$$n(t) = \begin{cases} 5e^{4t-4} & 0 \le t < 1 \\ -3t+8 & 1 \le t < 2 \end{cases}$$

Period $= T = 2$

$\therefore \quad f_0 = \frac{1}{2}$

$$c_0 = \frac{1}{T}\int_{t_0}^{t_0+T} s(t)\,dt$$

$$= \frac{1}{T}\int_0^2 s(t)\,dt$$

$$= \frac{1}{2}\left(\int_0^1 5e^{4t-4}\,dt + \int_1^2 -3t+8\,dt\right)$$

$$= \frac{1}{2}\left(\left[\frac{5e^{4t-4}}{4}\right]_0^1 + \left[\frac{-3t^2}{2}+8t\right]_1^2\right)$$

$$= \frac{1}{2}\left(\left[\left(\frac{5e^{4(1)-4}}{4}\right) - \left(\frac{5e^{4(0)-4}}{4}\right)\right] + \left[\left(\frac{-3(2)^2}{2}+8(2)\right) - \left(\frac{-3(1)^2}{2}+8(1)\right)\right]\right)$$

$\frac{4}{2}$ $-6+16$

$$= \frac{1}{2}\left(\left[\frac{5}{4} - \frac{5e^{-4}}{4}\right] + \left[10 - 8 + 1.5\right]\right) \quad \cancel{1}$$

$$= \frac{5}{8} - \frac{5}{8e^4} + \frac{3.5}{2}$$

$$= \frac{19}{8} - \frac{5}{8e^4}$$

$$= \frac{19-5e^{-4}}{8}$$

$$\therefore \quad c_0 = \frac{19-5e^{-4}}{8}$$

$$\approx 2.3635$$

$\cancel{4}\ 7 \quad \cancel{48}\ 17$

$$C_n = \frac{1}{T}\int_{t_0}^{t_0+T} s(t)e^{-j2\pi f_0 n t}\,dt$$

$$C_n = \frac{1}{2}\int_0^2 s(t)e^{-j\pi n t}\,dt$$

let $j\pi n = p$

$\therefore e^{2p} = 1$ and $e^{-2p} = 1$

$$C_n = \frac{1}{2}\int_0^2 s(t)e^{-pt}\,dt$$

$$C_n = \frac{1}{2}\left(\int_0^1 5e^{4t-4}e^{-pt}\,dt + \int_1^2 (-3t+8)e^{-pt}\,dt\right)$$

Let $x = \int_0^1 5e^{4t-4}e^{-pt}\,dt$

and $y = \int_1^2 (-3t+8)e^{-pt}\,dt$

$$x = \int_0^1 5e^{4t-4} e^{-pt}\,dt$$

$$= \int_0^1 5e^{4t-4-pt}\,dt$$

$$= \left[\frac{5e^{4t-4-pt}}{4-p}\right]_0^1$$

$$= \left[\frac{5e^{4(1)-4}e^{-p(1)}}{4-p}\right] - \left[\frac{5e^{4(0)}e^{-4}e^{-p(0)}}{4-p}\right]$$

$$= \left(\frac{5e^{-p}}{4-p}\right) - \frac{5e^{-4}}{4-p}$$

$$= \frac{5e^{-p} - 5e^{-4}}{4-p}$$

$$\boxed{y = \int_1^2 (-3t+8)e^{-pt}\,dt}$$

Parts

$uv - \int v\,du$

Let $u = -3t+8$ and $\frac{dv}{dt} = e^{-pt}$

$\frac{du}{dt} = -3$

and ~~$\frac{dv}{dt} = e^{-pt}$~~

$v = \frac{e^{-pt}}{-p}$

$$y = \left[(-3t+8)\left(\frac{e^{-pt}}{-p}\right)\right]_1^2 - \int \frac{e^{-pt}}{-p} \times -3\,dt$$

$$= \left[(-3t+8)\left(\frac{e^{-pt}}{-p}\right)\right]_1^2 + 3\int \frac{e^{-pt}}{-p}\,dt$$

$$= \left[(-3t+8)\left(\frac{e^{-pt}}{-p}\right)\right]_1^2 + 3\left[\frac{e^{-pt}}{p^2}\right]_1^2$$

$$y = \left[(-3(2)+8)\left(\frac{e^{-p(2)}}{-p}\right) - (-3(1)+8)\left(\frac{e^{-p}}{-p}\right)\right] + 3\left[\frac{e^{-pt}}{p^2}\right]_1^2$$

$$= \left[(2)\left(\frac{1}{-p}\right) - (5)\left(\frac{e^{-p}}{-p}\right)\right] + 3\left[\frac{e^{-p(2)}}{p^2} - \frac{e^{-p(1)}}{p^2}\right]$$

$$= \frac{2}{-p} + \frac{5e^{-p}}{+p} + 3\left[\frac{1}{p^2} - \frac{e^{-p}}{p^2}\right]$$

$$= \frac{2}{-p} + \frac{5e^{-p}}{p} + \frac{3 - e^{-p}}{p^2} = \frac{2 - 5e^{-p}}{-p} + \frac{3 - 3e^{-p}}{p^2}$$

