QUIZ1.Forms_of_LPP_and_Graphical_method _TUT4

Thursday, January 28, 2021 09:55

QUIZ1.Forms_of_LPP_...

MATB61 TUT04

Nick Huang **Quiz1 Forms of LPP and Graphical Method** Jan.28 2021 Week 3

Instuction: You will have 30 minutes to finish quiz and will have 5 minutes to submit the quiz to Crowdmark after the quiz has finished. You will need to keep your cameras on during the quiz and submission times. One hand-held calculator is allowed. Electronic devices, online calculators, notes and other aids are not allowed. Violation of the instruction can be considered as an academic misconduct, and will be reported to the instructor and the department immediately.

Question

Suppose a company has 10 employees (not all employees need to work) and is planning to sell some products A and B. Each product A will bring $10 per minutes profit but requires one employee to supervise. Each product B will bring $5 per minutes profit but requires two employees to supervise. Suppose the company requires that the number of products A should be at most 4 more than the number of products B. For example, if the number of product B is 3, then the number of product A can be at most 7. How many each products A and B should be planned to sell in order to maximize the profit per minute? What is the maximized profit?

$x \le y+4$

$x-y \le 4$

1. (5 marks) Set up a standard linear programming model for the question and write it in matrix notation.
2. (5 marks) Solve the problem using graphical method. Clearly indicate the feasible region, the optimal solution and the maximized profit.

Note: You need to show all your works as required by the question and reasoning to earn full marks. Unjustified answer will not be given partial marks.

1

1. Define x = the number of products A to be sold, y = the number of products B to be sold

2

$$\begin{aligned} \max\ & z = 10x + 5y \\ \text{s.t}\ & x + 2y \le 10 \\ & x - y \le 4 \\ & x \ge 0,\ y \ge 0 \end{aligned}$$

This is a standard LPP.

matrix notation:

1

$$\begin{aligned} \max\ & z = \vec{c}^T\vec{x} \\ \text{s.t}\ & A\vec{x} \le b \\ & \vec{x} \ge 0 \end{aligned}$$

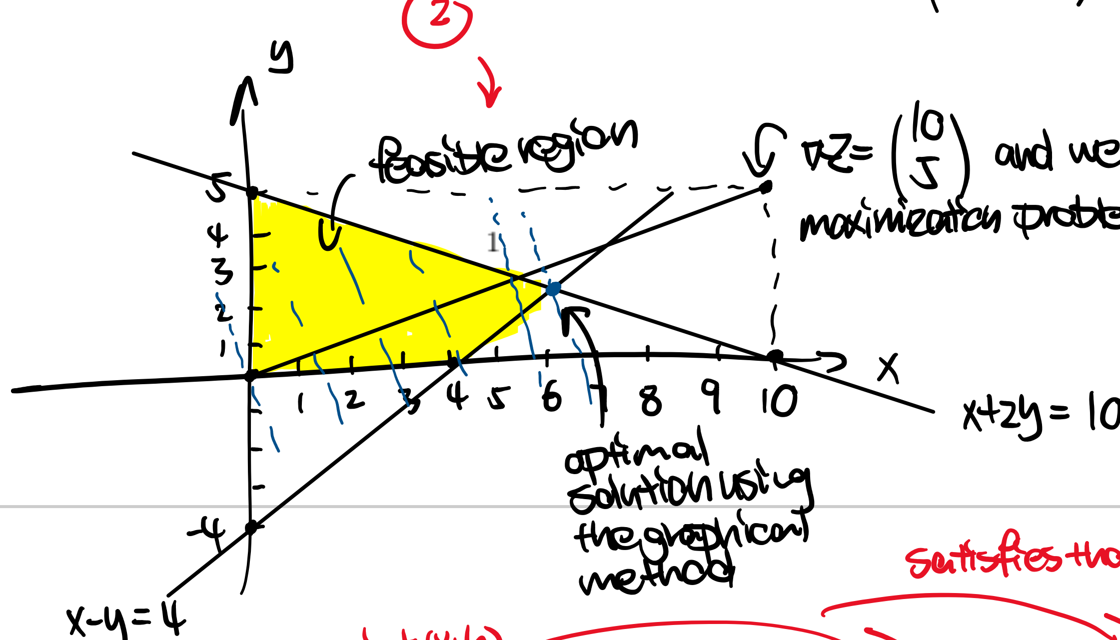
1

where $c = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$, $\vec{x} = \begin{pmatrix} x \\ y \end{pmatrix}$

$A = \begin{pmatrix} 1 & 2 \\ 1 & -1 \end{pmatrix}$ $b = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$

2

2.



$\nabla z = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$ and we have a maximization problem

1

point (x,y) satisfies those

An optimal solution occurs at the intersection of the lines $x+2y=10$, $x-y=4$

1

$$\begin{cases} x+2y = 10 & (1) \\ x-y = 4 & (2) \end{cases}$$

$(1)-(2)$: $3y = 6$

$y = 2$

sub $y=2$ into (1)

$x = 10-2y = 10-4 = 6$

s.t $(x,y) = (6,2)$ is an optimal solution

1

$z = 10x+5y = 10 \cdot 6 + 5 \cdot 2 = 60 + 10 = 70$ is the maximized profit