LaTeX exercise

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May 2, 2022

In this exercise we are given the code snippet:

```
static double ex(double x){
if(x<0)return 1/ex(-x);
if(x>1.0/8)return Pow(ex(x/2),2); // explain this
return 1+x*(1+x/2*(1+x/3*(1+x/4*(1+x/5*(1+x/6*(1+x/7*(1+x
    /8*(1+x/9*(1+x/10)))))))));
}
```

The formal definition of the exponential functions is

$$\exp x = \sum_{k=0}^{\infty} \frac{x^k}{k!} = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots \tag{1}$$

The factorial makes the higher order terms diminish rather quickly, and we can in good conscience approximate the power series as the first 11 terms,

$$\exp(x) \approx \mathrm{ex}(x) := 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \dots + \frac{x^{10}}{10!}. \tag{2}$$

If we factor x out we get

$$\mathrm{ex}(x) = 1 + x\left(1 + \frac{x}{2} + \frac{x^2}{6} + \dots + \frac{x^9}{10!}\right) \tag{3}$$

Doing this multiple times gets us

$$\mathrm{ex}(x) = 1 + x\left(1 + \frac{x}{2}\left(1 + \frac{x}{3}\left(1 + \dots\right)\right)\right). \tag{4}$$

Our approximation in eq. (2) is good for x close to 0, but for higher x it diverges from $\exp(x)$ as can be seen in fig. 1. To remedy this we decide to only use the approximation when the argument is $x < 1/8$. For $x > 1/8$ we can use the property

$$\exp(x) = \exp\left(\frac{x}{2}\right)^2 \approx \mathrm{ex}\left(\frac{x}{2}\right)^2. \tag{5}$$

Doing this trick multiple times lets us evaluate $\mathrm{ex}(x)$ at any x. Finally to avoid writing things twice we use $1/\mathrm{ex}(-x)$ for $x < 0$. The method evaluated from $x = -10$ to $x = 10$ is plotted in fig. 2.

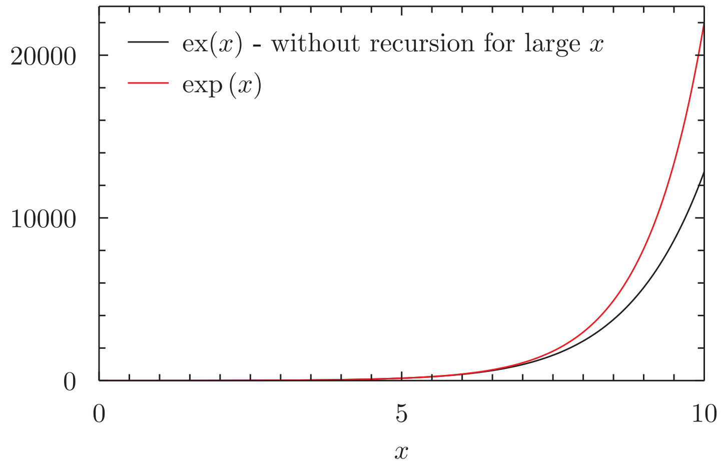

Figure 1: ex(x) diverges from exp(x) at large x.

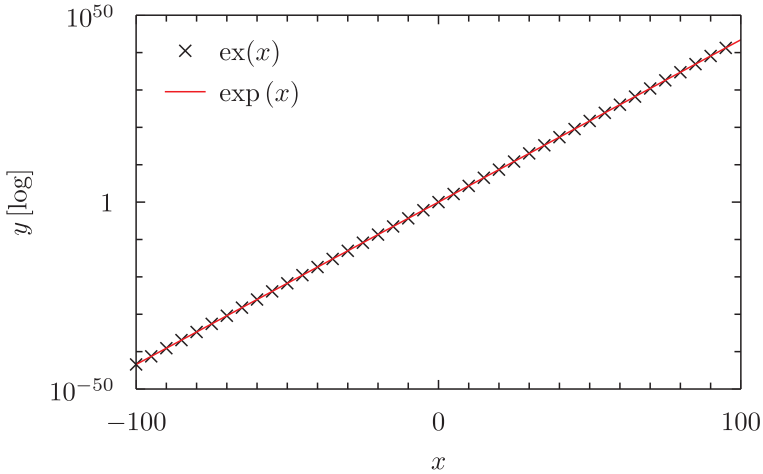

Figure 2: Logarithmic plot of the ex(x)-method against exp(x).