

Nonlinear synaptic interaction as a computational resource in the Neural Engineering Framework

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ABSTRACT: Nonlinear interaction in the dendritic tree is known to be an important computational resource in biological neurons. Yet, high-level neural compilers – such as the Neural Engineering Framework (NEF), or the predictive coding method published by Denève et al. in 2013 – tend not to include conductance-based nonlinear synaptic interactions in their models, and so do not exploit these interactions systematically. In this study, we extend the NEF to include synaptic computation of nonlinear multivariate functions, such as controlled shunting, multiplication, and the Euclidean norm. We present a theoretical framework that provides sufficient conditions under which nonlinear synaptic interaction yields a similar precision compared to traditional NEF methods, while reducing the number of layers, neurons, and latency in the network. The proposed method lends itself to increasing the computational power of neuromorphic hardware systems and improves the NEF’s biological plausibility by mitigating one of its long-standing limitations, namely its reliance on linear, current-based synapses. We perform a series of numerical experiments with a conductance-based two-compartment LIF neuron model. Preliminary results show that nonlinear interactions in conductance-based synapses are sufficient to compute a wide variety of nonlinear functions with performance competitive to using an additional layer of neurons as a nonlinearity.

The NEF [1] is a neural compiler that converts high-level mathematical descriptions to functional spiking networks. Populations of neurons represent vectors \vec{x} , while connections between populations compute functions $f(\vec{x})$ by projecting (i.e., decoding) the high-dimensional activity $\vec{a}(\vec{x})$ of a pre-population onto a low-dimensional representational space $f(\vec{x}) = D^f \vec{a}(\vec{x})$. The decoder D^f that best implements the desired function f can be computed ahead of time, since every neuron is assigned an encoding function $J(\vec{x}) = \alpha \langle \vec{x}, \vec{e} \rangle + J_0$, which maps the represented \vec{x} to somatic currents.

Crucially, since the NEF assumes current-based synapses, this encoding function is linear: populations may only represent linear combinations of vectors decoded from each pre-population. For example, if two pre-populations represent x and y , a common post-population may only represent additively superimposed functions $f_1(x) + f_2(y)$ (fig. 1a), but not multivariate, nonlinear $\phi(x, y)$. Such functions can only be computed if all input variables are represented in a middle-layer (fig. 1b).

In contrast, conductance-based synapses introduce nonlinear interactions between their inputs, which should in theory allow for a population to encode nonlinear functions across the vectors decoded from its various pre-synaptic populations.

In this work, we develop such a framework that systematically exploits conductance-based interactions, to yield populations that encode nonlinear multivariate functions across inputs (fig. 1c). This eliminates the need for a middle-layer, significantly increases the computational power per neuron, and improves the biological plausibility of the NEF by explicitly introducing nonnegative excitatory and inhibitory conductance synapses.

We denote the output spike rate of a neuron with conductance-based excitatory and inhibitory synapses, connected to two pre-populations representing quantities x, y as

$$G[J] = G[H(g_E^1(x) + g_E^2(y), g_I^1(x) + g_I^2(y))],$$

where $G[J]$ is the current-based neuron response curve, and $H(g_E, g_I)$ is a translation function modelling nonlinear interactions between synapses. The conductance functions $g_{E,I}^i(x)$ are decoded from the i th pre-population¹, i.e. $g_{E,I}^i(x) = \langle \vec{w}_{E,I}^i, \vec{a}^i(x) \rangle$ with nonnegative weight-vector $\vec{w}_{E,I}^i$.

The synaptic nonlinearity H can be separated from the neuronal nonlinearity by applying the inverse neuron-response function G^{-1} , which in the

¹Pre-synaptic populations can be split into excitatory and inhibitory neurons to accommodate Dale’s principle.

