

PROGRAMMATIC SYMBOLIC CIRCUIT ANALYSIS



OUTLINE

- Motivation
- Methodology, The programmatic approach
- Basic use, Circuit objects
- The ELAB class
- Modified Nodal Analysis by example
- Key features, Symbolic analysis, Transmuting
- Digitizing the standard toolset
- The project in action
- Conclusion and future opportunities



MOTIVATION

Complicated problems

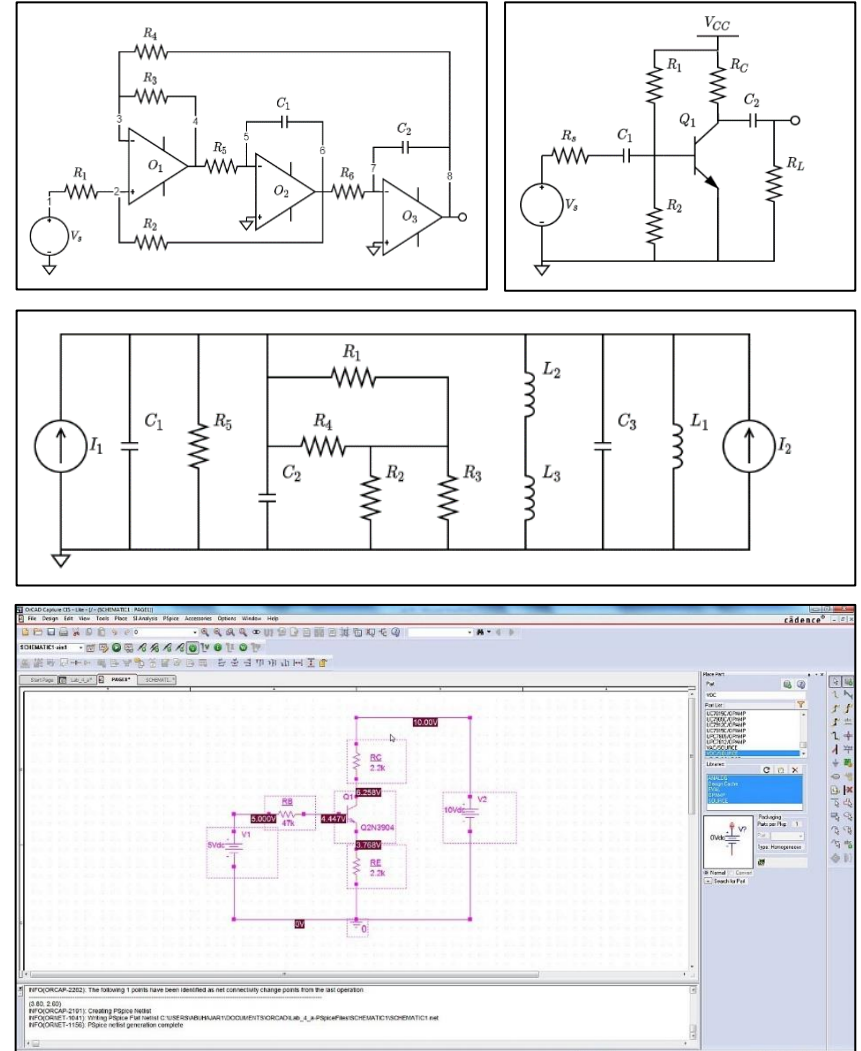
- Time-consuming
- Repetitive in nature

Heavy software

- Hard to install and run
- Unfamiliar environment
- Focus on numerical simulation, Assumes knowledge of values
- Limited understanding to be gained

Symbolic vs Numerical Analysis

- Symbolic analysis increases understanding but is time-consuming.
- Numerical analysis is near-instant but provides to no semantic understanding.



NEW APPROACH

Inspiration

- The field of data science, control systems, etc.
- Use pre-existing generalized tools
- Practically no standalone software

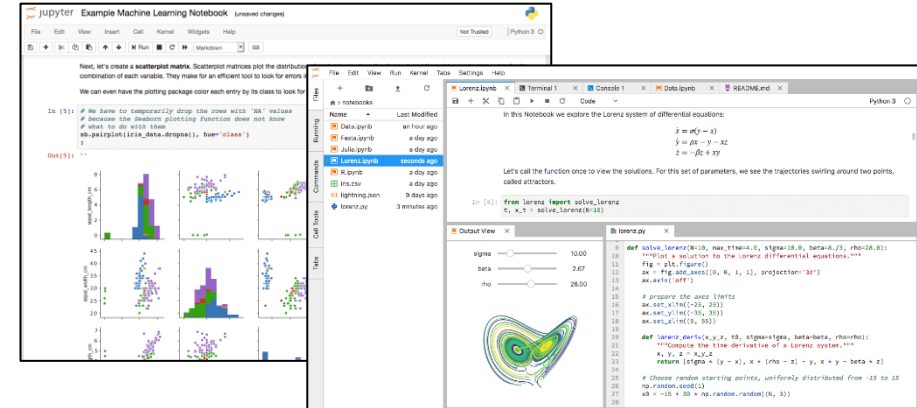
The “programmatic” approach

- A general skill with infinite versatility
- Unbound by isolated software
- Choice of environment

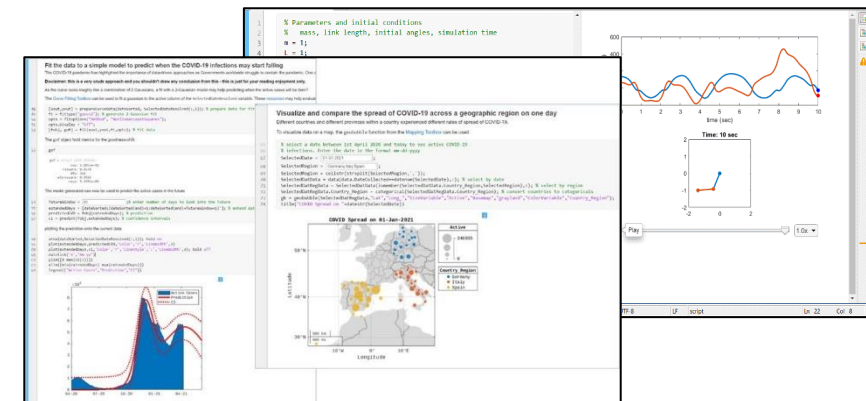
Why a MATLAB toolbox?

- Live script, formatted output
- Symbolic math toolbox
- Pre-existing systems analysis tools
- Familiarity, rapid prototyping
- The industry standard, very fast