3.*

$\boxed{4*}$

$$c_n = \frac{1}{2}\left(x + y\right)$$

$$c_n = \frac{1}{2}\left(\left(\frac{5e^{-p}-5e^{-4}}{4-p}\right) + \left(\frac{2-5e^{-p}}{-p} + \frac{3-3e^{-p}}{p^2}\right)\right)$$

$$c_n = \frac{1}{2}\left(\left(\frac{5e^{-j\pi n}-5e^{-4}}{4-j\pi n}\right) + \left(\frac{2-5e^{-j\pi n}}{-j\pi n} + \frac{3-3e^{-j\pi n}}{(j\pi n)^2}\right)\right)$$

$$(j\pi n)^2 \updownarrow -(\pi n)^2$$

$$c_n = \frac{1}{2}\left(\left(\frac{5e^{-p}-5e^{-4}}{4-p}\right) + \left(\frac{5pe^{-p}-2p+3-3e^{-p}}{p^2}\right)\right)$$

$$c_n = \frac{1}{2}\left(\left(\frac{5e^{-p}-5e^{-4}}{4-p}\right) + \left(\frac{2p+3e^{-p}-5pe^{-p}-3}{(\pi n)^2}\right)\right)$$

$$c_n = \frac{1}{2}\left(\left(\frac{5e^{-p}-5e^{-4}}{4-p}\right) + \left(\frac{2p+3e^{-p}-5pe^{-p}-3}{(\pi n)^2}\right)\right)$$ 5 *

$p = j\pi n$

$$e^{-p} = \begin{cases} 1 & n \in \text{even} \\ -1 & n \in \text{odd} \end{cases}$$

$$c_1 = \frac{1}{2}\left(5(-1) - 5e \right.$$

$$c_n = \frac{1}{2}\left(\frac{(5e^{-p}-5e^{-4})(4+p)}{(4-p)(4+p)} + \frac{2p+3e^{-p}-5pe^{-p}-3}{(\pi n)^2}\right)$$

$$c_n = \frac{1}{2}\left(\frac{20e^{-p}-20e^{-4}+5pe^{-p}-5pe^{-4}}{16+(\pi n)^2} + \frac{2p+3e^{-p}-5pe^{-p}-3}{(\pi n)^2}\right)$$

$$c_1 = \frac{1}{2}\left(\frac{20(-1)-20e^{-4}+5j\pi(-1)-5e^{-4}j\pi}{16+(\pi n)^2} + \frac{2j\pi+3(-1)+5j\pi-3}{(\pi n)^2}\right)$$

6 *

$$C_1 = \frac{1}{2}\left(\frac{-20-20e^{-4}}{16+(\pi n)^2} + \frac{-6}{(\pi n)^2}\right) + \left(\frac{5j\pi - 5e^{-4}j\pi}{16+(\pi n)^2} + \frac{7j\pi}{(\pi n)^2}\right)$$

$$C_1 = -0.6976 + 0.8049i$$

$$C_3 = \frac{1}{2}\left(\left(\frac{-20-20e^{-4}}{16+(\pi\times 3)^2} + \frac{-6}{(\pi\times 3)^2}\right) + \frac{-5e^{-4}j\pi\times 3 + 5j\pi\times 3}{16+(\pi n)^2} + \frac{2j\pi\times 3 + 5\times j\pi\times 3}{(\pi n)^2}\right)$$

$$C_3 = -0.13092 + 0.14257j$$

$$C_5 = \frac{1}{2}\left(\left(\frac{-20-20e^{-4}}{16+(5\pi)^2} + \frac{-6}{(5\pi)^2}\right) + \frac{-5e^{-4}j\pi\times 5 + 5j\pi\times 5}{16+(\pi\times 5)^2} + \frac{2j\pi\times 5 + 5j\pi\times 5}{(5\pi)^2}\right)$$

$$= -0.0509 + 0.0706j$$

$$c_n = \frac{1}{2}\left(\left(\frac{20-20e^{-4}}{16+(\pi n)^2}\right) + \left(\frac{5j\pi n - 5j\pi n e^{-4}}{16+(\pi n)^2} + \frac{2j\pi n - 5j\pi n}{(\pi n)^2}\right)\right)$$

7 *

$$c_2 = \cancel{0.17695}\ \frac{1}{2}\left(\left(\frac{20-20e^{-4}}{16+(\pi\times 2)^2}\right) + \left(\frac{5j\pi\times 2 - 5j\pi e^{-4}\times 2}{16+(2\pi)^2} + \frac{-3j\pi\times 2}{(2\pi)^2}\right)\right)$$

$$= 0.17695 + 0.0392j$$

$$c_4 = \frac{1}{2}\left(\left(\frac{20-20e^{-4}}{16+(\pi\times 4)^2}\right) + \left(\frac{5j\pi\times 4 - 5j\pi e^{-4}\times 4}{16+(\cancel{4}\pi\times 4)^2} + \frac{-3j\pi\times 4}{(4\pi)^2}\right)\right)$$

$$= 0.05644 + 0.0580j$$