Recall that

- A map $T : V \to W$ is called an isomorphism if it is both injective and surjective.
- A map $T : V \to W$ is called injective if $T(v_1) = T(v_2) \implies v_1 = v_2$ for all $v_1, v_2 \in V$.
- A map $T : V \to W$ is called surjective if for all $w \in W$, there exists $v \in V$ such that $T(v) = w$.

Questions

1. Let $S : U \to V$, $T : V \to W$ be linear transformations. Suppose that S is an isomorphism. Show the following
   (a) $T \circ S$ is injective if and only if T is injective
   (b) $T \circ S$ is surjective if and only if T is surjective

2. Let V, W be n-dimensional vector spaces and $T : V \to W$ be linear transformation. Show that the following statements are equivalent:

   (a) T is an isomorphism

   (b) For every bases α, β of V, W respectively, the matrix $[T]_\alpha^\beta$ is invertible.

   (c) There exists bases α, β of V, W respectively, the matrix $[T]_\alpha^\beta$ is invertible.