Jupyter Notebooks / Python for Data Science



Live Scripts / MATLAB for control systems, statistics



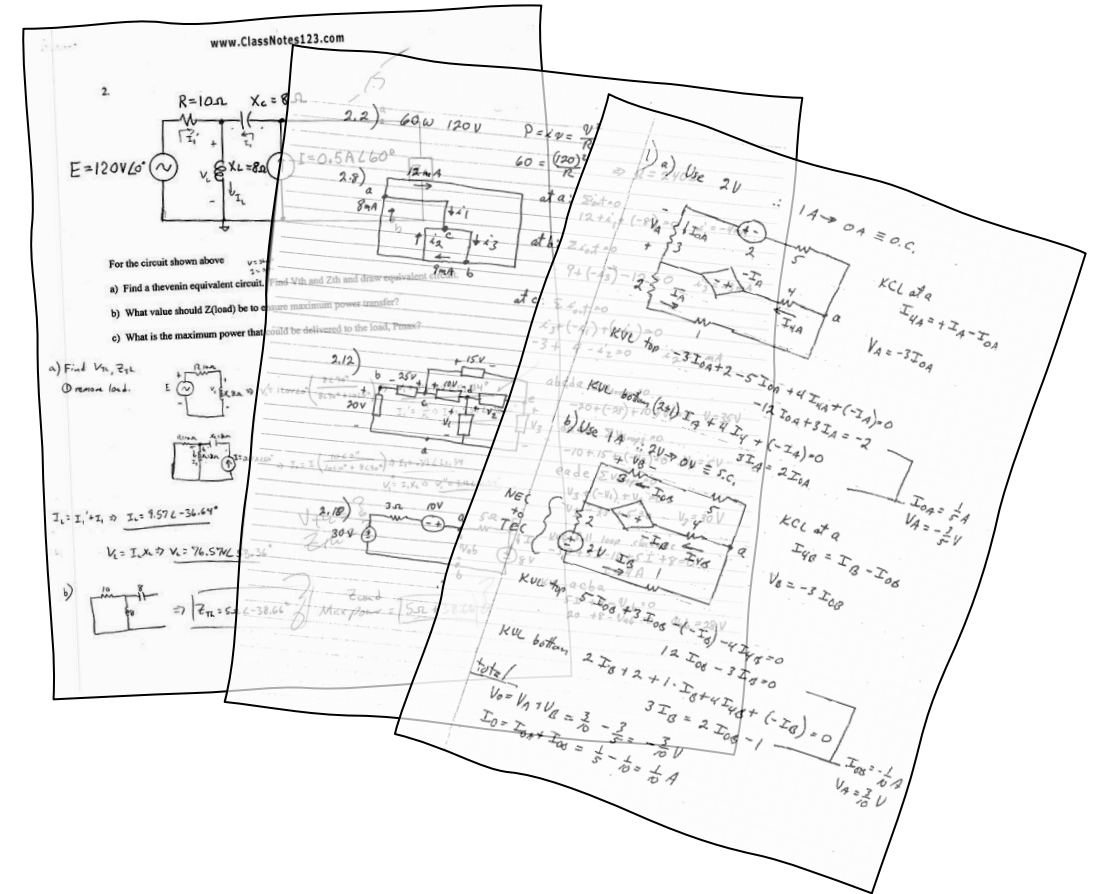
THE KEY IDEA

ELABorate

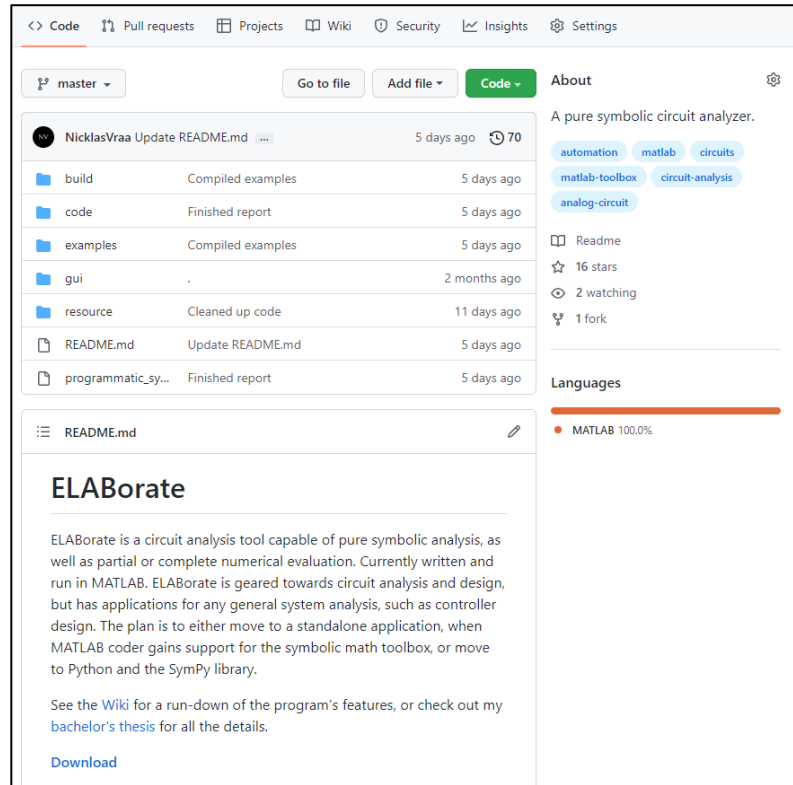
Electronic Laboratory

Why?

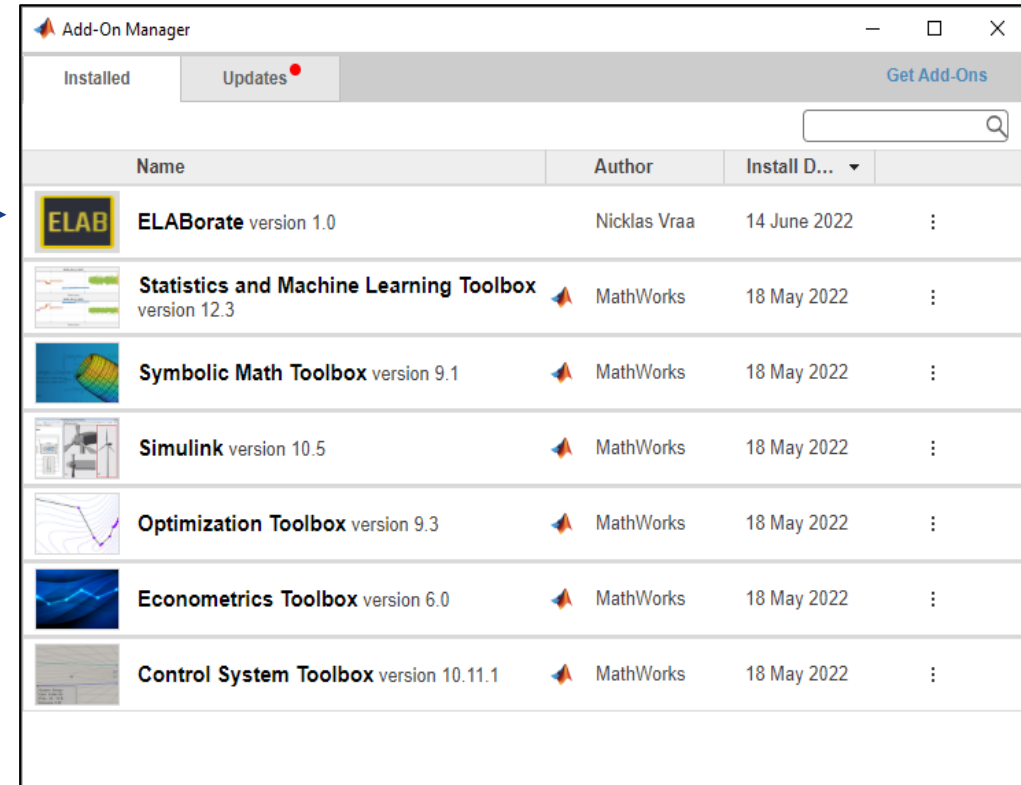
- Digitalizing our tools and research subjects
- Symbolically, we are still stuck with pen-and-paper










THE RESULTS



A screenshot of a GitHub repository page for 'ELABorate'. The repository is owned by 'NicklasVraa' and is in the 'master' branch. The file list shows a 'build' directory, 'code' directory, 'examples' directory, 'gui' directory, and 'resource' directory. The 'README.md' file is selected, showing the project description: 'A pure symbolic circuit analyzer.' The description mentions that ELABorate is a circuit analysis tool capable of pure symbolic analysis, as well as partial or complete numerical evaluation. It is currently written and run in MATLAB. The plan is to either move to a standalone application, when MATLAB coder gains support for the symbolic math toolbox, or move to Python and the SymPy library. A 'Download' link is visible at the bottom.

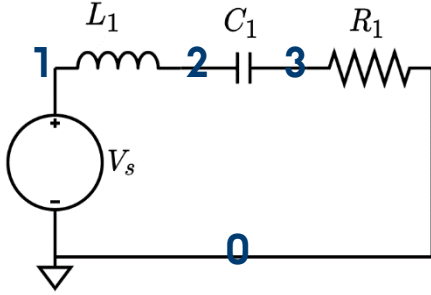


A screenshot of the MATLAB Add-On Manager window. The 'Installed' tab is selected, showing a list of installed add-ons. The list includes ELABorate version 1.0, Statistics and Machine Learning Toolbox version 12.3, Symbolic Math Toolbox version 9.1, Simulink version 10.5, Optimization Toolbox version 9.3, Econometrics Toolbox version 6.0, and Control System Toolbox version 10.11.1. The 'Updates' tab is also visible, indicating that there are updates available for some of the installed add-ons.

Name	Author	Install D...	
 ELABorate version 1.0	Nicklas Vraa	14 June 2022	:
 Statistics and Machine Learning Toolbox version 12.3	MathWorks	18 May 2022	:
 Symbolic Math Toolbox version 9.1	MathWorks	18 May 2022	:
 Simulink version 10.5	MathWorks	18 May 2022	:
 Optimization Toolbox version 9.3	MathWorks	18 May 2022	:
 Econometrics Toolbox version 6.0	MathWorks	18 May 2022	:
 Control System Toolbox version 10.11.1	MathWorks	18 May 2022	:



BASIC USE



rlc_series.txt				
1	Vs	1	0	DC 5
2	L1	1	2	1
3	C1	2	3	0.0001
4	R1	3	0	1000
5				

```
rlc_circuits.mlx
```

```
1 circuit = Circuit('circuits/rlc_series.txt');
```

Constructor in Circuit.m class

```
circuit =  
  Circuit with properties:  
  
    file_name: 'circuits/rlc_series.txt'  
    num_nodes: 3  
    num_elements: 4  
        list: 'Vs 1 0 DC 5+R1 3 0 1000+...'  
    Indep_VSs: [1x1 Indep_VS]  
    Indep_ISs: [0x0 Indep_IS]  
    Resistors: [1x1 Resistor]  
    Inductors: [1x1 Inductor]  
    Capacitors: [1x1 Capacitor]  
    Ideal_OpAmps: [0x0 Ideal_OpAmp]  
    Generic_zs: [0x0 Impedance]  
        VCVSs: [0x0 VCVS]  
        CCCSs: [0x0 CCCS]  
        VCCSs: [0x0 VCCS]  
  
    :
```

