

Defining a system symbolically.

```
syms s K;  
gain = K;  
feedback = 1;  
plant = 1/(s*(s+1)*(s+2)); % Open-loop.  
sys = gain * plant + feedback; % Closed-loop.
```

Characteristic equation representation of the system.

```
coeffs = ELAB.sd2ce(sys, true);
```

Original expression:

$$\frac{K}{s(s+1)(s+2)} + 1$$

Expanded form:

$$\frac{K}{s^3 + 3s^2 + 2s} + 1$$

Set equal to 0, then simplify:

$$s^3 + 3s^2 + 2s + K$$

Collect terms of equal power:

$$s^3 + 3s^2 + 2s + K$$

Routh array for stability analysis.

```
RA = ELAB.routh(coeffs, true);
```

Routh Array

$$\begin{pmatrix} 1 & 2 \\ 3 & K \\ 2 - \frac{K}{3} & 0 \\ K & 0 \end{pmatrix}$$

Critical points, where system transitions between stable and unstable.

```
ELAB.critical(RA,true);
```

Solving equations for param:

$$2 - \frac{K}{3} = 0$$

$$K = 0$$

For stability, param (K) = ]6;0[

From s^2 auxiliary equation:

$$(3s^2 + 6 = 0 \quad 3s^2 = 0)$$

Solved for s:

$$\begin{pmatrix} -\sqrt{2}i & 0 \\ \sqrt{2}i & 0 \end{pmatrix}$$

Breakaway points for root locus plot.

```
ELAB.breakaway(sys, gain, true);
```

Solving closed-loop for gain:

$$K = -s(s+1)(s+2)$$

Expanding expression:

$$K = -s^3 - 3s^2 - 2s$$

Derivative of gain of s:

$$-3s^2 - 6s - 2$$

Solving  $\text{diff}(\text{gain}) = 0$  for s:

$$\left( s = -\frac{\sqrt{3}}{3} - 1 \quad s = \frac{\sqrt{3}}{3} - 1 \right)$$

Sub s in 1st equation, solving for gain.

Breakaway points [s, gain]:

-1.5774	-0.3849
-0.4226	0.3849

```
plant = ELAB.sd2tf(plant); % Convert symbolic to tf-object for rlocus.  
rlocus(plant);
```



