

Question 1

- a. $f(n) = n$ and $g(n) = (n + 1) / 2$.
 $f/g \rightarrow \lim_{n \rightarrow \infty} n / (n+1)/2 = 2$, $f(n) = \text{Big Theta}(g(n))$
- b. $f(n) = n^2$ and $g(n) = n^2 + 6n$.
 $\lim_{n \rightarrow \infty} n^2 / (n^2 + 6n) = 1$, $f(n) = \text{Big Theta}(g(n))$
- c. $f(n) = \log n$ and $g(n) = \log n^2$.
 $\lim_{n \rightarrow \infty} \log n / \log n^2 = 1/2$, $f(n) = \text{Big Theta}(g(n))$
- d. $f(n) = 2^n$ and $g(n) = 2^{2n}$.
 $\lim_{n \rightarrow \infty} 2^n / 2^{2n} = 0$, $f(n) = o(g(n))$
- e. $f(n) = 5n$ and $g(n) = 4n^{3/2}$.
 $\lim_{n \rightarrow \infty} 5n / 4n^{3/2} = 1/\infty = 0$, $f(n) = o(g(n))$

Question 2

- a. $(n + 1)^3$ is $O(n^3)$.
 $\lim_{n \rightarrow \infty} (n+1)^3 / n^3 = 1$, $f(n) = \text{Big Theta}(g(n))$
 Growing slower so $f(n) = O(g(n))$, $f(n) = (n^3)$
- b. 2^{n+1} is $O(2^n)$.
 $\lim_{n \rightarrow \infty} 2^{n+1} / 2^n = 2$, same as above, $f(n) = O(g(n))$, $f(n) = O(2^n)$
- c. n is $o(n \log n)$.
 $\lim_{n \rightarrow \infty} n / n \log n = 0$, $f(n) = o(g(n)) \rightarrow f(n) = O(n \log n)$
- d. n^2 is $\omega(n)$.
 $\lim_{n \rightarrow \infty} n^2 / n = \infty$, $f(n) = \omega(g(n))$
- e. $n^3 \log n$ is $\Omega(n^3)$.
 $\lim_{n \rightarrow \infty} n^3 \log n / n^3 = \infty$, $f(n) = \omega(n^3) \rightarrow g(n) = o(f(n)) \rightarrow f(n) = \Omega(n^3)$

Question 3

- a. Algorithm 1:

```

s = 0
for i = 1 to n
    s = s + i
O(n)

```

- b. Algorithm 2:

```
p = 1
for i = 1 to 2 * n
    p = p * i
```

$O(3n) \rightarrow O(n)$

c. Algorithm 3:

```
p = 1
for i = 1 to n ** 2
    p = p * i
 $O(n^3)$ 
```

d. Algorithm 4:

```
s = 0
for i = 1 to 2 * n
    for j = 1 to i
        s = s + i
 $O(4n^3) \rightarrow O(n^3)$ 
```

e. Algorithm 5:

```
s = 0
for i = 1 to n ** 2
    for j = 1 to i
        s = s + i
 $O(n^5)$ 
```