THE CIRCUIT CONSTRUCTOR

Data

```
classdef Circuit < Base_System
% Circuit model, utilizing the Element subclasses.

properties (...)

properties (Access = {?ELAB, ?Analyzer, ?Modeller, ?Transmuter, ?Visualizer}) (...)

methods
% Methods pertaining to this circuit.

function obj = Circuit(file_name)
% Constructor of the circuit class.

    % Parse the text file at the given path (file_name).
    obj.file_name = file_name;
    file = fopen(file_name);

    % Convert to cell matrix, based on line number and spaces.
    netlist = textscan(file, '%s %s %s %s %s %s %s %s', 'CollectOutput', 1);
    obj.netlist = {netlist{1}(:,1) netlist{1}(:,2) netlist{1}(:,3) ...
        netlist{1}(:,4) netlist{1}(:,5) netlist{1}(:,6) ...
        netlist{1}(:,7) netlist{1}(:,8)};

    fclose(file);

    % Split cell matrix into vectors.
    [Name, N1, N2, arg3, arg4, arg5, arg6, arg7] = obj.netlist{:};

    % Convert node values from string-type to number-type.
    N1 = str2double(N1); N2 = str2double(N2);

    % Find number of elements by number of entries in the vectors.
    obj.num_elements = length(Name);
    obj.num_nodes = length(unique([N1; N2])) - 1;

    % This creates compound element arrays and handles setup.
    obj.update();
end
```

```
% Creating element objects based the parsed netlist information.
for i = 1:obj.num_elements

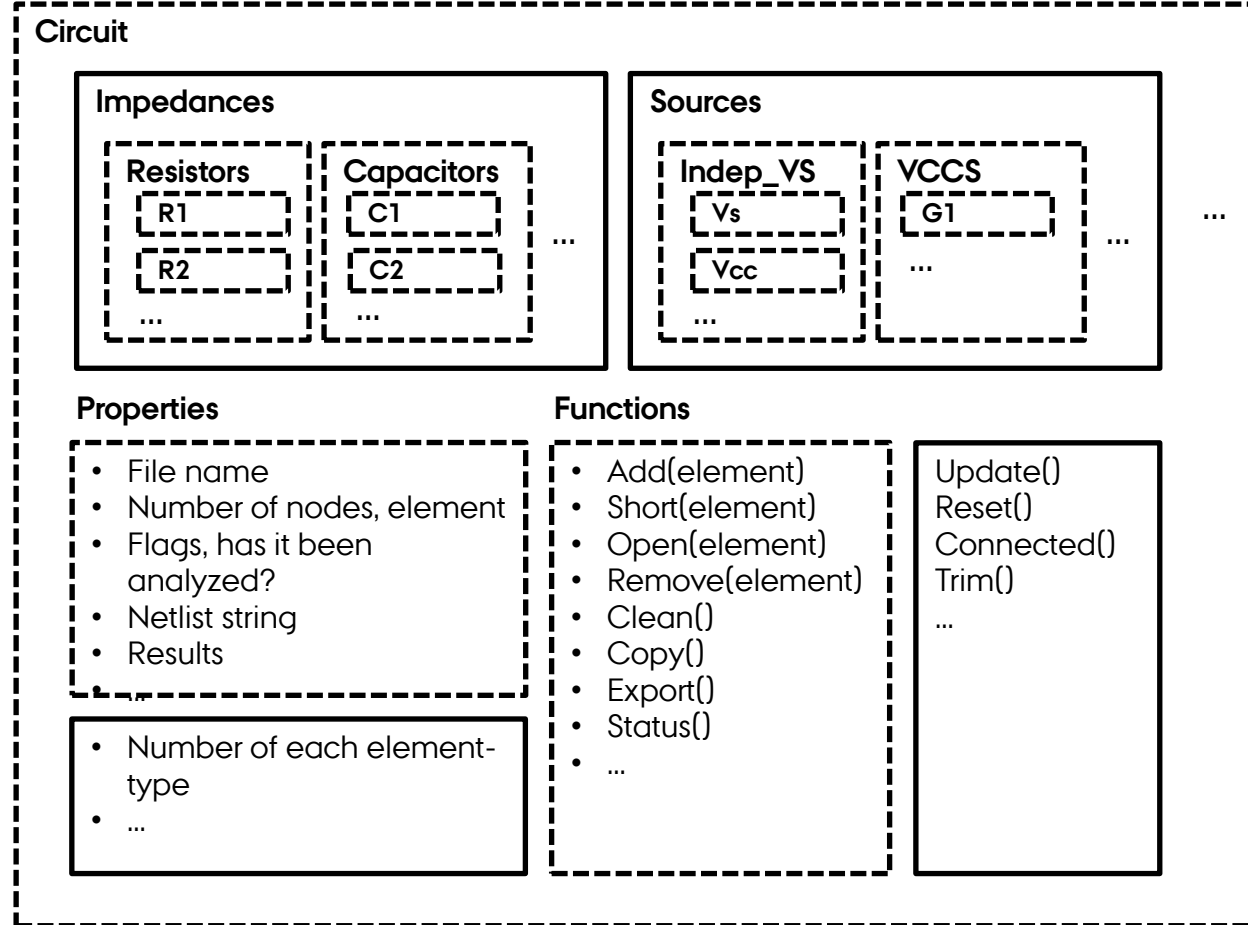
    % Element id is always in the same place.
    id = Name{i}(1:end);

    % First letter in ID tells the type of element.
    switch id(1)
        case {'V'}
            obj.Indep_VSs(end+1) = Indep_VS(id, N1(i), N2(i), arg3{i}, arg4{i});
        case {'I'}
            obj.Indep_ISs(end+1) = Indep_IS(id, N1(i), N2(i), arg3{i}, arg4{i});
        case {'R'}
            obj.Resistors(end+1) = Resistor(id, N1(i), N2(i), arg3{i});
        case {'L'}
            obj.Inductors(end+1) = Inductor(id, N1(i), N2(i), arg3{i});
        case {'C'}
            obj.Capacitors(end+1) = Capacitor(id, N1(i), N2(i), arg3{i});
        case {'Z'}
            obj.Impedances(end+1) = Impedance(id, N1(i), N2(i), arg3{i});
        case {'E'}
            obj.VCVSs(end+1) = VCVS(id, N1(i), N2(i), str2double(arg3{i}), ...
                str2double(arg4{i}), arg5{i});
        case {'G'}
            obj.VCCSs(end+1) = VCCS(id, N1(i), N2(i), str2double(arg3{i}), ...
                str2double(arg4{i}), arg5{i});
        case {'H'}
            obj.CCVSs(end+1) = CCVS(id, N1(i), N2(i), arg3{i}, arg4{i});
        case {'F'}
            obj.CCCSs(end+1) = CCCS(id, N1(i), N2(i), arg3{i}, arg4{i});
        case {'O'}
            obj.Ideal_OpAmps(end+1) = Ideal_OpAmp(id, N1(i), N2(i), ...
                str2double(arg3{i}));
        case {'Q'}
            obj.BJT_s(end+1) = BJT(id, N1(i), N2(i), str2double(arg3{i}), arg4{i});
        case {'M'}
            obj.MOSFETs(end+1) = MOSFET(id, N1(i), N2(i), ...
                str2double(arg3{i}), arg4{i});
    end
end
```



THE CIRCUIT OBJECT

```
1 % Part of ELABorate, all rights reserved. [...]  
3  
4 classdef Circuit < Base_System  
5 % Circuit model, utilizing the Element subclasses.  
6  
7 properties [...]  
31  
32 properties (Access = {?ELAB, ?Analyzer, ?Modeller, ?Transmuter, ?Visualizer}) [...]  
55  
56 methods  
57 % Methods pertaining to this circuit.  
58  
59 function obj = Circuit(file_name) [...]  
127  
128 function export(obj, name) [...]  
136  
137 function copy = clone(obj) [...]  
145  
146 function status(obj) [...]  
157  
158 function update_netlist(obj) [...]  
169  
170 function short(obj, X) [...]  
193  
194 function open(obj, X) [...]  
200  
201 function clean(obj) [...]  
226  
227 function add(obj, X) [...]  
281  
282 function remove(obj, X) [...]  
305  
306 end  
307 methods(Access = {?ELAB, ?Analyzer, ?Modeller, ?Transmuter, ?Visualizer})  
308 % Methods only available to ELABorate modules.  
309  
310 function connected = get_connected(obj, X, node) [...]  
327  
328 function update(obj) [...]  
334  
335 function reset(obj) [...]  
349  
350 function trim(obj) [...]  
397  
398 end  
399 methods(Access = private)  
400 % Methods only available to this circuit object.  
401  
402 function update_num_nodes(obj) [...]  
413  
414 function update_nums(obj) [...]  
438  
439 function update_arrays(obj) [...]  
449  
450 end  
451 methods(Static)  
452 % Methods shared among objects of this class.  
453  
454 function connected = check_elem_array(Xs, Y, node) [...]  
470  
471 end
```



