

## Learning Objective

Understand how and when to apply the technique of integration by parts to solve a given integration. The 'how' part is easy given the algorithm, but the 'when' part is more abstract and you will get some sense of it after looking at various examples starting from the basic ones. You will also review some trigonometry identities when working on the examples.

## Integration by parts

To solve an integration of the form  $\int f(x)dx$ , the technique of integration by parts might be helpful. The technique of integration by parts said:

$$\int u dv = uv - \int v du$$

Here is an example:

$$\int x e^x dx$$

Let

$$u = x \text{ and } dv = e^x dx$$

then

$$du = dx \text{ and } v = e^x$$

Using the technique of integration by parts, we have the following

$$\begin{aligned} \int x e^x dx &= \int u dv \\ &= uv - \int v du \\ &= x e^x - \int e^x dx \\ &= x e^x - e^x + c \end{aligned}$$

Another example that can be solved using integration by parts is

$$\int x^2 \sin(x) \cos(x) dx$$

## Questions

1.  $\int x e^{2x} dx$  This is a similar problem as the one given above. Try using the technique of integration by parts and solve the integral. Next, try to solve it again using the setup  $u = e^x$  and  $dv = x dx$ , what is the difficulty here?



6.  $\int x \cos(x) dx$  Instead of  $e^x$ , we have  $\cos(x)$  now. What is similar when solving this using the technique?

7.  $\int x \sin(x) \cos(x) dx$  Which trigonometry identity might be useful here?

8. so how can we solve  $\int x^2 \sin(x) \cos(x) dx$

9. Let try something else  $\int x \sqrt{4 - x^2} dx$

10. **Bonus:** Now how about  $\int_0^2 x^3 \sqrt{4-x^2} dx$ ? Try to solve it using integration by parts following the similar idea. You will need to change the choice of  $u$  and  $v$  to make everything 'nice'. One can also solve this using integration by substitution with  $x = 2\sin(\theta)$ ,  $0 \leq \theta \leq \frac{\pi}{2}$  and  $u = \cos(\theta)$  after.

## Further questions for practice

Here are some more practice questions which you should already know (at least with some idea) how to solve using the technique of integration by parts. The technique of integration by substitution is also very useful when combining with trigonometry functions, but we will not discuss here.

1.  $\int x^2 e^{-x} dx$
2.  $\int x^3 e^{x^2} dx$
3.  $\int x^2 \cos(2x) dx$
4.  $\int 2x^2 \cos^2 x dx$