MATB24 Quiz2, tut0022

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MATB24 Quiz2,…

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(1) [3 marks] Give a complete definition, or mathematical characterization of the word in bold. Let V be an F-vector space.

- A **linear combination** of $v_1, v_2, \ldots, v_k$ in V

A linear combination of $v_1, v_2, \ldots, v_k$ in V is $a_1v_1 + \ldots + a_kv_k$, where $a_1, \ldots, a_k \in \mathbb{F}$

(2) [5 marks] True or False? Justify by a short proof or a counter example.

- Any set of vectors which contains the zero vector is linear dependent.

We're considering the set $\{0, v_1, v_2, \ldots, v_n\}$

True, for any vectors $v_1, v_2, \ldots v_n$

$a_0 \cdot 0 + a_1 \cdot v_1 + a_2 \cdot v_2 + \ldots + a_n v_n = 0$ (it depends on what I means to your vector v)

we can choose $a_0 = 1$, and all other $a_i = 0$ for $i \neq 0$

s.t by def, they are linearly dependent.

(3) [7 marks] Carefully prove the given statement:

- Prove that $span(v_1, v_2) = span(v_1, 2v_1 + 5v_2)$

($\subseteq$) let $V \in span(v_1, v_2)$, $V = a_1v_1 + a_2v_2$, $a_1, a_2 \in \mathbb{F}$ (given)

If $a_1 = 2$, $a_2 = 3$ $\Rightarrow V = 2v_1 + 3v_2$

$2 = k_1 + 2k_2$

$3 = 5k_2$

Don't stop here

WTS $V = k_1v_1 + k_2(2v_1 + 5v_2)$, where $k_1, k_2 \in \mathbb{F}$

s.t we want $a_1v_1 + a_2v_2 = k_1v_1 + k_2(2v_1 + 5v_2)$

$= (k_1 + 2k_2)v_1 + 5k_2v_2$.

i.e we need $a_1 = k_1 + 2k_2$, $a_2 = 5k_2$ (*)

$$\begin{cases} a_1 = k_1 + 2k_2 \Leftarrow k_1 = a_1 - \frac{2a_2}{5} \\ a_2 = 5k_2 \Rightarrow k_2 = \frac{a_2}{5} \end{cases}$$

s.t $k_1 = a_1 - \frac{2a_2}{5} \in \mathbb{F}$, $k_2 = \frac{a_2}{5} \in \mathbb{F}$

and by construction, (closed under addition and multiplication)

$span(v_1, 2v_1 + 5v_2) \ni k_1v_1 + k_2(2v_1 + 5v_2) = a_1v_1 + a_2v_2 = V$

($\supseteq$) let $V \in span\{v_1, 2v_1 + 5v_2\} \Rightarrow V = a_1v_1 + a_2(2v_1 + 5v_2)$

WTS $V \in span\{v_1, v_2\}$

$V = (a_1 + 2a_2)v_1 + (5a_2)v_2$

$\in \mathbb{F}$ $\Rightarrow V \in span\{v_1, v_2\}$

s.t $span\{v_1, 2v_1 + 5v_2\} = span\{v_1, v_2\}$ ■