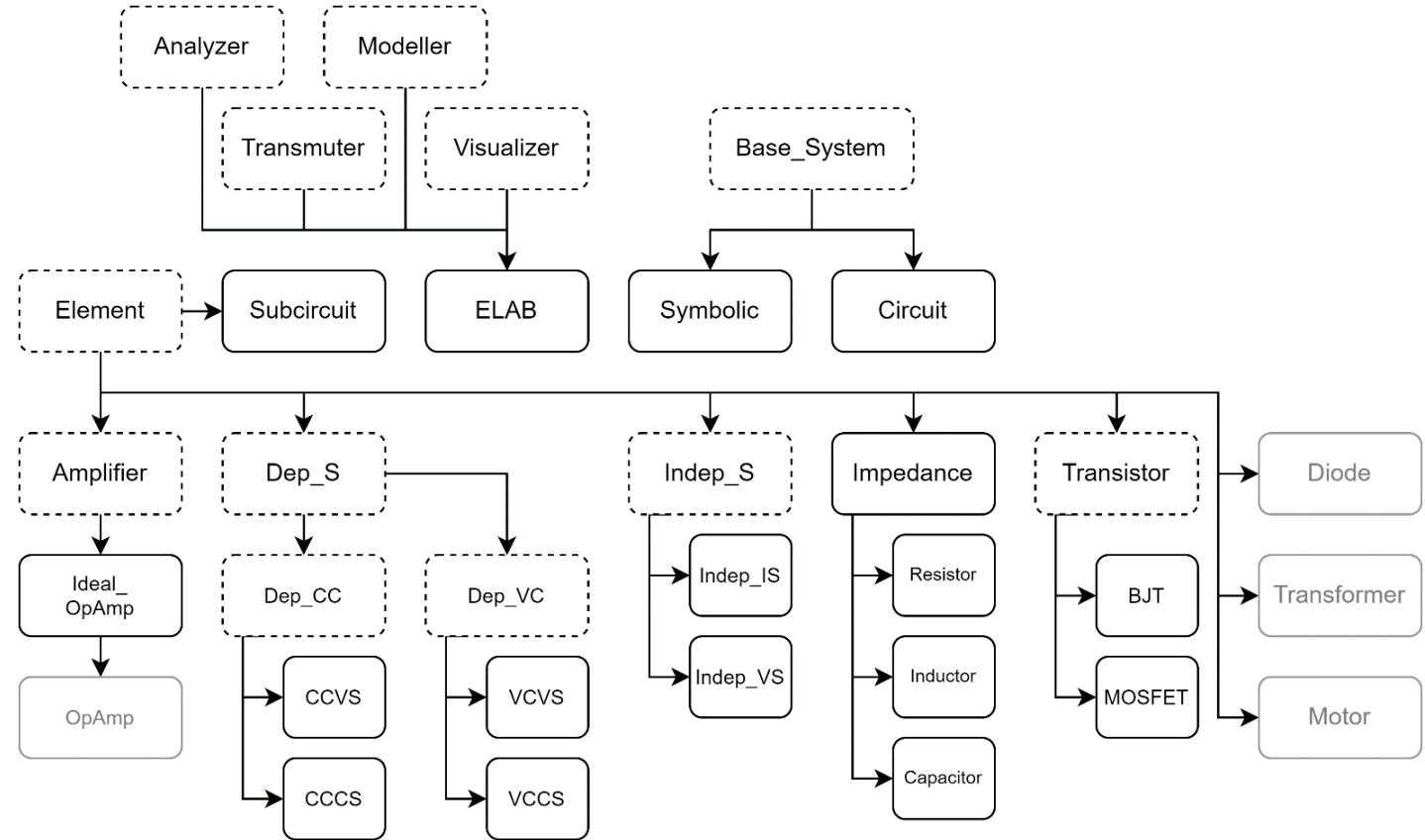
INHERITANCE

The Intent

- Have the program be communal
- Open-source, easily extended by third parties

Solution

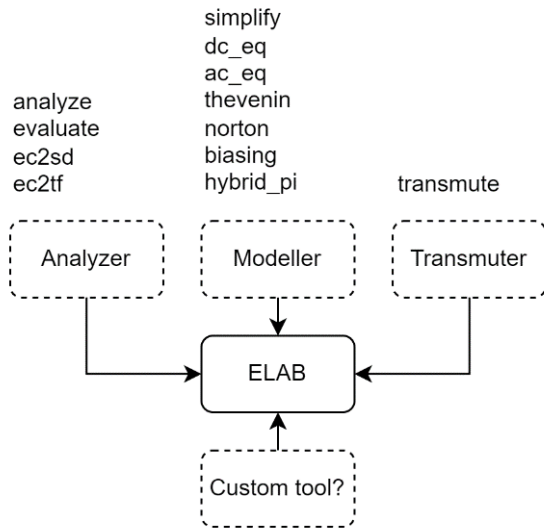
- Rely on a purely object-oriented approach
- Modularity
- Strong inheritance tree
- Limit access to core



User need only interact with Circuit and ELAB



THE ELAB CLASS



classdef ELAB < ... & Custom

Modeller

```

classdef Modeller
    % A collection of functions, specifically for safely altering the
    % circuit object or even converting the circuit to another form.

    methods(Static)

        function obj = simplify(obj, z_eq) [***]

        function obj = dc_eq(obj) [***]

        function obj = ac_eq(obj) [***]

        function obj = remove_sources(obj) [***]

        function obj = thevenin(obj, load) [***]

        function obj = norton(obj, load) [***]

        function orig = biasing(obj) [***]

        function obj = hybrid_pi(obj, freq, early) [***]

        function bool = is_same_type(X1, X2) [***]

        function bool = is_parallel(X1, X2) [***]

        function bool = is_series(obj, X1, X2) [***]

    end

    methods(Static, Access = private)

        function done = simplify_parallel(obj, z_eq) [***]

        function done = simplify_series(obj, z_eq) [***]

        function pair = find_parallel(~, Xs) [***]

        function pair = find_series(obj, Xs) [***]

        function rename(obj) [***]

        function rename_eqs(Xs) [***]

        function bool = check_shared(Xs, X1, X2, node) [***]

        function nodes = unique_nodes(X1, X2) [***]

        function [v_th, Z_th] = equivalent(obj, load) [***]

    end
end
  
```

Analyzer

```

classdef Analyzer
    % A collection of functions designed to facilitate symbolic
    % and numerical analysis of a given circuit object.

    methods(Static)

        function analyze(obj) [***]

        function evaluate(obj) [***]

        function out = ec2sd(obj, node_in, node_out) [***]

        function out = ec2tf(obj, node_in, node_out) [***]

        function RA = routh(poly_coeffs, show, epsilon) [***]

        function cr = critical(RA, show) [***]

        function bp = breakaway(sys, gain, show) [***]

        function [ord, type] = order_type(G, show) [***]

        function [Kp, Kv, Ka] = static_error_K(G, show) [***]

        function dp = dominant(G) [***]

        function [wn, zeta] = damp(G, show) [***]

        function N = period(x) [***]

        function out = observable(in) [***]

        function out = controllable(in) [***]

        function out = jordan(in) [***]

    end

    methods(Static, Access = private)

        function find_v_and_i(obj) [***]

    end
end
  
```

Transmuter

```

classdef Transmuter
    % A collection of functions designed to transmute a system into
    % another form, that may be more helpful for analysis or design.

    methods(Static)

        function out = transmute(in, type_in, type_out, show) [***]

        function out = sd2tf(in, show) [***]

        function out = zd2tf(in, show) [***]

        function out = tf2sd(in, show) [***]

        function out = tf2ss(in, show) [***]

        function out = ss2tf(in, show) [***]

        function out = tf2zp(in, show) [***]

        function out = zp2tf(in, show) [***]

        function out = de2sd(in, show) [***]

        function out = sd2de(in, show) [***]

        function out = sd2ce(in, show) [***]

        function out = ce2sd(in, show) [***]

        function out = sd2td(in, show) [***]

        function out = td2sd(in, show) [***]

        function out = nd2zd(in, show) [***]

    end
end
  
```

```

classdef ELAB < Analyzer & Modeller & Visualizer & Transmuter
    % The combination of the main classes of the ELABorate project.
  
```



ANALYZER - MODIFIED NODAL ANALYSIS

The Basic MNA Algorithm (for a computer)

- Build a Linear Time-Invariant system

$$Ax = z$$

$$A = \begin{bmatrix} G & B \\ C & D \end{bmatrix} \quad x = \begin{bmatrix} v \\ j \end{bmatrix} \quad z = \begin{bmatrix} i \\ e \end{bmatrix}$$

For the intermediate matrices comprising **A**:

- **G** is a diagonal matrix holding the sum of conductances of each circuit element connected between nodes i and j .
- **B** is an $n \times m$ matrix denoting the location of each voltage source. For entry $b_{i,j}$ a 0 means no voltage source, whereas a ± 1 means a voltage source. The sign indicates its orientation.
- **C** is an $m \times n$ matrix and simply the transpose of **B** so long as there are no dependent sources.
- **D** is an $m \times m$ matrix of zeros.

For the intermediate matrices comprising **x**:

- **v** is a vector holding the n unknown node voltages, excluding the ground voltage assumed to be 0.
- **j** is a vector holding the m unknown currents running through each voltage source in the circuit.