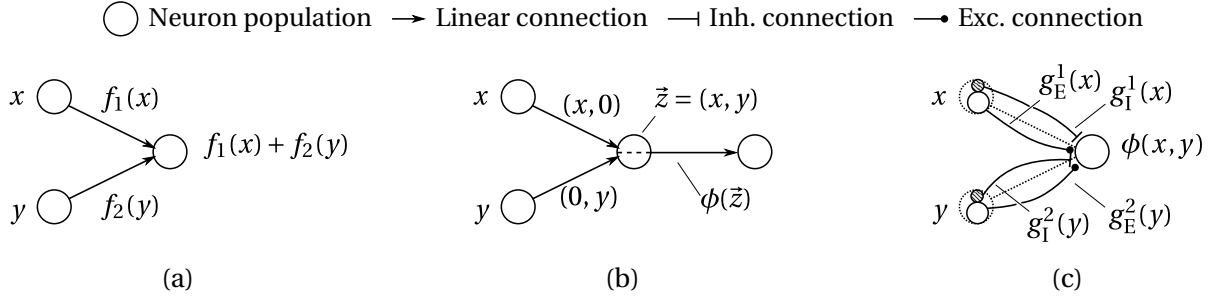


Figure 1: (a) Additivity in the NEF, (b) intermediate population for the computation of nonlinear multivariate functions, (c) exploiting nonlinear synaptic interaction.

case of LIF-based neurons is given in closed form. Applying G^{-1} to the left- and right-hand side of the above equation and replacing J with the encoding function for the desired $\phi(x, y)$ results in

$$\alpha \langle \phi(x, y), \vec{e} \rangle + J_0 = H((g_E^1(x) + g_E^2(y), g_I^1(x) + g_I^2(y))).$$

Thus, as a sufficient and necessary condition for the approximation of a multivariate nonlinear goal function ϕ , it must be possible to find nonnegative weight vectors $\vec{w}_{E,I}^i$ for the individual $g_{E,I}^i$, such that the above equation holds. Note that the computation of $\phi(x, y)$ solely relies on synaptic nonlinearities and does not exploit any of the post-population’s neural nonlinearities. Consecutive functions can be decoded from the post-population without loss of precision.

In our experiments we use an extension of the LIF neuron model that adds a compartment representing distal input to the dendritic tree [2, 3]. The dendritic compartment is resistively coupled to the soma, preventing spikes generated in the somatic compartment from destructively interfering with the nonlinearity in the dendritic portion of the model neuron. As per Kirchhoff’s laws, the somatic current can be modelled with little error (RMSE < 1% compared to empirical data) as a rational function

$$J = H(g_E, g_I) = \frac{a_0 + a_1 g_E - a_2 g_I}{b_0 + b_1 g_E + b_2 g_I},$$

where the parameters a_i, b_i are obtained by fitting empirical measurements of spike activity. Combining the previous two equations results in a nonnegative optimization problem w.r.t. $\vec{w}_{E,I}^i$.

Table 1 shows preliminary experiment results, in which we measure the errors of tuning-curve fits exploiting the nonlinear interaction of conductance-based synapses in comparison to results obtained

Table 1: Normalized RMSE between decoded output and $\phi(x, y)$. Mean over 50 runs, 100 neurons per population, 400 training samples $(x, y) \in [0, 1]^2$, L2-regularisation $\lambda = 10^{-3}$. (I, II) are the same setup with no intermediate population (fig. 1c) and gradient descent-based weight optimization. (I) uses conductance-based nonlinear synapses, (II-III) do not. (III) standard NEF network with intermediate population (fig. 1b), least-squares training, total of 200 neurons in middle- and output-layer.

FUNCTION	$\phi(x, y)$	COND.	CURRENT-BASED	
		(I)	(II)	(III)
Sum	$x + y$	1.44 %	1.20 %	0.94 %
Product	$x \cdot y$	6.85 %	23.25 %	4.18 %
Shunting	$x / (1 + 10y)$	8.71 %	31.09 %	14.81 %
Norm	$\sqrt{x^2 + y^2}$	1.80 %	5.49 %	1.27 %
Arctan	$\text{atan2}(x, y)$	4.12 %	8.66 %	6.35 %
Half-Max	$x \cdot (x > y)$	24.09 %	31.59 %	25.38 %

with various network configurations/training methods using standard current-based synapses. The conductance-based neurons (I) compute a variety of functions better than the same number of current-based neurons (II) and similarly to twice as many current-based neurons (III).²

- [1] Eliasmith and Anderson. *Neural Engineering: Computation, Representation, and Dynamics in Neurobiological Systems*. MIT press, 2003.
- [2] Koch. *Biophysics of Computation: Information Processing in Single Neurons*. Oxford University Press, 1999.
- [3] Vu et al. “The mechanism of tonic inhibition of crayfish escape behavior: distal inhibition and its functional significance”. In: *Journal of Neuroscience* 13.10 (1993).

²Source code available at <https://github.com/ctn-waterloo/cosyne2018-synaptic-computation>.