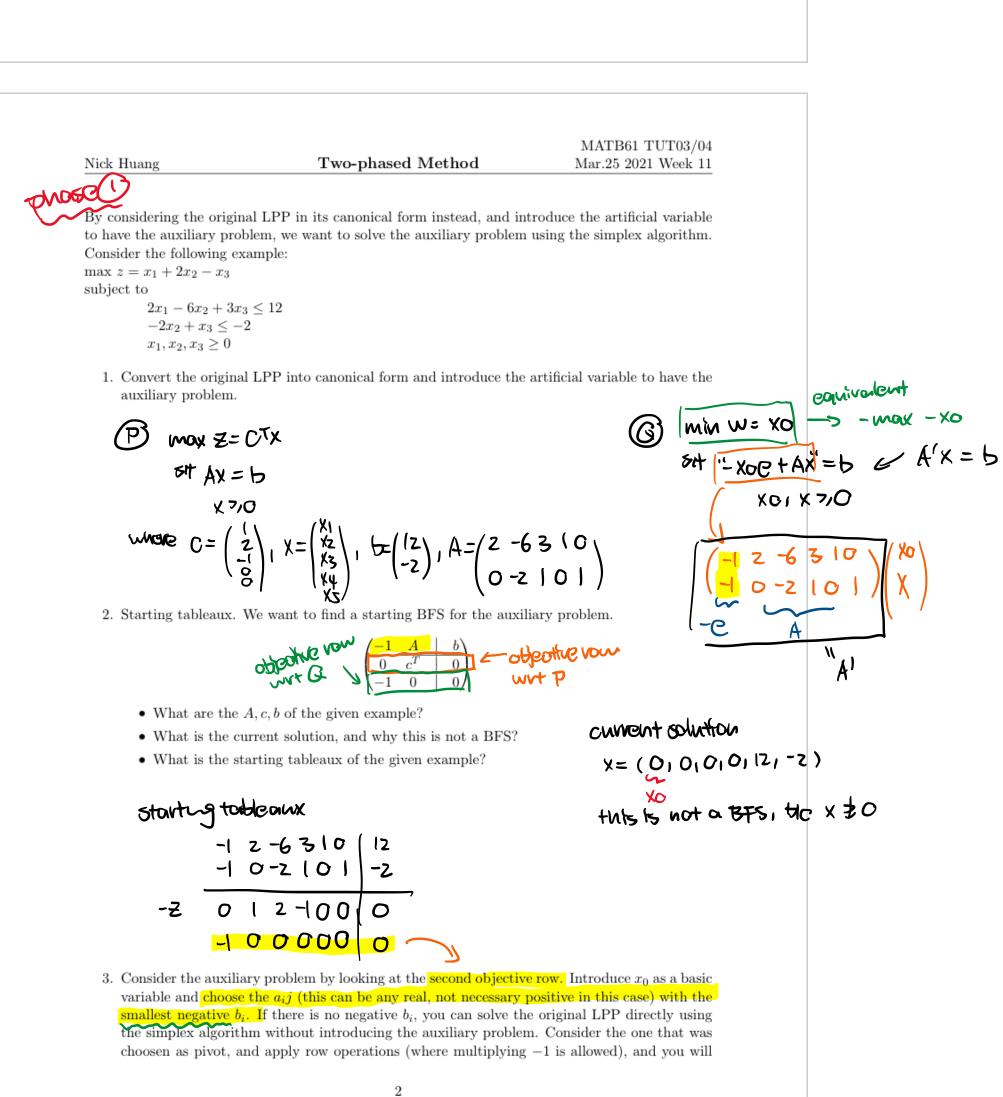
Sit AX = b

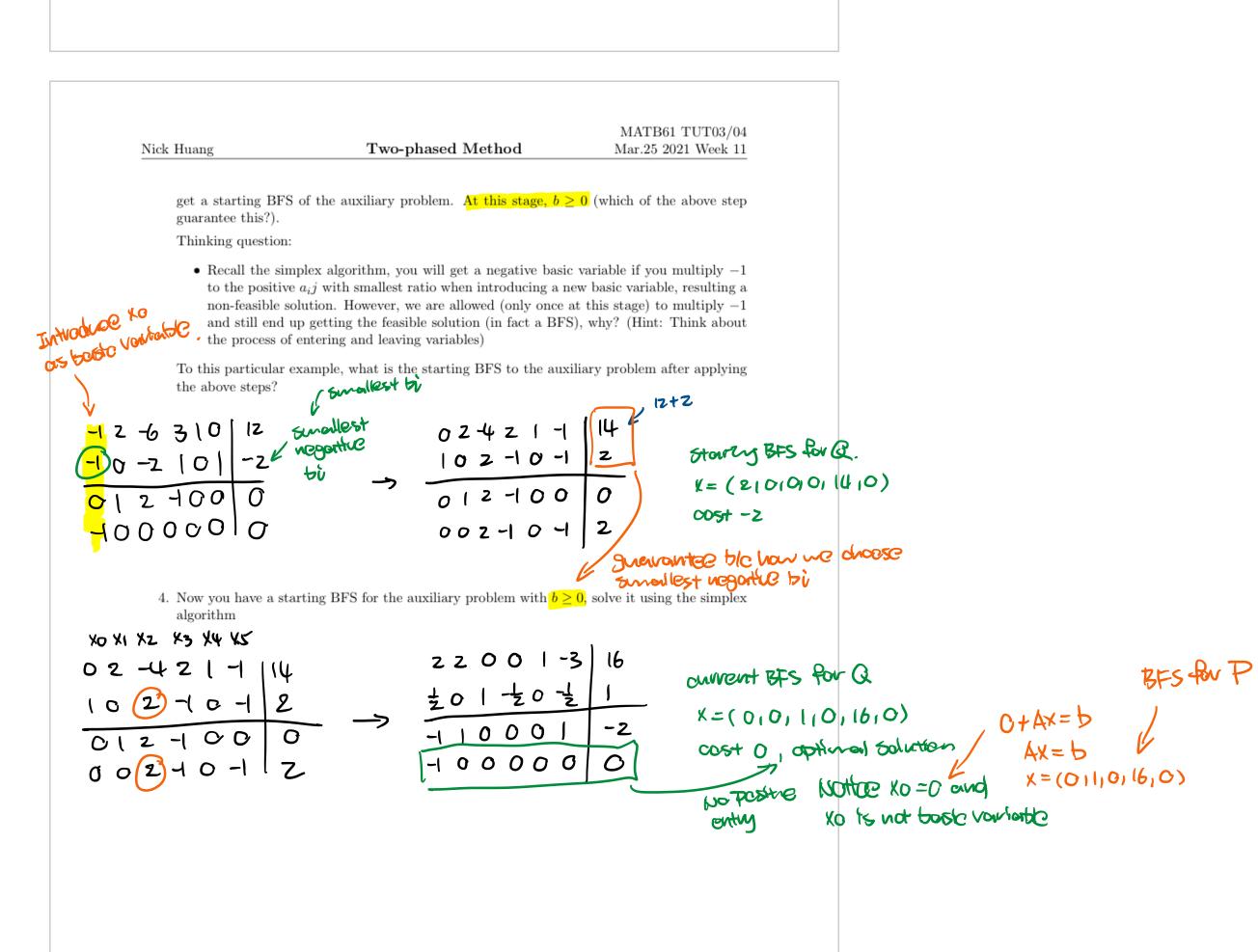
X 7/0

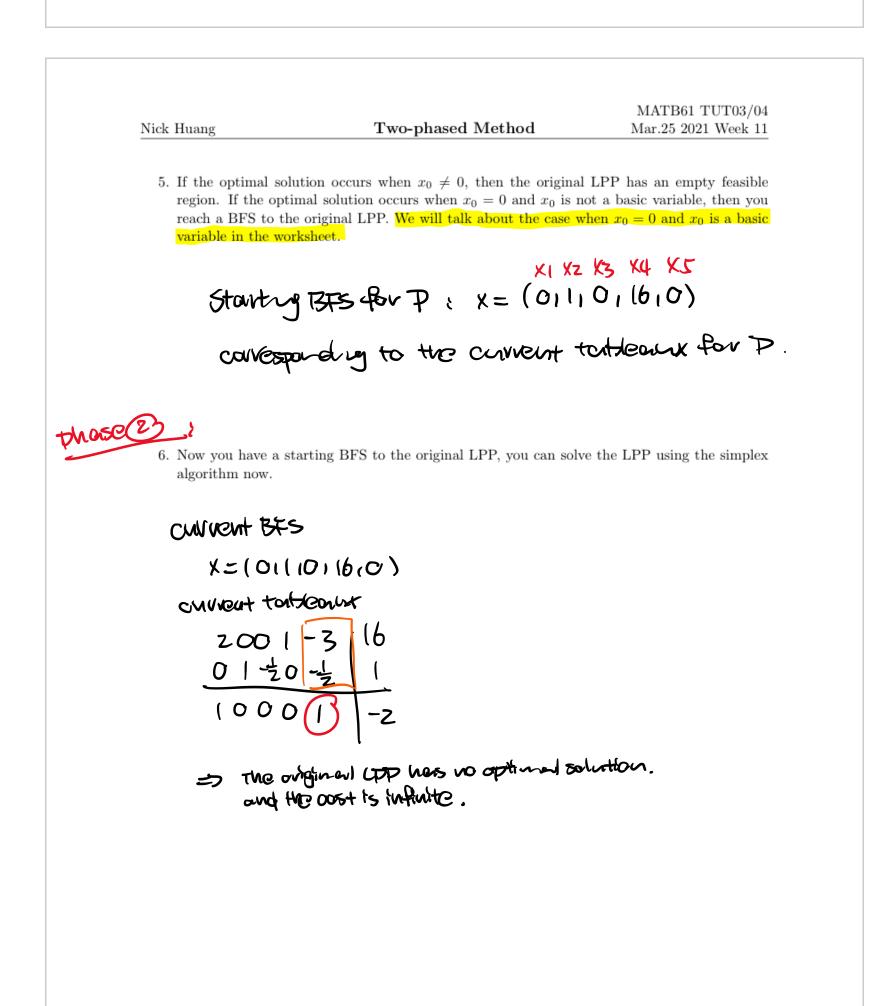
BFS HC Some

Original ore 1 when

X0 = 0 Two-phased Method Given an arbitrary LPP in its standard form, where $b \in \mathbb{R}^m$. Stondard form Max the objective function $z = c_1x_1 + ... + c_nx_n$ MON S=OTX subject to the Constraints $a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \le b_1$ ort AX & b $a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \le b_2$ X 70 $a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \le b_m$ $x_1, x_2, ..., x_n \ge 0$ to flud a stanty Consider the following auxiliary problem by introducing an artificial variable x_0 . min the **objective function** $w = x_0$ (Q) min w= Xo subject to the Constraints $-x_0 + a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \le b_1$ et - XOE HAX= P or - xoc + Ax & b $-x_0 + a_{21}x_1 + a_{22}x_2 + \dots + a_{2n}x_n \le b_2$ X = (x0) x) $-x_0 + a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \le b_m$ $x_0, x_1, x_2, ..., x_n \ge 0$ The connection between the auxiliary problem and the original problem is as follow. You will be given the following thinking questions in the worksheet. One could observe that the auxiliary problem must have an optimal solution given the con-If yes, $\exists x_1 \text{ sit}$ $Ax = b \text{ and } x \neq 0$ Plok $\vec{x} = (0, x)$ straints. Why? (Hint: $x_0 \ge 0$ and this is a minimization problem.) • If the auxiliary problem has an optimal solution $x_0 \neq 0$, then the original LPP has an empty feasible region. Why? (Hint: Think about the original LPP in canonical form, and the auxiliary problem be the one followed from the canonical LPP, if there is an x satisfying $Ax = b, x \ge 0$ in the original LPP, then the optimal solution for the auxiliary problem would will be aptimal have $x_0 = 0$ where you can find the optimal solution given x. How?) sin with xo to Q. • If the auxiliary problem has an optimal solution $x_0 \neq 0$, then there is a way to find a starting BFS which we will discuss next.







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