For the intermediate matrices comprising **z**:

- **i** is a vector holding the n sum of all current sources going into any given node.
- **e** is a vector simply holding the values of all independent current sources.

```
function analyze(obj)
% Symbolic analysis of given circuit object.

if obj.symbolically_analyzed
    return
end

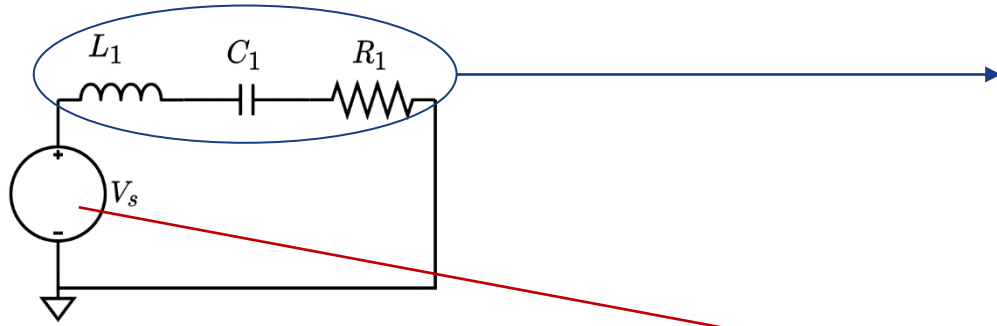
tic;

% Array allocation.
G = cell(obj.num_nodes, obj.num_nodes); [G{:}] = deal('0');
B = cell(obj.num_nodes, obj.num_VSs); [B{:}] = deal('0');
C = cell(obj.num_VSs, obj.num_nodes); [C{:}] = deal('0');
D = cell(obj.num_VSs, obj.num_VSs); [D{:}] = deal('0');
i = cell(obj.num_nodes, 1); [i{:}] = deal('0');
e = cell(obj.num_VSs, 1); [e{:}] = deal('0');
j = cell(obj.num_VSs, 1); [j{:}] = deal('0');
v = compose('v_%d', (1:obj.num_nodes)');

% Building circuit equations:
num_vs_parsed = 0;
```



MNA- EXAMPLE



$$A = \begin{bmatrix} G & B \\ C & D \end{bmatrix} \quad x = \begin{bmatrix} v \\ j \end{bmatrix} \quad z = \begin{bmatrix} i \\ e \end{bmatrix}$$

Algorithm

```

For all impedances
  If impedance is generic or resistor, set  $exp = 1$ 
  Else if impedance is capacitor, set  $exp = 1/s$ 
  Else if impedance is inductor, set  $exp = s$ 

  If anode is ground
    Set  $g_{j,j} = g_{j,j} + exp$ 
  Else if cathode is grounded
    Set  $g_{i,i} = g_{i,i} + exp$ 
  Else
    Set  $g_{i,i} = g_{i,i} + exp$ ,  $g_{j,j} = g_{j,j} + exp$ ,  $g_{i,j} = g_{i,j} + exp$ ,  $g_{j,i} = g_{j,i} + exp$ 
    
```

```

For all independent voltage sources
  If anode is not grounded
    Set  $b_{i,p} = b_{i,p} + 1$ ,  $c_{p,i} = c_{p,i} + 1$ 
  If cathode is not grounded
    Set  $b_{j,p} = b_{j,p} - 1$ ,  $c_{p,j} = c_{p,j} - 1$ 
  Add parsed id's to  $e$  as  $V_{id}$  and to  $j$  as  $I_{v_{id}}$ 
    
```

i, j = anode, cathode
 p = # processed v-sources

A =

$$A = \begin{bmatrix} \frac{1}{L_1 s} & -\frac{1}{L_1 s} & 0 \\ -\frac{1}{L_1 s} & C_1 s + \frac{1}{L_1 s} & -C_1 s \\ 0 & -C_1 s & C_1 s + \frac{1}{R_1} \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

*

x =

$$x = \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ I_{V_s} \end{bmatrix}$$

=

z =

$$z = \begin{bmatrix} 0 \\ 0 \\ 0 \\ V_s \end{bmatrix}$$

Solve for x

$$x = A^{-1}z$$

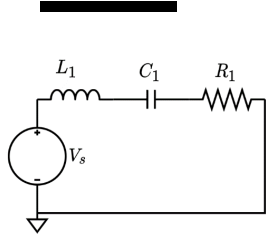
solutions =

$$\begin{bmatrix} v_1 = V_s \\ v_2 = \frac{V_s (C_1 R_1 s + 1)}{C_1 L_1 s^2 + C_1 R_1 s + 1} \\ v_3 = \frac{C_1 R_1 V_s s}{C_1 L_1 s^2 + C_1 R_1 s + 1} \\ I_{V_s} = -\frac{C_1 V_s s}{C_1 L_1 s^2 + C_1 R_1 s + 1} \end{bmatrix}$$

Circuit is fully defined.
 Evaluation is easy.



KEY FEATURES



```
ELAB.analyze(circuit)
```

Symbolic analysis successful (0.300184 sec).

```
ELAB.ec2sd(circuit, 1, 3)
```

Symbolic transfer function calculated successfully (4.291000e-03 sec).

ans =

$$\frac{v_3}{v_1} = \frac{C_1 R_1 s}{C_1 L_1 s^2 + C_1 R_1 s + 1}$$

```
TF = ELAB.ec2tf(circuit, 1, 3)
```

Numerical evaluation successful (0.0717553 sec).

Transfer function object created successfully (1.429922e-01 sec).

TF =

$$\frac{1000 s}{s^2 + 1000 s + 10000}$$

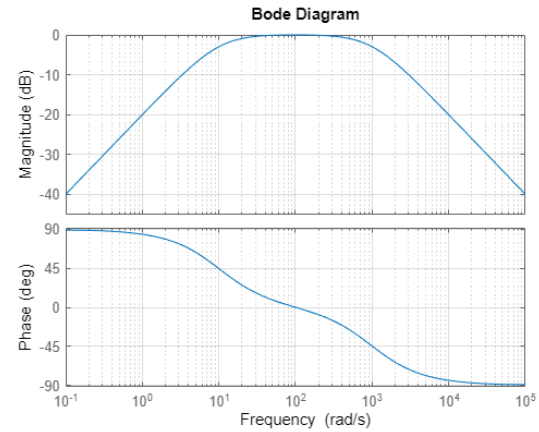
Continuous-time transfer function.

```
pole(TF), zero(TF) ...
```

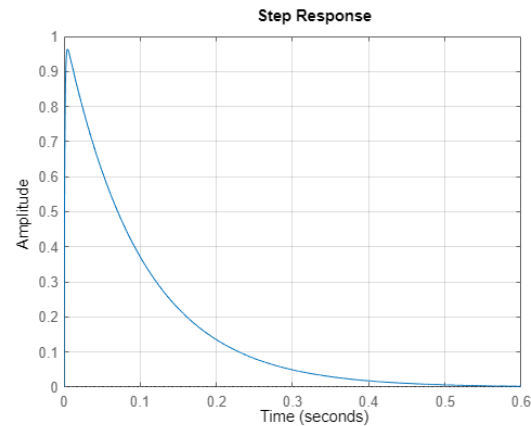
ans = 2×1
-989.8979
-10.1021

ans = 0

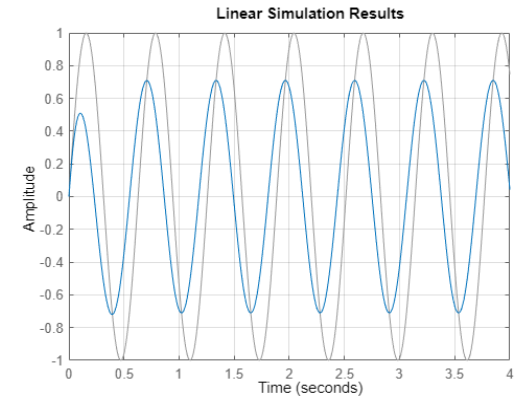
```
bode(TF); grid on;
```



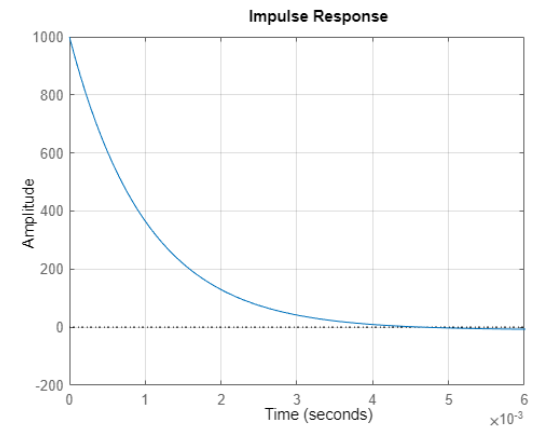
```
step(TF); grid on;
```



```
t = 0:0.01:4; % Arbitrary interval.  
u = sin(10*t); % Arbitrary signal.  
lsim(TF,u,t); grid on
```



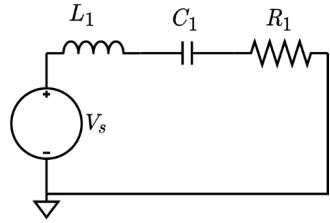
```
impz(TF); grid on;
```



The Visualizer class should improve these outputs

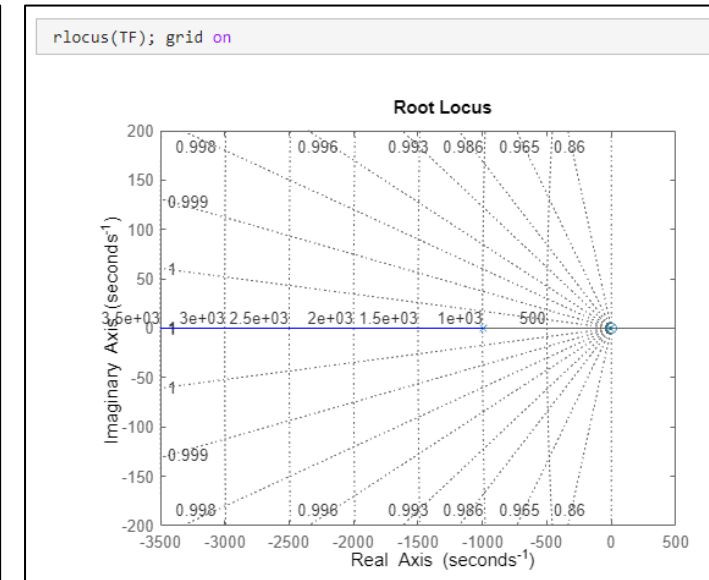
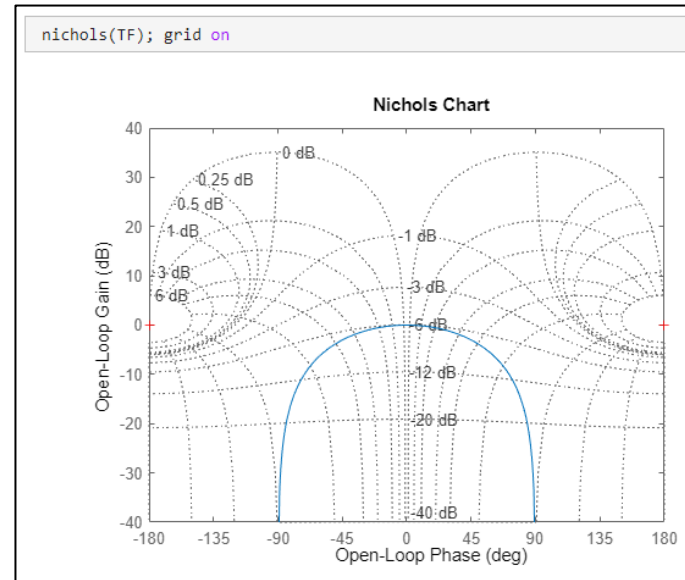
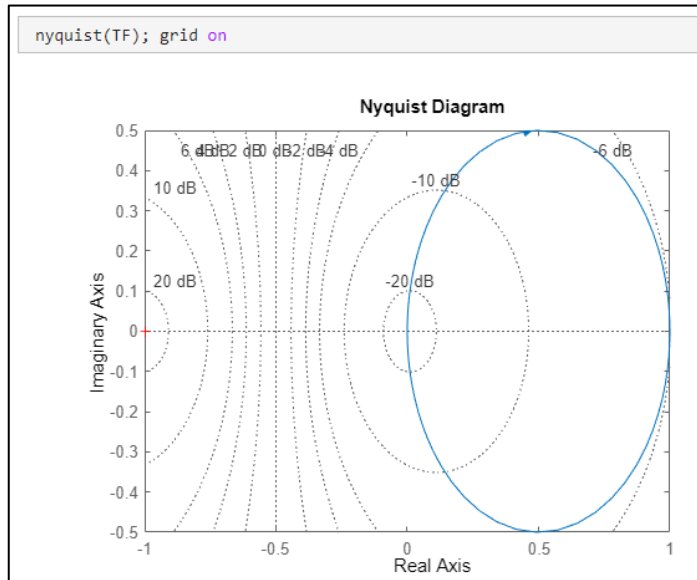
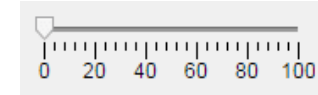


KEY FEATURES - II



rlc_series.txt				
1	Vs	1	0	DC 5
2	L1	1	2	1
3	C1	2	3	0.0001
4	R1	3	0	1000
5				

The Visualizer class should be able to manipulate the numerical values for dynamic graphs



Can be done for any circuit file



TRANSMUTING

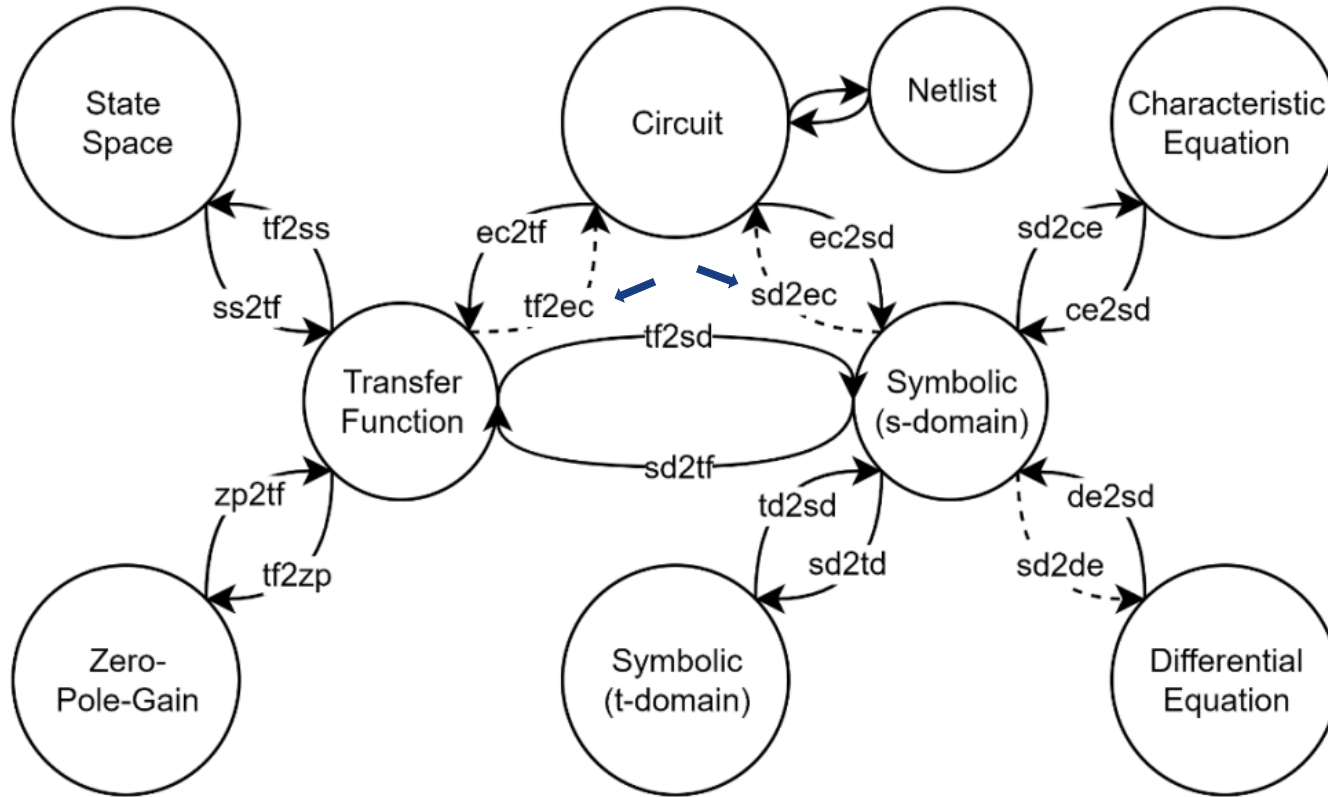


Figure 4 - Map of how the transmuter converts from one domain to another. Arrows correspond to functions.

```
syms s t y(t) Y
in = ODE(diff(y(t),2)+3*diff(y(t))+2*y(t) == 0, [0.1, 0.05]);
ELAB.transmute(in, 'de', 'td', true)
```

Diff. equation

$$\frac{\partial^2}{\partial t^2} y(t) + 3 \frac{\partial}{\partial t} y(t) + 2 y(t) = 0$$

Laplace transform:

$$2Y - \frac{s}{10} + 3Ys + Ys^2 - \frac{7}{20} = 0$$

Solve for tf:

$$\frac{2s+7}{20s^2+60s+40}$$

In s-domain

$$\frac{2s+7}{20s^2+60s+40}$$

Partial fraction decomp:

$$\frac{1}{4(s+1)} - \frac{3}{20(s+2)}$$

Inverse Laplace:

$$\frac{e^{-t}}{4} - \frac{3e^{-2t}}{20}$$

ans =

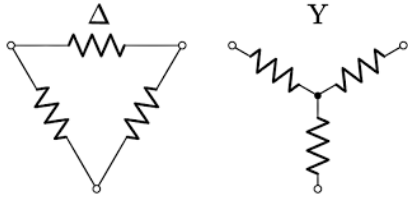
$$\frac{e^{-t}}{4} - \frac{3e^{-2t}}{20}$$

- 'sd' = Symbolic s-domain.
- 'td' = Symbolic t-domain.
- 'tf' = Transfer function.
- 'ss' = State space.
- 'zp' = Zero-pole-gain.
- 'de' = Differential equation.
- 'ce' = Characteristic equation.
- 'ec' = Electric circuit.

DIGITIZING CIRCUIT INTUITION

Detection

- is_parallel
- is_series
- is_same_type
- is_delta
- is_wye

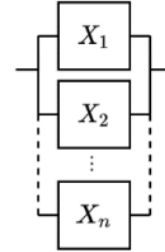


ID	Series equivalent	Parallel equivalent
R	$R1+R2$	$(R1*R2)/(R1+R2)$
C	$(C1*C2)/(C1+C2)$	$C1+C2$
L	$L1+L2$	$(L1*L2)/(L1+L2)$
V	$V1+v2$	$V1$ (only when $v1=v2$)
I	Not allowed	$I1+I2$

Detecting Parallels

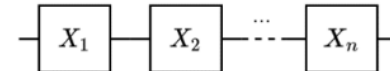
Let x_1 and x_2 be circuit elements

```
If  $z_1$ 's anode =  $z_2$ 's anode and  $z_1$ 's cathode =  $z_2$ 's cathode  
or  $z_1$ 's anode =  $z_2$ 's cathode and  $z_1$ 's cathode =  $z_2$ 's anode  
  Return true  
Else  
  Return false
```



Detecting Series

```
If parallel  
  Return false  
Else  
  If  $z_1$ 's anode =  $z_2$ 's anode or  $z_1$ 's anode =  $z_2$ 's cathode  
    Let shared be  $z_1$ 's anode  
  Else if  $z_1$ 's cathode =  $z_2$ 's anode or  $z_1$ 's cathode =  $z_2$ 's cathode  
    Let shared be  $z_1$ 's cathode  
  Else  
    Return false  
  If anything else is connected to shared  
    Return false
```



Return true



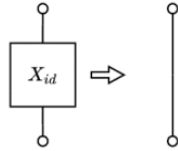
DIGITIZING CIRCUIT MANIPULATION

Shorting

Given an element x
Find node n connected to x with the lowest number.

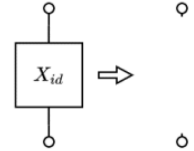
For all terminals T_1 of x
Find all elements Y connected to t_1
For all y in Y
For all terminals T_2 of y
If t_2 is equal to t_1
Change t_2 to n

Remove x from circuit
Update global properties of the circuit



Opening

Given an element x
Remove x from circuit
Let Y be all elements in the circuit
Initialize a list L of items to be removed



Trimming

For all y in Y
Let $sum = 0$
For all terminals t of y
If node n at t is not ground
Let Z be y 's connected to n
 $sum += \text{length of } Z$

If $sum = 0$
Append Z to L

Any elements not connected to anything else

For all y in Y
Let $found = false$
For all terminals t_1 of y
For all terminals t_2 of y excluding t_1 itself
If t_2 is not equal to t_1
Set $found = true$
Break loop

If not $found$
Append y to L

Remove all elements in L from circuit

Update global properties of the circuit

Any elements only connected to themselves

Cleaning

For the number of nodes in the circuit
Let k be the current node value
Let L be an empty list

For all elements in the circuit
Add all element terminal values to L

Let m be the smallest value in L which is also above k

For all elements in the circuit
Let X be the current element

For all terminals of X
Let t be the current terminal
If $t = m$
Set $t = k+1$

Update global properties of the circuit



MODELLING – HIGHER LEVEL MANIPULATION

Takes advantage of `short()`, `open()`, `simplify()`, etc.

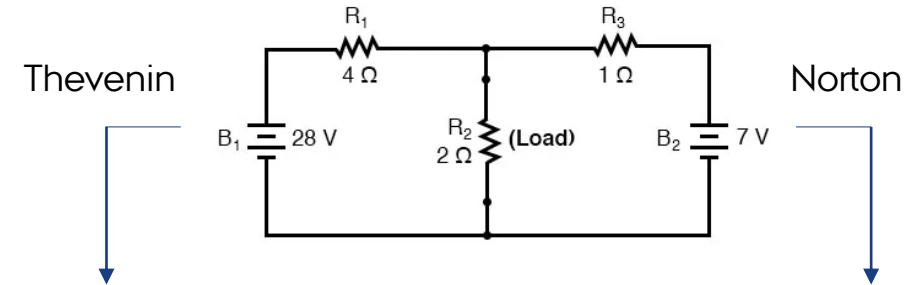
- `dc_eq`, `ac_eq`
- `thevenin`, `norton`
- `biasing`, `hybrid_pi`

Finding a Thevenin or Norton equivalent circuit is a widely used tool for analyzing circuit. Thevenin- and Norton-equivalents share the same initial approach.

1. **Open** the element being considered the load.
 2. **Analyze** the circuit, finding the voltage across the open gap.
 3. **Open** all voltage-sources and **short** all current-sources.
 4. **Simplify** the circuit to a single impedance.
- For the Thevenin equivalent circuit:
 - a. **Add** a voltage source with the value found in step 2.
 - b. **Add** the impedance found in step 4 in series with the voltage source.
 - c. **Add** back the load element.
 - For the Norton equivalent circuit:
 - a. Apply Ohm's law: $I_{Norton} = V_{step\ 2} / Z_{step\ 4}$
 - b. **Add** a current source with the value found in step a.
 - c. **Add** the impedance found in step 4 in parallel with the current source.
 - d. **Add** back the load element.

The words in bold will be standalone functions.

Point: Higher-level manipulation is easily implemented when following this approach



```
circuit = Circuit('circuits/th_no_equivalents.txt');
ELAB.thevenin(circuit, circuit.Resistors(2));
```

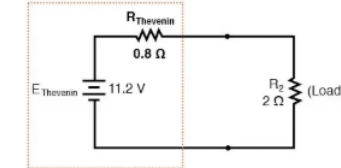
Symbolic analysis successful (0.237976 sec).

Numerical evaluation successful (0.0397994 sec).

circuit.list

```
ans =
'Vth 1 0 DC 56/5
Zth 1 2 4/5
R2 2 0 2
'
```

Thevenin Equivalent Circuit



```
circuit = Circuit('circuits/th_no_equivalents.txt');
ELAB.norton(circuit, circuit.Resistors(2));
```

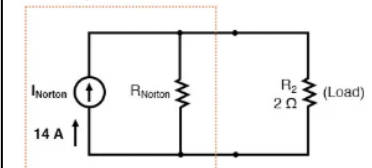
Symbolic analysis successful (0.245657 sec).

Numerical evaluation successful (0.0372693 sec).

circuit.list

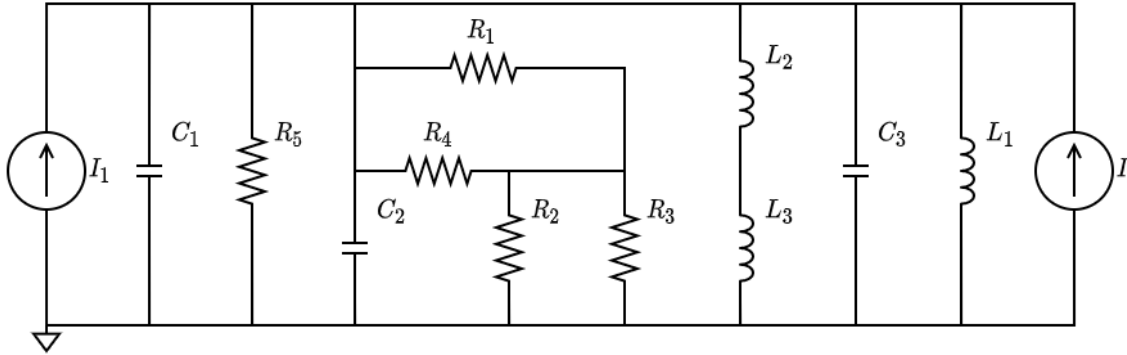
```
ans =
'Ino 1 0 DC 14
Zno 1 0 4/5
R2 1 0 2
'
```

Norton Equivalent Circuit



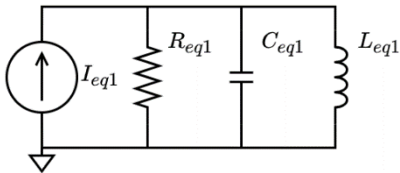
IN ACTION – SIMPLIFYING

```
complex.txt
I1 1 0 AC 5
I2 1 0 AC 10
R1 1 2 1000
R2 2 0 2000
R3 2 0 2000
R4 1 2 3000
R5 1 0 1000
C1 1 0 0.000002
C2 0 1 0.000003
C3 1 0 0.000002
L1 3 1 0.02
L2 3 0 0.03
L3 1 0 0.2
```



```
ELAB.simplify(circuit);
circuit.list
```

```
ans =
'I_eq1 1 0 AC 15
R_eq1 1 0 7000/11
L_eq1 1 0 0.04
C_eq1 1 0 0.000025'
```



Notice

- Complexity is arbitrary – $O(N)$ time, $N = \text{\#pairs}$
- Layout is arbitrary – Only connections matter
- Works numerically, symbolically and mixed
- Always appropriate naming
- Can treat all impedances as same type
- Output is simply another circuit object

```
circuit = Circuit('circuits/multiple_eqs.txt');
circuit.list
```

```
ans =
'Vs 1 0 DC Vs
R1 1 2 R1
R2 2 3 R2
R3 4 5 R3
R4 4 5 R4
R5 5 0 R5
C1 3 4 C1'
```

```
ELAB.simplify(circuit);
circuit.list
```

```
ans =
'Vs 1 0 DC Vs
Req1 2 1 R1+R2
Req2 3 0 R5+(R3*R4)/(R3+R4)
C1 2 3 C1'
```

```
circuit = Circuit('circuits/eq_impedance.txt');
circuit.list
```

```
ans =
'Vs 1 0 AC Vs
R1 2 3 3
R2 4 0 8
L1 2 4 0.2
C1 1 2 0.002
C2 3 0 0.01'
```

```
ELAB.evaluate(circuit)
```

Symbolic analysis successful (0.502269 sec).

Numerical evaluation successful (0.211514 sec).

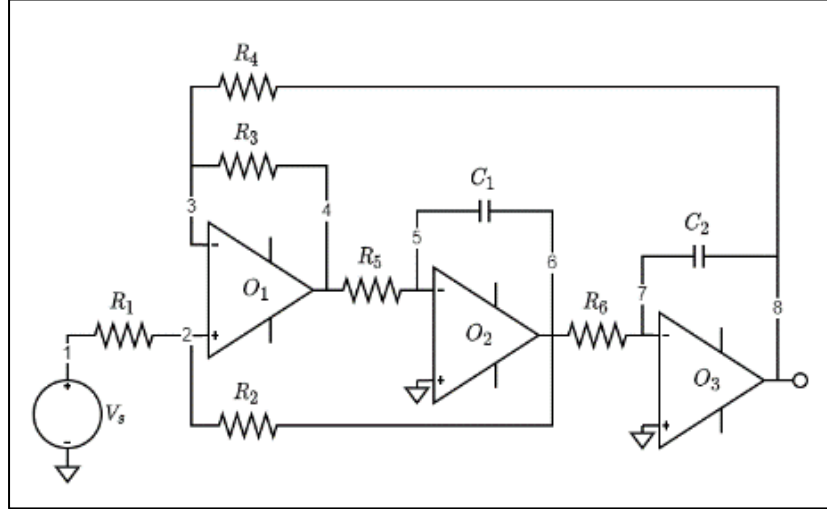
```
ELAB.simplify(circuit, true);
circuit.list
```

```
ans =
'Vs 1 0 AC Vs
Zeq1 1 0 0.002*s+(2405.0*s+8.0*s^2+1500.0)/(1100.0*s^2+500.0)'
```



IN ACTION – COMPLEX FILTER ANALYSIS

```
state_variable_filter.txt x +
Vs 1 0 AC
R1 1 2 1000
R2 2 6 19000
R3 3 4 10000
R4 3 8 10000
R5 4 5 16000
R6 6 7 16000
C1 5 6 10e-9
C2 7 8 10e-9
O1 2 3 4
O2 0 5 6
O3 0 7 8
```



```
ELAB.ec2sd(circuit,1,8)
```

Symbolic transfer function calculated successfully (5.072700e-03 sec).

ans =

$$\frac{v_8}{v_1} = \frac{R_2 (R_3 + R_4)}{R_1 R_3 + R_2 R_3 + C_2 R_1 R_3 R_6 s + C_2 R_1 R_4 R_6 s + C_1 C_2 R_1 R_4 R_5 R_6 s^2 + C_1 C_2 R_2 R_4 R_5 R_6 s^2}$$

```
bode(ELAB.ec2tf(circuit,1,4)); grid on; hold on;
```

Symbolic analysis successful (1.07754 sec).

Numerical evaluation successful (0.241102 sec).

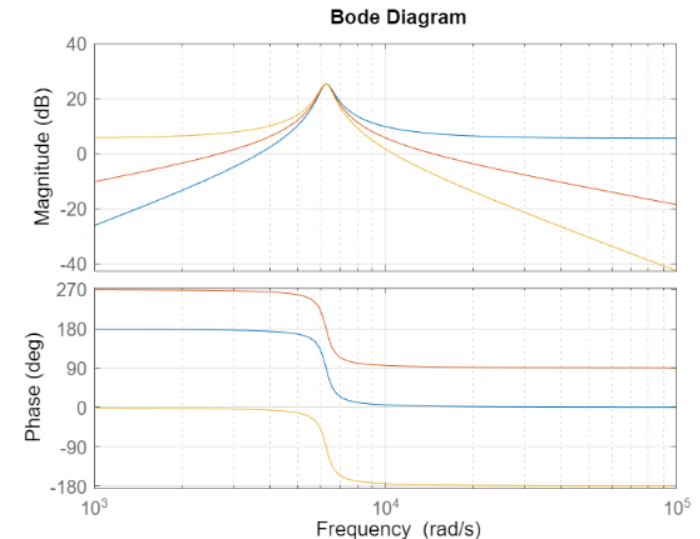
Transfer function object created successfully (2.675553e-01 sec).

```
bode(ELAB.ec2tf(circuit,1,6));
```

Transfer function object created successfully (2.028830e-02 sec).

```
bode(ELAB.ec2tf(circuit,1,8)); hold off;
```

Transfer function object created successfully (1.757560e-02 sec).



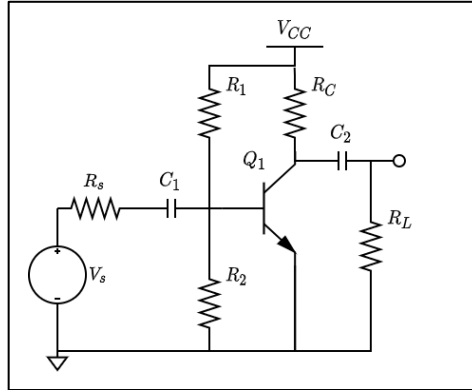
MATLAB does automatic simplification, if the symbols match



IN ACTION – TRANSISTOR MODELLING

Pre-modelling

```
bjt_complex_amp.txt
V_cc 4 0 DC
V_s 1 0 AC
R_s 1 2
R_1 3 4
R_2 3 0
R_C 4 5
R_L 6 0
C_1 2 3
C_2 5 6
Q_1 3 5 0
```



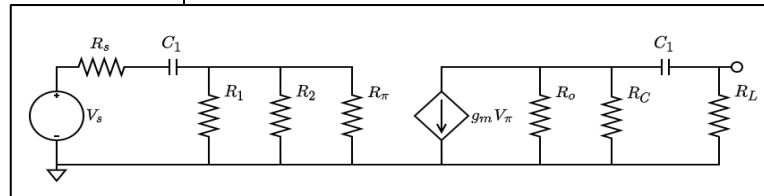
Notice

- Will handle an arbitrary number of transistors
- Biasing calculations are possible

Post-modelling

```
ELAB.hybrid_pi(circuit, 'lf', true);
circuit.list
```

```
ans =
'V_s 1 0 AC V_s
R_s 1 2 R_s
R_1 3 0 R_1
R_2 3 0 R_2
R_C 0 4 R_C
R_L 5 0 R_L
R_pi_Q_1 3 0 R_pi_Q_1
R_o_Q_1 4 0 R_o_Q_1
C_1 2 3 C_1
C_2 4 5 C_2
G_Q_1 4 0 3 0 G_Q_1'
```



Post-simplification

```
ELAB.simplify(circuit);
circuit.list
```

```
ans =
'V_s 1 0 AC V_s
R_s 1 2 R_s
R_L 5 0 R_L
R_eq1 0 4 (R_C*R_o_Q_1)/(R_C+R_o_Q_1)
R_eq2 3 0 (R_1*R_2*R_pi_Q_1)/(R_1*R_2+R_1*R_pi_Q_1+R_2*R_pi_Q_1)
C_1 2 3 C_1
C_2 4 5 C_2
G_Q_1 4 0 3 0 G_Q_1'
```

```
circuit.Resistors(3).resistance
```

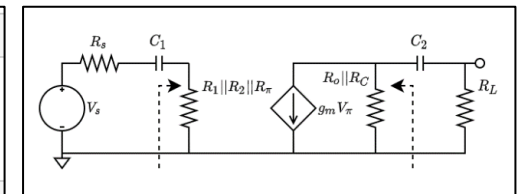
```
ans =
```

$$\frac{R_C R_{o,Q_1}}{R_C + R_{o,Q_1}}$$

```
circuit.Resistors(4).resistance
```

```
ans =
```

$$\frac{R_1 R_2 R_{\pi,Q_1}}{R_1 R_2 + R_1 R_{\pi,Q_1} + R_2 R_{\pi,Q_1}}$$



Input- and output resistance

FUTURE WORK - CONCLUSION

In-progress

- Delta-wye simplification
- Transformers, diodes
- Direct text input

Possibilities

- Computer vision (object detection), Image to netlist
- Graph theory for simplification?
- CircuiTikZ, schematic output

Scrapped

- GUI

New development!

[PeerJ Computer Science](#)


Lcapy: symbolic linear circuit analysis with Python

Research article

Computer Aided Design

Computer Education

Scientific Computing and Simulation

Michael Hayes 

February 18, 2022

▼ Author and article information

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